SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)	–	1(3)	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)	–	–	**1(2)**
3.	Matrices	2(2)	1(2)	–	–	**3(4)**
4.	Determinants	1(1)	–	–	1(5)*	**2(6)**
5.	Continuity and Differentiability	–	1(2)	2(6)#	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)	1(3)	–	**3(9)**
7.	Integrals	1(1)*	1(2)*	1(3)	–	**3(6)**
8.	Application of Integrals	1(1)	1(2)	1(3)	–	**3(6)**
9.	Differential Equations	1(1)*	1(2)*	1(3)*	–	**3(6)**
10.	Vector Algebra	1(1)*	1(2)*	–	–	**2(3)**
11.	Three Dimensional Geometry	2(2)# + 1(4)	–	–	1(5)*	**4(11)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	4(4)#	2(4)	–	–	**6(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Solve the differential equation $\sin\left(\frac{dy}{dx}\right) = a$.

OR

Solve the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$.

2. Check whether the function $f : z \to z$, defined by $f(x) = x^2 + 5 \ \forall x \in z$ is one-one or not.

3. A line makes angles α, β and γ with the coordinate axes. If $\alpha + \beta = 90°$, then find the value of angle γ.

OR

Find the distance of the plane $5x - y + 6z - 12 = 0$ from the origin.

4. If $\begin{bmatrix} a+b & 2 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}'$, then find the value of a.

5. If $|\vec{a} - \vec{b}| = |\vec{a}| = |\vec{b}| = 1$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

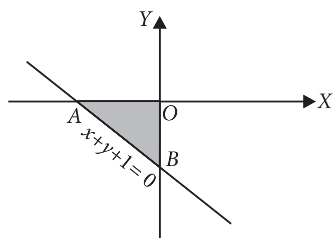
6. Let $A = \{1, 2, 3\}$. Check whether the relation $\{(1,1),(2,2),(3,3),(1,2),(2,1)\}$ is an equivalence relation on A or not.

7. Evaluate : $\int \frac{dx}{\sqrt{x^2 - 3x + 2}}$

OR

Evaluate : $\int_0^1 \left\{e^x + \sin\frac{\pi x}{4}\right\} dx$

8. Find the area of the shaded region, shown in the given figure.



9. A bag contains 12 balls, of which 5 are red and 7 are blue. If 2 balls are drawn at random then find the probability of getting atleast 1 blue ball.

OR

If A and B are two independent events such that $P(B) \neq 0$, then find $P(A \mid B)$.

10. If A and B are symmetric matrices of the same order, then show that $AB + BA$ is a symmetric matrix.

11. The probability distribution of a random variable X is given below :

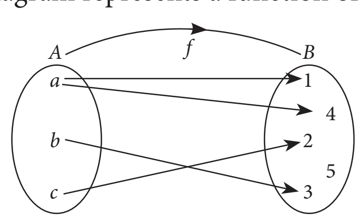
X	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

Find the value of k.

12. If the equation of a line is $\frac{2x-5}{4} = \frac{y+4}{3} = \frac{6-z}{6}$, then find the direction cosines of a line parallel to this line.

13. If $P(\text{not } E) = 0.36$ and $P(F \mid E) = 0.5$, then find $P(E \cap F)$.

14. Check whether the following arrow diagram represents a function or not.



15. If A and B are two independent events, then show that the probability of occurrence of atleast one of A and B is given by $1 - P(A')\,P(B')$.

16. The value of the determinant of a matrix A of order 3×3 is 4. Find the value of $|5A|$.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. Two cars A and B are running at the speed more than allowed speed on the road along the lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k})$, respectively.

Based on the above answer the following :

(i) The cartesian equation of the line along which car A is running, is

(a) $\frac{x+1}{3} = \frac{y+1}{-1} = \frac{z-1}{0}$ (b) $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$

(c) $\frac{x-1}{3} = \frac{y-1}{0} = \frac{z+1}{-1}$ (d) None of these

(ii) The direction cosines of line along which car A is running, are

(a) 3, –1, 0 (b) – 3, –1, 0 (c) $\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}, 0$ (d) $\frac{-3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}, 0$

(iii) The direction ratios of line along which car B is running, are

(a) 3, 0, 2 (b) 2, 3, 0 (c) 2, 0, 3 (d) 0, 3, 2

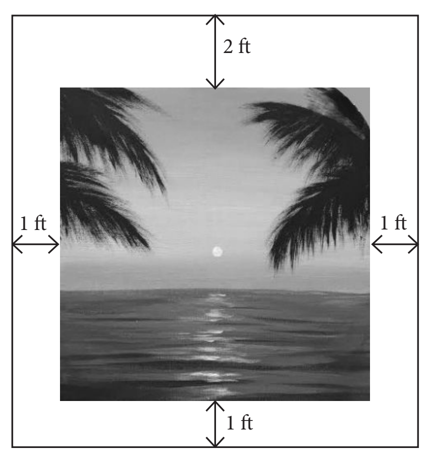
(iv) The shortest distance between the gives lines is

(a) 4 units (b) $2\sqrt{3}$ units (c) $3\sqrt{2}$ units (d) 0 units

(v) The cars will meet with an accident at the point

(a) (–1, 0, 4) (b) (4, 0, –1) (c) (4, –1, 0) (d) does not exist

18. Neeta has a rectangular painting having a total area of 24 ft^2 which includes a border of 1 ft on the left, right, bottom and a border of 2 ft on the top inside it.



