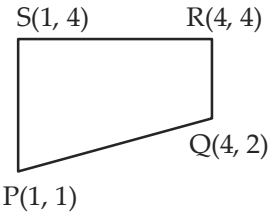
Chapter 3

Geometry & Mensuration

1. In the Cartesian plane four points P, Q, R, S have co-ordinates (1, 1), (4, 2), (4, 4) and (1, 4) respectively, The area of the quadrilateral PQRS is *[1995]*

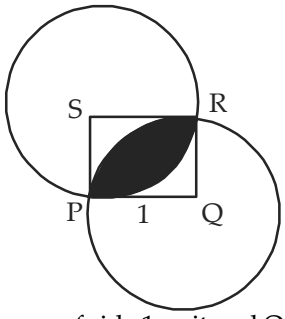


(a) 9
(b) 7.5
(c) 4.5
(d) impossible to find unless diagonals are known the lengths of the

2. A rectangular sump of dimension 6m × 5m × 4m is to be built by using bricks to make the outer dimension 6.2 m × 5.2 m × 4.2 m. Approximately how many bricks of size 20 cm × 10 cm × 5 cm are required to build the sump for storing water? *[1995]*

(a) 15408 (b) 3000
(c) 15000 (d) 30000

3. Consider the figure given below:

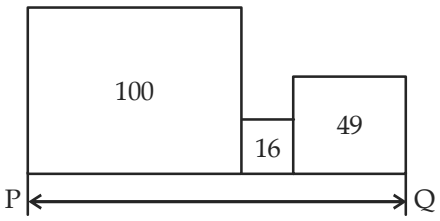


PQRS is a square of side 1 unit and Q, S are the centres of the two circles. The area of the shaded portion is *[1995]*

(a) $\frac{\pi}{2}$ (b) $\frac{1}{2}$

(c) $\frac{\pi}{4}-\frac{1}{2}$ (d) $\frac{\pi}{2}-1$

4. The following figure contains three squares with areas of 100, 16 and 49 sq. units respectively laying side by side as shown. By how much should the area of the middle square be reduced in order that the total length PQ of the resulting three squares is 19? *[1996]*

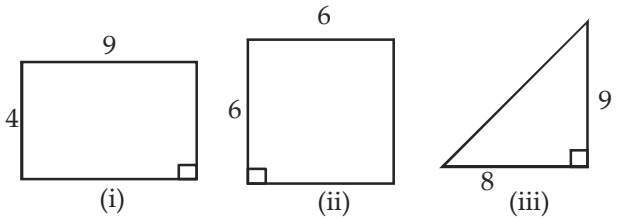


(a) 12 (b) 4
(c) 2 (d) 2

5. A rectangle has perimeter of 50 metres. If its length is 13 metres more than its breadth, then its area is: *[1996]*

(a) 124 m^2 (b) 144 m^2
(c) 114 m^2 (d) 104 m^2

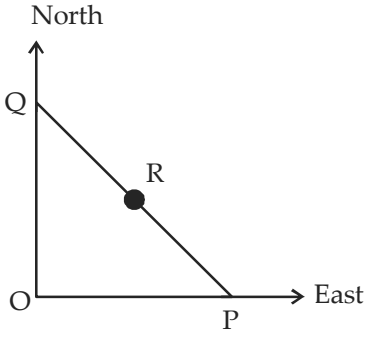
6. Consider the following figures: *[1996]*



Which one of the following conclusions can be drawn from these figures?

(a) The areas of the three figures are all different
(b) The areas of all the three figures are equal
(c) The perimeters of the three figures are equal
(d) The perimeters of figures I and II are equal

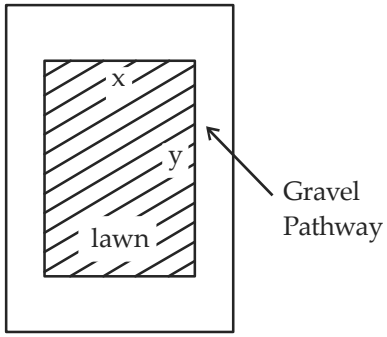
7. In the following figure *[1997]*



P is 300 km eastward of *O* and *Q* is 400 km north of *O*. *R is* exactly in the middle of *Q* and *P*. The distance between *Q* and *R* is

