Class IX Session 2024-25
Subject - Mathematics
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. If x = $2 + \sqrt{3}$, then $x + \frac{1}{x}$ = **[1]**

    a) 4 b) -5

    c) -4 d) 5

2. For what value of ‘k’, x = 2 and y = -1 is a solution of x + 3y – k = 0? **[1]**

    a) 2 b) -2

    c) -1 d) 1

3. A(-6, 3) be a point on the graph. Draw $AL \perp x - axis$. The co-ordinates of L are **[1]**

    a) (-6, 3) b) (0, 0)

    c) (-6, 0) d) (0, -6)

4. In a histogram the area of each rectangle is proportional to **[1]**

    a) the class size of the corresponding class interval

    b) cumulative frequency of the corresponding class interval

    c) the class mark of the corresponding class interval

    d) frequency of the corresponding class interval

5. The equation x = 7 in two variables can be written as **[1]**

    a) 1.x + 1.y = 7 b) 1.x + 0.y = 7

    c) 0.x + 1.y = 7 d) 0.x + 0.y = 7

6. A and B have the same weight. If they gain weight by 3 kg, then **[1]**

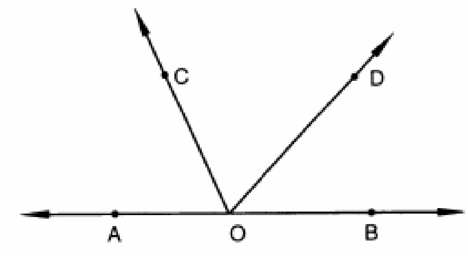
a) Weight of A $\neq$ Weight of B

b) Weight of A > Weight of B

c) Weight of A < Weight of B

d) Weight of A = Weight of B

7. In Fig. AOB is a straight line. If $\angle AOC + \angle BOD = 85°$, then $\angle COD =$ **[1]**



a) 100°

b) 85°

c) 90°

d) 95°

8. If one angle of a parallelogram is 24^o less than twice the smallest angle, then the measure of the largest angle of the parallelogram is **[1]**

a) 112^o

b) 68^o

c) 176^o

d) 102^o

9. If $a^2 + b^2 + c^2 = 30$ and a + b + c = 10, then the value of ab + bc + ca is **[1]**

a) 30

b) 25

c) 35

d) 40

10. Which of the following is not a solution of 2x – 3y = 12? **[1]**

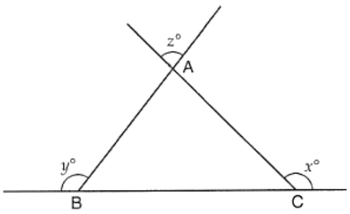
a) (0, -4)

b) (2, 3)

c) (6, 0)

d) (3, -2)

11. In figure, what is z in terms of x and y? **[1]**



a) x + y - 180°

b) x + y + 180°

c) x + y + 360°

d) 180° - (x + y)

12. In $\triangle ABC$, EF is the line segment joining the mid-points of the sides AB and AC. BC = 7.2 cm, Find EF. **[1]**

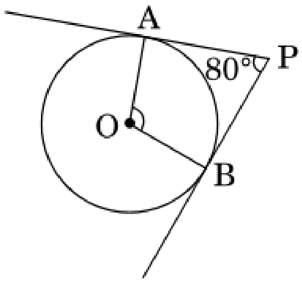
a) 2.6 cm

b) 3.5 cm

c) 3.6 cm

d) 3.4 cm

13. In Figure, if tangents PA and PB from an external point P to a circle with centre O, are inclined to each other at an angle of 80^o, then is equal to **[1]**



a) 100^o

b) 60^o

c) 50^o

d) 80^o

14. Read the statements carefully. **[1]**

Statement-1: The product of a rational and an irrational number is an irrational number.

Statement 2: Reciprocal of every rational number is a rational number.

a) Statement-1 is false but Statement-2 is true.

b) Statement-1 is true but Statement-2 is false.

c) Both Statement-1 and Statement-2 are false.

d) Both Statement-1 and Statement-2 are true.

15. The positive solutions of the equation ax + by + c = 0 always lie in the **[1]**

a) 3rd quadrant

b) 4th quadrant

c) 2nd quadrant

d) 1st quadrant

16. In ΔABC and ΔDEF its is given that $\angle B = \angle E$ and $\angle C = \angle F$ in order that $\Delta ABC \cong \Delta DEF$ we must have **[1]**

a) BC = EF

b) $\angle A = \angle D$

c) AB = DF

d) AC = DE

17. If $\left(3x + \frac{1}{2}\right)\left(3x - \frac{1}{2}\right) = 9x^2$ - p then the value of p is **[1]**

a) $\frac{1}{4}$

b) $-\frac{1}{4}$

c) 0

d) $\frac{1}{2}$

18. The volume of a spherical shell is given by **[1]**

a) $\frac{4}{3}\pi(R^3 - r^3)$

b) $\pi(R^3 - r^3)$

c) $\frac{4}{3}\pi(R^2 - r^2)$

d) $4\pi(R^3 - r^3)$

19. **Assertion (A):** The perimeter of a right angled triangle is 60 cm and its hypotenuse is 26 cm. The other sides of the triangle are 10 cm and 24 cm. Also, area of the triangle is 120 cm^2. **[1]**

Reason (R): $(\text{Base})^2 + (\text{Perpendicular})^2 = (\text{Hypotenuse})^2$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** There are infinite number of lines which passes through (2, 14). **[1]**

Reason (R): A linear equation in two variables has infinitely many solutions.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

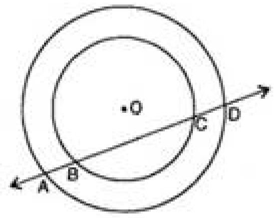
c) A is true but R is false. d) A is false but R is true.