Based on the above information, answer the following questions :

(i) If Neeta wants to paint in the maximum area, then she needs to maximize

(a) Area of outer rectangle

(b) Area of inner rectangle

(c) Area of top border

(d) None of these

(ii) It x is the length of the outer rectangle, then area of inner rectangle in terms of x is

(a) $(x+3)\left(\frac{24}{x}-2\right)$ (b) $(x-2)\left(\frac{24}{x}+3\right)$ (c) $(x-2)\left(\frac{24}{x}-3\right)$ (d) $(x-2)\left(\frac{24}{x}\right)$

(iii) Find the range of x.

(a) $(2, \infty)$ (b) (2, 8) (c) $(-\infty, 2)$ (d) (–2, 8)

(iv) If area of inner rectangle is maximum, then x is equal to

(a) 2 ft (b) 3 ft (c) 4 ft (d) 5 ft

(v) If area of inner rectangle is maximum, then length and breadth of this rectangle are respectively

(a) 2 ft, 3 ft (b) 3 ft, 4 ft (c) 1 ft, 2 ft (d) 2 ft, 4 ft

PART - B

Section - III

19. If $P(A)=\frac{3}{8}$, $P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{4}$, then find $P(\bar{A}|\bar{B})$ and $P(\bar{B}|\bar{A})$.

20. Evaluate : $\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}\,dx$

OR

Evaluate : $\int_0^1 \frac{2x}{1+x^2}dx$

21. Find the equation of the normal to the curve $y = x^2 + 4x + 1$ at the point where $x = 3$.

22. Find a matrix A such that $2A - 3B + 5C = O$, where $B=\begin{bmatrix}-2 & 2 & 0\\ 3 & 1 & 4\end{bmatrix}$ and $C=\begin{bmatrix}2 & 0 & -2\\ 7 & 1 & 6\end{bmatrix}$.

23. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

OR

If the angle between $\hat{i}+\hat{k}$ and $\hat{i}+\hat{j}+a\hat{k}$ is $\frac{\pi}{3}$, then find the values of a.

24. Find the principal value of $\cot^{-1}\left[\frac{\sqrt{1-\sin\frac{\pi}{2}}+\sqrt{1+\sin\frac{\pi}{2}}}{\sqrt{1-\sin\frac{\pi}{2}}-\sqrt{1+\sin\frac{\pi}{2}}}\right]$.

25. Let X and Y be two events such that $P(X)=\frac{1}{3}, P(Y)=\frac{4}{15}$ and $P(Y|X)=\frac{2}{5}$. Then find $P(X'|Y)$

26. Find the area bounded by the parabola $y = 2x - x^2$ and x-axis.

27. Determine the constants a and b such that the function $f(x)=\begin{cases} ax^2+b, & \text{if } x>2\\ 2, & \text{if } x=2\\ 2ax-b, & \text{if } x<2\end{cases}$ is continuous at $x = 2$.

28. Solve the differential equation $(x-1)\frac{dy}{dx}=2xy$.

OR

Solve the differential equation $5\frac{dy}{dx}=e^x y^4$.

Section - IV

29. Evaluate : $\int \frac{3x+1}{\sqrt{5-2x-x^2}}dx$

30. Show that the function $f(x) = |x - 3|, x \in R$ is continuous but not differentiable at $x = 3$.

31. Find the solution of the differential equation $(x^2 - 2x + 2y)\,dx + x\,dy = 0$.

OR

Solve : $e^x\sqrt{1-y^2}\,dx+\frac{y}{x}dy=0$

32. Find the area lying in the first quadrant bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$.

33. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f: A \to B$ is a function defined by, $f(x)=\frac{x-1}{x-2}$, show that f is one-one and onto.

34. Find the derivative of the function $y=\log\sqrt{\frac{1+\cos^2 x}{1-e^{2x}}}$ w.r.t. x. Also, find $y'\left(\frac{\pi}{2}\right)$.

OR

If $f(x) = [x]$, $-2 \le x \le 2$, then show that $f(x)$ is neither continuous nor differentiable at $x = 1$.

35. Find the minimum value of the function $f(x) = x \log x$.

Section - V

36. Using concept of inverse of matrix, find the matrix A satisfying the matrix equation $\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}\cdot A\cdot\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$.

OR

If $A=\begin{bmatrix}2 & -3 & 5\\3 & 2 & -4\\1 & 1 & -2\end{bmatrix}$, then find A^{-1}.

Using A^{-1}, solve the following system of equations :

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

37. Solve the following LPP graphically.

Maximize $Z=\frac{2x}{25}+\frac{y}{10}$

Subject to constraints

$x \ge 2000$, $y \ge 4000$

and $x + y \le 12000$

OR

Solve the following problem graphically.

Minimize $Z = x - 7y + 190$

subject to constraints

$x + y \le 8$

$x + y \ge 4$

$x \le 5$

$y \le 5$

and $x \ge 0$, $y \ge 0$

38. Find the vector equations of the planes through the intersection of the planes $\vec{r}\cdot(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})=0$, which are at a unit distance from the origin.

OR

Find the coordinates of the foot of the perpendicular drawn from the point $(2, 3, 4)$ to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also, find the perpendicular distance from the given point to the line.

