Inverse Trigonometric Functions

Que 1: $If\ sin(sin^{-1}\frac{1}{5} + cos^{-1}x) = 1, find\ x$ ***Marks :(2)***

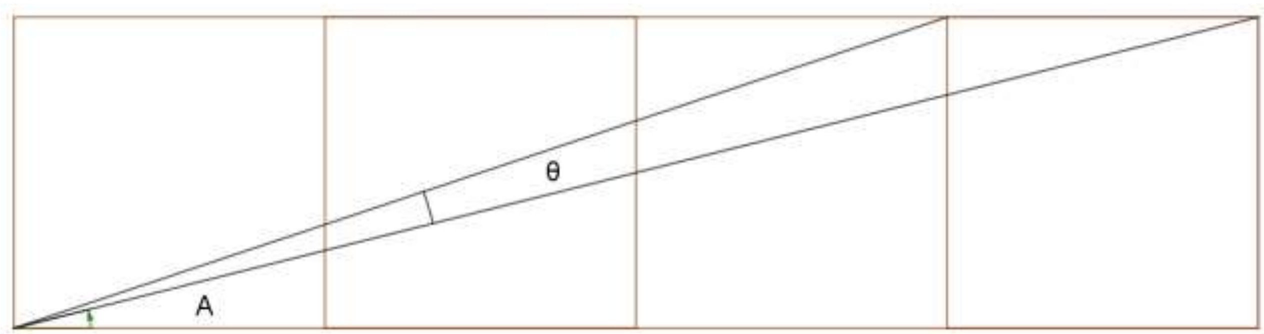
Ans:

$$(sin^{-1}\frac{1}{5} + cos^{-1}x) = sin^{-1}1 = \frac{\pi}{2}$$
$$sin^{-1}\frac{1}{5} = \frac{\pi}{2} - cos^{-1}x = sin^{-1}x$$
$$hence\ x = 1/5$$

Que 2: (i) If $tan^{-1}(tanx) = x$ **, then which of the following is the value of x.**

(a) 1.6 (b) 2 (c) -1.3 (d) -1.7

Consider the figure having 4 identical squares. ***Marks :(4)***



(ii) Evaluate $tanA$

(iii) Find the value of the angle θ

Ans:

(i) (c) -1.3

(ii) $tanA = \frac{1}{4}$

(iii)

$$\theta = tan^{-1}\left(\frac{1}{3}\right) - tan^{-1}\left(\frac{1}{4}\right)$$

$$= tan^{-1}\left(\frac{\frac{1}{3}-\frac{1}{4}}{1+\frac{1}{3}\times\frac{1}{4}}\right)$$
$$= tan^{-1}\left(\frac{1}{13}\right)$$

Que 3: (i) Which of the following is true? ***Marks :(4)***

$$(a) sin^{-1}x + cos^{-1}x = \frac{\pi}{2}$$
$$(b) tan^{-1}x - cot^{-1}x = \frac{\pi}{2}$$
$$(c) sec^{-1}x + cosec^{-1}x = \frac{\pi}{2}$$
$$(d) cos^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{3}$$

(ii) Find x , if $2sin^{-1}x - cos^{-1}x = \frac{\pi}{2}$

Ans: (i) (a)

(ii)

$$2sin^{-1}x - cos^{-1}x - 2cos^{-1}x + 2cos^{-1}x = \frac{\pi}{2}$$
$$2(sin^{-1}x + cos^{-1}x) - 3cos^{-1}x = \frac{\pi}{2}$$
$$\pi - 3cos^{-1}x = \frac{\pi}{2}$$
$$cos^{-1}x = \frac{\pi}{6} \Longrightarrow x = \frac{\sqrt{3}}{2}$$

OR

$$2sin^{-1}x - \left(\frac{\pi}{2} - sin^{-1}x\right) = \frac{\pi}{2}$$
$$\Rightarrow 3sin^{-1}x = \pi \Rightarrow sin^{-1}x = \frac{\pi}{3} \Rightarrow x = \frac{\sqrt{3}}{2}$$

Que 4: ***Marks :(3)***

(i) Which of the following is the value of $cos^{-1}\frac{4}{3}$

$(a)\frac{\pi}{5}$

$(b)1.8$

$(c)\frac{\pi}{7}$

$(d) Does\ not\ exixts$

(ii) If $tan^{-1}(-x) + cos^{-1}(-\frac{1}{2}) = \frac{\pi}{2}$ **, then find the value of x.**

Ans:

(i) (d)