(a) 250 km (b) 300 km
(c) 350 km (d) $250\sqrt{2}$ km

8. A rectangular plot of lawn shown in the figure has dimensions 'x' and 'y' and is surrounded by a gravel pathway of width 2 m. What is the total area of the Gravel Pathway? *[1997]*



(a) $2x + 2y + 4$ (b) $2x + 2y + 8$
(c) $4x + 4y + 8$ (d) $4x + 4y + 16$

9. If the numbers representing volume and surface area of a cube are equal, then the length of the edge of the cube in terms of the unit of measurement will be *[1997]*

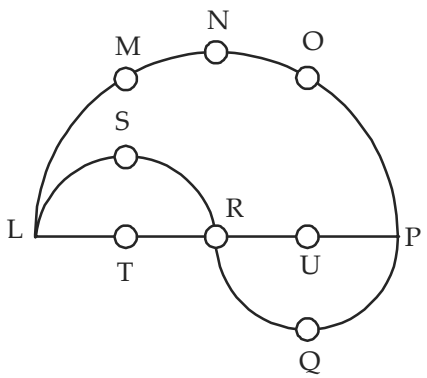
(a) 3 (b) 4
(c) 5 (d) 6

10. The length, breadth and height of a room are ℓ, b and h respectively. The perimeter of the ceiling expressed as a percentage of the total area of the four walls, is *[1997]*

(a) $100\,h\,\%$ (b) $\frac{100}{h}\%$

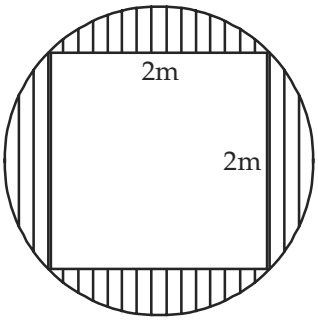
(c) $h\,\%$ (d) $\frac{h}{100}\%$

11. *LMNOP* is a semicircle with centre at R and diametre LP. LSR and RQP are also semi circles with centres at T and U respectively and diameters LR = RP = $\frac{1}{2}$ LP. The ratio of perimeter of *LMNOP* and *LSRQP* is *[1998]*



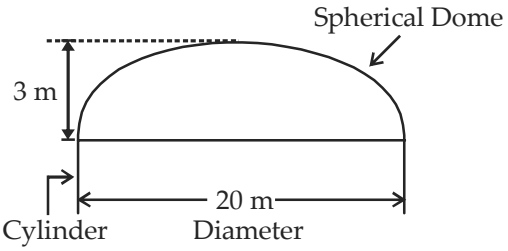
(a) 0.75 : 1 (b) 1 : 1
(c) 1 : 0.75 (d) 1.25 : 1

12. A square pond has 2 m sides and is 1m deep. If it is to be enlarged, the depth remaining the same, into a circular pond with the diagonal of the square as diameter as shown in the figure, then what would be the volume of earth to be removed? *[1998]*



(a) $(2\pi - 4)\ m^3$ (b) $(4\pi - 4)\ m^3$
(c) $(4\pi - 2)\ m^3$ (d) $(2\pi - 2)\ m^3$

13. The surface area of spherical dome-shaped roof of a cylindrical water tank shown in the figure is *[1999]*



(a) $60\ \pi\ m^2$ (b) $109\ \pi\ m^2$
(c) $120\ \pi\ m^2$ (d) $300\ \pi\ m^2$

14. A hemispherical bowl is filled to the brim with a beverage. The contents of the bowl are transferred into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both bowl and cylinder, then the volume of the beverage in the cylindrical vessel will be *[1999]*

(a) $66\frac{2}{3}\%$
(b) 78.5%
(c) 100%
(d) More than 100% (that is, some liquid will still be left in the bowl)

15. A man is standing on the 6m long pole whose length of shadow is 8m. If the length of his shadow is 2.4m, what is the height of the man ? *[1999]*