SOLUTIONS

1. We have, $\sin\left(\frac{dy}{dx}\right) = a$

$\Rightarrow \frac{dy}{dx} = \sin^{-1} a \quad \Rightarrow dy = \sin^{-1} a\, dx$

On integrating both sides, we get

$\int dy = \int \sin^{-1} a\, dx$

$\Rightarrow y = x \cdot \sin^{-1} a + c$

OR

We have, $\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}}$

On integrating both sides, we get

$\int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$

$\Rightarrow \sin^{-1} y = -\sin^{-1} x + c$

$\Rightarrow \sin^{-1} y + \sin^{-1} x = c$

2. Since, $f(1) = f(-1) = 6$, therefore f is not one-one.

[$\because$ For f to be one-one distinct elements should have distinct images]

3. We know that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2\alpha + \cos^2(90° - \alpha) + \cos^2\gamma = 1$ [$\because \alpha + \beta = 90°$]

$\Rightarrow \cos^2\alpha + \sin^2\alpha + \cos^2\gamma = 1 \Rightarrow 1 + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 0 \Rightarrow \cos\gamma = 0 \Rightarrow \gamma = 90°$.

OR

Required distance $= \frac{|-12|}{\sqrt{5^2 + (-1)^2 + 6^2}} = \frac{12}{\sqrt{62}}$

4. Given, $\begin{bmatrix} a+b & 2 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}'$

$\Rightarrow \begin{bmatrix} a+b & 2 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 2 \end{bmatrix}$

On comparing corresponding elements of the matrices, we get $a + b = 6$ and $b = 2 \Rightarrow a = 4$

5. Given, $|\vec{a} - \vec{b}| = |\vec{a}| = |\vec{b}| = 1$

Now, $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} \Rightarrow 1 = 1 + 1 - 2|\vec{a}||\vec{b}|\cos\theta$

(where θ is angle between $\vec{a}$ and $\vec{b}$)

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

OR

Given, $|\vec{a}| = |\vec{b}|, \theta = 60°$ and $\vec{a}\cdot\vec{b} = \frac{9}{2}$

Now, $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\Rightarrow \cos 60° = \frac{9/2}{|\vec{a}|^2} \Rightarrow \frac{1}{2} = \frac{9/2}{|\vec{a}|^2}$

$\Rightarrow |\vec{a}|^2 = 9 \Rightarrow |\vec{a}| = 3 \therefore |\vec{a}| = |\vec{b}| = 3$

6. We have, $A = \{1, 2, 3\}$

and let $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

Since, for each $a \in A$, we have $(a, a) \in R$.

Also, $(a, b) \in R \Rightarrow (b, a) \in R$

and $(1, 2) \in R, (2, 1) \in R \Rightarrow (1, 1) \in R$

Thus, R is reflexive, symmetric and transitive .

Hence, R is an equivalence relation.

7. We have, $\int \frac{dx}{\sqrt{x^2 - 3x + 2}} = \int \frac{dx}{\sqrt{\left(x^2 - 3x + \frac{9}{4}\right) - \frac{1}{4}}}$

$= \int \frac{dx}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} = \log\left|\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2}\right| + C$

OR

We have, $\int_0^1 \left\{e^x + \sin\frac{\pi x}{4}\right\} dx$

$= \left[e^x\right]_0^1 + \frac{4}{\pi}\left[-\cos\frac{\pi x}{4}\right]_0^1 = e - 1 - \frac{4}{\sqrt{2}\pi} + \frac{4}{\pi}$

8. Required area $= \left|\int_{-1}^{0} (-1 - x)\,dx\right|$

$= \left|\left[\left(-x - \frac{x^2}{2}\right)\right]_{-1}^{0}\right| = \left|\frac{-1}{2}\right| = \frac{1}{2}$ sq. unit

9. Required probability $= 1 - P$ (getting no blue ball)

$= 1 - \frac{{}^5C_2}{{}^{12}C_2} = 1 - \frac{10}{66} = \frac{56}{66} = \frac{28}{33}$

OR

Clearly $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\cdot P(B)}{P(B)} = P(A)$

10. $(AB + BA)^T = (AB)^T + (BA)^T$

$= B^T A^T + A^T B^T = BA + AB = AB + BA$

($\because A^T = A$ and $B^T = B$)

Hence, $AB + BA$ is a symmetric matrix.

11. Clearly, $\Sigma P(X) = 1$

$\Rightarrow \frac{5}{k}+\frac{7}{k}+\frac{9}{k}+\frac{11}{k}=1 \Rightarrow \frac{32}{k}=1 \Rightarrow k = 32$

12. The equation of line is $\frac{2x-5}{4}=\frac{y+4}{3}=\frac{6-z}{6}$

or $\frac{x-\frac{5}{2}}{2}=\frac{y+4}{3}=\frac{z-6}{-6}$.

$\therefore$ Direction ratios of line are < 2, 3, – 6 >.

$\therefore$ Direction cosines are

$$\frac{2}{\sqrt{2^2+3^2+(-6)^2}}, \frac{3}{\sqrt{2^2+3^2+(-6)^2}}, \frac{-6}{\sqrt{2^2+3^2+(-6)^2}}$$

or $\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$

So, direction cosines of a line parallel to given line are $\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$.

13. Clearly, $P(E) = 1 - P(\bar{E}) = 1 - 0.36 = 0.64$

Now, $P(F \mid E) = 0.5$

$\Rightarrow \frac{P(E \cap F)}{P(E)} = 0.5 \Rightarrow P(E \cap F) = P(E) \times 0.5$

$= 0.64 \times 0.5 = 0.32$

14. As $f(a)$ is not unique, therefore f is not a function.

15. P(atleast one of A and B)

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\, P(B)$ [$\because$ A, B are independent]

$= P(A) + P(B)\,[1 - P(A)] = [1 - P(A')] + P(B)\, P(A')$

$= 1 - P(A') + P(B)\, P(A') = 1 - P(A')\,[1 - P(B)]$

$= 1 - P(A')\, P(B')$

16. Given, A is a 3×3 matrix and $|A| = 4$

$\Rightarrow |5A| = 5^3 \cdot |A| = 125 \times 4 = 500.$

17. (i) (b) : The line along which car A is running, is $\vec{r} = \hat{i}+\hat{j}-\hat{k}+\lambda(3\hat{i}-\hat{j})$, which can be rewritten as