Section B

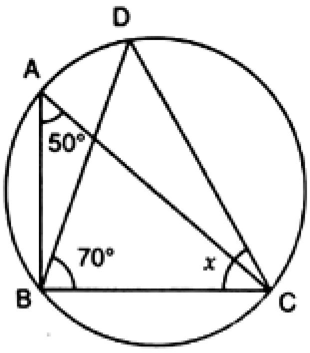
21. Prove the exterior angle formed by producing a side of a cyclic quadrilateral is equal to the interior opposite angle. **[2]**

22. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle. **[2]**

23. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD. **[2]**

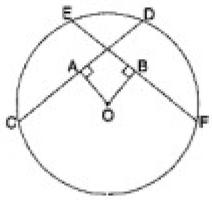


24. If O is the centre of the circle, find the value of x in the given figure: **[2]**



OR

In figure, OA and OB are respectively perpendiculars to chords CD and EF of a circle whose centre is O. If OA = OB, prove that $\overline{EC} \cong \overline{DF}$.



25. Write two solutions of the form x = 0, y = a and x = b, y = 0 : -4x + 3y = 12 **[2]**

OR

Express the linear equation y - 2 = 0 in the form ax + by + c = 0 and indicate the value of a, b and c in case.

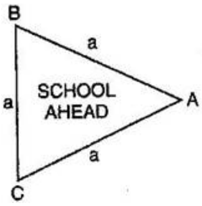
Section C

26. Rationalise the denominator: $\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}$ **[3]**

27. Factorize the polynomial: **[3]**

$8a^3 - b^3 - 12a^2b + 6ab^2$

28. A traffic signal board, indicating SCHOOLAHEAD is an equilateral triangle with side a Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board? **[3]**

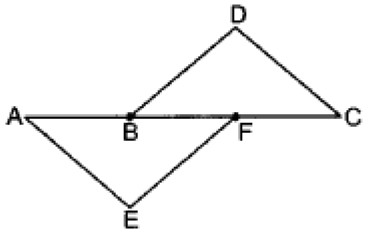


OR

Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs7 per m^2.

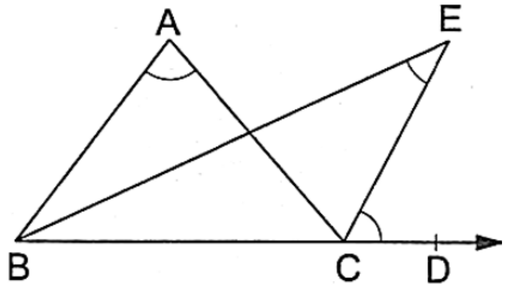
29. A cylinder, a cone and a sphere are of the same radius and same height. Find the ratio of their curved surface. **[3]**

30. In given figure, it is given that AB = CF, EF = BD and ∠AFE = ∠CBD. Prove that △AFE ≅ △CBD. **[3]**

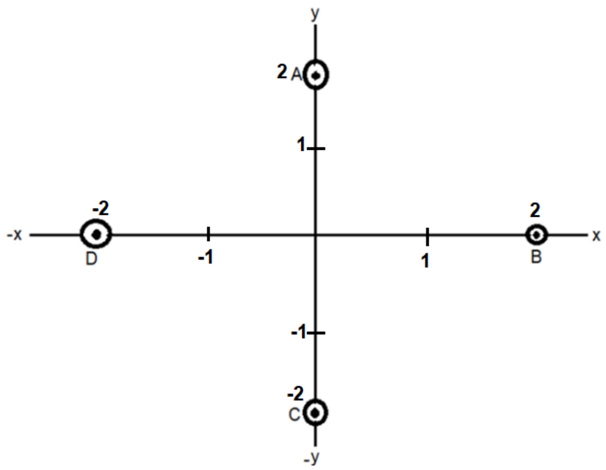


OR

In the given figure, the side BC of △ABC has been produced to a point D. If the bisectors of ∠ABC and ∠ACD meet at point E then prove that $\angle BEC = \frac{1}{2}\angle BAC$.



31. In fig. write the Co-ordinates of the points and if we join the points write the name of fig. formed. Also write Co-ordinate of intersection point of AC and BD. **[3]**



Section D

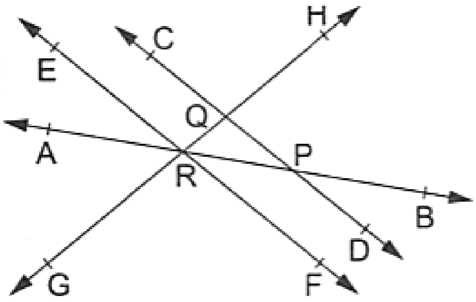
32. If $p = \frac{3-\sqrt{5}}{3+\sqrt{5}}$ and $q = \frac{3+\sqrt{5}}{3-\sqrt{5}}$, find the value of $p^2 + q^2$. **[5]**

OR

Find the values of a and b if $\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$.

33. In the adjoining figure, name: **[5]**

i. Two pairs of intersecting lines and their corresponding points of intersection
ii. Three concurrent lines and their points of intersection
iii. Three rays
iv. Two line segments

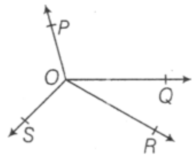


34. If is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ZYP, find ∠XYQ and reflex ∠QYP. **[5]**

OR

In the given figure, OP, OQ, OR and OS are four rays. Prove that

∠POQ + ∠ROQ + ∠SOR + ∠POS = 360°.