(a) 1.4 m (b) 1.6 m
(c) 1.8 m (d) 2.0 m

16. If the angle of triangle are in the ratio of 4 : 3 : 2, then the triangle *[1999]*

(a) is obtuse angled triangle
(b) has one angle greater than 80°
(c) is a right triangle
(d) is acute angled triangle

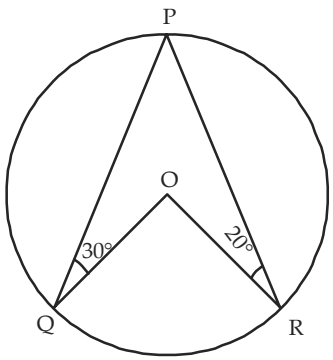
17. At a given time, two players are standing on a play-field. The cartesian coordinates of their locations are (20, 60) and (–40, –20) units. What is the distance between the players? *[1999]*

(a) 60 units (b) 80 units
(c) 100 units (d) 140 units

18. The area of an ellipse is twice that of a circle. The major diameter of the ellipse is twice that of the minor diameter. The radius of the circle is *[1999]*

(a) 50% of minor diameter of the ellipse
(b) 50% of major diameter of the ellipse
(c) minor diameter of the ellipse
(d) major diameter of the ellipse

19.



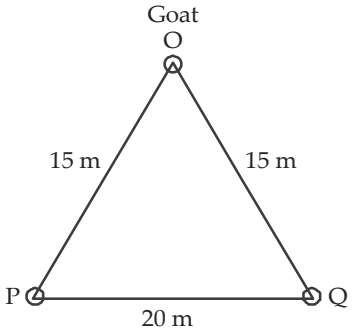
In the given figure $\angle OQP = 30$ and $\angle ORP = 20$., then $\angle QOR$ is equal to *[2000]*

(a) 100° (b) 120°
(c) 130° (d) 140°

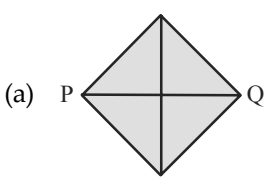
20. Which one of the following has a greater perimeter than the rest? *[2000]*

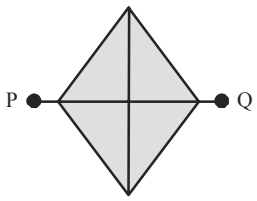
(a) A square with an area of 36 sq.cm
(b) An equilateral triangle with a side of 9 cm
(c) A rectangle with 10 cm as length and 40 sq cm as area
(d) A circle with a radius of 4 cm

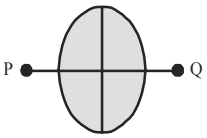
21. A goat is tied to two poles P and Q with ropes that are 15 meters long. P and Q are 20 metres apart as shown in the given diagram: *[2000]*

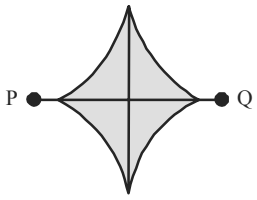


Which one of the following shaded portions indicates the total area over which the goat can graze?

(a) 

(b) 

(c) 

(d) 

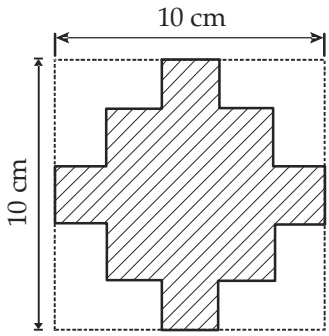
22. What is the maximum number of pieces of 5 cm × 5 cm × 10 cm of cake that can be cut from a big cake of 5cm × 30 cm × 30 cm size? *[2000]*

(a) 10 (b) 15
(c) 18 (d) 30

23. A rectangular water tank measures 15m × 6m at top and is 10m deep. It is full of water. If water is drawn out lowering the level by 1 meter, how much of water has been drawn out? *[2000]*

(a) 90000 litres (b) 45000 litres
(c) 4500 litres (d) 900 litres

24. In the given figure, all line segments of the shaded portion are of same length and at right angles to each other. The same can be out of a board of side 10 cm. What is the area of the shaded portion? *[2000]*