$(x\hat{i}+y\hat{j}+z\hat{k}) = (1+3\lambda)\hat{i}+(1-\lambda)\hat{j}-\hat{k}$

$\Rightarrow x = 1+3\lambda,\ y = 1-\lambda,\ z = -1$

$\Rightarrow \frac{x-1}{3} = \lambda,\ \frac{y-1}{-1} = \lambda,\ z+1 = 0$

Thus, the required cartesian equation is

$$\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$$

(ii) (c) : Clearly, D.R.'s of the required line are <3, –1, 0>

$\therefore$ D.C.'s are $< \frac{3}{\sqrt{3^2+(-1)^2+0^2}}, \frac{-1}{\sqrt{3^2+(-1)^2+0^2}}, 0 >$

i.e., $< \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}, 0 >$

(iii) (c) : The line along which car B is running, is $\vec{r} = 4\hat{i}-\hat{k}+\mu(2\hat{i}+3\hat{k})$, which is parallel to the vector $2\hat{i}+3\hat{k}$

$\therefore$ D.R.'s of the required line are <2, 0, 3>

(iv) (d) : Here, $\vec{a}_1 = \hat{i}+\hat{j}-\hat{k}$, $\vec{a}_2 = 4\hat{i}-\hat{k}$, $\vec{b}_1 = 3\hat{i}-\hat{j}$ and $\vec{b}_2 = 2\hat{i}+3\hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i}-\hat{j}$

and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = -3\hat{i}-9\hat{j}+2\hat{k}$

Now, $(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2) = (3\hat{i}-\hat{j})\cdot(-3\hat{i}-9\hat{j}+2\hat{k})$

$= -9 + 9 = 0$

Hence, shortest distance between the given lines is 0.

(v) (b) : Since, the point (4, 0, –1) satisfy both the equations of lines, therefore point of intersection of given lines is (4, 0, –1).

So, the cars will meet with an accident at the point (4, 0, –1).

18. (i) (b) : In order to paint in the maximum area, Neeta needs to maximize the area of inner rectangle.

(ii) (c) : Let x be the length and y be the breadth of outer rectangle.

$\therefore$ Length of inner rectangle $= x - 2$

and breadth of inner rectangle $= y - 3$

$\therefore A(x) = (x - 2)(y - 3)$ [$\because xy = 24$ (given)]

$= (x-2)\left(\frac{24}{x}-3\right)$

(iii) (b) : Dimensions of rectangle (outer/inner) should be positive.

$\therefore x - 2 > 0$ and $\frac{24}{x} - 3 > 0$

$\Rightarrow x > 2$ and $x < 8$

$\therefore$ Range of x is (2, 8).

(iv) (c) : We have, $A(x) = (x - 2)\left(\frac{24}{x}-3\right)$

$\Rightarrow A'(x) = (x-2)\left(\frac{-24}{x^2}\right)+\left(\frac{24}{x}-3\right) = \frac{48}{x^2}-3$

and $A''(x) = \frac{-96}{x^3}$

For $A(x)$ to be maximum or minimum, $A'(x) = 0$

$\Rightarrow -3+\frac{48}{x^2} = 0 \Rightarrow x = \pm\sqrt{\frac{48}{3}} = \pm 4$

$\therefore x = 4$ [Since, length can't be negative]

Also, $A''(4) = \frac{-96}{4^3} < 0$

Thus, at $x = 4$, area is maximum.

(v) (a) : If area of inner rectangle is maximum, then Length of inner rectangle $= x - 2 = 4 - 2 = 2$ ft

And breadth of inner rectangle $= y - 3 = \frac{24}{x} - 3$

$= \frac{24}{4} - 3 = 6 - 3 = 3$ ft

19. We have, $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$

$\Rightarrow P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$

$\Rightarrow P(\overline{A} \cap \overline{B}) = 1 - \{P(A) + P(B) - P(A \cap B)\}$

$\Rightarrow P(\overline{A} \cap \overline{B}) = 1 - \left\{\frac{3}{8} + \frac{1}{2} - \frac{1}{4}\right\} = \frac{3}{8}$

Also, $P(\overline{A}) = 1 - P(A) = \frac{5}{8}$ and $P(\overline{B}) = 1 - P(B) = \frac{1}{2}$

Now, $P\left(\overline{A} \mid \overline{B}\right) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{\frac{3}{8}}{\frac{1}{2}} = \frac{3}{4}$

and $P\left(\overline{B} \mid \overline{A}\right) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{A})} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$

20. Let $\tan^{-1} x = \theta \Rightarrow x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

Now, $x = 0 \Rightarrow \theta = 0$ and $x = 1 \Rightarrow \theta = \frac{\pi}{4}$

$\therefore \quad I = \int_0^1 \frac{x\tan^{-1} x}{(1+x^2)^{3/2}} dx = \int_0^{\pi/4} \frac{\theta\tan\theta}{\sec^3\theta} \sec^2\theta\, d\theta$

$= \int_0^{\pi/4} \theta\sin\theta\, d\theta = [-\theta\cos\theta]_0^{\pi/4} - \int_0^{\pi/4} (-\cos\theta)\, d\theta$

$= [-\theta\cos\theta]_0^{\pi/4} + [\sin\theta]_0^{\pi/4} = \frac{4-\pi}{4\sqrt{2}}$

