6.12 Combination of Thin Lenses in Contact

Consider a simple optical system that consists of two thin lenses L_1 and L_2 in contact and placed on a common axis. Their focal lengths are f_1 and f_2 respectively. For such an optical system, we assume that the image formed by the first lens becomes the object for a second lens, and we get final image due to the system. We now derive formula for focal length of this equivalent lens as follows.

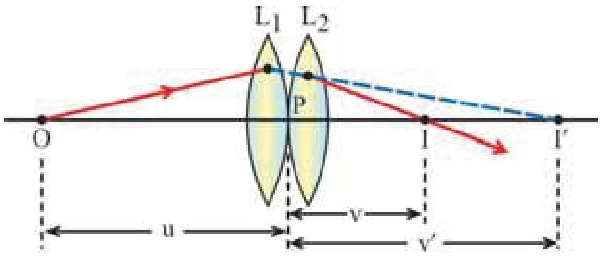


Figure 6.20 Combination of Thin Lenses

From the figure 6.20, consider a case of point like object (O) whose final image (I) is formed due to two thin lenses in contact.

Using Gauss' formula for lens L_1, $-\frac{1}{u} + \frac{1}{v'} = \frac{1}{f_1}$ (6.12.1)

For lens L_2, $-\frac{1}{v'} + \frac{1}{v} = \frac{1}{f_2}$ (6.12.2)

Adding these equations, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2}$ (6.12.3)

If we assume that the final image is formed by a single equivalent lens of the focal length f, then

$$\frac{1}{f} = \frac{-1}{u} + \frac{1}{v} \quad (6.12.4)$$

$$\therefore \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} \quad (6.12.5)$$

$$\text{or } f = \frac{f_1 \cdot f_2}{(f_1 + f_2)} \quad (6.12.6)$$

Equation (10.12.5) or (10.12.6) is the algebraic relation between f_1, f_2 and f. While using them to find equivalent focal length for different combinations of lenses, proper sign convention should be adopted.

If there are n number of thin lenses in contact, equivalent focal length of them is given by,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + ... + \frac{1}{f_n} \quad (6.12.7)$$

Lenses with Separation : If two thin lenses are not in contact, but having some separation d, then equivalent focal length can be written as,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2} \quad (6.12.8)$$

Also, $d - (f_1 + f_2)$ is known as the **optical interval** between the two lenses.

Power :

But $\frac{1}{f_1} = P_1 =$ power of lens L_1

$\frac{1}{f_2} = P_2 =$ power of lens L_2

Therefore, from equation (6.12.8), equivalent power of the combitation is

$$P = P_1 + P_2 + ... + P_n \quad (6.12.9)$$

Lateral Magnification :

For two-lens system lateral magnification due to lens L_1 is $m_1 = \frac{v'}{u}$.

That due to lens L_2 is $m_2 = \frac{v}{v'}$.

If resultant magnification is m, then

$$m = \frac{v}{u} = \frac{v'}{u} \times \frac{v}{v'}$$

$$m = m_1 \times m_2$$

For n number of lenses, $m = m_1 \times m_2 \times .... \times m_n$ (6.12.10)

Equation (6.12.10) suggestes that in order to improve magnification one may use combination of lenses (e.g., compound microscope).

6.13 Combination of Lens and Mirror

The combination of lenses are important for achieving proper magnification, focussing of image at a desired point, etc. Similarly combinations of lenses and mirrors are also useful. We consider one such combination of convex mirror and convex lens.

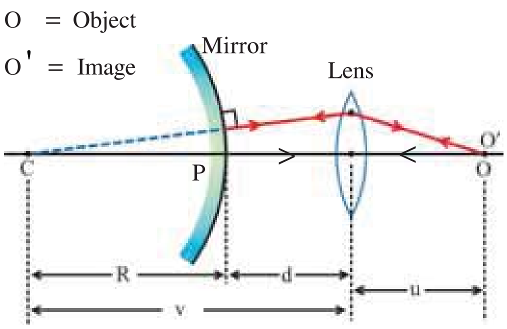


Figure 6.21 Focal Length of a Convex Mirror Using a Convex Lens

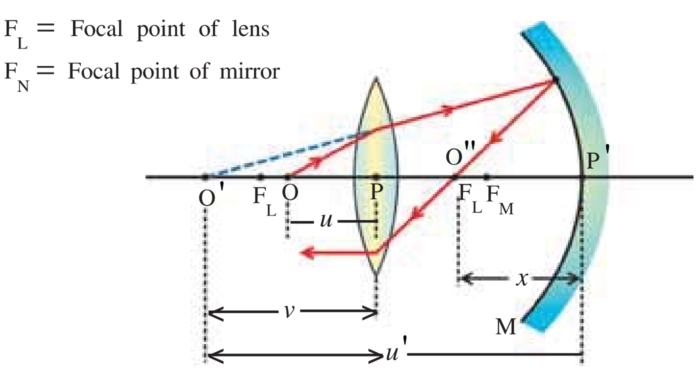
As shown in the figure 6.21, image (O') is formed on the same side of the object. For a given object distance (u), we adjust the mirror distance (d) from the lens in such a way that the image is formed at the object position itself (i.e., parallax between an object and image is removed). In this case, rays incident on to the mirror will be normal to the mirror. In absense of the mirror the image would have been formed at C. Its distance from the lens is v. Since rays falling on the mirror are normal, point C is the centre of curvature for the mirror.

Thus, by measuring v and d, we can find focal length of the mirror as,

$$f = \frac{R}{2} = \frac{1}{2}(v - d).$$

Illustration 9 : A converging lens of focal length 15 cm and a converging mirror of focal length 20 cm are placed with their principal axes coinciding. Point object is placed at a distance 12 cm from the lens. Refracted ray from the lens gets reflected from the mirror, and again refracted by the lens. It is found that the final ray coming out of the lens is parallel to the principal axis. Find the distance between the mirror and the lens.

Solution :



Applying Gauss' formula to lens,

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\therefore v = \frac{u \cdot f}{u + f} = \frac{(-12)\times(15)}{-12+15}$$

(Using cartesian sign convention)

$= -60$ cm.

Negative sign indicates that image (O$'$) is virtual. This image works as an object for the mirror. For mirror, object distance,

$u' = O'O'' + O''P' = (PO' + PO'') + O''P'$

$= (60 + 15) + x = (75 + x)$ cm (for mirror, $PO' = v$ is taken positive)

Since image due to mirror is obtained at O$''$, its distance from the mirror is x.

Applying Gauss' formula to the mirror,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore \frac{1}{-(75+x)} + \frac{1}{-x} = \frac{1}{-f}$$

Simplifying , $\dfrac{(75+2x)}{(75+x)\cdot x} = \dfrac{1}{20}$

$\therefore x^2 + 35\cdot x - 1500 = 0$

$\therefore x = 25$ cm or $x = -60$ cm.

Thus, physically acceptable solution is 25 cm. Therefore, distance of the mirror from the lens is $= 25 + 15 = 40$ cm.

Illustration 10 : Distance between an object and a screen is d. Prove that for a thin convex lens, there are two positions for the object at which image can be obtained on the screen, and under certain condition only. Derive the condition for the same. When will the image not be formed ?

Solution : Suppose the object distance is u,

$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

For a convex lens u is negative. So,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

But, $u + v = d$ (given)

$\therefore v = d - u$

$$\therefore \frac{1}{d-u} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore \frac{u+d-u}{u(d-u)} = \frac{1}{f}$$

$\therefore u^2 - ud + fd = 0$

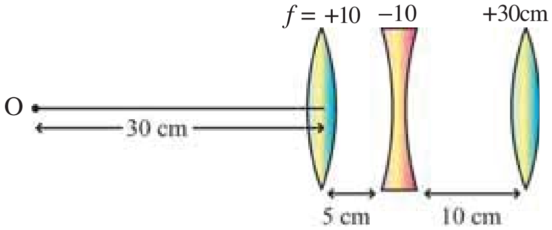
This is the quadratic equation for variable u. It's roots are as given below :

$$u = \frac{d \pm \sqrt{d^2 - 4fd}}{2}$$

Thus, if $d > 4f$, two values of u are possible and if $d < 4f$, u will be a complex number and hence the image will not be formed.

Illustration 11 : Decide the position of the image formed by the given combination of lenses.

Solution : For the image formed by first lens.



$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f}$$

$$\therefore \frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10}$$

$$\therefore v_1 = 15 \text{ cm}$$

Thus image formed by the first lens is formed at 15 cm distance on the right-hand side. This image is on the right-hand side of the second lens at 15 − 5 = 10 cm distance and so it acts as a virtual object for the second lens.

Now for the second lens,

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\therefore \frac{1}{v_2} - \frac{1}{10} = -\frac{1}{10}$$

$$\therefore v_2 = \infty$$

This distance v_2 (= ∞) is the object distance for the third lens. So, the third image formed due to it should be on the principal focus of the third lens. Thus, as the focal length of the third lens is 30 cm, the final image is formed at 30 cm distance on the right side of the third lens.

Illustration 12 : For a thin lens prove that when the heights of the object and the image are equal, object distance and image distance are equal to $2f$.

Solution : Here, $|h| = |h'|$

$\therefore |v| = |u|$

Using the equation for lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-u} = \frac{1}{f}$$

$$\therefore \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore \frac{2}{v} = \frac{1}{f}$$

$$\therefore v = 2f$$

$$\therefore u = v = 2f$$

Here, the points at $2f$ distance on both the sides of the lens are called **conjugate focl.**

Illustration 13 : Two converging lenses of powers 5D and 4D are placed 5 cm apart. Find the focal length and power of this combination.

Solution : Focal length of first lens, $f_2 = \frac{1}{5} = 0.2 \text{ m} = 20 \text{ cm}$

Focal length of second lens, $f_2 = \frac{1}{4} = 0.25$ m $= 25$ cm

Distance between two lenses, $d = 5$ cm

Now, equivalent focal length of this combination is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$f = \frac{f_1 \cdot f_2}{(f_1 + f_2) - d} = \frac{20 \times 25}{(20+25)-5} = 12.5 \text{ cm}$$

And equivalent power is given by,

$P = (P_1 + P_2) - d.P_1P_2$

$= (5 + 4) - (0.05) \times (5)(4)$ (d is written in meter)

$\therefore$ P = 8D or $P = \frac{1}{f} = \frac{1}{0.125} = 8D$

6.14 Refraction and Dispersion of Light due to a Prism

As shown in the figure 6.22, the cross-section perpendicular to the rectangular surface of a prism is shown. A ray PQ of monochromatic light is incident at point Q on the surface AB. According to Snell's law, it is refracted and travels along the path QR. Thus, it deviates from the incident direction by an amount δ_1. This ray QR is incident on the surface AC at point R, and emerging out as a ray RS. It suffers a deviation δ_2. By extending the incident ray PQ to PQE, total deviation between the incident and the emergent ray is found. When the emergent ray RS is extended backword it meets PE at D. Angle between the incident ray and the emergent ray is called the angle of deviation, δ.

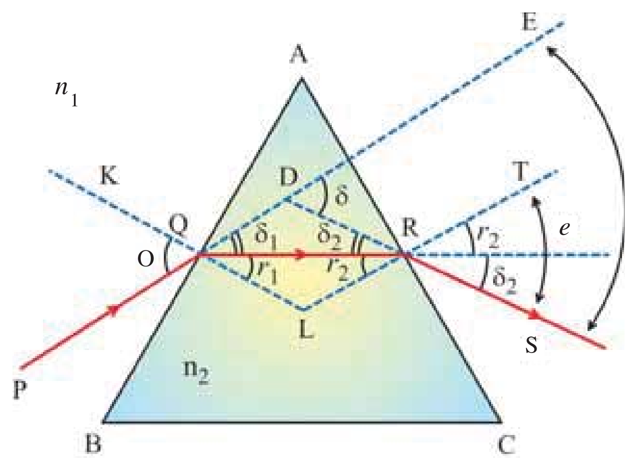


Figure 6.22 Refraction Due to Prism

From figure 6.22, in □AQLR, ∠AQL and ∠ARL are right angles.

$\therefore m\angle A + m\angle QLR = 180°$ (6.14.1)

and for ΔQLR, $r_1 + r_2 + m\angle QLR = 180°$ (6.14.2)

Comparing above equations,

$r_1 + r_2 + m\angle QLR = m\angle A + m\angle QLR$

$\therefore r_1 + r_2 = A$ (6.14.3)

For ΔDQR, $\angle EDR \equiv \angle EDS = \delta$ is the exterior angle. Therefore,

$\delta = \angle DQE + \angle DRQ$

$\therefore \delta = \delta_1 + \delta_2$ (6.14.4)

But $\delta_1 + r_1 = i$ ($\because$ vertically opposite angles)

$\therefore \delta_1 = i - r_1$ (6.14.5)

Similarly, $\delta_2 = e - r_2$ (6.14.6)

$\therefore \delta = (i - r_1) + (e - r_2) = (i + e) - (r_1 + r_2)$

Using equation (6.14.3)

$\delta = i + e - A$ or $i + e = A + \delta$ (6.14.7)

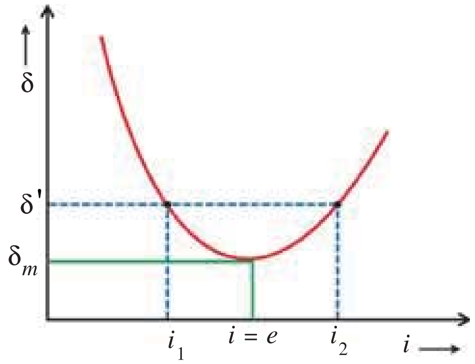


Figure 6.23 Variation of Deviation with Angle of Incidence

Equation (6.14.7) gives the relation between angle of deviation, angle of incidence and angle of emergence and the prism angle. It is known as an **equation for prism.**

It is clear from the above equation that the angle of deviation depends on the angle of incidence. For the sake of understanding, the graph of the measured values of angle of deviation against corresponding angle of incidence for an equilateral prism is shown in the figure 6.23.

We can see from the graph that for two values of angle of incidence (i_1 and i_2) angle of deviation δ is same. This can be understood from the reversibility of the rays.

If the incident ray is SR instead of PQ, then the refracted ray will follow exactly the reverse path, i.e., SRQP, and the emergent ray becomes PQ. In this case also, however, the angle of deviation remains the same. But for a particular value of angle of deviation there exists only one value of angle of incidence. And experimentally, it is found that this angle of deviation is minimum (δ_m). In the condition of minimum deviation of the incident ray the angle of deviation is called the angle of minimum deviation (δ_m). In this situation it is found that $i = e$.

From equation (6.14.7),

$$\delta_m = i + i - A = 2i - A$$

$$i = \frac{A+\delta_m}{2} \quad (6.14.8)$$

Applying Snell's law at pont Q,

$$n_1 \sin i = n_2 \sin r_1 \quad (6.14.9)$$

At point R, considering SR as the incident ray,

$$n_1 \sin e = n_2 \sin r_2$$

As $i = e$

$$\therefore\ n_1 \sin i = n_2 \sin r_2 \quad (6.14.10)$$

From equations (6.14.9) and (6.14.10)

$$\therefore\ r_1 = r_2 \quad (6.14.11)$$

From equation (6.14.3), and let $r_1 = r_2 = r$,

$$r + r = A$$

$$\therefore\ r = \frac{A}{2} \quad (6.14.12)$$

Substituting the values of (6.14.8) and (6.14.12) in either in (6.14.9) or (6.14.10), This gives,

$$\therefore\ n_1 \sin\left(\frac{A+\delta_m}{2}\right) = n_2 \sin\left(\frac{A}{2}\right)$$

$$\text{or}\quad \frac{n_2}{n_1} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad (6.14.14)$$

If the prism is kept in air, i.e. $n_1 = 1$ and $n_2 = n$,

$$\therefore\ n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad (6.14.15)$$

Equation (6.14.15) shows that value of δ_m depends on the angle of prism. The refractive index of the material of the prism and the medium in which prism is kept.

For equilateral prism, when δ is minimum, refracted ray (QR) through the prism is **parallel to the base BC** of the prism. Equation (6.14.15) is of practical importance to measure refractive index of the material of the prism.

Case : The prisms with small angle of prism are called thin prisms. For such prisms, angle of deviation is also small. In this case equation (6.14.15) gives

$$\delta_m = A(n - 1) \tag{6.14.16}$$

Dispersion :

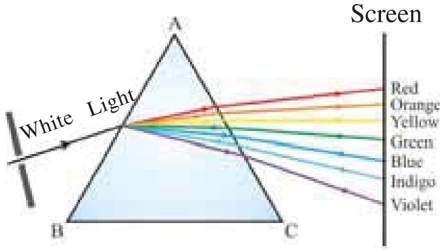


Figure 6.24 Dispersion of White Light

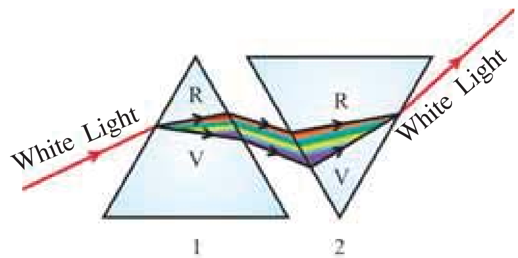


Figure 6.25 Dispersion and Recombination of White Light

As shown in figure 6.24, when a beam of white light or sun light passes through a prism the emergent light is made up of different colours. To understand this phenomenon, Newton has arranged two identical prisms as shown in figure 6.25. A ray of white light is incident on the prism-1, and emergent ray from the prism-2 is observed. It is found that this emergent ray is also white. This experiment explains that the first prism disperses the colours of white light, while the second prism brings them together.

The phenomenon in which light gets divided into its constituent colours is known as **dispersion of light.**

It is found that for the visible part of the electromagnetic spectrum violet colour has the maximum refractive index and red colour has the lowest. From equation (6.14.16), corresponding minimum angle of deviation through the same prism is

$\delta_v = A(n_v - 1)$

and $\delta_r = A(n_r - 1)$

It is now clear that as $n_v > n_r$, $\delta_v > \delta_r$.