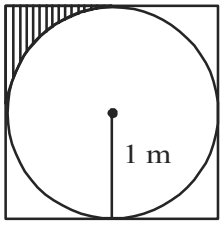


(a) 36 cm^2 (b) 48 cm^2
(c) 52 cm^2 (d) 64 cm^2

25. A rectangular piece of iron sheet measuring 50 cm by 100 cm is rolled into cylinder of height 50 cm. If the cost of painting the cylinder is ` 50 per square metre, then what will be the cost of painting the outer surface of the cylinder? *[2000]*

(a) ₹ 25.00
(b) ₹ 37.50
(c) ₹ 75.00
(d) ₹ 87.50

26.

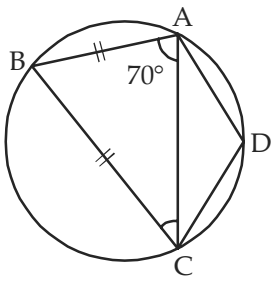


A circle of 1 *m* radius is drawn inside a square as shown in figure given above. What is the area of the shaded portion (in m^2) ? *[2001]*

(a) $(4-\pi)$ (b) $\left(1-\frac{\pi}{2}\right)$

(c) $\left(\frac{1}{4}-\frac{\pi}{4}\right)$ (d) $\left(1-\frac{\pi}{4}\right)$

27. In the above figure, ABCD is a cyclic quadrilateral, AB = BC and $\angle BAC = 70°$, then $\angle ADC$ is *[2001]*



(a) 40° (b) 80°

(c) 110° (d) 140°

28. The length of the longest pole that can be placed in a room 12 m long, 9 m wide and 8 m high, is *[2002]*

(a) 12 m (b) 14 m

(c) 17 m (d) 21 m

29. Consider the volumes of the following: *[2002]*

1. A parallelopiped of length 5 cm, breadth 3 cm and height 4 cm
2. A cube of each side 4 cm
3. A cylinder of radius 3 cm and length 3 cm.
4. A sphere of radius 3 cm.

The volumes of these in the decreasing order is

(a) 1, 3, 2, 4 (b) 4, 2, 3, 1

(c) 1, 2, 3, 4 (d) 4, 3, 2, 1

30. A big rectangular plot of area 4320 m^2 is divided into 3 square shaped smaller plots by fencing parallel to the smaller side of the plot. However some area of land was still left as a square could not be formed. So, 3 more square shaped plots were formed by fencing parallel to the longer side of the original plot such that no area of the plot was left surplus. What are the dimensions of the original plot? *[2005]*

(a) 160 m × 27 m (b) 240 m × 15 m

(c) 120 m × 36 m (d) 135 m × 32 m

31. The diameters of two circular coins are in the ratio of 1: 3. The smaller coin is, made to roll around the bigger coin till it returns to the position from where the, process of rolling started. How many times the smaller coin rolled around the bigger coin? *[2010]*

(a) 9 (b) 6

(c) 3 (d) 1.5

32. A village having a population of 4000 requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. The water of this tank will last for *[2011 - II]*

(a) 2 days (b) 3 days

(c) 4 days (d) 5 days

33. Two glasses of equal volume are respectively half and three-fourths filled with milk. They are then filled to the brim by adding water. Their contents are then poured into another vessel. What will be the ratio of milk to water in this vessel? *[2011 - II]*

(a) 1 : 3 (b) 2 : 3

(c) 3 : 2 (d) 5 : 3

34. A gardener increased the area of his rectangular garden by increasing its length by 40% and decreasing its width by 20%. The area of the new garden *[2014 - II]*

(a) has increased by 20%.

(b) has increased by 12%.

(c) has increased by 8%.

(d) is exactly the same as the old area.

35. In a plane, line X is perpendicular to line Y and parallel to line Z; line U is perpendicular to both lines V and W; line X is perpendicular to line V. *[2015-I]*

Which one of the following statements is correct?

(a) Z, U and W are parallel.