35. The lengths of 62 leaves of a plant are measured in millimetres and the data is represented in the following table: **[5]**

Length (in mm)	Number of leaves
118 - 126	8
127 - 135	10
136 - 144	12
144 - 153	17
154 - 162	7
163 - 171	5
172 - 180	3

Draw a histogram to represent the data above.

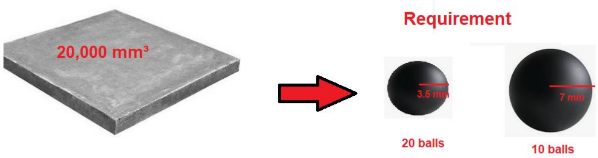
Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

In Agra in a grinding mill, there were installed 5 types of mills. These mills used steel balls of radius 5 mm, 7 mm, 10 mm, 14 mm and 16 mm respectively. All the balls were in the spherical shape.

For repairing purpose mills need 10 balls of 7 mm radius and 20 balls of 3.5 mm radius. The workshop was having 20000 mm^3 steel.

This 20000 mm^3 steel was melted and 10 balls of 7 mm radius and 20 balls of 3.5 mm radius were made and the remaining steel was stored for future use.



i. What was the volume of one ball of 3.5 mm radius? (1)

ii. What was the surface area of one ball of 3.5 mm radius? (1)

iii. What was the volume of 10 balls of radius 7 mm? (2)

OR

How much steel was kept for future use? (2)

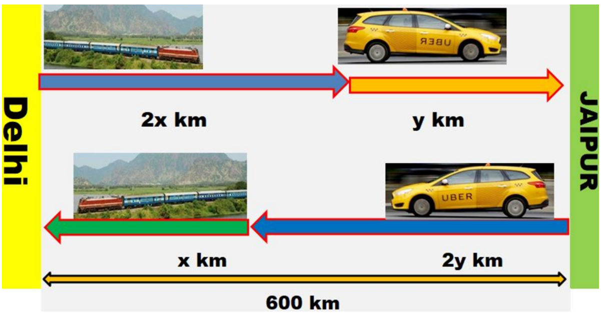
37. **Read the following text carefully and answer the questions that follow:** **[4]**

Ajay lives in Delhi, The city of Ajay's father in laws residence is at Jaipur is 600 km from Delhi. Ajay used to travel this 600 km partly by train and partly by car.

He used to buy cheap items from Delhi and sale at Jaipur and also buying cheap items from Jaipur and sale at Delhi.

Once From **Delhi to Jaipur** in forward journey he covered 2x km by train and the rest y km by taxi.

But, while returning he did not get a reservation from Jaipur in the train. So first 2y km he had to travel by taxi and the rest x km by Train. From Delhi to Jaipur he took 8 hrs but in returning it took 10 hrs.



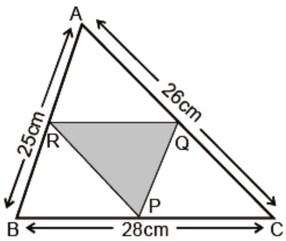
i. Write the above information in terms of equation. (1)
ii. Find the value of x and y? (1)
iii. Find the speed of Taxi? (2)

OR

Find the speed of Train? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

There is a Diwali celebration in the DPS school Janakpuri New Delhi. Girls are asked to prepare Rangoli in a triangular shape. They made a rangoli in the shape of triangle ABC. Dimensions of $\triangle$ABC are 26 cm, 28 cm, 25 cm.



i. In fig R and Q are mid-points of AB and AC respectively. Find the length of RQ. (1)
ii. Find the length of Garland which is to be placed along the side of $\triangle$QPR. (1)
iii. R, P and Q are the mid-points of AB, BC, and AC respectively. Then find the relation between area of $\triangle$PQR and area of $\triangle$ABC. (2)

OR

R, P, Q are the mid-points of corresponding sides AB, BC, CA in ΔABC, then name the figure so obtained BPQR. (2)

Solution

Section A

1. **(a)** 4

   Explanation: $x + \frac{1}{x}$

   $\Rightarrow \frac{x^2+1}{x}$

   now, put x = $2 + \sqrt{3}$

   we have,

   $\frac{(2+\sqrt{3})^2+1}{2+\sqrt{3}}$

   $\Rightarrow \frac{4+3+2(2\sqrt{3})+1}{2+\sqrt{3}}$

   $\Rightarrow \frac{8+4\sqrt{3}}{2+\sqrt{3}}$

   $\Rightarrow \frac{4(2+\sqrt{3})}{2+\sqrt{3}}$

   = 4

2. **(c)** -1

   Explanation: For finding value of 'k', we put x = 2 and y = -1 iin a equation x + 3y – k = 0

   x+3 y-k=0

   2+3(-1)=k

   2-3=k

   k=-1

3. **(c)** (-6, 0)

   Explanation: Since AL perpendicular to x-axis,

   So ,point L lies on x-axis, and we know that for any point on x-axis y-ordinate is zero.

   So,we have L= (—6 , 0)

4. **(d)** frequency of the corresponding class interval

   Explanation: A histogram is a display of statistical information that uses rectangles to show the frequency of data items in successive numerical intervals of equal size. In the most common form of histogram, the independent variable is plotted along the horizontal axis and the dependent variable is plotted along the vertical axis.