OR

Let $I = \int_0^1 \frac{2x}{1+x^2} dx$

Put $1 + x^2 = t \Rightarrow 2x dx = dt$

Also, $x = 0 \Rightarrow t = 1$ and $x = 1 \Rightarrow t = 2$

$\therefore \quad I = \int_1^2 \frac{dt}{t} = [\log|t|]_1^2$

$= \log 2 - \log 1 = \log 2 \qquad [\because \log 1 = 0]$

21. When $x = 3$, we have $y = (3^2 + 4 \times 3 + 1) = 22$

So, the point of contact is (3, 22)

Now, $y = x^2 + 4x + 1$ ... (i)

$\Rightarrow \frac{dy}{dx} = 2x + 4$ and $\left(\frac{dy}{dx}\right)_{(3,\,22)} = (2 \times 3 + 4) = 10$

$\therefore$ Equation of the normal at (3, 22) is

$\frac{y-22}{x-3} = \frac{-1}{10} \Rightarrow x + 10y - 223 = 0$

22. Given, $2A - 3B + 5C = O$

$\Rightarrow 2A = 3B - 5C \Rightarrow A = \frac{1}{2}[3B - 5C]$ ...(i)

Now, $3B - 5C = 3\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

$= \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} - \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix}$

$= \begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$

From (i), we get $A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$.

23. Given, $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times \vec{0}$

$\Rightarrow \vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0} \Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$

Again, $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \vec{b} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{b} \times \vec{0}$

$\Rightarrow \vec{b} \times \vec{a} + \vec{b} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0} \Rightarrow \vec{a} \times \vec{b} = \vec{b} \times \vec{c}$

Thus, we get $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

OR

We have, $\cos\frac{\pi}{3} = \frac{(\hat{i} + \hat{k}) \cdot (\hat{i} + \hat{j} + a\hat{k})}{\sqrt{2}\sqrt{1+1+a^2}}$

$\left(\because \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}\right)$

$\Rightarrow \frac{1}{2} = \frac{1+a}{\sqrt{2}\sqrt{2+a^2}} \Rightarrow \frac{1}{4} = \frac{(1+a)^2}{2(2+a^2)}$

$\Rightarrow 2 + a^2 = 2(1 + a^2 + 2a) \Rightarrow a^2 + 4a = 0 \Rightarrow a = 0, -4$

24. We have, $\cot^{-1}\left[\frac{\sqrt{1-\sin\frac{\pi}{2}} + \sqrt{1+\sin\frac{\pi}{2}}}{\sqrt{1-\sin\frac{\pi}{2}} - \sqrt{1+\sin\frac{\pi}{2}}}\right]$

$= \cot^{-1}\left[\frac{0+\sqrt{2}}{0-\sqrt{2}}\right] \qquad \left[\because \sin\frac{\pi}{2} = 1\right]$

$= \cot^{-1}(-1)$

$= \frac{3\pi}{4} \qquad \left[\because \cot\frac{3\pi}{4} = \cot\left(\pi - \frac{\pi}{4}\right) = -\cot\frac{\pi}{4} = -1\right.$

$\left.\text{and } \frac{3\pi}{4} \in (0, \pi)\right]$

25. Since, $P(Y \mid X) = \frac{P(X \cap Y)}{P(X)}$, so we have

$P(X \cap Y) = P(Y|X) \cdot P(X) = \frac{2}{5} \cdot \frac{1}{3} = \frac{2}{15}$

Now, $P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{\frac{2}{15}}{\frac{4}{15}} = \frac{1}{2}$

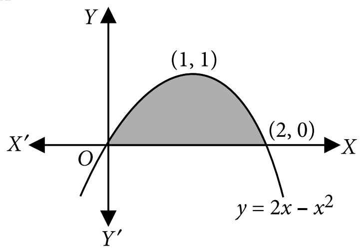
$\therefore \quad P(X'|Y) = 1 - P(X|Y) = 1 - \frac{1}{2} = \frac{1}{2}$

26. The bounded region is as shown in figure.

Curve is $y = 2x - x^2$

$\Rightarrow y - 1 = -(x - 1)^2$

is a downward parabola with vertex (1, 1)



$\therefore$ Required area

$= \int_0^2 (2x - x^2)dx = \left[x^2 - \frac{x^3}{3}\right]_0^2 = 4 - \frac{8}{3} - 0 = \frac{4}{3}$ sq. units.

27. We have, R.H.L. (at $x = 2$)

$= \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} ax^2 + b = 4a + b$

L.H.L. (at $x=2$) $= \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2ax - b) = 4a - b$

and $f(2) = 2$

Since, $f(x)$ is continuous at $x = 2$.

$\therefore \quad 4a + b = 2$ and $4a - b = 2$

Solving, we get $a = \frac{1}{2}$, $b = 0$

Thus, $f(x)$ is continuous at $x = 2$ if $a = \frac{1}{2}$ and $b = 0$.