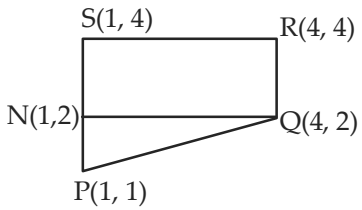
(b) X, V and Y are parallel.

(c) Z, V and U are all perpendicular to W.

(d) Y, V and W are parallel.

HINTS & SOLUTIONS

1. (b)



Area of the Quadrilateral = Area ($\square NQRS$) + Area ($\triangle PQN$)

Area ($\square$NQRS) = NQ × QR

$$NQ = \sqrt{(4-1)^2 + (2-2)^2} = 3$$

$$QR = \sqrt{(4-4)^2 + (4-2)^2} = 2$$

Area ($\square NQRS$) = 3 × 2 = 6

$$\text{Area } (\triangle PQN) = \frac{1}{2} \times NP \times NQ$$

$$NP = \sqrt{(1-1)^2 + (2-1)^2} = 1$$

$$NQ = \sqrt{(4-1)^2 + (2-2)^2} = 3$$

$$\text{Area } (\triangle PQN) = \frac{1}{2} \times 3 \times 1 = 1.5$$

$\therefore$ Net area = 6 + 1.5 = 7.5

2. (a) Let the number of bricks be n.

Now, Volume of outer dimension of sump – volume of sump = volume needed to be built by the bricks.

= (6.2 × 5.2 × 4.2 – 6 × 5 × 4)

= 15.408 m^3.

Volume of each brick = 0.2 × 0.1 × 0.05

$= 10 \times 10^{-4} m^3 = 1 \times 10^{-3} m^3$.

$\therefore$ $(1 \times 10^{-3})n = 15.408$

$$n = \frac{15.408}{1 \times 10^{-3}} = 15408$$

3. (d) Let us take the lower circle first.

$$\text{Area of the arc PRQ} = \frac{90^\circ}{360^\circ}(\pi r^2)$$

$$= \frac{1}{4}\pi(1)^2 = \frac{\pi}{4}$$

$$\text{area } (\Delta PRQ) = \frac{1}{2}(1)(1) = \frac{1}{2}$$

$\therefore$ Area of shaded portion of the lower circle

= area of the arc PRQ – area (ΔPRQ)

$$= \frac{\pi}{4} - \frac{1}{2} = \frac{\pi - 2}{4}$$

Since, both the circles are having the same radius, so they are symmetrical.

$\therefore$ Net shaded Area $= 2\left(\frac{\pi-2}{4}\right) = \frac{\pi-2}{2} = \frac{\pi}{2} - 1$

4. (a) Let the final length of the side of the smaller square be a.

Now, $a + 10 + 7 = 19$

$a = 19 - 17 = 2$

$\therefore$ Area of the smaller square = $(2)^2 = 4$

$\therefore$ Decrease in the area of the smaller square = 16 – 4

= 12 sq. units.

5. (c) Let the breadth be b.

Then, length = $b + 13$

Perimeter = $50 = 2(l + b)$

$2(b + 13 + b) = 50$

$b = 6$ m

$l = 6 + 13 = 19$ m

$\therefore$ Area = length × breadth

= 19 (6) = 114 m^2

6. (b) Area of (rectangle) = 9 × 4 = 36

Area of (square) = 6 × 6 = 36

$$\text{Area (triangle)} = \frac{1}{2} \times 9 \times 8 = 36$$

$\therefore$ Area of all the three figures are equal.

7. (a) $QP = \sqrt{(QO)^2 + (OP)^2} = \sqrt{(400)^2 + (300)^2} = 500$

$\therefore$ $QR = \frac{QP}{2} = 250$ km.

8. (d) Length of the lawn including Gravel pathway

$= x + 2 + 2 = x + 4$

Breadth of the lawn including Gravel pathway

$= y + 2 + 2 = y + 4$

$\therefore$ Area of the lawn including of Gravel pathway

$= (x + 4)(y + 4)$

$\therefore$ Area of Gravel pathway = Area of the lawn including Gravel pathway – Area of the lawn.