5. **(b)** 1.x + 0.y = 7

   Explanation: The equation x = 7 in two variables can be written as exactly 1.x + 0.y = 7

   because it contain two variable x and y and coefficient of y is zero as there is no term containing y

   in equation x = 7

6. **(d)** Weight of A = Weight of B

   Explanation: Let the weights of A and B be x kgs. If both of them gain weight by 3 kgs, their new weight would be 'x + 3' kgs. According to Euclid's axiom if equals are added in equals, then whole are equal.

   Hence, Weight of A = Weight of B.

7. **(d)** 95°

   Explanation: Given,

   AOB = Straight line

$\angle$AOC + $\angle$BOD = 85°

$\angle$AOC + $\angle$COD + $\angle$BOD = 180° (Straight line)

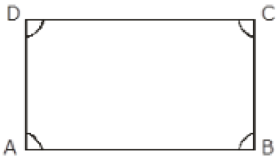
85° + $\angle$COD = 180°

$\angle$COD = 95°

8. **(a)** 112°

Explanation:

Let angles of parallelogram are $\angle$A, $\angle$B, $\angle$C, $\angle$D



Let smallest angle = $\angle$A

Let largest angle = $\angle$B

= $\angle$B = 2$\angle$A – 24° ...(i)

$\angle$ A + $\angle$B = 180° [adjacent angle of parallelogram]

So, $\angle$A + 2$\angle$A -24° = 180°

= 3$\angle$A = 180° + 24° = 204°

= $\angle$A = $\frac{204^o}{3}$ = 68°

= $\angle$B = 2 $\times$ 68° - 24° = 112°

9.

(c) 35

Explanation: Using identity

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow$ ab + bc + ca = $\frac{(a+b+c)^2-\left(a^2+b^2+c^2\right)}{2}$

$\Rightarrow$ ab + bc + ca = $\frac{(10)^2-(30)}{2}$

$\Rightarrow$ ab + bc + ca = $\frac{100-30}{2}$

$\Rightarrow$ ab + bc + ca =$\frac{70}{2}$

$\Rightarrow$ ab + bc + ca = 35

10.

(b) (2, 3)

Explanation: We have to check (2, 3) is a solution of 2x – 3y = 12 if (2, 3) satisfy the equation then (2, 3) solution of 2x – 3y = 12

LHS = 2x - 3y

2 $\times$ 2 - 3 $\times$ 3

4 - 9 = -5

RHS = -5

LHS $\neq$ RHS

So (2, 3) is not a solution of 2x - 3y = 12

11. **(a)** x + y - 180°

Explanation: From figure

$\angle$A = z°

$\angle$ACB = 180 - z°

$\angle$ABC = 180 - y°

Now, in $\triangle$ABC

$\angle$A + $\angle$B + $\angle$C = 180°

$\Rightarrow$ z° + 180 - y° + 180° - x° = 180°

$\Rightarrow$ z° = x° + y°- 180°

12.

(c) 3.6 cm

Explanation: E and F are midpoints of sides AB and AC. By midpoint theorem, EF is parallel to BC and EF is $\frac{1}{2}$ of BC.

So, EF = $\frac{1}{2}$ of (7.2) = 3.6 cm

13. **(a)** 100^{o}

Explanation: By angle sum proporty for qud.

$\angle A + \angle B + \angle O + \angle P$ = 360.

also OA $\perp$ AP and OB $\perp$ PB.

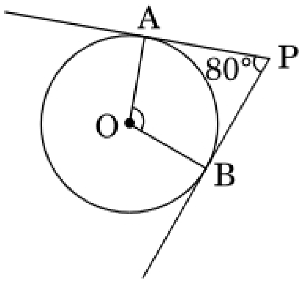
($\because$ OA and PB are radius and tangent of circe)

$\therefore$ 90 + 90 + $\angle O + \angle P$ = 360

$\angle O + 80$ = 360 - 180

$\angle O$ = 180 - 80

$\angle O$ = 100^{o}



14.

(b) Statement-1 is true but Statement-2 is false.

Explanation: Statement-1 is true and statement-2 is false as 0 is a rational number but $\frac{1}{0}$ is not defined.

15.

(d) 1st quadrant

Explanation: The positive solutions of the equation ax + by + c = 0 always lie in the 1st quadrant

Because in 1st quadrant both x and y have positive value.

16. **(a)** BC = EF

Explanation: In ΔABC and ΔDEF

$\angle B = \angle E$ and $\angle C = \angle F$

For congruence, BC = EF

Therefore by AAS axiom

$\Delta ABC \cong \Delta DEF$

17. **(a)** $\frac{1}{4}$

Explanation: $\left(3x + \frac{1}{2}\right)\left(3x - \frac{1}{2}\right)$ = $9x^2$ - p

$\Rightarrow (3x)^2 - \left(\frac{1}{2}\right)^2$ = $9x^2$ - p

$\Rightarrow 9x^2 - \frac{1}{4}$ = $9x^2$ - p

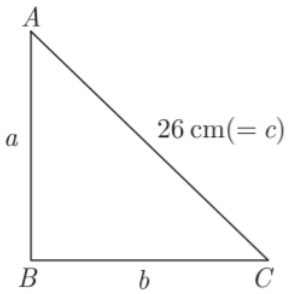
$\Rightarrow p = \frac{1}{4}$

18. **(a)** $\frac{4}{3}\pi(R^3 - r^3)$

Explanation: The volume of a spherical shell is given by $\frac{4}{3}\pi\left(\mathrm{R}^3 - r^3\right)$ where R = Larger radius and r = smaller radius

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:



a + b + c = 60

a + b + 26 = 60

a + b = 34 ...(i)