(ii)

$$-tan^{-1}(x) + \pi - \frac{\pi}{3} = \frac{\pi}{2}$$

$$tan^{-1}x = \frac{\pi}{6} \Longrightarrow x = \frac{1}{\sqrt{3}}$$

Que 5: ***Marks :(6)***

(i) If $cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)$ **, where** $xy < 1$ **, then show that** $y = \frac{1-x}{1+x}$

(ii) Hence show that the function $f(x) = \frac{1-x}{1+x}$ **is inverse of itself.**

Ans: (i)

$$2tan^{-1}x + 2tan^{-1}y = \frac{\pi}{2}$$

$$tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4}$$

$$\frac{x+y}{1-xy} = 1$$

$$x + y = 1 - xy \Longrightarrow y = \frac{1-x}{1+x}$$

(ii)

$$fof(x) = f\left(\frac{1-x}{1+x}\right)$$

$$= \frac{1-\left(\frac{1-x}{1+x}\right)}{1+\left(\frac{1-x}{1+x}\right)} = x$$

Que 6: ***Marks :(3)***

(i) Which of the following can be the domain of the function $f(x) = cos^{-1}x$

$(a)[-1, 1]$
$(b)R$
$(c)[-2, 1]$
$(d)[-\frac{\pi}{2}, \frac{\pi}{2}]$

(ii)If $cos^{-1}x > sin^{-1}x$ **, then find the interval in which x lies.**

Ans:

(i) (a) $[-1, 1]$

(ii)

$$cos^{-1}x > \frac{\pi}{2} - cos^{-1}x$$
$$2cos^{-1}x > \frac{\pi}{2}$$
$$cos^{-1}x > \frac{\pi}{4}$$
$$\implies x \in [-1, \frac{1}{\sqrt{2}})$$

Que 7: ***Marks :(4)***

(i) Which of the following is the value of $sin^2(sin^{-1}0.7)$ **?**

(a) 0.7 (b) - 0.7 (c) 0.49 (d)0.14

(ii) Find the value of $tan^2(sec^{-1}2) + cot^2(cosec^{-1}3)$

Ans:

(i) (c) 0.49 $(sin(sin^{-1}0.7))^2 = (0.7)^2 = 0.49$

(ii) $tan^2(sec^{-1}2) + cot^2(cosec^{-1}3)$

$$= sec^2(sec^{-1}2) - 1 + cosec^2(cosec^{-1}3) - 1$$
$$= 4 - 1 + 9 - 1 = 11$$

Que 8: ***Marks :(6)***

$$(sin^{-1}\frac{1}{5}+cos^{-1}x)=sin^{-1}1=\frac{\pi}{2}$$
$$sin^{-1}\frac{1}{5}=\frac{\pi}{2}-cos^{-1}x=sin^{-1}x$$
$$hence\ x=1/5$$

Ans: (i)

$$cos\left(sin^{-1}\frac{2}{3}\right)$$
$$=cos\left(cos^{-1}\sqrt{1-\frac{4}{9}}\right)=\frac{\sqrt{5}}{3}$$

(ii) $cos^{-1}x+cos^{-1}y=\frac{\pi}{2}$

$$cos(cos^{-1}x+cos^{-1}y)=cos\frac{\pi}{2}$$
$$cos(cos^{-1}x)cos(cos^{-1}y)-sin(cos^{-1}x)sin(cos^{-1}y)=0$$
$$xy-\sqrt{1-x^2}\sqrt{1-y^2}=0$$
$$x^2y^2=(1-x^2)(1-y^2)$$
$$x^2y^2=1-x^2-y^2+x^2y^2$$
$$x^2+y^2=1$$

(iii)

$$x^2+y^2=1 \Longrightarrow \frac{144}{x^2}+\frac{256}{x^2}=1$$
$$\Longrightarrow x^2=400 \Longrightarrow x=20$$

Que 9: ***Marks :(4)***

(i) Evaluate [-0.3],where [x] denotes the greastest integer function of x.

(ii) Which of the following is the domain of the function $f(x)=sin^{-1}x$?

$(a)[-1,1]$ $(b)(-1,0)$ $(c)[-1,0]$ $(d)\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

(iii) Write the domain of the function $g(x)=sin^{-1}[x]$

Ans:

(i) [-0.3] = -1

(ii) (a) [-1,1]

(iii) Since the domain of $sin^{-1}x$ is [-1,1], the possible value of [x] = {-1, 0,1}

$$\begin{aligned}[x] &= -1, \quad -1 \leq x < 0 \\ &= 0, \quad\quad 0 \leq x < 1 \\ &= 1, \quad\quad 1 \leq x < 2\end{aligned}$$

Therefore the domain of $g(x) = sin^{-1}[x]$ is [-1,2).