Thus, deviation of violet colour is more compared to the deviation of red colour.

The total angle through which the spectrum is spread is called as the angular dispersion. It is defined as,

$$\theta = \delta_v - \delta_r = (n_v - n_r) \cdot A \tag{6.14.17}$$

For example, the spectrum obtained by a prism made up of flint glass is wider, more dispersed and more detailed as compared to the one obtained by common crown glass.

Illustration 14 : For a prism, angle of prism is 60° and it's refractive index is 1.5, find (1) angle of incidence corresponding to the angle of minimum deviation and (2) angle of emergence for angle of maximum deviation.

Solution : (1) For minimum deviation,

$r_1 = r_2$ and $A = r_1 + r_2$

$\therefore$ A = $2r_1$

or $r_1 = \frac{A}{2} = \frac{60}{2} = 30°$

Now $n = 1.5$ and

$n = \frac{\sin i}{\sin r_1}$

$\therefore n \sin r_1 = \sin i$

$\therefore 1.5 \times \sin 30° = \sin i$

$\therefore 1.5 \times 0.5 = \sin i$

$\therefore i_1 = 48° \ 35'$

(2) For maximum deviation, $i = 90°$

$\therefore 1.5 = \frac{\sin 90°}{\sin r_1} \quad \therefore r_1 = 41° \ 48'$

$\therefore r_2 = A - r_1 = 60 - 41° \ 48' = 18° \ 12' \ (\because r_1 + r_2 = A)$

$1.5 \sin r_2 = \sin e \ (\because n \sin r_2 = \sin e)$

$\therefore 1.5 \times \sin 18° \ 12' = \sin e$

$\therefore \sin e = 0.4685$

$\therefore e = 27° \ 56'$

Illustration 15 : An equilateral prism is kept in air and for a particular ray, angle of minimum deviation is 38°. Calculate the angle of minimum deviation if the prism is immersed in water. Refractive index of water is 1.33.

Solution : $\frac{n_g}{n_a} = \frac{\sin\left(\frac{60+38}{2}\right)^\circ}{\sin 30^\circ}$

Taking $n_a = 1$,

$n_g = \frac{\sin 49^\circ}{\sin 30^\circ} = 1.509$

When prism is immersed in water,

$$\frac{n_g}{n_\omega} = \frac{\sin\left(\frac{60+\delta_m}{2}\right)^\circ}{\sin 30^\circ}$$

But $n_\omega = 1.33$

$$\therefore \frac{1.509}{1.33} = \frac{\sin\left(\frac{60+\delta_m}{2}\right)^\circ}{0.5}$$

$$\therefore \sin\left(\frac{60+\delta_m}{2}\right)^\circ = 0.5679$$

$$\therefore \frac{60+\delta_m}{2} = 34° \ 36'$$

$$\therefore \delta_m = 9° \ 12'$$

6.15 Scattering of Light

Light scattering is one of the two major physical processes that contribute to the visible appearance of most of routine objects, the other being absorption. Broadly, scattering can be classified either as elastic or inelastic. Natural occurrence like, colour of sky during sunrise or sunset and during day time, colour of clouds can be understood by elastic scattering of light due to atmospheric atoms, molecules, water droplets, etc. Light falling on such particles is absorbed by them and immediately radiated in different amount in different directions. As a result, part of the intensity of light ray is diverted to different directions in different proportions.

It is found that the intensity of scattered light depends on the ratio (α) of the size of the particle (i.e. its diameter, for spherical particles) and wavelength of the light.

If $\alpha << 1$: Scattering is known as Rayleigh scattering

$\therefore$ $\alpha \approx 1$: Scattering is known as Mie-scattering.

$\therefore$ $\alpha >> 1$: Geometric scattering.

6.15.1 Rayleigh Scattering : If the size of the particle which scatters the light is smaller than the wavelength of the incident light, the scattering is known as **Rayleigh scattering.**

Lord **Rayleigh** showed theoretically that the intensity of scattering is inversely proportional to the fourth power of the wavelenght of light. Since the wavelength of blue light is 1.7 times smaller than the red light. So, the intensity of scattered blue light is 8 to 9 times more than the intensity of scattered red light. Thus, intense scattered-blue light is responsible for the sky to be bluish.

Another consequence of Rayleigh seattering is the appearance of reddish colour of the sun either at the sunrise or at the sunset

As shown in figure 6.26, at the sunrise or sunset, light from the sun has to travel relatively more distance to reach the observer on the earth as compared to the noon-time. During the passage of light in the atmospheric light of smaller wavelengths scatter more. Hence, only light with high wavelengths (i.e., reddish or yellowish-red) can reach to the observer substantially. Thus, the sun appears reddish. However, if we see vertically upward, sky appears blue. This effect is maximum in the direction perpendicular to the incident light. The same is the reason for reddish full-moon while rising or setting.

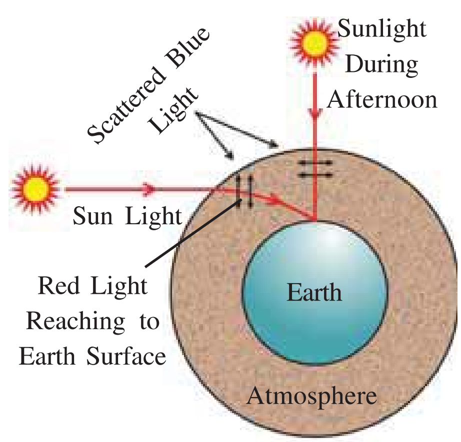


Figure 6.26 Seattering of Sun Light Due to Atmosphere

It is found that the intensity of the Rayleigh scattered light increases rapidly as the ratio α increases. Further, the intensity of Rayleigh scattered light is identical in the forward and reverse directions.

6.15.2 Mie-Scattering : If the size of scatterer particles are slightly larger than the wave length of the light, scattering is known as Mie-scattering. It was studied by Gustav Mie in 1908. It is found that as the size of the particle increases, the proportion of diffused scattering also increases. Since water droplets in the cloud have size comparable to wavelength of light, scattering of sun light through clouds is diffused scattering. It is independent of incident wavelengths. Hence, all colours scatter equally, and the clouds appear white. Unlike Rayleigh scattering, Mie-scattering is observed in larger amount in the forward direction than in the reverse direction. Also, as the particle size increases, more amount of the light is scattered in the forward direction.

For information only : The Mie-scattering shows that if the size of the particle lies between two wavelengths of light, then the light having more wavelength is scattered more than the light with smaller wavelength. If dust clouds have such size then the rising sun and moon or setting sun and moon would be seen blue or greeen !

However, such a situation rarely occurs. In the 19th century when the Volcano Krakotoa in Indonesia erupted and in 1950 when there were extensive forest fires in East-Canada and North-East USA, such situation took place.

If earth had no atmosphere, the sky would have been blackish, and stars would have been visible even during day time ! This becomes reality at or above 20 km from the earth surface.

In presense of high pollution in the atmosphere, the sky appears greyish and hazy instead of blue.

6.15.3 Raman-Scattering : The Raman effect was first reported by Indian Nobel laureate C. V. Raman. This inelastic scattering of light was also predicted by Adolf Smekal in 1923. Hence, this effect is also known as **Smekal-Raman effect**.

When a strong beam of visible or ultraviolet light is incident on gas, liquid or transparent solid, a small fraction of light is scattered in all directions. It is found that the scattered light spectrum is made up of lines of incident wavelength (Rayleigh lines) and weak additional lines of changed wave lenghts. These additional lines due to inealstic scattering are called **Raman lines.** Raman lines are found symmetrically on both sides of the central Rayleigh lines. Raman lines with low frequencies (or higher wavelengths) are known as **Stokes lines,** and the one on higher frequency (or low wavelength) sides are known as **Antistokes lines**.

Raman lines are the characteristics of the material.

Raman scattering is the most versatile technique to study characteristics of the material, different excitations in the materials, in optical amplifiers, to study biological organisms and human tissues, etc.

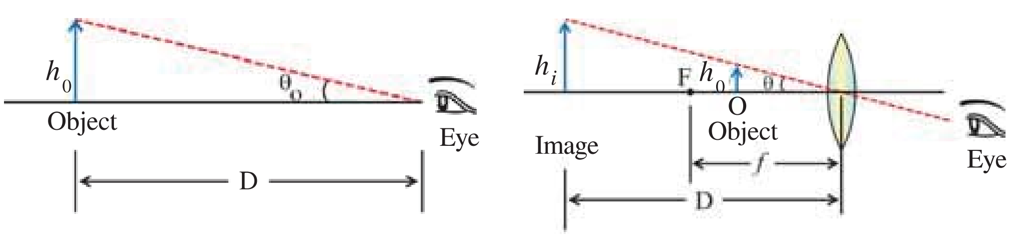
6.16 Optical Instruments : The purpose of most optical instruments is to enable us to see the object better. They are made up of combination of refracting and/or reflecting devices such as lenses, mirrors and prisms. They can be divided into two groups : instruments forming **real images** (e.g., projectors) and instruments forming imaginary images (e.g., microscopes and telescopes).

We first study simple microscope.

6.16.1 Simple Microscope : Suppose we want to see a microscopic object clearly and magnified.

The least distance at which a small object can be seen clearly with comfort is known as **near point** (D) or **distance of most distinct vision**. For normal eye this distance is 25 cm.

Suppose a linear object with height h_0 is kept at near point (i.e., $u \equiv D = 25$ cm) from eye. Let it subtend an angle θ_0 with the eye (See figure 6.27 (a)).



(a) (b)

Figure 6.27 Simple Magnifier

Now, if object is kept within the focal length (f) of a convex lens such that its virtual, erect and magnified image is formed at a distance equal to the near point. Since the lens is very close to eye, angle (θ) subtended by the object with lens and eye are almost identical.

The angular magnification is defined as

$$m' = \frac{\tan\theta}{\tan\theta_0} \approx \frac{\theta}{\theta_0} \text{ (for small } \theta \text{ and } \theta_0) \quad (6.16.1)$$

Also, from figures 6.27 (a) and (b),

$$\tan\theta_0 \approx \theta_0 = \frac{h_0}{D}$$

$$\text{and } \tan\theta \approx \theta_0 = \frac{h_i}{D}$$

$$\therefore \; m' = \frac{h_i}{h_0} \quad (6.16.2)$$

But for convex lens, linear magnification,

$$|m| = \frac{v}{u}$$

$$|m| = \frac{D}{u} \quad (6.16.3)$$

Using Gauss' formula,

$$-\frac{1}{(-u)} + \frac{1}{(-D)} = \frac{1}{f} \text{ (for this image } v = D \text{ is negative)}$$

$$\therefore \; \frac{1}{u} = \frac{1}{f} + \frac{1}{D} = \frac{D+f}{D \cdot f}$$

$$\therefore \; u = \frac{Df}{D+f} \quad (6.16.4)$$

Using (6.16.3) in equation (6.16.4),

$$|m| = 1 + \frac{D}{f} \quad (6.16.5)$$

When the image is at a very large distance

$$|m| \approx \frac{D}{f} \quad (6.16.6)$$

Combinedly equations (6.16.5) and (6.16.6) suggest that the value of m should be between $\frac{D}{f}$ and $\left(1+\frac{D}{f}\right)$

6.16.2 Compound Microscope : We have seen that in a simple microscope magnifying power depends on $\frac{D}{f}$. Thus, we tempted to use a convex lens with small focal length in order to improve magnification. It is found, however, that by reducing the value of focal length, image becomes distorted. Thus, very large and clear image is not possible with a simple microscope. But if magnified image due to one simple microscope is used as an object for another simple microscope, then we get very enlarged image. This is the basic principle of a compound microscope.

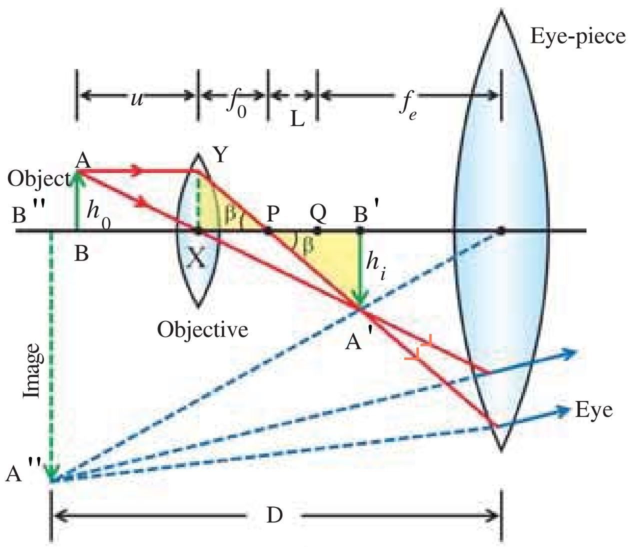


Figure 6.28 Compound Microscope

The lens kept near the object is known as **objective,** while the one nearer to eye is known as **eye-piece.** Distance between the second focal point (P) of the objective and the first focal point (Q) of the eye-piece is known as **tube-length (L)** of the microscope.

It is clear from the figure that the image obtained by the objective is real, inverted and magnified. This image acts as an object for the eye-piece. Eye-piece works as a simple microscope and gives a virtual and highly magnified final image (A''B'').

The image due to objective is observed close to the focal point of an eye-piece. Due to this reason final image is formed at a considerable large distance.

Magnification : Magnification due to the objective,

$$m_0 = \frac{h_i}{h_0} \tag{6.16.7}$$

From ΔXYP and ΔPA'B', respectively,

$$\tan\beta = \frac{XY}{PX} = \frac{h_0}{f_0} \Rightarrow h_0 = f_0 \cdot \tan\beta$$

and $\tan\beta = \frac{A'B'}{PB'} \approx \frac{h_i}{PQ}$ ($\because$ Q and B' are very close to each other)

$$\therefore h_i = PQ \cdot \tan\beta = L \cdot \tan\beta$$

$$\therefore m_0 = \frac{L}{f_0} \tag{6.16.8}$$

Magnification due to eye-piece,

$$m_e = \left(\frac{D}{f_e} + 1\right) \text{ (See Equation (6.16.5)} \tag{6.16.9}$$

Resultant magnification of a compound microscope is (Equation (6.12.10)),

$$m = m_0 \times m_e$$

$$= \frac{L}{f_0} \times \left(\frac{D}{f_e} + 1\right) \tag{6.16.10}$$

In practice, eye-piece is so adjusted that image A'B' falls very close to its focus Q. Thus, image obtained by eye-piece will be at very large distance (D). Thus, above equation can be written as,

$$m \approx \frac{L}{f_0} \times \frac{D}{f_e} \tag{6.16.11}$$

In order to have large magnification, tube length (L) of the microscope should be kept large.

Illustration 16 : An object is 10 mm from the objective of a compound microscope. The lenses are 30 cm apart and the intermediate image is 50 mm from the eyepiece. What overall magnification is produced by the instrument ?

Solution : From the figure 6.28, applying Gauss's formula to the objective,

$$\frac{1}{-u} + \frac{1}{v} = \frac{1}{f_0} \qquad (1)$$

where v = image distance due to objective lens $\approx f_0$ + L (as Q and B' are very close to each other).

Since image due to objective is formed at 50 mm from the eye-piece, and distance between two lenses is 30 cm = 300 mm (given), image distance from objective

v = 300 − 50 = 250 mm

From equation (1),

$$\frac{-1}{-10} + \frac{1}{250} = \frac{1}{f_0} \text{ (using sign convention)}$$

$$\therefore f_0 = \frac{250 \times 10}{(250+10)} = 9.62 \text{ mm} \approx 10 \text{ mm}$$

Since $v \simeq f_0$ + L $\Rightarrow$ L = 250 − 10 = 240 mm

Final image is always close to the object,

D ≈ (object distance for objective) + (distance between two lenses)

= 10 + 300 = 310 mm

For eye-piece, Gauss' equation,

$$\frac{1}{-u} + \frac{1}{v} = \frac{1}{f_e}$$

$$\therefore \frac{1}{f_e} = \frac{-1}{-50} + \frac{1}{-310} \text{ (For virtual image, } v = -D)$$

$$= \frac{-310+50}{(50 \times 310)}$$

$\therefore |f_e| = 59.6 \approx 60$ mm

thus, resultant magnification is

$$m = \frac{L}{f_0} \times \frac{D}{f_e} = \frac{240}{10} \times \frac{310}{60} = 124$$

Note : Since the final image obtained at a distance 31 cm from the eye-piece is greater than the near-point distance, it can be seen comfortably.

6.16.3 Astronomical Telescope : After observing minute objects using a microscope, now it's time to observe very huge celestial bodies which are crores of kilometers away. Such bodies, in spite of being huge and very far from each other, they are seen to be small and very close to each other by our naked eyes (for example, stars). For observing such objects an Astronomical Telescope is used. It's ray diagram is shown in figure 6.29.

In this telescope two convex lenses are kept in such a way that their principal axis coincide. The lens facing the object is called objective and the lens near the eye is known as eye-piece. Here, the diameter and the focal length of the objective are greater than that of the eye-piece.

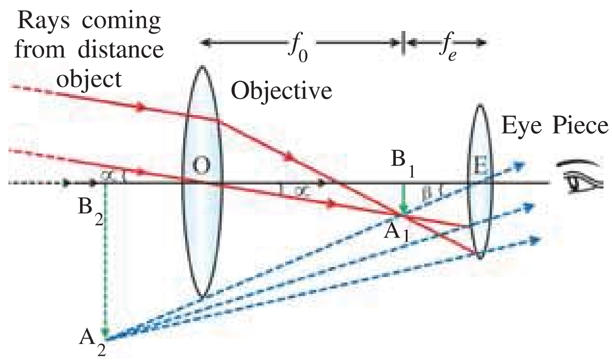


Figure 6.29 Astronomical Telescope

The eye-piece can move to and fro in the telescope-tube. When the telescope is focussed on a distant object, parallel rays coming from this object form a real, inverted, and small image

A_1B_1 on the second principal focus of the objective. This image is the object for the eye-piece. Eye-piece is moved to and fro to get the final and magnified inverted image A_2B_2 of the original object at a certain distance.