Now, $26^2 = a^2 + b^2$ ...(ii)

Squaring (1) both sides, we get

$(a + b)^2 = (34)^2$

$a^2 + b^2 + 2ab = 34 \times 34$

$(26)^2 + 2ab = 1156$ [From (ii)]

2ab = 1156 - 676

2ab = 480

ab = 240 ...(iii)

Now, $a + \frac{240}{a} = 34$ [From (i) and (iii)]

$a^2 - 24a - 10a + 240 = 0$

a(a - 24) - 10(a - 24) = 0

a = 10, 24

Now, other sides are 10 cm and 24 cm

$s = \frac{26+10+24}{2} = 30$ cm

Area of triangle = $\sqrt{30(30-26)(30-10)(30-24)}$

$= \sqrt{30 \times 4 \times 20 \times 6} = 120$ cm^2

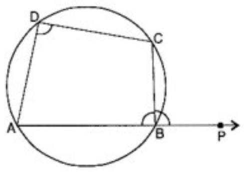
20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Through a point infinite lines can be drawn. Through (2, 14) infinite number of lines can be drawn. Also a line has infinite points on it hence a linear equation representing a line has infinite solutions.

Section B

21. Given: ABCD is a cyclic quadrilateral whose side AB is produced to P to formed exterior ∠CBP.



To prove: ∠ CBP. = Interior opposite ∠ ADC

Proof : ∵ ABCD is a cyclic quadrilateral

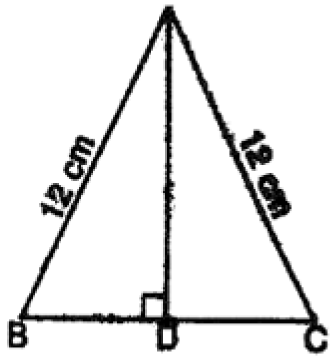
∴ ∠ADC+ ∠ABC = 180° 180°

∵ Opposite angles of a cyclic quadrilateral are supplementary

Also, ∠ABC + ∠CBP = 180° ...(2) |Linear Pair Axiom

From (1) and (2), we have

∠ABC + ∠CBP = ∠ABC + ∠ABC

22. 

a = 12 cm, b = 12 cm

Perimeter = 30 cm

a + b + c = 30

⇒ 12 + 12 + c = 30

⇒ 24 + c = 30

⇒ c = 30 – 24

⇒ c = 6 cm

s = $\frac{30}{2}$ cm = 15 cm

$\therefore$ Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

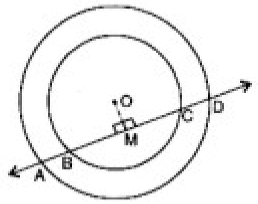
$= \sqrt{15(15-12)(15-12)(15-6)}$

$= \sqrt{15(3)(3)(9)} = 9\sqrt{15}$ cm^2

23. Given: A line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D.

To prove : AB = CD

Construction : Draw $OM \perp BC$



Proof: $\because$ The perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore$ AM = DM

and BM = CM

Subtracting (2) from (1), we get

AM - BM = DM - CM

$\Rightarrow$ AB = CD

24. We have, $\angle$BAC = 50°

$\angle$DBC = 70°

Therefore, $\angle$BDC = $\angle$BAC = 50° ... (Angles on same segment)

In triangle BDC, by angle sum property

$\angle$BDC + $\angle$BCD + $\angle$DBC = 180°

50° + x + 70° = 180°

120° + x = 180°

x = 60°.

OR

Given: In figure, OA and OB are respectively perpendiculars to chords CD and EF of a circle whose centre is O. OA = OB.

To prove: $\overline{EC} \cong \overline{DF}$

Proof: OA = OB |Given

$\therefore CD = EF$ |$\because$ Chords equidistant from the centre are equal

$\Rightarrow \overline{CD} \cong \overline{EF}$ |$\because$ If two chords of a circle are equal, then their corresponding arcs are congruent

$\Rightarrow \overline{CD} - \overline{ED} \cong \overline{EF} - \overline{ED}$

$\Rightarrow \overline{CE} \cong \overline{DF}$

$\Rightarrow \overline{EC} \cong \overline{DF}$

25. Given, -4x +3y =12 .....(1)

Put value of x = 0 in equation (1), we get

⇒ 0 + 3y = 12

⇒ y = 4

Thus, **x = 0 and y = 4 is a solution**

put value of y = 0 in equation (1), we get

⇒ -4x + 0 = 12

⇒ x = -3

Thus, **x = -3 and y = 0 is a solution**

OR

We need to express the linear equation y - 2 = 0 in the form ax + by + c = 0 and indicate the values of a, b and c.

y - 2 = 0 can also be written as 0·x + 1·y - 2 = 0.

We need to compare the equation 0·x + 1·y - 2 = 0 with the general equation ax + by + c = 0, to get the values of a, b and c.

Therefore, we can conclude that a = 0, b = 1 and c = -2

Section C

26. $\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}$

$\frac{1}{(\sqrt{7}+\sqrt{6})-\sqrt{13}} \times \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})+\sqrt{13}}$

= $\frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})^2-\sqrt{13}^2}$ [∵$a^2 - b^2 = (a + b)(a - b)$]

= $\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{(7+6+2\sqrt{42})-13}$

= $\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{13+2\sqrt{42}-13}$

= $\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2\sqrt{42}}$

= $\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2\sqrt{42}} \times \frac{\sqrt{42}}{\sqrt{42}}$

= $\frac{\sqrt{7\times42}+\sqrt{6\times42}+\sqrt{13\times42}}{2(\sqrt{42})^2}$

= $\frac{\sqrt{7\times7\times6}+\sqrt{6\times6\times7}+\sqrt{546}}{2\times42}$

= $\frac{7\sqrt{6}+6\sqrt{7}+\sqrt{546}}{84}$

27. $8a^3 - b^3 - 12a^2b + 6ab^2$

The expression $8a^3 - b^3 - 12a^2b + 6ab^2$ can also be written as $= (2a)^3 - (b)^3 - 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$

$= (2a)^3 - (b)^3 - 3 \times 2a \times b\,(2a - b)$.