$= (x + 4)(y + 4) - xy = 4x + 4y + 16$

9. (d) Let the length of the edge be a

Now, $a^3 = 6a^2$

$a = 6$

10. (b) Total area of four walls = $2h(l+b)$

Perimeter of the ceiling $= 2l + 2b = 2(l + b)$

Perimeter of the ceiling expressed in percentage of the Total area of the four walls = $\frac{2(l+b) \times 100}{2h(l+b)}\% = \frac{100}{h}\%$

11. (b) Let $LR = R$

Perimeter of $LMNOP = \frac{2\pi R}{2} = \pi R$

Perimeter of $LSRQP$ = Perimeter of LSR + Perimeter of RQP

$$= \frac{\pi R}{2} + \frac{\pi R}{2} = \pi R$$

$\therefore$ Perimeter $\left(\frac{LMNOP}{LSRQP}\right) = \frac{\pi R}{\pi R} = 1:1$

12. (a) Diagonal of the square $= \sqrt{(2)^2 + (2)^2} = 2\sqrt{2}$

Radius of the circle $= \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\therefore$ Volume to be removed
= (Volume of the circle – Volume of the square)

$$= \left\{\left[\pi\left(\sqrt{2}\right)^2 \times 1\right] - (2 \times 2 \times 1)\right\} = 2\pi - 4$$

13. (b) Let Radius of the sphere = r m

$x = y = 10$ m

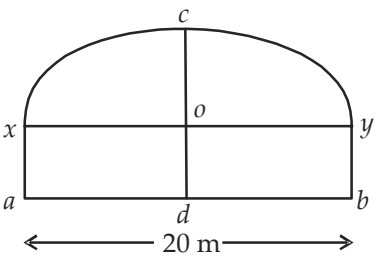
$d = 2r - 3$

$\therefore$ $x \times y = c \times d$

$\Rightarrow$ $10 \times 10 = 3(2r - 3)$

$\Rightarrow$ $6r - 9 = 100$, $\Rightarrow 6r = 109$

$\therefore$ $r = \frac{109}{6}$ m



$\therefore$ Surface area of spherical dome

$$= \frac{4\pi\left(\frac{109}{6}\right)^2 \times 3}{2\left(\frac{109}{6}\right)} = 109\pi \text{m}^2$$

14. (c) Let the radius of hemispherical bowl = r

$\therefore$ Volume of hemispherical bowl = $\frac{2}{3}\pi r^3$

Let the height of cylindrical vessel = h

$$r = h\left(1 + \frac{50}{100}\right)$$

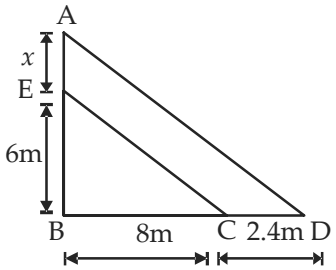
$$h = \frac{2}{3}r$$

Volume of cylindrical vessel = $\pi r^2 (2r/3) = \frac{2}{3}\pi r^3$

Hence, volume of the beverage in the cylindrical vessel

$$= \frac{(2/3)\pi r^3}{(2/3)\pi r^3} \times 100\% = 100\%$$

15. (c)



In ΔABD and ΔEBC, $\frac{AB}{BD} = \frac{BE}{BC}$

$$\frac{6 + x}{10.4} = \frac{6}{8}$$

$$x = 1.8 \text{ m.}$$

Where, x = height of the man.

16. (d) Let the angles be $4x$, $3x$ and $2x$.

$$4x + 3x + 2x = 180°$$

$$x = 20°$$

$\therefore$ angles are 80°, 60° and 40°.