28. We have, $(x - 1)\frac{dy}{dx} = 2xy$

$\Rightarrow (x - 1)dy = 2xydx \Rightarrow \frac{dy}{dx} = \frac{2xy}{x-1}$

$\Rightarrow \int \frac{1}{y}dy = 2\int \frac{x}{x-1}dx = 2\int \frac{x-1+1}{x-1}dx$

$\Rightarrow \log|y| + C = 2[x + \log|x - 1|]$

OR

We have, $5\frac{dy}{dx} = e^x y^4 \Rightarrow \frac{5dy}{y^4} = e^x dx$

On integrating both sides, we get

$5.\int y^{-4}dy = \int e^x dx$

$\Rightarrow 5.\frac{y^{-3}}{(-3)} = e^x + C \Rightarrow \frac{-5}{3y^3} = e^x + C$

29. Let $I = \int \frac{3x+1}{\sqrt{5-2x-x^2}}dx$

This integral is of the form $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$,

where $px + q$ is given by

$px + q = A\frac{d}{dx}(ax^2 + bx + c) + B$

Now, we write given integrand as

$3x + 1 = A\frac{d}{dx}(5 - 2x - x^2) + B$

$\Rightarrow 3x + 1 = A(-2 - 2x) + B$

$\Rightarrow 3x + 1 = -2Ax + (-2A + B)$

On equating the coefficients of x and constant term both sides, we get

$3 = -2A \Rightarrow A = -\frac{3}{2}$

and $1 = -2A + B \Rightarrow 1 = -2\left(-\frac{3}{2}\right) + B \Rightarrow B = -2$

$\therefore$ Given integral can be rewritten as

$$I = \int \frac{-\frac{3}{2}(-2-2x)}{\sqrt{5-2x-x^2}}dx + \int \frac{-2}{\sqrt{5-2x-x^2}}dx$$

$$= -\frac{3}{2}\left[\frac{(5-2x-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right] - \int \frac{2}{\sqrt{(\sqrt{6})^2 - (x+1)^2}}dx$$

$$= -\frac{3}{2}\frac{(\sqrt{5-2x-x^2})}{1/2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + C$$

$$= -3(\sqrt{5-2x-x^2}) - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + C$$

30. Given, $f(x) = |x - 3|$, $x \in R$.

$\therefore \quad f(3) = |3 - 3| = |0| = 0$

Now, $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} |x - 3|$

$= \lim_{x \to 3^-} (-(x-3)) = -(3-3) = 0$

and $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} |x - 3|$

$= \lim_{x \to 3^+} (x - 3) = 3 - 3 = 0$

Thus $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$

Thus, $f(x)$ is continuous at $x = 3$.

L.H.D. $= f'(3^-) = \lim_{h \to 0} \frac{f(3-h) - f(3)}{-h}$

$= \lim_{h \to 0} \frac{|3-h-3|-0}{-h} = \lim_{h \to 0} \frac{|-h|}{-h} = \lim_{h \to 0} \frac{h}{-h} = -1$

R.H.D. $= f'(3^+) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$

$= \lim_{h \to 0} \frac{|3+h-3|-0}{h} = \lim_{h \to 0} \frac{|h|}{h} = \lim_{h \to 0} \frac{h}{h} = 1$

$\Rightarrow$ L.H.D. $\neq$ R.H.D.
Thus, $f(x)$ is not differentiable at $x = 3$.

31. As $(x^2 - 2x + 2y)\,dx = -x\,dy$

$$\Rightarrow x\frac{dy}{dx} + 2y = 2x - x^2$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{2}{x}\right)y = 2 - x$$

I.F. $= e^{\int \frac{2}{x}dx} = x^2$

Now, required solution is

$$yx^2 = \int x^2(2-x)\,dx$$

$$\Rightarrow yx^2 = \frac{2x^3}{3} - \frac{x^4}{4} + C \Rightarrow y = \frac{2x}{3} - \frac{1}{4}x^2 + \frac{C}{x^2},$$

which is the required solution.

OR

We have, $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0$

$$\Rightarrow e^x\sqrt{1-y^2}\,dx = -\frac{y}{x}dy$$

$$\Rightarrow x e^x dx = -\frac{y}{\sqrt{1-y^2}}dy \Rightarrow \int \underset{\text{I}}{x}\,\underset{\text{II}}{e^x}\,dx = -\int \frac{y}{\sqrt{1-y^2}}dy$$

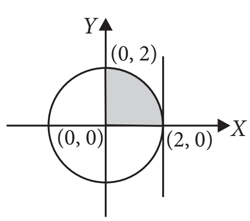
[Integrating both sides]

$$\Rightarrow xe^x - \int e^x dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}, \text{ where } t = 1 - y^2$$

$$\Rightarrow xe^x - e^x = \frac{1}{2}\left(\frac{t^{1/2}}{1/2}\right) + C \Rightarrow xe^x - e^x = \sqrt{t} + C$$

$\Rightarrow xe^x - e^x = \sqrt{1-y^2} + C$, which is the required solution.

32. Required area



$$= \int_0^2 \sqrt{4-x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= 0 + 2\sin^{-1}(1) - 0 = 2 \times \frac{\pi}{2} = \pi \text{ sq. units.}$$

33. Here, $f: A \to B$ is given by $f(x) = \dfrac{x-1}{x-2}$,

where $A = R - \{2\}$ and $B = R - \{1\}$

Let $f(x_1) = f(x_2)$, where $x_1, x_2 \in A$

(Clearly $x_1 \neq 2$ and $x_2 \neq 2$)

$$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$$

$\Rightarrow (x_1 - 1)(x_2 - 2) = (x_1 - 2)(x_2 - 1)$

$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - x_1 - 2x_2 + 2$

$\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.

Let $y \in B = R - \{1\}$ *i.e.*, $y \in R$ and $y \neq 1$ such that $f(x) = y$

$$\Leftrightarrow \frac{x-1}{x-2} = y \Leftrightarrow (x-2)y = x - 1$$

$\Leftrightarrow xy - 2y = x - 1 \Leftrightarrow x(y - 1) = 2y - 1$

$$\Leftrightarrow x = \frac{2y-1}{y-1} \quad \text{...(i)}$$

$\therefore\ f(x) = y$ for $x = \dfrac{2y-1}{y-1} \in A$ (as $y \neq 1$)

Hence, f is onto.