We obtain the expression for the magnifying power of a telescope, as follows.

Magnification of the telescope,

$$m = \frac{\text{Angle subtended by the final image with eye}}{\text{Angle subtended by the object with the objective or eye}} = \frac{\beta}{\alpha}$$

From figure 6.29

$$\text{Magnification, } m = \frac{\beta}{\alpha}$$

$$= \frac{A_1B_1}{f_e} \times \frac{f_0}{A_1B_1}$$

$$\therefore\ m = \frac{f_0}{f_e}$$

This equation shows that to increase the magnification of the telescope, focal length of the objective should be increased, and focal length of the eye-piece should be reduced. $f_0 + f_e$ is the **optical length** of the telescope. So, length of the tube $L \geq f_0 + f_e$.

If the focal length of the eye-piece is 1 cm and the focal length of the objective is 200 cm, magnification of the telescope would be 200. Using such a telescope, if the stars having angular distance $1'$ are observed, they would be seen at $200 \times 1' = 200' = 3.33°$ angular distance from each other.

For a telescope, light **gathering power** and **resolving power** (power to view two nearby objects distinctly) are very important.

Amount of light entering the objective of the telescope is directly proportional to the square of the diameter of the objective. Also, with increase in the diameter of the objective, resolving power also increases.

Image formed in this type of telescope is inverted. So if we see from the Earth we get an inverted view of the real scene. To get rid of this problem, an extra pair of inverting lenses in the terrestrial telescope are kept, so that the erect image of the distant object is obtained. Such a telescope is called a **terrestrial telescope**. However, Galileo had used a convex lens and a concave lens in such a telescope.

To get rid of the practical problem faced in obtaining high resolution and high magnification in refracting telescopes, mirrors are used in modern telescopes. Such a telescope is known as reflecting telescope. In such a telescope we can get rid of other problems like chromatic aberration and also spherical aberration, if a parabolic mirror is used.

(In chromatic aberration the edge of the image is seen multicoloured due to dispersion of light and in spherical aberration, image of a point like object is seen spread out).

Construction of the telescope made by Cassegrain (reflecting telescope) is shown in figure 6.30.

As shown in the figure, parallel rays coming from a distant object are incident on the reflecting surface of the primary concave mirror. The reflecting surface of the mirror is parabolic. The rays after getting reflected from this surface are focussed on the principal focus (F) of this mirror. (If the eye-piece is kept near F the image can be seen. But as F is inside the tube, it is difficult to place the eye-piece there.) Cassegrain placed a convex mirror. Rays reflected by the secondary mirror are focussed on the eye-piece after passing through the hole kept in the primary mirror. Diameter and focal length of the primary mirror are kept large in such telescope.

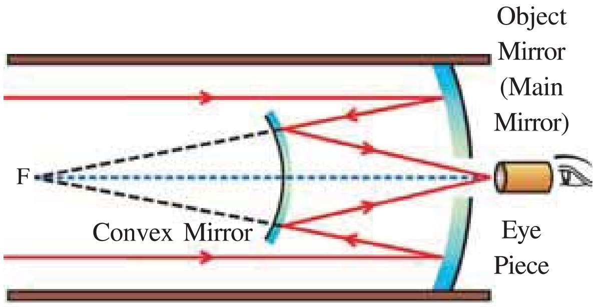


Figure 6.30 Reflecting Telescope

Binoculars used for bird watching or for viewing a cricket match are double telescopes. Here, the final image is erect. In the **binoculars** use of prisms reduces the size of the binoculars. Binoculars are so named because in them viewing is possible by both eyes.

6.16.4 Human Eye : Human eye is the best example of a natural optical appliance. See figure 6.31.

The ray entering the eye is first refracted in the cornea, yet the eye lens is the main factor "culprit" in this case. Due to this lens, inverted and real image is formed on the ratina. This image is processed in the human brain and as a final effect, we feel the image be erect.

Retina has two types of cells :

(1) Rods : These cells give the sensations of less intensity of light.

(2) Cones : These cells give the sensations of colour and high intensity of light.

In case of eye, distance between the retina and the lens is fixed. That is why focal length of the eye lens changes in such a way that the images of the object are always obtained on the retina. (Really, eye lens is smart lens). This becomes possible due to the ciliary muscles attached to the lens. It makes the lens thick or thin as per requirement.

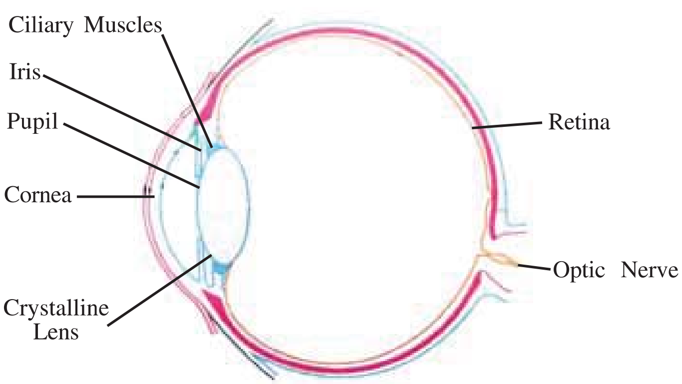
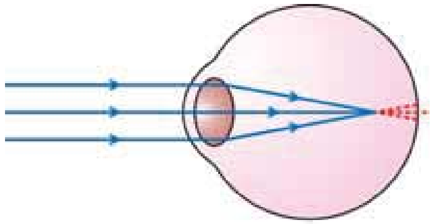


Figure 6.31 Human Eye (For Information Only)

The Iris controls the amount of light entering the eye. It does the work by controlling the size of the pupil. When we see the object kept on the side, lens of the eye rotates and brings the image on the central region of the retina, (fovea).

Defects of Vision : If the lens of eye cannot become thin as per requirement and remains thick only, then rays coming from far objects, which are parallel, undergo extra refraction as shown in figure 6.32, and get focussed in front of the retina. And therefore far off objects cannot be seen clearly. But the image of nearby objects is formed on the retina (figure 6.33). This type of defect is called **Near sightedness (myopia).**



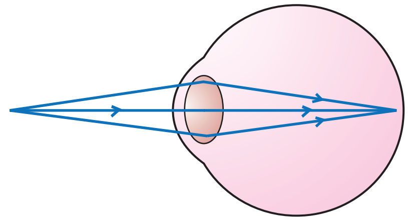


Figure 6.32 Image of Distance Object Falls in front of Retina

Figure 6.33 Image of Nearby Object Falls on the Retina

This defect can be corrected by using concave lens of proper focal length (figure 6.34).

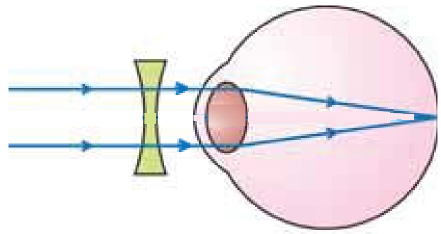
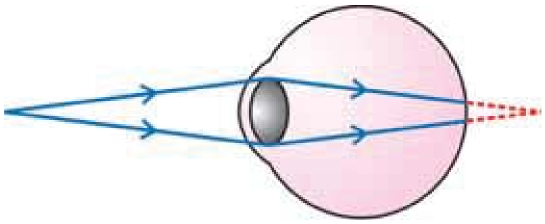


Figure 6.34 To Correct this Defect, Concave Lenses are Used

If the lens remains thin, does not become thick as per requirement, rays coming from a nearby object suffer less refraction and are focussed behind the retina. (figure 6.35). Such an image is not clear. Image of a distant object is formed on the retina only and can be seen clearly, but nearby objects cannot be seen clearly. This defect is called **far sightedness (hypermetropia)**. This type of defect is due to less convergence of rays. To correct this defect a convex lens of proper focal length is used (figure 6.36).



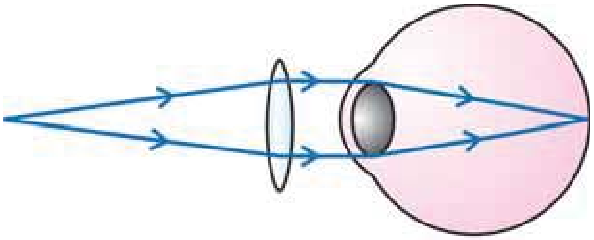


Figure 6.35 Hypermetropia

Figure 6.36 Convex Lens Between Object and Eye

Some people, if shown a wire gauge cannot see the vertical and horizontal both wires clearly, but any one is seen clearly. This defect is called **astigmatism**. If the curvature of the lens and the corena are not the same, this defect occurs. E.g., if a person can see horizontal wires but not vertical. Here, horizontal curvatures are same but vertical curvatures are not. So rays are refracted equally in the horizontal plane, but refraction in the vertical plane is not equal. As a result horizontal wires are seen clearly and vertical wires are not seen clearly. To get rid of this defect, cylindrical lens is used. In the above mentioned case a cylindrical lens of proper curvature and horizontal axis can be used to rectify the defect.

SUMMARY

1. For mirrors Gauss' equation is $\frac{1}{u} + \frac{1}{v} = \frac{2}{R} = \frac{1}{f}$, where u = object distance, v = image distance, R = radius of curvature and f = focal length.
2. Lateral magnification for mirrors is given by $m = \frac{h'}{h} = -\frac{v}{u}$
3. For a compound slab of different transparent media, general form of Snell's law is written as, $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \cdot \sin\theta_3 = ...$
4. Total internal reflection is used as reflectors, e.g. flint glass-prism may be used as high quality reflector. For glass–air interface, critical angle (C) is given by,

   $C = \sin^{-1}\left(\frac{1}{n}\right)$, where n = refractive index of glass.
5. Total internal reflection phenomenon is also used in optical fibres.
6. For thin lens : $\frac{-1}{u} + \frac{1}{v} = \left(\frac{n_2 - n_1}{n_1}\right) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ and $\frac{1}{f} = \frac{-1}{u} + \frac{1}{v}$
7. Since the principle of reversibility suggests that the object and image are conjugate to each other, interchanging the positions of an object, image distance can be determined.
8. Power of lenses in contact is given by

   $P = P_1 + P_2 + ...$
9. Magnification of lenses in contact is given by

   $m = m_1 \times m_2$ .. ..
10. Focal length of lenses in contact is given by

    $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + ...$
11. Prism equation is given by $\delta = i + e - A$. At minimum angle of deviation, $\delta_m = 2i - A$. For thin prisms, (A < <), $\delta_m = A(n - 1)$, where n = refractive index of the material of prism.
12. Scattering can be classified into two : elastic scattering (Rayleigh and Mie-Scattering) and inealstic scattering (e.g., Raman Scattering). If the size of the particle scattering light is smaller than the wavelength of the incident light, it is known as the Rayleigh scattering, if otherwise, it is known as the Mie-scattering.
13. Compound microscope can be thought of as made up of two cascaded simple microscopes, in which magnified image due to first simple microscope works as an object for the second.
14. For high resolution and magnification, curved mirrors are used in modern telescopes.
15. Retina has two types of cells : rods give the sensations of less intense light and cones give sensations of colour and high intense light
16. Defects of vision can be overcome by proper lenses.

EXERCISES

For the following statements choose the correct option from the given options :

1. An object is placed at a distance of 25 cm on the axis of a concave mirror, having focal length 20 cm. Find the lateral magnification of an image.

   (A) 2 (B) 4 (C) –4 (D) –2
2. A fish in a lake is at a 6.3 m horizontal distance from the edge of the lake. If it is just able to see a tree on the edge of the lake, its depth in the lake is .... m. Refractive index of the water is 1.33.

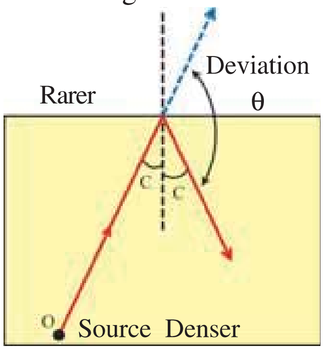
   (A) 6.30 (B) 5.52 (C) 7.5 (D) 1.55

3. For a thin convex lens when the heights of the object is double than its image, its object distance is equal to .... focal length of a lens is f.
(A) f (B) $2f$ (C) $3f$ (D) $4f$

4. A liquid of refractive index n is filled in a tank. A plane mirror is kept at the bottom of the tank. A point like object (P) is kept at a height h from the mirror on the liquid surface. An observer observes the object and its image in the vertically downward direction from top. How much distance will observer note between P and its image ?
(A) $2n \cdot h$ (B) $\frac{2h}{n}$ (C) $\frac{2h}{(n-1)}$ (D) $h\left(1+\frac{1}{n}\right)$

5. Depth of a well is 5.5 m and refractive index of water is 1.33. If viewed from the bottom, by how much height would the bottom of the well appear to be shifted up ?
(A) 5.5 m (B) 2.75 m (C) 4.13 m (D) 1.37 m

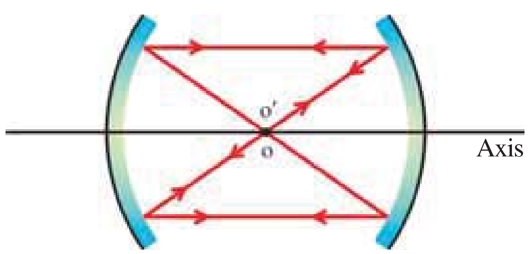
6. A ray of light is travelling from a denser medium to rarer medium. For these media, the critical angle is C. The maximum possible deviation of the ray is ....



(A) $\pi - 2$ (B) $\pi - 2c$
(C) 2C (D) $\frac{\pi}{2} + C$

[**Hint :** The situation at total reflection is shown in the figure.]

7. A point object O is placed midway between on the common axis of two concave mirrors of equal focal length. If the final image is formed at the position of the object, the separation between two mirrors is .... Focal length of mirrors is f.



(A) f (B) $2f$
(C) $\frac{3}{2}f$ (D) $\frac{1}{2}f$.

[**Hint :** A situation is depicted in the figure.]
[**Note :** Another possible situation for which object and its image coincide is when distance between two mirrors is $4f$.]

8. The focal length of a thin lens made from the material of refractive index 1.5 is 20 cm. When it is placed in a liquid of refractive index 1.33, its focal length will be .... cm.
(A) 80.81 (B) 45.48 (C) 60.25 (D) 78.23

9. A tank contains water upto a height of 30 cm and above it an oil up to another 30 cm height. .... cm shifts in the position of bottom of the tank is observed when viewed from the above. Refractive indices of water and oil are 1.33 and 1.28, respectively.
(A) 7.44 (B) 6.46 (C) 14.02 (D) 6.95

[**Hint :** From $\frac{h'}{h} = \frac{n_1}{n_2}$

$$\frac{h'-h}{h} = \frac{-\Delta h}{h} = \frac{n_1 - n_2}{n_2}$$

$$\therefore -\frac{\Delta h}{h} = \left(\frac{n_1}{n_2} - 1\right)$$ (Shift, Δh is negative because shift, $\Delta h = h - h'$)

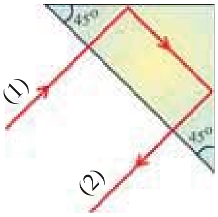
$$\therefore \text{ shift, } \Delta h = h \times \left(1 - \frac{n_1}{n_2}\right) = h \times \left(1 - \frac{1}{n_{21}}\right)$$]

10. For a thin plano convex glass lens with radius of curvature 20 cm, focal length is .... cm. Refractive index (n) of the material of the lens is 1.5 and it is kept in air

(A) 20 (B) 40 (C) 60 (D) 80

[**Hint :** For air − glass lens, $\frac{-1}{u} + \frac{n}{v} = \frac{1}{f} = \frac{(n-1)}{R}$]

11. For right-angled prism, ray-1 is the incident ray and ray-2 is the emergent ray, as shown in the figure. Refractive index of the prism is ....



(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$

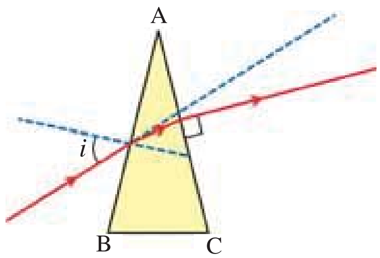
(C) $\frac{2}{\sqrt{3}}$ (D) $\sqrt{2}$

12. A ray of light is incident normally on the surface of an equileteral prism made up of material with refractive index 1.5. The angle of deviation is ....

(A) 30° (B) 45° (C) 60° (D) 75°

[**Hint :** For the present case use the formula $\sin C = \frac{1}{n}$ to understand the phenomenon.]

13. A ray is incident at an angle i on the surface of a prism with very small prism angle A, and emerges normally from the opposite surface. If the refractive index of the prism is μ the angle of incidence i is nearly equal to ....



(A) $\frac{A}{\mu}$ (B) $\frac{\mu A}{2}$

(C) $\frac{A}{2\mu}$ (D) μA

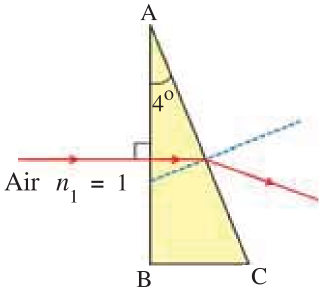
[**Hint :** Use the given figure.]

14. A small linear object of length b is placed on the axis of a concave mirror. The end of the object facing the mirror is at a distance u from the mirror. If f is the focal length of a mirror, the length of the object will be .... approximately.

(A) $b\left(\frac{u-f}{f}\right)^2$ (B) $b\left(\frac{f}{u-f}\right)$ (C) $\left(\frac{u-f}{f}\right)$ (D) $b\left(\frac{f}{u-f}\right)^2$

[**Hint :** Neglect b whenever necessary.]

15. A horizontal ray is incident on a right-angled prism with prism angle of 4°. If the refractive index of material of the prism is 1.5, angle of emergence is .... Use the given figure.