Using identity $(x - y)^3 = x^3 - y^3 - 3xy\,(x - y)$ with respect to the expression

$(2a)^3 - (b)^3 - 3 \times 2a \times b\,(2a - b)$, we get $(2a - b)^3$

Therefore, after factorizing the expression $8a^3 - b^3 - 12a^2b + 6ab^2, weget (2a - b)^3$

28. Let the Traffic signal board is ΔABC.

According to question, Semi-perimeter of ΔABC (s) =$\frac{a+a+a}{2} = \frac{3a}{2}$

Using Heron's Formula, Area of triangle ABC = $\sqrt{s\,(s-a)\,(s-b)\,(s-c)}$

=$\sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$

=$\sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}$

=$\sqrt{3\left(\frac{a}{2}\right)^4}$

=$\frac{\sqrt{3}a^2}{4}$

Now,If Perimeter of this triangle = 180 cm

⇒ Side of triangle (a) =$\frac{180}{3}$ = 60 cm

Using the above derived formula,

Area of triangle ABC

=$\frac{\sqrt{3}(60^2)}{4}$

=$15 \times 60\sqrt{3}$

=$900\sqrt{3}$cm^2

OR

We have, 2s = 50 m + 65 m + 65 m = 180 m

S = 180 ÷ 2 = 90 m

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{90(90-50)(90-65)(90-65)}$

$= \sqrt{90 \times 40 \times 25 \times 25} = 60 \times 25$

= 1500m^2.

Cost of laying grass at the rate of Rs7 per m^2 = Rs(1500 × 7) = Rs10,500.

29. Suppose r be the common radius of a cylinder, cone and a sphere.

height of the cylinder = Height of the cone = Height of the sphere $= 2r$

Let 'l' be the slant height of the cone. Then

l = $\sqrt{r^2+h^2}$ = $\sqrt{r^2+(2r)^2}$ = $\sqrt{5}$r

S_1 = Curved surface area of cylinder = $2\pi rh$

= $2\pi r \cdot 4\pi r^2$

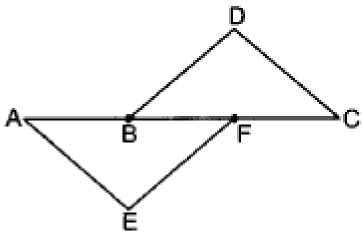
S_2 = Curved surface area of cone = $\pi rl = \pi r\sqrt{5}r = \sqrt{5}\pi r^2$

S_3 = Curved surface area of sphere = $4\pi r^2$

$S_1 : S_2 : S_3 = 4\pi r^2 : \sqrt{5}\pi r^2 : 4\pi r^2$

$\therefore S_1 : S_2 : S_3 = 4:\sqrt{5}:4$

30.



In triangles AFE and CBD (in above shown figure) , we have

AB = CF (Given)

Adding BF on both the sides, we get:-

AB + BF = CF + BF

or, AF = BC

Now in triangles AFE and CBD, we have AF= CB (Proved above)

$\angle$AFE = $\angle$CBD (Given)

and EF = BD (Given) .So, according to SAS congruency criteria of triangles;

$\triangle$AFE $\cong$ $\triangle$CBD Hence, proved.

OR

Side BC of $\triangle$ABC has been produced to D.

$\therefore \angle ACD = \angle BAC + \angle ABC$

$\Rightarrow \frac{1}{2}\angle ACD = \frac{1}{2}\angle BAC + \frac{1}{2}\angle ABC$

$\Rightarrow \angle ECD = \frac{1}{2}\angle BAC + \frac{1}{2}\angle ABC$ ...(i)

Again, side BC of AEBC has been produced to D

$\therefore \angle ECD = \angle CBE + \angle BEC$

$\Rightarrow \angle ECD = \frac{1}{2}\angle ABC + \angle BEC$

From (i) and (ii), we get

$\frac{1}{2}\angle ABC + \angle BEC = \frac{1}{2}\angle BAC + \frac{1}{2}\angle ABC$ [each equal to $\angle$ECD]

$\therefore \angle BEC = \frac{1}{2}\angle BAC$

31. i. The Co-ordinate of point A is (0, 2), B is (2, 0), C is (0, -2) and D is (-2, 0).

ii. If we joined them we get square.

iii. Co-ordinate of intersection point of AC and BD is (0, 0).