Thus, f is one-one and onto.

34. We have, $y = \log\sqrt{\dfrac{1+\cos^2 x}{1-e^{2x}}}$

$$= \frac{1}{2}[\log(1+\cos^2 x) - \log(1 - e^{2x})]$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1+\cos^2 x}\cdot\frac{d}{dx}(1+\cos^2 x) - \frac{1}{1-e^{2x}}\cdot\frac{d}{dx}(1-e^{2x})\right]$$

$$= \frac{1}{2}\left[\frac{2\cos x}{1+\cos^2 x}\cdot(-\sin x) + \frac{e^{2x}}{1-e^{2x}}\cdot 2\right]$$

$$= -\frac{\sin x\cos x}{1+\cos^2 x} + \frac{e^{2x}}{1-e^{2x}}.$$

Now, $y'\left(\dfrac{\pi}{2}\right) = 0 + \dfrac{e^{\pi}}{1-e^{\pi}} = \dfrac{e^{\pi}}{1-e^{\pi}}$

OR

$$\text{Given, } f(x) = \begin{cases} -2, & \text{if } -2 \le x < -1 \\ -1, & \text{if } -1 \le x < 0 \\ 0, & \text{if } 0 \le x < 1 \\ 1, & \text{if } 1 \le x < 2 \\ 2, & \text{if } 2 \le x \end{cases}$$

Clearly, $f(1) = 1$

$$\lim_{x\to 1^-} f(x) = \lim_{h\to 0^+} f(1-h) = \lim_{h\to 0^+}(0) = 0$$

$$\lim_{x\to 1^+} f(x) = \lim_{h\to 0^+} f(1+h) = \lim_{h\to 0^+}(1) = 1$$

$$\because \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x) = f(1)$$

$\therefore\ f(x)$ is not continuous at $x = 1$ and hence non differentiable at $x = 1$.

[$\because$ Every differentiable function is continuous]

35. We have, $f(x) = x\log x$

$\therefore\ f'(x) = 1 + \log x$ and $f''(x) = \dfrac{1}{x}$

Now $f'(x) = 0$, if $1 + \log x = 0$

$\Rightarrow \log x = -1 = -\log e$

$\Rightarrow \log x = \log(e^{-1}) = \log \frac{1}{e} \Rightarrow x = \frac{1}{e}$

When $x = \frac{1}{e}, f''(x) = \frac{1}{\left(\frac{1}{e}\right)} = e > 0$

$\therefore$ By the second derivative test, f is minimum at $x = \frac{1}{e}$

Minimum value of f at $x = \frac{1}{e}$ is

$$\frac{1}{e}\log\left(\frac{1}{e}\right) = \frac{1}{e}.\log(e^{-1}) = \frac{1}{e}.(-1)\log e = -\frac{1}{e}$$

36. Let $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$

Since, $|B| = (4 - 3) = 1 \neq 0$ and $|C| = (9 - 10) = -1 \neq 0$

$\therefore B^{-1}$ and C^{-1} exist.

Now, the given matrix equation becomes $BAC = I_2$

$\Rightarrow B^{-1}(BAC)C^{-1} = B^{-1}I_2C^{-1}$

$\Rightarrow (B^{-1}B)A(CC^{-1}) = B^{-1}(I_2C^{-1})$

$\Rightarrow I_2AI_2 = B^{-1}C^{-1}$

$\Rightarrow A = B^{-1}C^{-1}$ ...(i)

Now, adj $B = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

$\therefore B^{-1} = \frac{1}{|B|}\cdot \text{adj } B = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ $[\because |B| = 1]$

Again, adj $C = \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$

$\therefore C^{-1} = \frac{1}{|C|}\cdot \text{adj } C = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ $[\because |C| = -1]$

Now from (i), $A = B^{-1}C^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

OR

We have, $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2)$$

$= -6 + 5 = -1 \neq 0$

$\therefore A^{-1}$ exists.

Now, $A_{11} = 0, A_{12} = 2, A_{13} = 1, A_{21} = -1, A_{22} = -9, A_{23} = -5, A_{31} = 2, A_{32} = 23, A_{33} = 13$

$\therefore$ adj $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}\text{adj } A = (-1)\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations is

$2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3$

This system of equations can be written as $AX = B$,

where $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Since A^{-1} exists therefore solution given by

$$X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 1, y = 2$ and $z = 3$.

37. Given L.P.P. is Maximize $Z = \frac{2x}{25} + \frac{y}{10}$ ... (i)

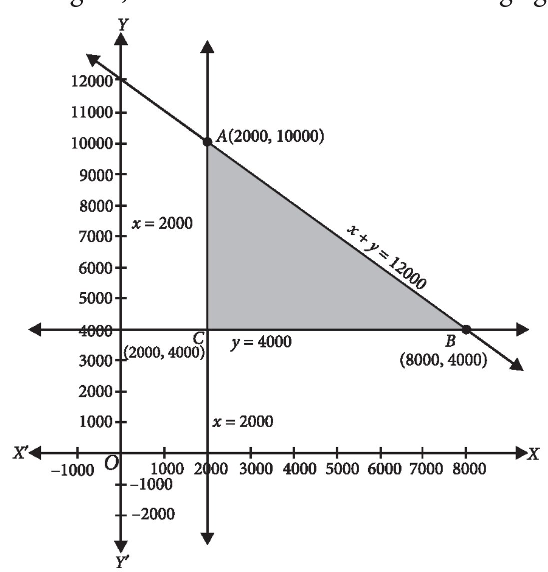
subject to constraints

$x \geq 2000$ ... (ii)

$y \geq 4000$ ... (iii)

and $x + y \leq 12000$ ... (iv)

On plotting inequalities (ii), (iii) and (iv), we get the shaded region, which is bounded in the following figure.