(A) 4° (B) 6°

(D) 10° (D) 0°

16. Which of the following is responsible for glittering of a diamond ?

(A) Interference (B) Diffraction (C) Total internal reflection (D) Refraction

17. The radii of curvature of both the sides of a convex lens are 15 cm and if the refractive index of the material of the lens is 1.5, then focal length of lens in air is .... cm

(A) 10 (B) 15 (C) 20 (D) 30

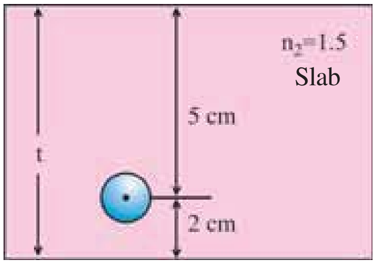
18. An image of an object obtained by a convex mirror is n times smaller than the object. If the focal length of lens is f, the object distance would be ....

(A) $\frac{f}{n}$ (B) $\frac{f}{(n-1)}$ (C) $(n - 1)f$ (D) nf

19. Time taken by the sunlight to pass through a slab of thickness 4 mm and refractive index 1.5 is .... sec.

(A) 2×10^{-8} (B) 2×10^{8} (C) 2×10^{-11} (D) 2×10^{11}

20. An air bubble in a glass slab with refractive index 1.5 is 5 cm deep when viewed from one face and 2 cm deep when viewed from the opposite face. The thickness of the slab is .... cm.



(A) 10.5 (B) 7

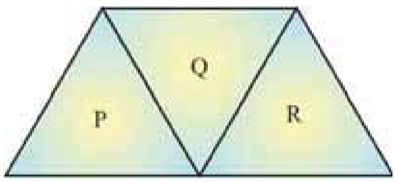
(C) 105 (D) 70

[**Hint :** Use $\frac{h'}{h} = \frac{n_2}{n_1}$]

21. The focal length of an equiconvex lens in air is equal to either of its radii of curvature. The refractive index of the material of the lens is ....

(A) $\frac{4}{3}$ (B) 1.5 (C) 2.5 (D) 0.8

22. A ray of light experiences minimum deviation by an equilateral prism P. Now two prisms Q and R made of the same material as that of P are arranged as shown in the figure. The ray of light will now experience, (The dimensions of P, Q and R are same.) ....



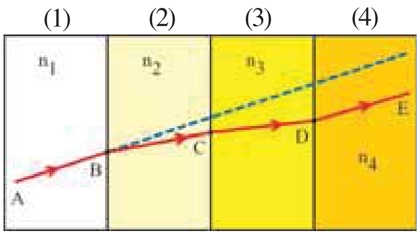
(A) larger deviation

(B) no deviation

(C) same deviation as that due to P

(D) total internal reflection

23. The refractive indices of four media, as shown in the figure, are n_1, n_2, n_3 and n_4. AB is an incident ray. DE, the emergent ray, is parallel to the incident ray AB, then ....



(A) $n_1 = n_2$ (B) $n_2 = n_3$

(C) $n_3 = n_4$ (D) $n_4 = n_1$

24. If the tube length of astronomical telescope is 105 cm and magnifying power is 20 for normal setting, then the focal length of the objective is .... cm.

(A) 10 (B) 20 (C) 25 (D) 100

[**Hint :** Optical length of astronomical telescope is given by $L \geq f_0 + f_e$]

25. The top sky looks blue in morning hours because, ...

(A) red light is absorbed (B) blue light is scattered the most

(C) sun radiates blue light only in the morning.

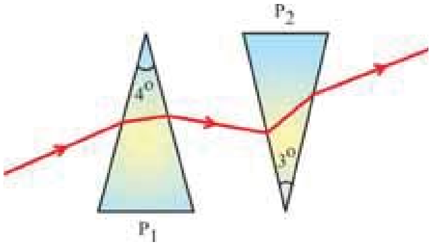
(D) blue light is absorbed by the sky

26. A defect of vision in which lines in one plane of an object appear in focus while those in another plane are out of focus is called ....
(A) astigmatism (B) distortion (C) myopia (D) hypermetropia

27. Stokes and antistokes lines observed in Raman scattering is due to .... of light.
(A) reflection (B) elastic scattering
(C) inelastic scattering (D) dispersion

28. A convex lens of focal length 10 cm is used as a simple microscope. When image of an object is obtained at infinite, magnification is .... Near point for normal vision is 25 cm.
(A) 1.0 (B) 2.5 (C) 0.4 (D) 25

29. As shown in the figure, thin prisms P_1 and P_2 are combined to produce dispersion without deviation. For prism P_1, angle of prism is 4^o and refractive index is 1.54. For prism P_2 angle of prism is 3^o. Refractive index of material of P_2 is ....



(A) 1.72 (B) 1.5
(C) 2.4 (D) 0.58

[**Hint :** For thin prism, $\delta = A(n - 1)$]

30. A spherical convex surface separates an object and image spaces of refractive index 1.0 and 1.5 respectively. If radius of curvature of the surface is 25 cm, its power is .... D.
(A) 13 (B) 33 (C) 3.3 (D) 1.3

[**Hint :** $\frac{-n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$ and $P = \frac{1}{f}$]

31. A light ray is incident at an angle 30^o with normal on a 3 cm thick plane slab of refractive index $n = 2.0$. The lateral shift of the incident ray is .... cm.
(A) 0.835 (B) 8.35 (C) 1.5 (D) 1.197

[**Hint :** Since incident angle θ_1 is not small, lateral shift, $x = \frac{t\sin(\theta_1 - \theta_2)}{\cos\theta_2}$]

ANSWERS

1. (C)	**2.** (B)	**3.** (C)	**4.** (B)	**5.** (D)	**6.** (B)
7. (B)	**8.** (D)	**9.** (C)	**10.** (B)	**11.** (D)	**12.** (C)
13. (D)	**14.** (D)	**15.** (B)	**16.** (C)	**17.** (B)	**18.** (C)
19. (C)	**20.** (A)	**21.** (B)	**22.** (C)	**23.** (D)	**24.** (D)
25. (B)	**26.** (A)	**27.** (C)	**28.** (B)	**29.** (A)	**30.** (D)
31. (A)					

Answer the following questions in brief :

1. What are paraxial rays ?
2. State Snell's Law.
3. What is total internal reflection ?
4. Light is incident normally on a glass slab with refractive index of 1.67. Find percentage reflected intenstity (I_r) compared to the incident intensity.
5. What is the use of cladding in the case of optical fibers ?

6. Define optical centre of a lens.
7. Write one advantage of using Newton's formula over lens-maker's formula.
8. Initially, two thin lenses were kept in contact. Now, if they are separated by d distance, what happens to the focal length of a combination ?
9. What are conjugate foci ?
10. Define near point or distance of most distinct vision.
11. What is the function of rods in retina ?

Answer the following questions :

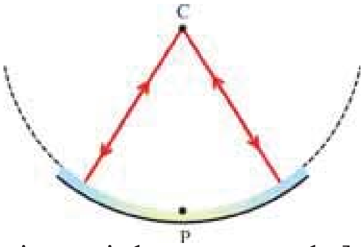
1. Obtain relation between focal length and raidus of curvature for convex mirror.
2. For concave mirror, derive the mirror formula.
3. Define lateral magnification for mirrors. Using cartesian sign convention, derive its relation with image distance and object distance.
4. Obtain an expression for lateral shift due to rectangular slab.
5. Explain the relation between real depth and the virtual depth.
6. Explain total internal reflection.
7. How right-angled prisms are useful as perfact reflecting surface ?
8. Explain how total internal reflection is useful in optical fibre.
9. For a spherically curved surface, derive the relation, $\frac{-n_1}{u} + \frac{n_2}{V} = \frac{(n_2 - n_1)}{R}$
10. Explain the image formation due to thin lens and derive $\frac{-1}{u} + \frac{1}{v} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ relation.
11. Derive lens-maker's formula for thin lens.
12. Derive Newton's formula for thin lens.
13. Explain conjugate points and conjugate distances.
14. Define lateral magnification for lenses. Obtain its relation to extra focal distances.
15. Derive the relation for effective focal length of an optical system made up of two thin lenses in contact.
16. Obtain the relation $f = \frac{1}{2}(v - d)$ for a convex mirror using a combination of convex mirror and convex lens.
17. Derive an equation $\delta = i + e - A$ for equilateral prism.
18. Using $\delta = i + e - A$ for equilateral prism obtain an equation for refractive index (n) of material of the prism.
19. Write note on Rayleigh scattering.
20. What is scattering ? Explian Raman Scattering.
21. Obtain an expression for magnification for simple microscope.
22. With diagram, derive an expression for magnification for compound microscope.
23. Write note on refracting telescopes.
24. What are reflecting telescopes ? What are the advantages of them over refracting telescopes ?
25. Discuss astigmatism defect of human eye.

Solve the following examples :

1. An object moves with uniform velocity (v_0) on the axis of a concave mirror. If it moves towards the mirror, show that when it is at a distance u from the mirror. The velocity of its image is given by $v_i = \left(\frac{R}{2u - R}\right)^2 v_0$, where R is radius of curvature of the mirror.

2. An image of a linear object due to a convex mirror is $\frac{1}{4}^{th}$ of the length of the object. If focal length of the mirror is 10 cm, find the distance between the object and the image. The linear object is kept perpendicular to the axis of the mirror. [**Ans :** 37.5 cm]

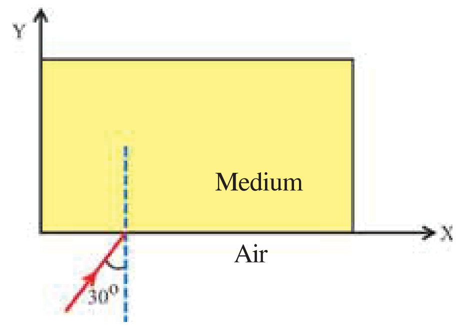
3. A concave mirror has been so placed on a table that its axis remains vertical. P and C are pole and centre of curvature respectively. When a point like object is placed at C, its real image is formed at C. If now, water is filled in mirror. What can be said about the position of the image?



[Ans : image is between c and p]

4. The diameter of the sun subtends an angle of 0.5° at the pole of the concave mirror. The radius of curvature of the mirror is 1.5 m. Find the diameter of the image of the sun. Consider the distance of sun from the mirror infinite. [**Ans :** 0.654 cm]

5. A ray, as shown in the figure, is incident at the angle of incidence 30° on the surface and travels in the medium. If the refractive index of the medium is given by $n(y) = 1.5 - ky$. Here, k is constant and it is equal to $0.25 m^{-1}$. At which value of y, will the ray becomes horizontal in the medium ? Here, y is in meter. [**Ans :** $y = 3$ m]

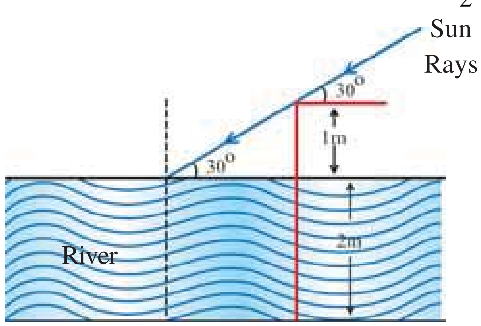


6. A narrow beam of light is incident on a glass plate of refractive index 1.6. It makes an angle 53° with normal to the interface. Find the lateral shift of the beam at the point of emergence, if thickness of the plate is 20 mm. Take sin53° = 0.8. [**Ans :** 9 mm]

7. A real image obtained by a concave mirror is 4 times begger than the object. If the object is displaced by 3 cm away from the mirror, the image size becomes 3 times the object size. Find the focal length of the mirror. [**Ans :** 36 cm]

8. The refractive index of material of a particular optical fibre is 1.75. At what maximum angle a ray can be made incident on it, so that it is totally internally reflected ? Consider air as an external medium with refractive index as 1.0. [**Ans :** $\frac{\pi}{2}$]

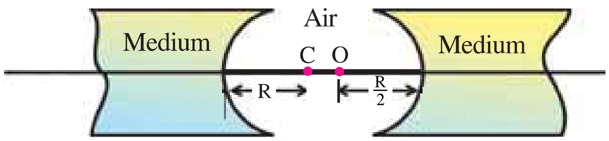
9. A level measuring post (a rod) has been kept in a river of 2 m depth vertically such that its 1 m portion remains outside the river. At this instant, the sun makes an angle of 30° with the horizontal. Find the length of the shadow of the level measuring post on the bottom of the river (see figure). The refravtive index of water is $\frac{4}{3}$.



[**Ans :** 3.44 m]

10. A vessel is fully filled with liquid having refractive index $\frac{5}{3}$. At the bottom of the vessel a point-like source of light is kept. An observer looks at the source of light from the top. Now, an opaque circular disc is kept on the surface of the water in such a way that its centre just rests above the light source. Now liquid is taken out from the bottom gradually. Calculate the maximum height of the liquid to be kept so that light source cannot be seen from outside. Radius of the disc is 1 cm. [**Ans :** 1.33 cm]

11. As shown in the figure, two concave refracting surfaces of equal radii of curvature (R) and refractive indices (n = 1.5) face each other in air (n = 1.0).



A point object (0) is placed midway in between the centre and one of the vertices of the refracting surfaces. Find the distance between image O' formed by one surface and image O'' formed by the other surface in terms of R.

[**Hint :** Use $\frac{-n_1}{u} + \frac{n_2}{v} = (n_2 - n_1)\frac{1}{R}$ for both the refracting surfaces.] [**Ans :** 0.114 R]

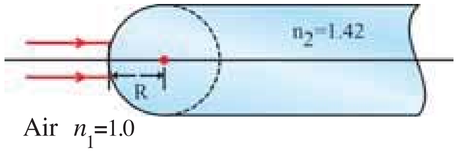
12. (1) If $f = +\ 0.5m$ calculate power of a lens.

(2) The radii of curvature of a convex lens are 10 cm and 15 cm. If its focal length is 12 cm, find the refractive index of the material of the lens.

(3) The focal length of a convex lens in air is 20 cm. What will be its focal length in water. The refractive index of water is 1.33 and that of glass is 1.5.

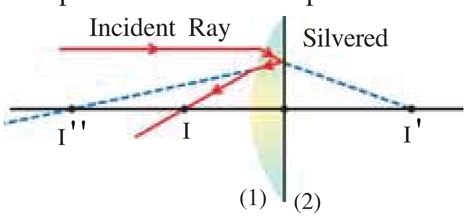
[**Ans :** (1) + 2 D (2) 1.5 (3) 78.2 cm]

13. One end of a cylindrical rod made from the material of refractive index 1.42 is hemispherical.



A narrow beam of parallel rays is incident as shown in the figure. At how much distance will this beam of ray be focussed from the hemispherical surface ? [**Ans :** 3.38 R]

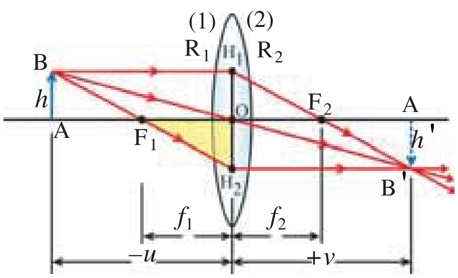
14. The plane surface of a plano convex lens of focal length 20 cm is silvered and made



reflecting, as shown in the figure. Find new focal length of the system.

[**Ans :** 10 cm]

15. Consider a general case of thin lens with first principal focal length (f_1) and second principal



focal length (f_2). Obtain the expression for magnification in terms of f_1 and f_2 as $\left(\frac{v - f_2}{f_1}\right)$. Also, for a special case of $f_2 = f_1 = f$, deduce Gauss' equation from the expression for the magnification. Use cartesian sign convention.

[**Hint :** From figure ΔBH_1H_2 and ΔF_1OH_2 are similar, and $\Delta B'H_2H_2$ and ΔF_2OH_1 are similar.]

7

Dual Nature of Radiation and Matter

7.1 Introduction

At the end of the nineteenth century, most physicists thought that the Newtonian laws governing the motion of material particles, thermodynamics and Maxwell's theory for electromagnetic waves are complete and fundamental laws of physics. They all together constitute "Classical Mechanics". Classical physics deals primarily with macroscopic phenomena. Most of the effects with which classical theory is concerned are either directly observable or can be made observable with relatively simple instruments. Thus, there is a close link between the world of classical physics and our sense of perception. Almost all known **macroscopic** problems were satisfactorily solved applying the laws of classical mechanics, and therefore scientists have turned their concentration to the study of atomic and subatomic (i.e. microscopic and submicroscopic) systems. Unlike macroscopic system, since these systems are inaccessible to direct observations, the experiments which have generated interest and curiosity studying some microscopic problems are worth mentioning here.

Study of the influence of an electric field to cathode rays by Jean Perin (1895), and experimental demonstration of negatively charged particles have discovered an electron. Just later, J. J. Thomson found the ratio of charge to mass ($\frac{e}{m}$ = 1.756×10^{11} C/kg) for an electron, while Milikan (1909) had estimated the charge of an electron ($e = 1.602 \times 10^{-19}$ C). It was also established that the smallest basic unit of matter is an atom, and it is electrically neutral. Wilhem Rontgen (1885) accidentally discovered X-rays and just few years later, Henry Bacquerel (1896) and Madam Curie (1898) with different compounds have discovered radio activity.

These were the few experiments which provided a foundation to perform series of different experiments yielding results which could not be explained by the laws of classical mechanics. The specific heats of solids and diatomic gases at very low temperatures, large electrical conductivities of metallic solids, structure of an atom and the characteristic wave lengths emitted or absorbed by different elements, the photoelectric effect, the study of black-body radiation were the notable problems which could not be understood in terms of classical mechanics.

For the resolution of the apparent paradoxes posed by these observations and certain other experimental facts, it became necessary to introduce new ideas quite foreign to commonsense concepts regarding the nature of matter and radiation.

Historically to understand how entirely new concepts were emerged, we study the difficulties in explaining the black-body radiation.