17. (c) $A = (20, 60)$ and $B = (-40, -20)$

$$AB = \sqrt{\left[20 - (-40)\right]^2 + \left[60 - (-20)\right]^2}$$

$$= \sqrt{3600 + 6400} = \sqrt{10000} = 100$$

18. (a) Let the minor diameter of ellipse = $2b$

Major diameter of ellipse = $2a = 2(2b)$

$$2a = 4b$$

$$a = 2b$$

Let the radius of circle = r

Now, $\pi ab = 2\pi r^2$

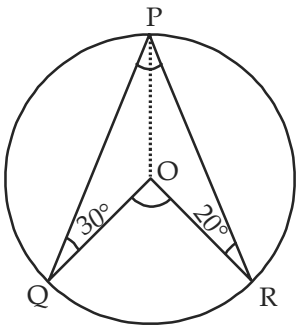
or $\pi (2b)b = 2\pi r^2$

$\therefore$ $r = b$

$\therefore$ Radius of circle = $\frac{1}{2}$ × minor diameter of ellipse

= 50% of minor diameter of an ellipse

19. (a)



In ΔPOQ, $\angle PQO = \angle OPQ = 30°$ ($\because$ $OP = OQ$ = radius)

In ΔPOR, $\angle ORP = \angle RPO = 20°$

$\angle RPQ = \angle OPQ + \angle RPO = 30° + 20° = 50°$

$\therefore$ $\angle QOR = 2\angle RPQ = 2(50°) = 100°$

($\because$ Angle at the centre = 2 × Angle of the Perimeter)

20. (c) (a) Side of the square = $\sqrt{36} = 6$ cm

Perimeter of the square = $4(6) = 24$ cm

(b) Perimeter of the triangle = $9 + 9 + 9 = 27$ cm

(c) Area of the rectangle = 40

$lb = 40$

$$b = \frac{40}{10} = 4$$

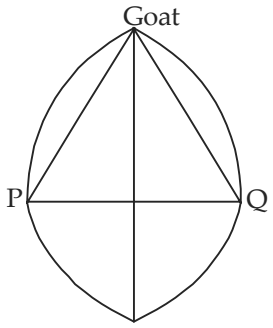
Perimeter = $2(l + b) = 2(4 + 10) = 28$ cm

(d) Perimeter of the circle = $2\pi r$

$= 2(3.14)(4) = 25.12$ cm

Clearly, Perimeter of the rectangle is maximum.

21. (c) When the goat is tied to pole P, then it will move in the form of an arc. Similarly, when it is tied to pole Q, again it will tranverse an arc. Only 'c' is showing the case.



22. (c) Let the maximum no of small pieces of cake be n.

Now, $n(5 \times 5 \times 10) = 5 \times 30 \times 30$

$$n = \frac{5 \times 30 \times 30}{5 \times 5 \times 10} = 18$$

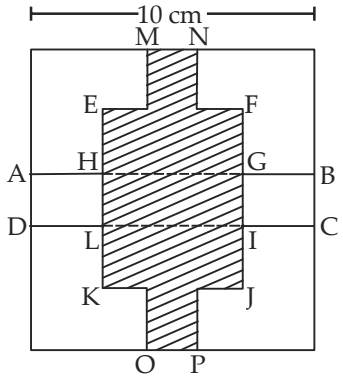
23. (a) Water drawn out

= Initial volume of the tank – Final volume of the tank

= $[15 \times 6 \times 10 - 15 \times 6 \times (10 - 1)]$

= $15 \times 6 \times (10 - 9) = 90$ m^3 $= 90000\ \ell$

24. (c)



Hence area of shaded portion = 2 (Area of square) + 2 (Area of rectangle EFGH IJKL) + Area of rectangle ABCD

= $2(4 + 6 \times 2) + 10 \times 2$

= 52 cm^2

25. (a) Let r and h be the radius and height of cylinder respectively.

Since the cylinder is made by rolling 100 cm side.

$$\therefore h = 50 \text{ cm} = \frac{1}{2}m \text{ and } 2\pi r = 100 \text{ cm}$$

$$\Rightarrow r = \frac{1}{2\pi} \text{ m}$$

$\therefore$ Curved surface area = $2\pi rh$

$$= 2\pi \times \frac{1}{2\pi} \times \frac{1}{2} = \frac{1}{2}m^2$$

Cost of painting = ₹ 50 per m^2

$$\therefore \text{Total cost of painting} = \frac{1}{2} \times 50 = ₹\ 25$$

26. (d) Since radius of circle = 1 m

$\therefore$ Side of square = 2m

So Area of circle = $\pi(1)^2 = \pi$m^2

and Area of square = $2^2 = 4$ m^2

$\therefore$ Area of shaded region = $\frac{1}{4}$ (Area of square – Area of circle)

$$= \frac{1}{4}(4 - \pi)\text{m}^2 = \left(1 - \frac{\pi}{4}\right)\text{m}^2$$

27. (d) In ΔABC, $AB = BC$ (isosceles triangle)