Now, we evaluate Z at the corner points $A(2000, 10000)$, $B(8000, 4000)$ and $C(2000, 4000)$.

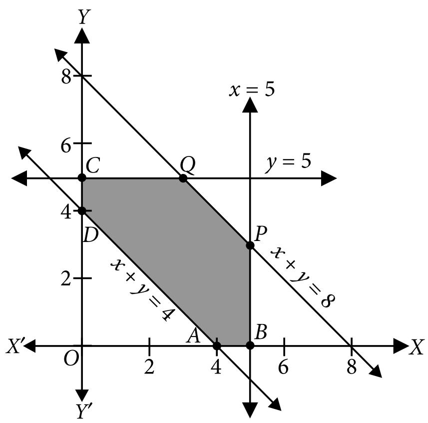
Corner Points	Value of $Z = \frac{2x}{25} + \frac{y}{10}$
$A(2000, 10000)$	1160 (Maximum)
$B(8000, 4000)$	1040
$C(2000, 4000)$	560

Clearly, maximum value of Z is 1160 at $x = 2000$ and $y = 10000$.

OR

The given problem is Minimize $Z = x - 7y + 190$ subject to $x + y \leq 8$, $x + y \geq 4$, $x \leq 5$, $y \leq 5$ and $x \geq 0$, $y \geq 0$

To solve this LPP graphically, let us first covert the inequations into equations and draw the corresponding lines.



The feasible regions $ABPQCDA$.

P is the point of intersection of $x = 5$ and $x + y = 8$ *i.e.*, $P(5, 3)$ and Q is the point of intersection of $y = 5$ and $x + y = 8$ *i.e.*, $Q(3, 5)$.

The corner points of the feasible region are $A(4, 0)$, $B(5, 0)$, $P(5, 3)$, $Q(3, 5)$, $C(0, 5)$ and $D(0, 4)$.

The value of the objective function $Z = x - 7y + 190$ at these points are

$Z(A) = 4 - 7(0) + 190 = 194$

$Z(B) = 5 - 7(0) + 190 = 195$

$Z(P) = 5 - 7(3) + 190 = 174$

$Z(Q) = 3 - 7(5) + 190 = 158$

$Z(C) = 0 - 7(5) + 190 = 155$

$Z(D) = 0 - 7(4) + 190 = 162$

Clearly, Z is minimum at $x = 0$, $y = 5$. The minimum value of Z is 155.

38. The equation of the planes through the intersection of the planes $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$ is

$[\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12] + \lambda[\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k})] = 0$

$\Rightarrow \vec{r} \cdot [(2 + 3\lambda)\hat{i} + (6 - \lambda)\hat{j} + 4\lambda\hat{k}] + 12 = 0$ ...(i)

These planes are at a unit distance from the origin. Therefore, length of the perpendicular from the origin on (i) is 1 unit.

$$\Rightarrow \frac{12}{\sqrt{(2 + 3\lambda)^2 + (6 - \lambda)^2 + 16\lambda^2}} = 1$$

$\Rightarrow 144 = (2 + 3\lambda)^2 + (6 - \lambda)^2 + 16\lambda^2$

$\Rightarrow 144 = 40 + 26\lambda^2 \Rightarrow 26\lambda^2 = 104$

$\Rightarrow \lambda^2 = 4 \Rightarrow \lambda = \pm 2$

Putting the values of λ in (i), we obtain

$\vec{r} \cdot (8\hat{i} + 4\hat{j} + 8\hat{k}) + 12 = 0$

or $\vec{r} \cdot (-4\hat{i} + 8\hat{j} - 8\hat{k}) + 12 = 0$

These equations can also be written as

$\vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) + 3 = 0$

or $\vec{r} \cdot (-\hat{i} + 2\hat{j} - 2\hat{k}) + 3 = 0$,

which are the equations of the required planes.

OR

Let M be the foot of the perpendicular drawn from the point $P(2, 3, 4)$ to the given line.

The coordinates of any point on the line

$$\frac{4 - x}{2} = \frac{y}{6} = \frac{1 - z}{3}$$

i.e. on the line $\frac{x - 4}{-2} = \frac{y}{6} = \frac{z - 1}{-3}$ are given by

i.e. $-2\lambda + 4, 6\lambda, -3\lambda + 1$

Let $M = (-2\lambda + 4, 6\lambda, -3\lambda + 1)$ ...(i)

$\therefore$ Direction ratios of PM are $-2\lambda + 2, 6\lambda - 3, -3\lambda - 3$

The direction ratios of the given line are $-2, 6, -3$

Since PM is perpendicular to the given line

$\therefore -2(-2\lambda + 2) + 6(6\lambda - 3) - 3(-3\lambda - 3) = 0$

$\Rightarrow 49\lambda - 13 = 0 \Rightarrow \lambda = \frac{13}{49}$

Putting $\lambda = \frac{13}{49}$ in (i), we get

$$M \equiv \left(\frac{170}{49}, \frac{78}{49}, \frac{10}{49}\right)$$

$\therefore$ Required length,

$$PM = \sqrt{\left(\frac{170}{49} - 2\right)^2 + \left(\frac{78}{49} - 3\right)^2 + \left(\frac{10}{49} - 4\right)^2} = \sqrt{\frac{44541}{2401}}$$

$= \frac{3}{7}\sqrt{101}$ units.

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