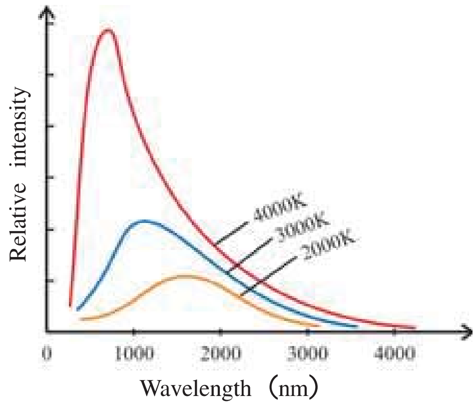


Figure 7.1 Relative Intensity as a Function of Wavelength

Black-body Radiation : In 1897, Lummer and Pringsheim measured the intensities of different wavelengths (i.e., intensity distribution) of black-body or cavity radiations, which is plotted in the figure 7.1.

Scientists were trying to explain these graphs using the laws of electromagnetic theory and thermodynamics.

On the thermodynamic grounds and by using ideas of electromagnetism, Wien gave an expression for energy density as, $u_\lambda = \frac{1}{\lambda^5} \cdot exp\left(-\frac{b}{\lambda \cdot T}\right)$; where b is cosnstant and T is absolute temperature. Such an equation can explain the experimental reuluts only for small wave lengths, but fails to explain the higher wavelengths intensity distribution.

Rayleigh and Jeans determined the number of normal modes of vibration for small intervals of wavelengths, considering the radiations as electromagnetic waves. Each normal mode corresponds to one harmonic oscillator. As the degrees of freedom for harmonic oscillator is two, according to equipartition law for energy, its kinetic energy is $k_B T$. Here, k_B is the Boltzmann constant. Based on this argument, they derived an equation for energy density as,

$$u_\lambda = \frac{8\pi k_B T}{\lambda^4} \qquad (7.1.1)$$

This equation can explain the energy distribution for large wavelengths only. Further, the total energy density (u_{tot}) covering all possible wavelengths must follow the Stefan-Boltzmann's law ($u_{tot} = \sigma \cdot T^4$; where σ = Stefan-Boltzmann's constant). But using equation (7.1.1), if we calculate the total energy density, i.e., $u_{tot} = \int_0^\infty \frac{8\pi k_B T}{\lambda^4} d\lambda$, we get infinite ($\infty$) answer ! This is called ultraviolet catastrophe. On the other hand, Wien's law requires (λ_{max}). T = constant, (b)is called wien's constant. (7.1.2.)

Here, λ_{max} is the wavelength corresponding to the peak value in the intensity distribution graph at that temperature.

Thus, all the attempts based on thermodynamics and electromagnetic theories failed to explain the entire energy distribution curves of black-body radiation.

7.2 Planck's Hypothesis for Radiation

The explanation of energy distribution curves of black-body or cavity radiation was given by Max Planck (1900) at the Academy of Science in Berlin.

He suggested – **"The walls of cavity emitting radiations are made of electric dipoles. According to their temperature, different dipoles oscillate with different frequencies and emit radiations of frequencies equal to frequencies of their oscillations."**

Now, according to the classical physics an oscillator may possess any amount of energy. That is, an oscillator may acquire continuously varying (from zero to maximum available) energy.

Planck presented a revolutionary idea that **"these microscopic oscillators may not possess any arbitrary energy as allowed by the laws of classical mechanics. If the vibrational frequency of such a microscopic oscillator is f, then it may possess energy given by,**

$$E_n = nhf, \qquad (7.2.1)$$

where n = 1, 2, 3 ..... Here h is known as Planck's universal constant. Thus, according to Planck, energy of such microscopic oscillator depends on its vibrational frequency. This is in contrast to classical oscillator, whose energy depends on its amplitude of oscillation, as per the well known equation $\frac{1}{2}kA^2$. Here, k is the force constant and A is amplitude.

Equation (7.2.1) also suggests that the energy of an oscillator of frequency f is hf, $2hf$, $3hf$, ..., etc. It cannot possess the fractional energy like $0.1hf$, $\frac{1}{2}hf$, $0.06hf$. Thus, energy of microscopic oscillator is an integral multiple of hf. In other words, the smallest quantum of energy of an oscillator of frequency f is 'hf'.

This smallest bundle or packet or quantum of energy is known as **photon.** When an oscillator emits radiation of frequency f, its energy decreases in integral multiple of hf. And quanta of energy hf are emitted. That is, energy is not emitted continuously but in the form of quanta. This phenomenon is known as the **quantization** of energy. (You have also studied the quantization of electric charge.) If an oscillator possesses energy $5hf$, meaning 5 quanta each with energy hf.

Based on his hypothesis Planck could successfully derive the equation of spectral emissive power for a perfect black-body radiation, which is given by

$W_f = \frac{2\pi f^2}{c^2} \times \frac{hf}{\left[e^{\left(\frac{hf}{k_B T}\right)} - 1\right]}$. Here, c = speed of light in vacuum, T = absolute temperature of a perfect balck body, k_B = Boltzmann's constant. (This equation is only for information.)

Above equation gives maximum energy density at the wavelength (λ_{max}) corresponding to Wien's law. Using the experimental values of Stefan-Boltzmann constant σ, Wien's constant b (see equation (7.1.2.)) and Boltzmann's constant k_B, value of Planck's constant (h) can be determined as

$$h = 6.625 \times 10^{-34} \text{ J.s}$$

It can be proved that in the limit $hf \rightarrow 0$, above equation correctly reproduces the classical value $k_B T$, predicted by the law of equipartition of energy. It appears, therefore, that the very small but **non-zero** value of constant 'h' is a measure of the failure of classical mechanics.

Only For Information : If quantum effects are to be observed, the frequency should be high enough so that $\frac{hf}{k_B T}$ becomes comparable to unity. For example, at room temperature (T $\approx$ 300 K), $\frac{hf}{k_B T} \approx \frac{1}{6}$ for $f = 10^{12}$ Hz. This shows that only when oscillator of at least this frequency or higher, quantum statistical effects become noticeable at room temperature.

7.3 Photoelectric Effect

7.3.1 Emission of Electrons : We know that metals have **free electrons.** However, these free electrons normally cannot come out of the metal surface. The reason is that electrons at the surface experience strong attractive inward force due to positive metallic ions; while virtually no attractive force from the outside. In other words, very close to the surface, potential energy of electrons increase with distance as compared to inside electrons. That is, a potential-barrier exists at the surface. Thus, to bring an electron out, some minimum amount of energy must be supplied to it. This minimum energy required to get emission of an electron is known as **work function** (ϕ_0) of the metal.

The work function of a metal depends on type of the metal, nature of its surface and its temperature.

To bring an electron out of the metal, required energy may be supplied by any of the following ways.

Thermionic Emission : In this method, current is passed through a filament so that it gets heated sufficiently (normally 2500–3000 K). Hence, free electrons in it gain enough energy and get emitted from the metal. Such kind of electron emission is observed in devices like diode, triode, T.V. tube (cathode ray tube), etc.

Field Emission or Cold Emission : When a metal is subjected to strong electric field of the order of $10^8 \ \frac{V}{m}$, electrons are pulled out of the metal surface.

Photo Electric Emission : When an electromagnetic radiation of enough high frequency is incident on a cleaned metallic surface, electrons can be liberated from the metal surface. This phenomenon is known as the **photoelectric effect** and the electrons so emitted are known as **photo electrons.** To have photo emission, the frequency of incident light should be more than some minimum frequency. This minimum frequency is called the **threshold frequency** (f_0). It depends on the type of the metal. For most of the metals (e.g. Zn, Cd, Mg) threshold frequency lies in the ultraviolet region of electromagnetic spectrum. But for alkali metal (Li, Na, K, Rb) it lies in the visible region.

7.3.2 Hertz's Experiment : The photoelectric effect was discovered accidentally in 1887 by H. Hertz, during his study on the phenomenon of emission of electromagnetic waves by means of spark discharge. In his experiment electromagnetic waves from the transmitter (antenna) induced a potential difference across the spark-gap, as evidence from the jumping spark across it. Hertz noticed that the sparks jumped more easily when the cathode was illuminated by ultraviolet light. This observation suggested that light facilitated the escape of charges from the metallic cathode across the spark-gap. Further, Hallwachs extended this experiment for zinc plate. He connected the negatively charged zinc plate with an electroscope. When this plate was irradiated with ultraviolet light, it was observed that negative charge on the plate decreased. Not only this, even when a neutral plate is irradiated with ultraviolet light it becomes positively charged, while postively charged plates became more positively charged. Hallwachs concluded that under the effect of ultraviolet light, negatively charged electrons are emitted from the zinc plate. These electrons are known as photoelectrons.

7.3.3 Lenard's experiment : The details of the photoelectric phenomenon were studied by P. Lenard, one of Hertz's students. The experimental arrangement to study the photoelectric effect is shown in the figure 7.2.

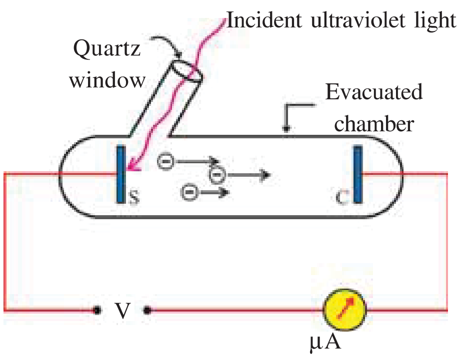


Figure 7.2 Experimental Arrangement to Study Photoelectric Effect

The ultraviolet light entering from quartz window is incident on the cleaned photosensitive surface S. C is the collector, while S is the cathode. C can be kept at different positive or negative voltages with respect to S.

The characteristies of photoelectric effect can be studied in reference to the frequency and the intensity of incident light, and also in terms of number of photoelectrons emitted and their maximum kinetic energy.

When the collector is positive with respect to S, the photo electrons are attracted to it and micro-ammeter registers a current. The amount of current passing through the ammeter gives an idea of the number of photoelectrons. At some value of positive potential difference, when all the emitted electrons are collected, increasing the potential difference further has no effect on the current.

When the collector is made negative with respect to S, the emitted electrons are repelled and only those electrons which have sufficient kinetic energy to overcome the repulsion may reach to the collector, and constitute current. So the current in ammeter falls. On making collector more negative, number of photoelectrons reaching the collector further decreases. For some specific negative potential of the collector, even the most energetic electrons are unable to reach collector, and photoelectric current becomes zero. It remains zero even if the potential is made further negative than the specific value of negative potential. This minimum specific negative potential of the collector with respect to the emitter (photo sensitive surface) at which photoelectric current becomes zero is known as the **stopping potential** (V_0) for the given surface. It is thus the measure of maximum kinetic energy ($\frac{1}{2}mv^2_{max}$) of the emitted photoelectrons. If charge and mass of an electron are e and m respectively,

$$\frac{1}{2}mv^2_{max} = eV_0 \tag{7.3.1}$$

Lenard performed further experiments by varying the intensity (brightness) of the incident light, and measured maximum K.E. and number of photoelectrons via the photoelectric current. He found that by increasing the intensity of the incident light, photoelectric current (i.e. the number of photoelectrons) increases but do not affect the K.E. of the emitted electrons. In the contrast, when he performed the experiment with different frequencies, higher than the threshold frequency of the incident light, changes the stopping potential (V_0) and thereby the K.E. of the emitted electrons, leaving photoelectric current unaltered. It was found that by increasing frequency V_0 and therefore maximum K.E. of the photoelectrons increase, and vice versa. It was also observed that the photo-electrons are emitted within 10^{-9} s after the light is incident.

In summary,

(1) The maximum K.E. of photoelectrons depend on the frequency of incident light, and does not depend on the intensity.

(2) The number of photoelectrons depend directly on the intensity of incident light.

(3) Photoelectric effect is always observed whenever incident light has frequency either equal to or greater than the thereshold frequency for the given surface irrespetive of the intensity.

(4) The phenomenon of photoelectric effect is spontaneous (takes about 10^{-9} sec.).

Above inferences can be depicted in the graphs below : (Figure 7.3 and 7.4)

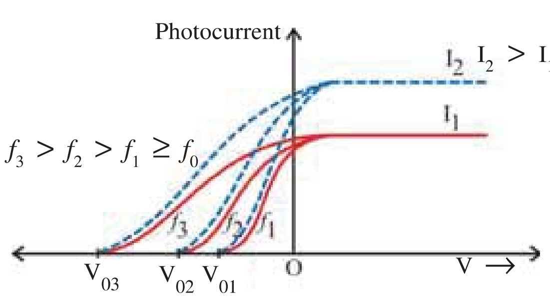


Figure 7.3 Variation of Photoelectirc Current

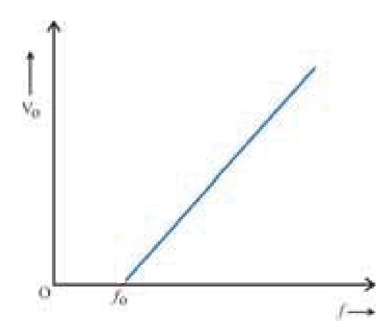


Figure 7.4 Variation of Stopping Potential with Frequency of Incident Light

7.3.4 Explanation from the wave theory of light : Above experimental results cannot be understood with the wave theory of light.

(1) According to the wave theory of light, energy and intensity of wave depend on its amplitude. Hence intense radiation has high energy and on increasing intensity, energy of photoelectrons should also increase. In contradiction to it, experimental results show that the energy of photoelectrons does not depend on the intensity of incident light.

According to the wave theory, energy of light has no relation to its frequency. Hence change in energy of photoelectrons with the change in frequency cannot be explained.

(2) Photoelectrons are emitted immediately (within the 10^{-9} s) on making light incident on the metal surface. Since the free electrons within the metal are withheld under the effect of certain forces, and to bring them out, energy must be supplied.

Now, if the incident energy is showing a wave nature, free electrons in metal get energy gradually and when accumulates energy at least equal to the work function then after they escape from the metal. Thus, electrons get emitted only sometime after the light is incident.

(3) According to the wave theory, less intense light is 'weak' in terms of energy. To liberate photoelectron with such light one has to wait long till electron gathers sufficient energy. Against that experiment shows immediate emission of electron even with diminutive intensity but of course, with sufficiently high frequency.

Thus, wave theory fails to explain the photoelectric effect.

7.3.5 Einstein's Explanation : Einstein gave a successful explanation of the photoelectric effect in 1905 for which he received the Nobel Prize in 1921.

Planck had assumed that emission of radiant energy takes place in the quantized form, the photon, but once emitted it propagates in the form of wave. Einstein further assumed that not only the emission, even the absorption of light takes place in the form of photons.

For Information Only : In the wave nature, the energy is supposed to be spread uniformly across the wave fronts, Einstein proposed that the light energy is not spread over wavefronts but is concentrated in small packets, the photons. He wrote : "According to the assumption considered here, when a light ray starting from a point is propagated, the energy is not continuously distributed over an ever increasing volume, but it consists of a finite number of energy quanta, localized in space, which move without being divided and which can be absorbed or emitted only as a whole."

Suppose frequency of incident light is f, hence energy of its photon is hf. When this photon is incident on the metal, during the interaction with an electron, it is totally absorbed if its frequency (and therefore energy) is greater than threshold frequency or otherwise does not lose energy at all.

As per the laws of classical mechanics (Newtonian mechanics and Maxwell's theory for electromagnetic waves) there is no reason to expect any sensitive frequency dependence of photon-electron interaction. (You will learn its detailed answer in advance course in physics, if you choose physics to shape your career.)

Now if f_0 is the threshold frequency the appropriate photon energy hf_0 will be equal to work function ϕ_0, and at that frequency the photoelectrons are emitted with the minimum (zero) kinetic energy. For frequency $f > f_0$, the maximum kinetic energy of emitted photoelectrons will be,

$$\frac{1}{2}mv^2_{max} = hf - \phi_0$$

From equations (7.3.1), $eV_0 = hf - hf_0$

$$\therefore V_0 = \frac{h}{e} \cdot f - \left(\frac{hf_0}{e}\right) \quad (7.3.2)$$

According to this equation the graph of V_0 versus f is a straight line with a slope $\frac{h}{e}$ and intercept on the X-axis at f_0. This is in excellent agreement with the experimental results shown in the Figure 7.4.

The intensity of light incident on surface is the light energy incident per unit surface area in unit time normal to the surface. According to photon theory (particle nature) of light, if n photons are incident per unit surface area in unit time, intensity of light is $I = nhf$, where hf is the energy of the photon of frequency f. Thus, according to photon theory, more the intensity of light more is the number of photons incident per second and hence more is the photoelectric current. Again showing an experimental trend.

Also, since the interaction between photon and electron takes place as the absorption as a whole or not at all, emission of photoelectron will be instant. Unlike wave nature, where electron has to wait till it gathers enough energy for escape.

Thus, experimental observations for photoelectric effect are reproduced by considering a particle (quantized) nature (photon) of light.

Following table shows work functions and corresponding threshold freqeuncy for some metals.

Table 7.1

Workfunctions and Threshold Frequencies (For information only)

Metal	ϕ_0 (in eV)	f_0 ($\times 10^{14}$ Hz)	Metal	ϕ_0 (in eV)	f_0 ($\times 10^{14}$ Hz)
Cs	1.9	4.60	Fe	4.5	10.89
K	2.2	5.32	Ag	4.7	11.37
Ca	3.2	7.74	Au	4.9	11.86
Cd	4.1	9.92	Ni	5.0	12.10
Al	4.2	10.16	Pt	6.4	15.49

Illustration 1 : Let an electron requires 5×10^{-19} joule energy to just escape from the irradiated metal. If photoelectron is emitted after 10^{-9} s of the incident light, calculate the rate of absorption of energy. If this process is considered classically, the light energy is assumed to be continuously distributed over the wave front. Now, the electron can only absorb the light incident within a small area, say 10^{-19} m^2. Find the intensity of illumination in order to see the photoelectric effect.

Solution : The rate of absorption of energy (power) is

$$P = \frac{E}{t} = \frac{5\times10^{-19}}{10^{-9}} = 5 \times 10^{-10} \frac{J}{s}$$

From the definition of the intensity of light,

$$I = \frac{\text{Energy}}{\text{time}\times\text{area}} = \frac{5\times10^{-10}}{10^{-19}} = 5 \times 10^{9} \frac{J}{s.m^2} \text{ (i.e., 500 billion } \frac{\text{Watt}}{m^2})$$

Since, practically it is impossibly high energy, which suggests that explanation of the photoelectric effect in classical term is not possible.