$\therefore$ $\angle BAC = \angle BCA = 70°$

Now, $\angle ABC + \angle BAC + \angle BCA = 180°$

$\angle ABC = 180° - (70° + 70°) = 40°$

As, $ABCD$ is a cyclic quadrilateral and sum of opposite angle pairs is equal to 180°

So, $\angle ADC + \angle ABC = 180°$

$\angle ADC = 180° - 40° = 140°$

28. (c) Length of the longest pole will be placed along the diagonal of the room

Length of the diagonal or pole

$$= \sqrt{(\text{length})^2 + (\text{breadth})^2 + (\text{height})^2}$$

$$= \sqrt{(12)^2 + (9)^2 + (8)^2} = 17 \text{ m.}$$

29. (d) Volume of Parallelopiped = $l \times b \times h$

$= 5 \times 3 \times 4 = 60$ cm^3

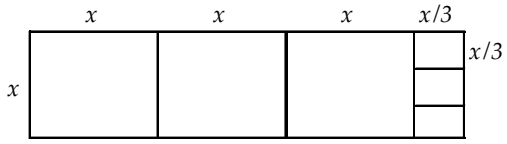
Volume of cube = $(\text{side})^3 = 4^3 = 64$ cm^3

Volume of cylinder = $\pi r^2 h = 3.14 \times 3^2 \times 3 = 84.78$ cm^3

Volume of sphere = $\frac{4}{3}\pi r^3 = 4 \times 3.14 \times 3^2 = 113.04$ cm^3

Then the correct decreasing order is 4, 3, 2, 1.

30. (c)



Required area $= 3(x)(x) + 3\left(\frac{x}{3}\right)\left(\frac{x}{3}\right)$

$$3x^2 + \frac{3x^2}{9} = 4320$$

$$x = 36 \text{ m}$$

Other dimension $= 3x + \frac{x}{3} = 3(36) + \frac{36}{3} = 120$ m

31. (c) Perimeter of bigger coin : Perimeter of smaller coin
$= 2\pi(3) : 2\pi(1) = 3 : 1$
Hence, smaller coin will roll 3 times around the bigger coin.

32. (b) Volume of tank $= 20 \times 15 \times 6 \text{ m}^3$
$= 20 \times 15 \times 6 \times 1000$ litre
$\therefore$ The water of the tank will last for

$$\frac{20 \times 15 \times 6 \times 1000}{150 \times 4000} \text{ days.}$$

i.e. The water of the tank will last for 3 days.

33. (d) Milk in 1st Glass $= \frac{1}{2}V$

Milk in 2nd Glass $= \frac{3}{4}V$

Now the 1st glass $= \frac{1}{2}V$

Now the 2nd glass $= \frac{1}{4}V$

When both glasses are mixed then the ratio of milk to water

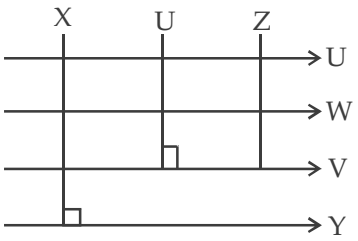
$$\frac{1}{2}+\frac{3}{4} : \frac{1}{2}+\frac{1}{4} = \frac{2+3}{4} : \frac{2+1}{4} = 5 : 3$$

34. (b) Let initial dimensions be, ℓ & b
$\therefore$ Final length is 1.4 l
Final breadth is 0.8 b
$\therefore$ Final area is $= 1.4\, l \times 0.8\, b$
$= 1.12\, lb$
$\therefore$ Area is increased by 12%.

Shortcut Method : $+40 - 20 + \frac{40 \times (-20)}{100}$

$= 20 - 8 = 12\%$
Therefore, the area of the new garden increased by 12%

35. (d)



From above diagram, it is clear that Y, V and W are parallel.

■ ◆ ■