Section D

32. $p = \frac{3-\sqrt{5}}{3+\sqrt{5}}$

$= \frac{3-\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$

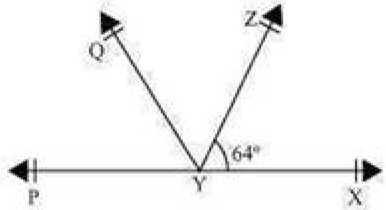
$= \frac{(3-\sqrt{5})^2}{3^2-\sqrt{5}^2}$
$= \frac{9+5-6\sqrt{5}}{9-5}$
$= \frac{14-6\sqrt{5}}{4}$
$= \frac{7-3\sqrt{5}}{2}$
$q = \frac{3+\sqrt{5}}{3-\sqrt{5}}$
$= \frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$
$= \frac{(3+\sqrt{5})^2}{3^2-\sqrt{5}^2}$
$= \frac{9+5+6\sqrt{5}}{9-5}$
$= \frac{14+6\sqrt{5}}{4}$
$= \frac{7+3\sqrt{5}}{2}$
$p^2 + q^2$
$= \left(\frac{7-3\sqrt{5}}{2}\right)^2 + \left(\frac{7+3\sqrt{5}}{2}\right)^2$
$= \frac{49+45-42\sqrt{5}}{4} + \frac{49+45+42\sqrt{5}}{4}$
$= \frac{94-42\sqrt{5}}{4} + \frac{94+42\sqrt{5}}{4}$
$= \frac{47-21\sqrt{5}}{2} + \frac{47+21\sqrt{5}}{2}$
$= \frac{47-21\sqrt{5}+47+21\sqrt{5}}{2}$
$= \frac{94}{2}$
= 47

OR

LHS
$= \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$
$= \frac{7+3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$
$= \frac{7\times3-7\sqrt{5}+3\sqrt{5}\times3-3\sqrt{5}\times\sqrt{5}}{3^2-\sqrt{5}^2} - \frac{7\times3+7\sqrt{5}-3\sqrt{5}\times3-3\sqrt{5}\times\sqrt{5}}{3^2-\sqrt{5}^2}$
$= \frac{21-7\sqrt{5}+9\sqrt{5}-15}{9-5} - \frac{21+7\sqrt{5}-9\sqrt{5}-15}{9-5}$
$= \frac{6+2\sqrt{5}}{4} - \frac{6-2\sqrt{5}}{4}$
$= \frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}$
$= \frac{0+4\sqrt{5}}{4}$
$= 0 + \sqrt{5}$
We know that,
$\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$
$0 + \sqrt{5} = a + b\sqrt{5}$
a = 0 and b = 1

33. i. $\overleftrightarrow{EF}$, $\overleftrightarrow{GH}$ and their corresponding point of intersection is R.
$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and their corresponding point of intersection is P.
ii. $\overleftrightarrow{AB}$, $\overleftrightarrow{EF}$, $\overleftrightarrow{GH}$ and their point of intersection is R.
iii. Three rays are: $\overrightarrow{RB}$, $\overrightarrow{RH}$, $\overrightarrow{RG}$
iv. Two line segments are: $\overline{RQ}, \overline{RP}$.

34. We are given that ∠XYZ = 64^o, XY is produced to P and YQ bisects ∠ZYP We can conclude the given below figure for the given situation:



We need to find ∠XYQ and reflex ∠QYP

From the given figure, we can conclude that ∠XYZ and ∠ZYP form a linear pair.

We know that sum of the angles of a linear pair is 180^o.

∠XYZ + ∠ZYP = 180^o

But ∠XYZ = 64^o

⇒ 64^o + ∠ZYP = 180^o

⇒ ∠ZYP = 116^o

Ray YQ bisects ∠ZYP,or

∠QYZ = ∠QYP = $\frac{116^o}{2}$ = 58^o

∠XYQ = ∠QYZ + ∠XYZ

= 58^o + 64^o = 122^o.

Reflex ∠QYP = 360° - ∠QYP

= 360^o - 58^o

= 302^o.

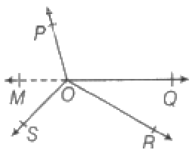
Therefore, we can conclude that ∠XYQ = 122^o and Reflex ∠QYP = 302^o

OR

Let us produce a ray OQ backwards to a point M, then MOQ is a straight line.

Now, OP is a ray on the line MOQ. Then, by linear pair axiom, we have

∠MOP + ∠POQ = 180^o ......(i)



Similarly, OS is a ray on the line MOQ. Then, by linear pair axiom, we have

∠MOS + ∠SOQ = 180^o ....(ii)

Also, ∠SOR and ∠ROQ are adjacent angles.

∴ ∠SOQ = ∠SOR + ∠ROQ ...(iii)

On putting the value of ∠SOQ from Eq.(iii) in Eq.(ii), we get

∠MOS + ∠SOR + ∠ROQ = 180^o ....(iv)

Now, on adding Eqs.(i) and (iv), we get

∠MOP + ∠POQ + ∠MOS + ∠SOR + ∠ROQ = 180^o + 180^o

⇒ ∠MOP + ∠MOS + ∠POQ + ∠SOR + ∠ROQ = 360^o ....(iv)

But ∠MOP + ∠MOS = ∠POS

Then, from Eq.(v), we get

∠POS + ∠POQ + ∠SOR + ∠ROQ = 360^o

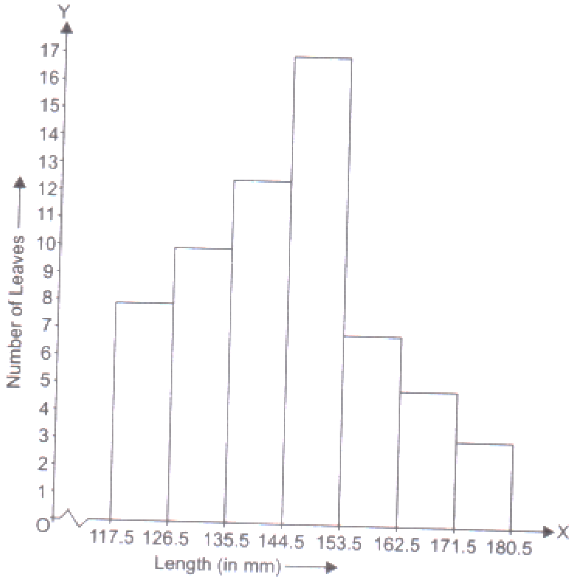
Hence proved.