Illustration 2 : Work function of metal is 2 eV. Light of intensity 10^{-5} W m^{-2} is incident on 2 cm^2 area of it. If 10^{17} electrons of these metals absorb the light, in how much time does the photo electric effect start ? Consider the waveform of incident light.

Solution : Intensity of incident light is 10^{-5} W m^{-2}.

$\therefore$ Energy incident on 1 m^2 area in 1 s is 10^{-5} J.

$\therefore$ Energy incident on area of 2 cm^2 = 2×10^{-4} m^2

$$= 2 \times 10^{-4} \times 10^{-5} = 2 \times 10^{-9} \text{ J}$$

This energy is absorbed by 10^{17} electrons.

$\therefore$ Average energy absorbed by each electron = $\dfrac{2 \times 10^{-9} \text{J}}{10^{17}}$ = 2×10^{-26} J

Now, electron may get emitted when it absorbs energy equal to the work function of its metal. In the given problem work function is 2 eV = $2 \times 1.6 \times 10^{-19}$ J. Thus, electron requires

$(2 \times 1.6) \times 10^{-19}$ J of energy to get emitted.

To absorb 2×10^{-26} J of energy, time required is 1 s, therefore to absorb energy $2 \times 1.6 \times 10^{-19}$ J, time required is,

$$t_e = \frac{2 \times 1.6 \times 10^{-19}}{2 \times 10^{-26}} = 1.6 \times 10^{7} \text{ s}$$

Note : If light is considered as wave, photo electron would not be emitted instantaneously as generally seen in the experiments.

7.4 Particle Nature of Light

The photons are considered as discrete amounts of energy (packets) with smallest being the *hf*. Thus, by nature itself the concept of photon involves the essence of radiation. So, can we consider photon as a real particle ? The compton effect, in which X-rays are scattered by the free electrons, gives the answer. To explain compton effect, photon was considered as a real particle just like a material particle. The way electron collides with any other matter particles, electron may also undergo same type of collision with photon. Also, this collision was considered to follow the laws of conservation of momentum and energy. Thus, as a result of the study of photoelectric effect and compton effect, following properties were attributed to a photon.

(1) Like a material particle, photon is also a real particle.

(2) Energy of a photon of frequency f is hf.

(3) Momentum of photon of frequency f is $\dfrac{hf}{c}$.

According to Einstein's special theory of relativity the relation between energy (E) and momentum (p) of a particle is given by,

$$E = \sqrt{p^2c^2 + m_0^2 \cdot c^4}, \text{ where } c = \text{ speed of light in vacuum and} \qquad (7.4.1)$$

m_0 = rest mass.

Mass of a particle moving with speed v as obtained from equation (7.4.1) is given by,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (7.4.2)$$

Since, in vacuum, photon moves with speed equals to speed of light, its rest mass

$$m_0 = m \times \sqrt{1-\frac{c^2}{c^2}} = 0$$

From equation (7.4.1),

$$E = p.c. \qquad (7.4.3)$$

$$\text{or } p = \frac{E}{c} = \frac{hf}{c} \qquad (7.4.4)$$

(4) Mass of a photon, $m = \frac{E}{c^2}$ ($\because$ E = mc^2); where m is given by (7.4.2).

(5) Like a real particle, photon interacts with other particles obeying the laws of conservation of energy and momentum.

For Information Only : To say that electromagnetic radiations propagate as "waves" on one side, at the same time say that in their interaction with matter they exchange energy and momentum as discrete particles (photons) appears contradictory. Let us understand the situation in more details.

Because these cannot be understood in terms of our classical ideas regarding "waves" and "particles". These can be understood only if we accept that :

(1) Light is emitted from a source as described as photons.

(2) Detector records light as discrete photon.

(3) Propagation of light from the source to the detector can be described in terms of "probability waves".

(4) When a "photon" detector is placed in the radiation field of electromagnetic waves, the number of photons detected over the area of the detector is proportional to the square of the amplitude of electromagnetic waves, but the detector interacts with the field as discrete photons.

Illustration 3 : If the efficiency of an electric bulb of 1 watt is 10%, what is the number of photons emitted by it in one second ? The wavelength of light emitted by it is 500 nm. $h = 6.625 \times 10^{-34}$ J s

Solution : As the bulb is of 1 W, if its efficiency is 100 %, it may emit 1 J radiant energy in 1 s. But here the efficiency is 10%, hence it emits $\frac{1}{10}$ J = 10^{-1} J radiant energy in 1 s.

Note : The efficiency of bulb is 10 %. It means it emits 10% of energy consumed in form of light and remaining 90 % is wasted in form of heat energy (due to the resistance of filament.)

$\therefore$ Radiant energy obtained from the bulb in 1 s. = 10^{-1} J

If it consists of n photons,

$$nhf = 10^{-1} \text{ J}$$

$$\therefore n = \frac{10^{-1}}{hf} = \frac{0.1}{6.625 \times 10^{-34} \times \frac{c}{\lambda}} = \frac{\lambda \times 10^{-1}}{6.625 \times 10^{-34} \times 3 \times 10^{8}} \qquad (\because f = \frac{c}{\lambda})$$

$\therefore\ n = \dfrac{0.1 \times 500 \times 10^{-9}}{6.625 \times 10^{-34} \times 3 \times 10^{8}}$ (∵ velocity of light, $c = 3 \times 10^8$ m s^{-1})

$\therefore\ n = 2.53 \times 10^{17}$ photons.

Illustration 4 : 11×10^{11} photons are incident on a surface in 10 s. These photons correspond to a wavelength of 10 Å. If the surface area of the given surface is 0.01 m^2, find the intensity of given radiations. Velocity of light is 3×10^8 m s^{-1}, $h = 6.625 \times 10^{-34}$ J.s.

Solution : Number of photons incident in 10 s = 11×10^{11}

$\therefore$ Number of photons incident in 1 s = 11×10^{10}

Now, these photons being incident on area 0.01 m^2

Number of photons being incident on 1 m^2 in 1 s,

$$n = \frac{11 \times 10^{10}}{0.01} = \frac{11 \times 10^{10}}{10^{-2}} = 11 \times 10^{12}$$

Energy associated with n photons,

$$= nhf = \frac{nhc}{\lambda} = \frac{11 \times 10^{10} \times 6.6 \times 10^{-34} \times 3 \times 10^{8}}{10 \times 10^{-10}} = 2.18 \times 10^{-3}$$

$\therefore$ Intensity of incident radiation = 2.18×10^{-3} W m^{-2}

Illustration 5 : A beam of photons of intensity 2.5 W m^{-2} each of energy 10.6 eV is incident on 1.0×10^{-4} m^2 area of the surface having work function 5.2 eV. If 0.5 % of incident photons emits photo-electrons, find the number of photons emitted in 1 s. Find minimum and maximum energy of these photo electrons.

1 eV = 1.6×10^{-19} J

Solution : Here, intensity of incident radiation is 2.5 W m^{-2}.

$\therefore$ Energy incident per 1 m^2 in 1 s = 2.5 J

$\therefore$ Radiant energy incident on area 1.0×10^{-4} m^2 in 1 s = $2.5 \times 1.0 \times 10^{-4} = 2.5 \times 10^{-4}$ J

Suppose there are n number of photons in this energy.

$\therefore\ nhf = 2.5 \times 10^{-4}$ (1)

but hf = energy of photon = 10.6 eV = $10.6 \times 1.6 \times 10^{-19}$ J

(∵ 1 eV = 1.6×10^{-19} J)

Replacing it in equation (1) and making n the subject of equation,

$$n = \frac{2.5 \times 10^{-4}}{hf} = \frac{2.5 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}}$$

As 0.50 % of these photons emits photo electrons,

$$\begin{bmatrix} 100 : 0.5 \\ n\ :\ ? \end{bmatrix}$$

$\therefore$ Number of photo electrons emitted in 1 sec is,

$$N = \frac{0.50 \times n}{100} = \frac{0.5 \times 2.5 \times 10^{-4}}{100 \times 10.6 \times 1.6 \times 10^{-19}}$$

$$= 7.37 \times 10^{11}\ s^{-1}$$

The minimum energy of photo electon is = 0 J. Such photo electrons spend all the energy gained from the photon against the work function.

Maximum energy of photo electorn :

$E = hf - \phi_0 = 10.6\ eV - 5.2\ eV \quad (\because\ hf = 10.6\ eV \text{ and } \phi_0 = 5.2\ eV)$

$= 5.4\ eV$

Illustration 6 : Radius of a beam of radiation of wavelength 5000 Å is 10^{-3} m. Power of the beam is 10^{-3} W. This beam is normally incident on a metal of work function 1.9 eV. What will be the charge emitted by the metal per unit area in unit time ? Assume that each incident photon emits one electron.

$h = 6.625 \times 10^{-34}$ J s

Solution : Power of the beam of light = 10^{-3} W

∴ Amount of energy incident in unit time = 10^{-3} J

If the number of photons corresponding to this energy is n,

$$nhf = nh\frac{c}{\lambda} = 10^{-3} \Rightarrow n = \frac{10^{-3} \times \lambda}{hc}$$

$$\therefore\ n = \frac{10^{-3} \times 5000 \times 10^{-10}}{6.625 \times 10^{-34} \times 3 \times 10^{8}} \qquad (\because\ \lambda = 5000\ \text{Å} = 5000 \times 10^{-10}\ m)$$

These photons are incident on the surface of radius 10^{-3} m in one second.

∴ Number of photons incident per unit area in one second,

$$n_1 = \frac{10^{-3} \times 5000 \times 10^{-10}}{6.625 \times 10^{-34} \times 3 \times 10^{8} \times \pi \times (10^{-3})^2}$$

Each photon emits one electron and charge on electron being $e = 1.6 \times 10^{-19}$ C

∴ amount of charge emitted per unit area in unit time.

$$Q = n_1 e = \frac{10^{-3} \times 5000 \times 10^{-10} \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34} \times 3 \times 10^{8} \times 3.14 \times 10^{-6}} = 128.6\ C$$

Illustration 7 : Work function of some metals are Na : 1.92 eV, K : 2.2 eV, Cd : 4.1 eV, Ni : 5 eV. A laser beam from He-Cd of wavelength 3300 Å is incident on it. From which of the metals photo electrons will be emitted, if the distance of the source is initially 1 m from the metals. If it is brought to the distance of 10 cm will there be any change in emission ?

$h = 6.625 \times 10^{-34}$ J s. $c = 3 \times 10^{8}$ m s^{-1}, 1 eV = 1.6×10^{-19} J.

Solution : For photo-electric effect to be observed, energy of each photon should be at least equal to or more than work function of the metal.

$$\therefore\ hf = h\frac{c}{\lambda} \geq \text{work-function, } \phi_0$$

$$\text{Energy of Incident radiation} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{3300 \times 10^{-10}}\ J = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{3300 \times 10^{-10} \times 1.6 \times 10^{-19}}$$

$(\because\ 1\ eV = 1.6 \times 10^{-19}\ J)$

Energy of Incident radiation = 3.76 eV

This result shows that the metal which has the work function 3.76 eV or less, may produce photoelectric effect. In the given list of metals Na and K may produce photoelectric effect, while in Cd or Ni this effect is not observed.

While the source is brought nearer, from 1 m to 10 cm, the intensity of incident light will of course increase, but its frequency will remain same. Hence, Na and K will emit more number of photo electrons and photo electric cuurent will increase, but still photo electric effect will not be seen in Cd and Ni.

Illustration 8 : U. V. light of wavelength 200 nm is incident on polished surface of Fe. Work function of the surface is 4.5 eV. Find, (1) stopping potential (2) maximum kinetic energy of photo-electrons (3) maximum speed of photo electrons.

$h = 6.625 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m s^{-1}, 1 eV $= 1.6 \times 10^{-19}$ J.

$m = 9.11 \times 10^{-31}$ kg

Solution : $eV_0 = \frac{1}{2} mv^2_{max} = hf - \phi_0 = \frac{hc}{\lambda} - \phi_0$

First we find $\frac{hc}{\lambda}$, to calculate V_0.

$$\frac{hc}{\lambda} = \frac{6.625\times10^{-34}\times3\times10^{8}}{200\times10^{-9}} = 9.94 \times 10^{-19} \text{ J} = 6.21 \ eV$$

Now, $eV_0 = \frac{hc}{\lambda} - \phi_0 = 6.21 - 4.5$ ($\because \phi = 4.5$ eV) $= 1.71$ eV

$\therefore V_0 = 1.71$ V

Now,

$$\therefore \frac{1}{2}mv^2_{max} = eV_0 = 1.71 \ eV = (1.71)\ (1.6 \times 10^{-19}) \text{ J} = 2.74 \times 10^{-19} \text{ J}$$

$$\therefore v^2_{max} = \left(\frac{2.74\times10^{-19}\times2}{9.11\times10^{-31}}\right) = 6.0 \times 10^{11}$$

$$\therefore v_{max} = 7.75 \times 10^{5} \text{ m s}^{-1}$$

Illustration 9 : A crystal of Cu emits 8.3×10^{10} $\frac{\text{photo-electrons}}{m^2 s}$. Atomic mass of Cu is 64 g mol^{-1} and its density is 8900 kg m^{-3}. Supposing that photo electrons are emitted from first five layers of atoms of Cu, will one electron be emitted per how many (average) atoms ? Consider the crystal to be a simple cubic lattice.

Solution : As the number of photo electrons are given as photo electrons/m^{2}s, consider the cube of crystal of length 1 m. Volume of such a crystal $= 1 \times 1 \times 1 = 1$ m^3. Now density is 8900 kg m^{-3}. Hence, the mass of such crystal is 8900 kg. As the atomic mass is 64 g mol^{-1}, number of atoms in 64×10^{-3} kg of Cu will be same as Avogadro number.

64×10^{-3} kg : 6.02×10^{23}

$\therefore$ 8900 kg : number of atoms (?)

$\therefore$ Number of atoms in 8900 kg of Cu, N = $\frac{6.02\times10^{23}\times8900}{64\times10^{-3}}$ (1)

These atoms form simple cubic lattice.

If in one row there are n number of atoms, in one layer there may be n^2 number of atoms.

In 5 layers number of atoms = $5n^2$

Note that total number of atoms in the given cube is n^3

$\therefore$ N = n^3

$\therefore$. From equation (1),

$$n^3 = \frac{6.02\times10^{23}\times8900}{64\times10^{-3}}$$

$$\therefore n = \left(\frac{6.02\times10^{23}\times8900}{64\times10^{-3}}\right)^{\frac{1}{3}} = 4.37 \times 10^9$$

$\therefore$ $5n^2 = 5 \times (4.37 \times 10^9)^2 = 9.55 \times 10^{19}$

$\therefore$ 8.3×10^{10} photo electrons are emitted from 9.55×10^{19} atoms.

If 8.3×10^{10} photo electrons are emitted from $5n^2$ atoms, from how many atoms one electron is emitted ?

8.3×10^{10} : $5n^2$

1 : ? (number of atoms)

$$\therefore \frac{5n^2}{8.3\times10^{10}} = \frac{9.55\times10^{19}}{8.3\times10^{10}}$$

$\therefore$ Number of atoms emitting one photo-electrons = 1.15×10^9

Illustration 10 : Light of 4560 Å of 1 mW is incident on photo-sensitive surface of Cs (Cesium). If the quantum efficiency of the surface is 0.5 %, what is the amount of photo-electric current produced ?

Solution : Meaning of light of 1 mW is that 1 mJ = 10^{-3} J of energy is being incident on the surface in 1 s. This light is being incident in the form of photons of energy hf. If n photons are incident.

$nhf = 10^{-3}$ (1)

Out of n photons only 0.5% photons emit photo-electrons, as the quantum efficiency is 0.5%.

Now, 0.5% of n, is

$$\begin{bmatrix} 100 : 0.5 \\ n : ? \end{bmatrix}$$

$\therefore$ Number of photo-electrons = $\frac{n\times0.5}{100}$

Photo electric current is produced by these electrons is being emitted in 1 s.

Photoelectric current, I = Number of photo–electrons emitted in 1 s × charge of electron

$\therefore\ I = \frac{n \times 0.5}{100} \times 1.6 \times 10^{-19}$ A (2)

But from equation (1),

$n = \frac{10^{-3}}{hf} = \frac{10^{-3}}{6.625 \times 10^{-34} \times \frac{c}{\lambda}}$ $\left(\because f = \frac{c}{\lambda}\right)$

$\therefore\ n = \frac{10^{-3} \times 4560 \times 10^{-10}}{6.625 \times 10^{-34} \times 3 \times 10^{8}} = 2.303 \times 10^{15}$

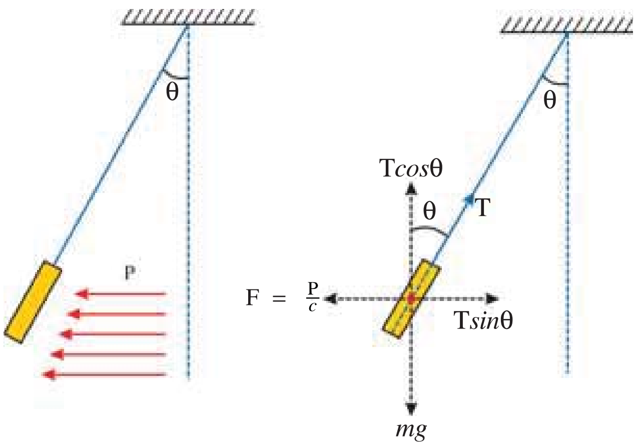
Replacing the value of n in equation (2),

$I = \frac{2.303 \times 10^{15} \times 0.5 \times 1.6 \times 10^{-19}}{100}$

$\therefore\ I = 1.84 \times 10^{-6}$ A = 1.84 μA

Illustration 11(a) : As shwon in the figure, light of energy P(joule) is incident on a small, flat strip of metal of mass m, suspended with the help of weightless string of length l in 1 s. All the energy incident on it is absorbed and the strip remains in equilibrium at an angle θ with respect to vertical. If the light is monochromatic, find angle θ.