35. The given table is in inclusive form. So, we will first convert it into an exclusive from as given below:

Length (in mm)	Number of leaves
117.5 – 126.5	8
126.5 – 135.5	10
135.5 – 144.5	12
144.5 – 153.5	17

153.5 – 162.5	7
162.5 – 171.5	5
171.5 – 180.5	3

A histogram for this table is shown in the figure given below:



Section E

36. i. The radius of the ball= 3.5 mm

Volume of the ball

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$

= 179.66 mm^3

ii. Radius of one ball = 3.5 cm

The surface area of one ball

$= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$

= 154 mm^2

iii. Radius of one ball= 7 cm

Thus volume of 10 balls of radius 7 mm

$= 10 \times \frac{4}{3}\pi r^3$

$= 10 \times \frac{4}{3} \times \frac{22}{7} \times 7^3$

=14373.3 mm^3

OR

Volume of 10 balls of 7 mm = 14373.3 mm^3

Volume of 1 ball of 3.5 mm = 179.66 mm^3

Volume of 20 balls of 3.5 mm = 179.66 $\times$ 20= 3593.33 mm^3

Total steel required to be melted = 14373.3 + 3593.33 = 17966 mm^3(Approx)

Thus steel left over = 20,000 - 17966 = 2034 mm^3

37. i. Delhi to Jaipur: 2x + y = 600

Jaipur to Delhi: 2y + x = 600

Let S_1 and S_2 be the speeds of Train and Taxi respectively, then

Dehli to Jaipur: $\frac{2x}{S_1} + \frac{y}{S_2} = 8$ ...(i)

Jaipur to Delhi: $\frac{x}{S_1} + \frac{2y}{S_2} = 10$ ...(ii)

ii. 2x + y = 600 ...(1)

x + 2y = 600 ...(2)

Solving (1) and (2) $\times$ 2

2x + y - 2x - 4y = 600 - 1200

$\Rightarrow$ - 3y = - 600

$\Rightarrow$ y = 200

Put y = 200 in (1)

2x + 200 = 600

$\Rightarrow$ x = $\frac{400}{2}$ = 200

iii. We know that speed = $\frac{Distance}{Time} \Rightarrow$ Time = $\frac{Distance}{Speed}$

Let S_1 and S_2 are speeds of train and taxi respectively.

Delhi to Jaipur: $\frac{2x}{S_1} + \frac{y}{S_2}$ = 8 ...(i)

Jaipur to Delhi: $\frac{x}{S_1} + \frac{2y}{S_2}$ = 10 ...(ii)

Solving (i) and (ii) $\times$ 2

$\Rightarrow \frac{2x}{S_1} + \frac{y}{S_2} - \frac{2x}{S_1} - \frac{4y}{S_2}$ = 8 - 20 = -12

$\Rightarrow \frac{-3y}{S_2}$ = -12

We know that y = 200 km

$\Rightarrow S_2 = \frac{3\times200}{12}$ = 50 km/hr

Hence speed of Taxi = 50 km/hr

OR

We know that x = 200 km

Put S_2 = 50 km/hr ...(i)

$\frac{400}{S_1} + \frac{200}{50}$ = 8

$\Rightarrow \frac{400}{S_1}$ = 8 - 4 = 4

$\Rightarrow S_1 = \frac{400}{4}$ = 100 km/hr

Hence speed of Train = 100 km/hr

38. i. We know that line joining mid points of two sides of triangle is half and parallel to third side.

Hence RQ is parallel to BC and half of BC.

RQ = $\frac{28}{2}$ = 14 cm

Length of RQ = 14 cm

ii. By mid-point theorem we know that line joining mid points of two sides of triangle is half and parallel to third side.

PQ = $\frac{AB}{2} = \frac{25}{2}$ = 12.5 cm

QR = $\frac{BC}{2} = \frac{28}{2}$ = 14 cm

RP = $\frac{AC}{2} = \frac{26}{2}$ = 13 cm

Length of garland = PQ + QR + RP = 12.5 + 14 + 13 = 39.5 cm

Length of garland = 39.5 cm.

iii. As R and P are mid-points of sides AB and BC of the triangle ABC, by mid point theorem, RP || AC Similarly, RQ || BC and PQ || AB. Therefore ARPQ, BRQP and RQCP are all parallelograms. Now RQ is a diagonal of the parallelogram ARPQ, therefore, $\triangle$ARQ $\cong$ $\triangle$PQR Similarly $\triangle$CPQ $\cong$ $\triangle$RQP and $\triangle$BPR $\cong$ $\triangle$QRP So, all the four triangles are congruent.

Therefore Area of $\triangle$ARQ = Area of $\triangle$CPQ = Area of $\triangle$BPR = Area of $\triangle$PQR

Area $\triangle$ABC = Area of $\triangle$ARQ + Area of $\triangle$CPQ + Area of $\triangle$BPR + Area of $\triangle$PQR

Area of $\triangle$ ABC = 4 Area of $\triangle$PQR

$\triangle$PQR = $\frac{1}{4}$ar(ABC)

OR

As R and Q are mid-points of sides AB and AC of the triangle ABC. Similarly, P and Q are mid points of sides BC and AC by mid-point theorem, RQ || BC and PQ || AB. Therefore BRQP is parallelogram.