Solution : When electromagnetic radiations are incident on a surface, force is produced due to pressure. Here, P joule of energy is incident in 1 s. If this radiation is made of photons and n photons are incident in 1 s,



$nhf = P$ (1)

Now, momentum of each photon, $p = \frac{hf}{c}$ (2)

Replacing the value of hf from (1) in (2),

$p = \frac{P}{nc}$

$\therefore$ momentum of n photons $= np = \frac{P}{c}$

The strip gains this much momentum every second.

$\therefore$ Rate of change of momentum $= \frac{P}{c}$ = Force

$\therefore\ F = \frac{P}{c}$ (3)

This force is shown in the Figure.

As the strip is in equilibrium, equating their vertical and horizontal components,

$\left.\begin{aligned} T\cos\theta &= mg \\ \text{and } T\sin\theta &= \frac{P}{c} \end{aligned}\right\}\ \therefore\ \tan\theta = \frac{P}{cmg} \Rightarrow \theta = \tan^{-1}\left(\frac{P}{cmg}\right)$

Illustration 11(b) : If the strip is slightly displaced from its state of equilibrium, find the period of it's simple harmonic oscillations.

Solution : Here, effective gravitational acceleration = $\vec{g_e} = \frac{\vec{P}}{mc} + \vec{g}$

$$\therefore \ |\vec{g_e}| = \sqrt{\left(\frac{P}{mc}\right)^2 + g^2}$$

$$\text{Now, } T = 2\pi\sqrt{\frac{l}{g_e}} = \sqrt{\frac{l}{\sqrt{\left(\frac{P}{mc}\right)^2 + g^2}}}$$

$$\therefore \ T = 2\pi\left[\frac{l}{\left\{\left(\frac{P}{mc}\right)^2 + g^2\right\}^{\frac{1}{2}}}\right]^{\frac{1}{2}}$$

7.5 Photocell

A Photocell (which is also known as electric eye) is a technological application of the photoelectric effect. In some photocells single layer of photosensitive material is used. A schematic diagram of a typical photocell is shown in the figure 7.5.

The wall of the photocell is made of glass or quartz. When the light (of suitable frequency) is incident on the photosensitive surface, a photocurrent of few micro ampere is normally obtained. When intensity of incident light is changed the photo electric current also changes. Using this property of photocell, control systems are operated and the intensity of light can be measured.

They are used in light meters, photographic camera, electric bell, burglar alarm, fire alarm. In astronomy, they are used to study the spectra of stars and their temperatures.

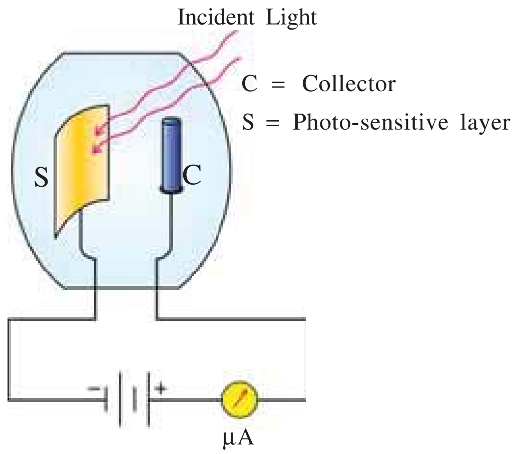


Figure 7.5 A Photocell

7.6 Matter Waves - Wave Nature of Particles

The photoelectric and compton effect have confirmed that light behaves as a collection of particles and not as a wave. At the same time, we also know that the phenomena of diffraction, interference and polarization can be understood only when light behaves as wave. This is a paradox of the existence of two quite different (the wave and the particle) nature of the same physical quantity (light). One possibility is to suppose that light propagates in the form of wave but assumes particle character at the instant of absorption or emission (i.e. during the interaction with matter). This explanation suggests that radiation shows dual nature; (continuous) wave-like extended and (discrete) quantized particle behaviour under the suitable conditions.

According to the theory of relativity, Lorentz transformation for a change of reference frame requires that relation like between E and f must necessarily hold for momentum (p) and wave-vector (k). Since for photon rest mass (m_0) is zero, its momentum is given by (see equation 7.4.4),

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \quad (\because\ c = f\lambda) \tag{7.6.1}$$

$$\text{or } \lambda = \frac{h}{p} \tag{7.6.2}$$

Based on this requirement, in 1924, Louis de Broglie arugued that if light (which consists of waves according to classical mechanics) can sometimes behave like particles. Then it should be possible for matter (which consists of particles according to classical picture) to exhibit wave-like behaviour under favourable circumstances. **"Nature should be symmetric with respect to radiation and particles."** The dual nature of radiation and particle must be a part of some general law of nature. That is, radiation and matter both show dual nature : particle and wave.

Thus, according to de Broglie, equation (7.6.2) is also true for material particles. For a particle with mass m and moving with a speed v (i.e., momentum, $p = mv$), when showing wave nature, corresponding wavelength can be found by using equation (7.6.2), as

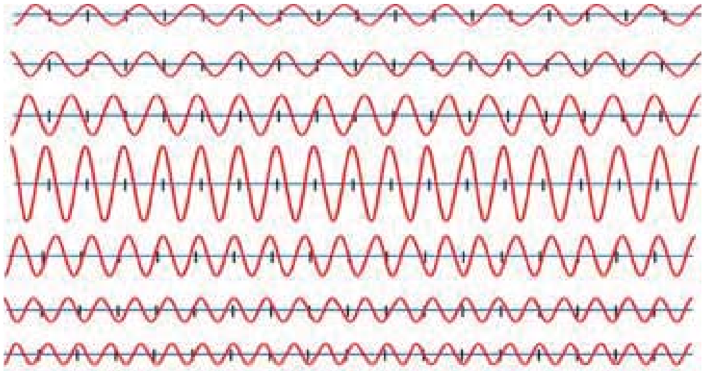
$$\lambda = \frac{h}{mv} \tag{7.6.3}$$

This wavelength is known as de Broglie wavelength of the particle. We must remember that it is not that any kind of wave is attached to the matter particle. Under some circumstances, the behaviour of the particle can be explained by its wave nature.

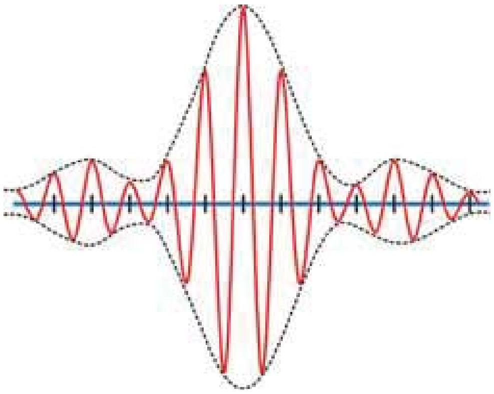
Actually, the concept of matter particle as a wave was well supported by Erwin Schroedinger (1926) through his differential wave equation. He showed that this wave equation (for matter waves associated to particles) together with some physically-required conditions leads to quantized (discrete) nature of various physical quantities which supports the wave nature of particles. While the experimental evidences for matter as a wave were due to Davisson-Germer experiment, (which we will study in the next section), Kikuchi's diffraction experiment and Thomson's experiment showing associated de Broglie waves of electrons.

However, the most serious problem raised by the discovery of the wave nature of matter concerns the very basic definition of a 'particle'. Classically, particle means a point-like object endowed with a precise position and momentum. The de Broglie's hypothesis, which also supports wave-like (i.e. an extended in space) behaviour of matter, questions about how to measure accurately position and momentum of a material particle.

A pure harmonic wave extending in space obviously cannot represent point-like particle. This suggests that the wave activity of a wave representing a particle must be limited to (or nearby to) the space occupied by the particle. For this reason an idea of wave packet (i.e. a wave which is confined to a small region of space) is introduced.



(a) Harmonic Waves with Slightly Differing Wave Lengths.



(b) Amplitude Variation Due to Superposition of Harmonic Waves.

Figure 7.6 Construction of Wave Packat

We know that when many harmonic waves with slightly varying wave lengths are superposed (Hey, don't forget superposition principle), non-zero displacement of resulting wave is limited to small part of the space (See figure 7.6). In this sense, it would seem reasonable to suppose that the particle is within the region of the packet. Further, the probability of finding the particle is more in a region in which the displacement of the resulting wave is greater. If we use a single harmonic wave to represent a particle, the probability of finding a particle anywhere from $-\infty$ to $+\infty$ is equal. (This is because amplitude of a harmonic wave is finite and equal everywhere.) In other words, the position of the particle becomes totally uncertain. But, since the harmonic wave has unique wave length (λ), according to equation (7.6.3), its momentum is unique and certain.

If the concept of wave-packet (a group of superimposing waves of different wave lengths) is used to represent particle, position of the particle is more certain and is proportional to the size of the wave-packet. But as several waves of different wave lengths are used to represent a particle, its momentum is no longer unique and becomes uncertain.

Thus, the fundamental dual nature of radiation and particle introduces uncertainty in the simultaneous measurement of physical quantities.

Heisenberg's Uncertainty Principle : According to Heisenberg's uncertainty principle, if the uncertainty in the x-coordinate of the position of a particle is Δx and uncertainty in the x-component of its momentum is Δp (i.e. in one dimension) then

$$\therefore \; \Delta x \cdot \Delta p \geq \frac{h}{2\pi} \geq \hbar \text{ (Read as } h \text{ cut or } h \text{ cross).} \qquad (7.6.4)$$

Now, if $\Delta x \to 0$ then $\Delta p \to \infty$

and $\Delta p \to 0$ then $\Delta x \to \infty$

Similarly, one finds uncertainty principle associated in measuring energy of a particle and time as,

$$\therefore \; \Delta E \cdot \Delta t \geq \hbar \qquad (7.6.5)$$

Only for Information : We discussed about the probability of the particle to be at a definite point. In fact the wave functions representing a particle can be mathematically obtained in the form of solutions of typical differential equations (Schroedinger's equation). These wave functions may be real or complex according to the situation. According to Max Born, the probability of finding a particle at any point in the space in one dimension is proportional to the square of the magnitude of such a wave function ($|\psi|^2 = \psi^*\psi$). Hence, we have to deal with such probabilities while discussing about microscopic particles. This branch of physics is called **wave mechanics**.

You might have noted that the approach of physics based on quantum mechanics is not deterministic like classical physics.

So, for a microscopic particle like an electron, it is meaningless to question whether it is a particle or a wave. Actually it is neither a wave nor a particle. It is more fundamental physical reality whose behaviour can be understood with particle mechanics in some situation and with wave mechanics in the other. The mathematical studies developed in reference to the wave and particle nature are merely two disciplines to understand the nature.

Noted writer Margenau compares the question : "wave or particle ?" with the question "what is the colour of an egg of an elephant ?" This question is meaningful only if an egg of an elephant exists !

Illustration 12 : Find the certainty with which one can locate the position of (1) a bullet of mass 25 g and (2) an electron, both moving with a speed 500 m/s, accurate to 0.01 %. Also, draw inferences based on your results. Mass of an electron is 9.1×10^{-31} kg.

Solution : (1) Uncertainty in measurement of momentum of a bullet is 0.01% of its exact value. i.e., Δp = 0.01% of mv.

$$= \left(\frac{0.01}{100}\right) \times (25 \times 10^{-3}) \times (500)$$

$$= 1.25 \times 10^{-3} \text{ kg m s}^{-1}$$

Therefore, corresponding uncertainty in the determination of position is

$$\therefore \Delta x = \frac{\hbar}{\Delta p} \text{ (using equation 7.5.4)}$$

$$= \frac{6.625 \times 10^{-34}}{2 \times 3.14 \times (1.25) \times 10^{-3}} \times 10^{-3} \qquad (\because \hbar = \frac{h}{2\pi})$$

$$= 8.44 \times 10^{-32} \; m.$$

Conclusion : The value of Δx is too small compared to the dimension of the bullet, and can be neglected. That is, position of the bullet is determined accurately.

(2) Uncertainty in measurement of momentum of an electron is

$$\therefore \Delta p = \left(\frac{0.01}{100}\right) \times (9.1 \times 10^{-31}) \times (500) = 4.55 \times 10^{-32} \text{ kgms}^{-1}$$

Corresponding uncertainty in position is

$$\Delta x = \frac{6.625 \times 10^{-34}}{2 \times 3.14 \times 4.55 \times 10^{-32}} = 0.23 \times 10^{-2} \text{ m} = 2.3 \text{ mm}$$

Conclusion : Uncertainty in position for an electron (2.3 mm) is too large compared to the dimension of an electron, when it is assumed to be as a particle. Consequently, the concept of an electron as a tiny particle does not hold.

7.7 Davisson-Germer Experiment

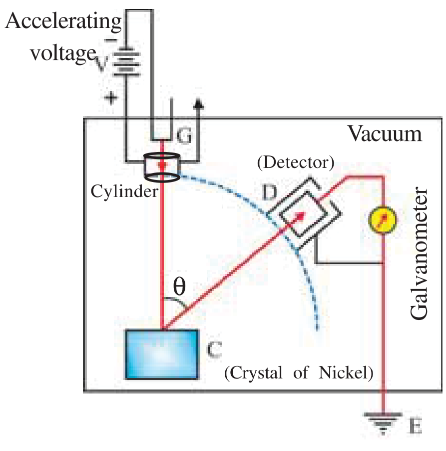


Figure 7.7 Arrangement for Davisson-Germer Experiment

Till 1927, De Broglie's hypothesis did not get any experimental confirmation. In 1927, two scientists named Davisson and Germer performed series of experiments at Bell laboratory to study scattering of electron by a piece of Nickle placed in vacuum.

The device used by them is shown in figure 7.7.

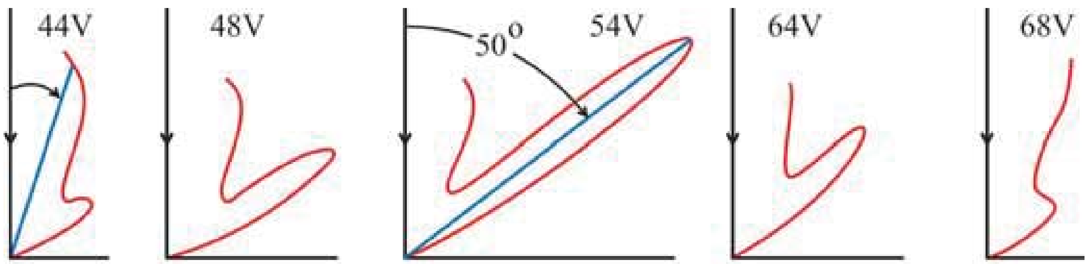
Here, G is the electron gun having tungsten filament coated with barium oxide. Filament is heated with L. T. (Low Tension = low p.d.). Hence, it emits electrons.

Now, these electrons can be accelerated under appropriate electric field produced by H. T. (High Tension). These electrons pass through a cylinder having a small hole and form a thin beam of electrons which is incident on a piece of Nickel and get scattered by it (in fact by its atoms). To detect the electrons scattered in different directions a detector D is arranged which can be moved on a circular scale as shown in figure 7.7. The output current from this detector passes through a galvanometer. The amount of current represents the number of electrons scattered in that direction.

According to classical physics, number of electrons scattered in different directions does not depend much on the angle of scattering. Also, this number hardly depends on the energy of incident electrons. Davisson and Germer tested these predictions of classical physics using the piece of Nickel as the scatterer.

During one of their experiments the bottle filled with liquefied air burst and the surface of the piece of Nickel was damaged. They heated the piece of Nickel to a high temperature and then cooled it to level its surface. Again when the experiment was repeated they found "something unusual". They found that the results of diffraction of electrons by Nickel are similar to the diffraction of X-rays by a crystal. This can happen only if electrons act as waves. This happened because when the piece of Nickel was heated and then cooled it was converted into a single crystal.

In this experiment the intensity of electron beam scattered at different angles of scattering, can be measured for the given accelerating voltage. Angle of scattering (θ) is the angle between the incident beam and scattered beam of electrons. The graphs of intensity $\rightarrow \theta$ for the observations taken by Davisson and Germer between 44 V to 68 V are shown qualitatively in figure 7.8.



(a) (b) (c) (d) (e)

Figure 7.8 Results for Davisson Germer Experiment

The graphs indicate that the number of electrons scattered at a specific angle of scattering is maximum for the given accelerating voltage. See the graph of 54 V carefully. Here, the number of electrons scattered at an angle of 50^o is found to be maximum. These experimental results can be understood if the electrons are considered as the waves having de Broglie wavelength and if we accept that electrons are scattered just as X-rays by a crystal. The interatomic distance of Nickel is known. With this information and using the equation of scattering wavelength of electron can be obtained experimentally.

If the accelerating voltage is V and charge of an electron is e, energy of electron is

$$\frac{1}{2}mv^2 = eV$$

$$\therefore \; m^2v^2 = 2meV$$

$$\therefore \; mv = \sqrt{2meV}$$

But wavelength, $\lambda = \frac{h}{mv}$

$$\therefore \; \lambda = \frac{h}{\sqrt{2meV}} \qquad (7.7.1)$$

In above equation substituting V = 54 V, $h = 6.625 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C, we get $\lambda = 1.66 \times 10^{-10}$ m. The value of λ obtained in the experiment was 1.65×10^{-10} m. **Thus, accidentally it was proved that an electron behaves as wave also.**

For Information Only : The development of quantum physics is very interesting. This is the magnificent knowledge of mankind struggling to know the nature. Not only that but it is the confluence of rivers like science, mathematics and philosophy. Diving in it we realize how magnificent is the nature !

Illustration 13 : Suppose you are late in reaching the school, and you are going at the speed of 3.0 m s^{-1}. If your mass is 60 kg. assuming that you are a particle find your de Broglie wavelength $h = 6.625 \times 10^{-34}$ J S.

Solution : $p = mv = 60 \times 3.0 = 1.8 \times 10^2$ kg m s^{-1}

Now, $\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{1.8 \times 10^2} = 3.68 \times 10^{-36}$ m

Note : This wavelength is even smaller than the radius of the nucleus ($\sim 10^{-15}$ m) by 10^{-21} times. If you want to make your wave properties "regular", your mass should be reduced to unimaginable level.

Illustration 14 : A proton falls freely under gravity of Earth. What will be its de Broglie wavelength after 10 s of its motion ? Neglect the forces other than gravitational force.

$g = 10$ m s^{-2}, $m_p = 1.67 \times 10^{-27}$ kg, $h = 6.625 \times 10^{-34}$ J s

Solution : From $v = v_0 + gt$,

$v = gt$

$\therefore$ momentum, $p = m_p v = m_p gt$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{m_p gt}$$

$$\therefore \lambda = \frac{6.625 \times 10^{-34}}{1.67 \times 10^{-27} \times 10 \times 10}$$

$$\therefore \lambda = 3.96 \times 10^{-9} \text{ m} = 39.6 \text{ Å}$$

Illustration 15 : An electron is at a distance of 10 m from a charge of 10 C. Its total energy is 15.6×10^{-10} J. Find its de Broglie wavelength at this point.

$h = 6.625 \times 10^{-34}$ J s; $m_e = 9.1 \times 10^{-31}$ kg; $k = 9 \times 10^9$ SI,

$e = 1.6 \times 10^{-19}$ C

Solution : Potential energy of an electron, $U = -k\frac{(q)(e)}{r}$

$$\therefore U = -\frac{9 \times 10^9 \times 10 \times 1.6 \times 10^{-19}}{10}$$

$$\therefore U = -14.4 \times 10^{-10} \text{ J} \quad (1)$$

Now total energy E = Kinetic energy K + Potential energy U

$\therefore$ K = E − U

$= 15.6 \times 10^{-10} + 14.4 \times 10^{-10}$

$\therefore$ K = 30 × 10^{-10} J

But, $K = \frac{p^2}{2m_e}$

$\therefore p = \sqrt{2Km_e}$

$$\lambda = \frac{h}{\sqrt{2Km_e}} = \frac{6.625\times10^{-34}}{\sqrt{2\times30\times10^{-10}\times9.1\times10^{-31}}}$$

$\therefore \lambda = 8.97 \times 10^{-15}$ m

Illustration 16 : Compare energy of a photon of X-rays having 1 Å, wavelength with the energy of an electron having same de Broglie wavelength. $h = 6.625 \times 10^{-34}$ J s; $c = 3 \times 10^8$ m s^{-1}; $m_e = 9.1 \times 10^{-31}$ kg

Solution : For photon,

Energy, $E_p = hf = \frac{hc}{\lambda}$ $\lambda = 1$ Å $= 10^{-10}$ m

$$\therefore E_p = \frac{6.625\times10^{-34}\times 3\times10^{8}}{10^{-10}} = 19.87 \times 10^{-16} \text{ J}$$

For an electron;

Energy, $E_e = \frac{p^2}{2m}$

According to de Broglie relation, $p = \frac{h}{\lambda}$

$$\therefore E_e = \frac{h^2}{\lambda^2(2m)} = \frac{(6.625\times10^{-34})^2}{(10^{-10})^2\times2\times9.1\times10^{-31}} = 2.41 \times 10^{-17} \text{ J}$$

$$\therefore \frac{E_p}{E_e} = \frac{19.87\times10^{-16}}{2.41\times10^{-17}}$$

$$\therefore \frac{E_p}{E_e} = 82.4$$

Thus, energy of photon is 82.4 times the energy of electron having same wavelength.

Illustration 17 : Wavelength of an electron having energy E is $\lambda_0 = \frac{h}{\sqrt{2mE}}$, where m is the mass of an electron. Find the wavelength of the electron when it enters in X-direction in the region having potential V(x). If we imagine that due to the potential, electron enters from one medium to another, what is the refractive index of the medium ?

Solution : Energy of electron in the region having potential

E = (Kinetic energy)K + (Potential energy)U

$\therefore E = \frac{p^2}{2m} - eV(x)$

$$\therefore \; p = [2m(E + eV(x))]^{\frac{1}{2}}$$

$$\therefore \; \lambda = \frac{h}{p} = \frac{h}{[2m(E + eV(x))]^{\frac{1}{2}}}$$

Now, refractive index $= \dfrac{\lambda_0}{\lambda} = \dfrac{[2m(E + eV(x))]^{\frac{1}{2}}}{(2mE)^{\frac{1}{2}}}$ $\left(\because \; \lambda_0 = \dfrac{h}{\sqrt{2mE}}\right)$

$$\therefore \text{ Refractive index } = \left[\frac{E + eV(x)}{E}\right]^{\frac{1}{2}}$$

Illustration 18 : Consider the radius of a nucleus to be 10^{-15} m. If an electron is assumed to be in such nucleus, what will be its Energy ?

Mass of electron = 9.1×10^{-31} kg; $h = 6.625 \times 10^{-34}$ J s

Solution : As the electron acts as a wave in this situation, the maximum uncertainty in its position.

$\therefore \; \Delta x = 2r = 2 \times 10^{-15}$ m r = radius of the nucleus = 10^{-15} m

Now, according to Heisenberg's principle

$$\therefore \; \Delta x \cdot \Delta p \approx \frac{h}{2\pi}$$

$$\therefore \; \Delta p \approx \frac{h}{2\pi \Delta x} = \frac{6.625 \times 10^{-34}}{2 \times 3.14 \times 2 \times 10^{-15}} = 0.5274 \times 10^{-19}$$

Now, if this uncertainty is (approximately) taken as the momentum ($p \approx \Delta p$), energy of electron

$$E = \frac{p^2}{2m}$$

$$= \frac{(0.5274 \times 10^{-19})^2}{2 \times 9.1 \times 10^{-31}} \text{ J} = \frac{(0.5274 \times 10^{-19})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}} \text{ eV} = 9.55 \times 10^3 \text{ MeV}$$

Now, the binding energy of a nucleus is several MeV. As compared to it the energy of an electron in the nucleus is very large. Hence, electron can not reside in a nucleus.

Illustration 19 : Find the wave packet formed due to the superposition of two harmonic waves represented by $y_1 = A \sin(\omega t - kx)$ and $y_2 = A \sin[(\omega + d\omega)t - (k + dk)x]$

Solution : According to the principle of superposition,

$$y = y_1 + y_2$$

$$= A \sin(\omega t - kx) + A \sin[(\omega + d\omega)t - (k + dk)x]$$

Using the relation $\sin A + \sin B = 2 \sin\left(\dfrac{A+B}{2}\right) \cdot \cos\left(\dfrac{A-B}{2}\right)$

$$y = 2A \cos\left(\frac{xdk - td\omega}{2}\right) \cdot \sin\left[(\omega t - kx) + \left(\frac{td\omega - xdk}{2}\right)\right]$$

As amplitude of the wave packet = $2A \cos\left(\dfrac{xdk - td\omega}{2}\right)$, it depends both on the position and time.

SUMMARY

1. Difficulties by the classical theoretical explanation of certain experimental observations like, energy distribution in black-body radiation, stability of an electrically neutral atom and its spectra, specific heats of solids and diatomic molecules at low temperatures, etc., have forced scientists to think totally differently.
2. Planck with his revolutionary idea that energy of microscopic oscillating dipoles is quantized to hf. And total energy is always an integral multiple of the smallest quantum of energy (hf), the photon. Here, h is known as the Planck's constant. The photon possesses all the properties of a material particle.
3. Planck's hypothesis could solve black-body radiation problem successfully.
4. To bring an electron out of the metal, some minimum amount of energy must be supplied to an electron, which is known as work function of the metal. The work function depends on the type of metal, nature of its surface and its temperature.
5. Corresponding to work function minimum frequency required to eject photoelectron is known as the threshold frequency.
6. Dependence of photoelectric current on the intensity of incident light, value of maximum kinetic energy of an emitted photoelectron on frequency of incident light and not on its intensity, instantaneous (within 10^{-9} sec) emission of photoelectrons cannot be explained by the wave nature of light.
7. Assuming light as a particle, Einstein could solve the mystery of the photoelectric effect. His photoelectric equation $\frac{1}{2}mv^2{}_{max} = eV_0 = hf - \phi_0$ is in accordance with the energy conservation law.
8. Photoelectric effect and Compton effect have confirmed the dual nature of radiation.
9. On the symmetry argument, de Broglie had further proposed dual nature for material particles. Which was supported by experimental observations (e.g., Davisson-Germer experiment) as well as by theoretical calculations (e.g., Schroedinger wave equation).
10. This confirms the dual (particle and wave) nature for both radiation and matter particles.
11. The non-zero value of Planck's constant (h) alongwith Heisenberg's uncertainty principle measures the inadequacy of the classical mechanics.

EXERCISES

For the following statements choose the correct option from the given options :

1. Cathode rays ....
   (A) are the atoms moving towards the cathod.
   (B) are electromagnetic waves.
   (C) are negative ions travelling from cathode to anode.
   (D) are electrons emitted by cathode and travelling towards anode.
2. Which of the following statement is not true for a photon ?
   (A) Photon produces pressure (B) Photon has energy hf.
   (C) Photon has momentum $\frac{hf}{c}$ (D) Rest mass of photon is zero
3. The velocity of photon emitted in photo-electric effect depends on the properties of photosensitive surface and..........
   (A) frequency of incident light (B) state of polarization of incident light
   (C) time for which the light is incident (D) intensity of incident light
4. Photoelectric effect represents that
   (A) electron has a wave nature (B) light has a particle nature
   (C) (1) and (2) both (D) none of the above

5. De Broglie wavelength of a particle moving with velocity 2.25×10^8 m s^{-1} is same as the wavelength of photon. The ratio of kinetic energy of the particle to the energy of photon is..........

Velocity of light = 3×10^8 m s^{-1}

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{7}{8}$

6. Energy of photon is $E = hf$ and its momentum is $p = \frac{h}{\lambda}$, where λ is the wavelength of photon. With this assumption speed of light wave is ....

(A) $\frac{p}{E}$ (B) $\frac{E}{p}$ (C) Ep (D) $\left(\frac{E}{p}\right)^2$

7. Wavelength λ_A and λ_B are incident on two identical metal plates and photo electrons are emitted. If $\lambda_A = 2\lambda_B$, the maximum kinetic energy of photo electrons is ....

(A) $2K_A = K_B$ (B) $K_A < \frac{K_B}{2}$ (C) $K_A = 2K_B$ (D) $K_A > \frac{K_B}{2}$

8. Cathode rays travelling in the direction from east to west enter in an electric field directed from north to south. They will deflect in ....

(A) east (B) west (C) south (D) north

9. If photoelectric effect is not seen with the ultraviolet radiations in a given metal, photo electrons may be emitted with the ....

(A) infrared waves (B) radio waves (C) X-rays (D) visible light

10. Photons of energy 1 eV and 2.5 eV successively illuminate a metal whose work function is 0.5 eV, The ratio of maximum speed of emitted electron is ....

(A) 1 : 2 (B) 2 : 1 (C) 3 : 1 (D) 1 : 3

11. When frequencies f_1 and f_2 are incident on two identical photo sensitive surfaces, maximum velocities of photo electrons of mass m are v_1 and v_2, hence ....

(A) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$ (B) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

(C) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$ (D) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

12. A proton and an α-particle are passed through same potential difference. If their initial velocity is zero, the ratio of their de Broglie's wavelength after getting accelerated is.\

(A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) $2\sqrt{2}$: 1

13. Mass of photon in motion is ....

(A) $\frac{c}{hf}$ (B) $\frac{h}{\lambda}$ (C) hf (D) $\frac{hf}{c^2}$

14. Wavelength of an electron having energy 10 keV is .... $\mathring{A}$.

(A) 0.12 (B) 1.2 (C) 12 (D) 120

15. If the momentum of an electron is required to be same as that of wave of 5200 $\mathring{A}$ wavelength, its velocity should be .... m s^{-1}.

(A) 10^3 (B) 1.2×10^3 (C) 1.4×10^3 (D) 2.8×10^3

16. The uncertainty in position of a particle is same as it's de Broglie wavelength, uncertainty in its momentum is ....

(A) $\frac{\hbar}{\lambda}$ (B) $\frac{2\hbar}{3\lambda}$ (C) $\frac{\lambda}{\hbar}$ (D) $\frac{3\lambda}{2\hbar}$

17. A proton and electron are lying in a box having unpenetrable walls, the ratio of uncertainty in their velocities are .... [m_e = mass of electron and m_p = mass of proton.]

(A) $\frac{m_e}{m_p}$ (B) $m_e \cdot m_p$ (C) $\sqrt{m_e \cdot m_p}$ (D) $\sqrt{\frac{m_e}{m_p}}$

18. When α-particles are accelerated under the p.d. of V volt, their de Broglie's wavelength is .... Å [Mass of α-particle is 6.4×10^{-27} kg and its charge is 3.2×10^{-19} C.]

(A) $\frac{0.287}{\sqrt{V}}$ (B) $\frac{12.27}{\sqrt{V}}$ (C) $\frac{0.103}{\sqrt{V}}$ (D) $\frac{1.22}{\sqrt{V}}$

19. De Broglie wavelength of a proton and α-particle is same, ... physical quantity should be same for both.

(A) velocity (B) energy (C) frequency (D) momentum

20. To reduce de Broglie wavelength of an electron from 10^{-10} m to 0.5×10^{-10} m, its energy should be ....

(A) increased to 4 times (B) doubled
(C) halved (D) decreased to fourth part

21. The de-Broglie wavelength of a proton and α–particle is same. The ratio of their velocities will be ....

[α–particle is the He-nucleus, having two protons and two neutrons. Thus, its mass $m_\alpha \approx 4m_p$; where m_p is the mass of the proton.]

(A) 1 : 4 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1

22. The de-Broglie wavelength associated with a particle with rest mass m_0 and moving with speed of light in vacuum is ....

(A) $\frac{h}{m_0 c}$ (B) 0 (C) ∞ (D) $\frac{m_0 c}{h}$

23. An image of sun is formed by convex lens of focal length 40 cm on the metal surface of a photoelectric cell, and a photoelectric current I is produced. If now another lens with half the focal length but with same diameter is used to focus the sun image, on the photoelectric cell, photoelectric current becomes ....

(A) $\frac{I}{4}$ (B) 2 I (C) I (D) $\frac{I}{2}$

24. In quantum mechanics, a particle ....

(A) can be regarded as a group of harmonic waves.
(B) can be regarded as a single wave of definite wave-length only
(C) can be regarded as only a pair of two harmonic waves
(D) is a point-like object with mass.

25. Which of the following physical quantity has the dimension of planck constant (h) ?

(A) Force (B) Angular momentum
(C) Energy (D) Power

ANSWERS

1. (D)	2. (A)	3. (A)	4. (B)	5. (B)	6. (B)
7. (B)	8. (D)	9. (C)	10. (A)	11. (A)	12. (D)
13. (D)	14. (A)	15. (C)	16. (A)	17. (A)	18. (C)
19. (D)	20. (A)	21. (D)	22. (B)	23. (C)	24. (A)
25. (B)					

Answer the following questions in brief :

1. What is photon ?
2. What is ultraviolet catastrophe ?
3. Write Planck's hypothesis to explain energy distribution for cavity radiation.
4. Write Planck's revolutionary idea to explain energy distribution for cavity radiation.
5. Define work function of metal.
6. On which factors work function of metal depends ?
7. What is thermionic emission ?
8. Define field emission.
9. Give definition of photoelectric emission.
10. What is threshold frequency ? On which factor does threshold frequency depend ?
11. What is stopping potential ?
12. Which physical quantity can be infered from the knowledge of shopping potential ?
13. On what factor does the stopping potential depend ?
14. Write de Broglie hypothesis.
15. Define wave packet.
16. State Heisenberg's Uncertainty principle.
17. Write the conclusion of Davisson-Germer's experiment.
18. If the threshold wave length of Na element is 6800 $\overset{o}{A}$, find its work function in eV.
19. Calculate the energy of photon in eV for a radiation of wavelength 5000 $\overset{o}{A}$?

Answer the following questions :

1. Write the characteristics of photoelectric emission.
2. How wave theory fails to explain the experimental results of photoelectric effect ?
3. Explain Einstein's explanation for photoelectric effect.
4. Write the properties of a photon.
5. Write a short note on photo cell.
6. Explain the experimental arrangement of Davisson-Germer experiment.
7. Explain the conclusions of Davisson-Germer experiment.
8. Calculate the maximum kinetic energy (eV) of a photo electron for a radiation of wave length 4000 $\overset{o}{A}$ incident on a surface of metal having work function 2 eV ?
9. A light beam of 6000 $\overset{o}{A}$ wavelength and 39.6 w/m^2 intensity is incident on a metal surface. If 1 % photon of the incident photon comitts the photo electron, calculate the number of photo electron emitted per second ?

Solve the following examples :

1. A small piece of Cs (work function = 1.9 *e*V) is placed 22 cm away from a large metal plate. The surface charge density on the metal plate is 1.21×10^{-9} C m^{-2}. Now, light of 460 nm wavelength is incident on the piece of Cs. Find the maximum and minimum energies of photo electrons on reaching the plate. Assume that no change occurs in electric field produced by the plate due to the piece of Cs.

   [**Ans. :** Minimum energy = 29.83 *e*V, Miximum energy = 30.63 *e*V)

2. Threshold wavelength of tungsten is 2.73×10^{-5} cm. Ultraviolet light of wavelength 1.80×10^{-5} cm is incident on it. Find, (1) threshold frequency, (2) work function (3) maximum kinetic energy (in joule and *e*V) (4) stopping potential and (5) maximum and minimum velocity of an electron.

   [**Ans. :** (1) $f_0 = 1.098 \times 10^{15}$ Hz $\approx 1.1 \times 10^{15}$ Hz, (2) $\phi = 4.54$ eV, (3) $K_{max} = 3.76 \times 10^{-19}$ J = 2.35 eV, (4) $V_0 = 2.35$ V, (5) $v_{max} = 9.09 \times 10^5$ m $s^{-1} \approx 9.1 \times 10^5$ m s^{-1}, $V_{min} = 0$ m s^{-1})

3. Wavelength of light incident on a photo-sensitive surface is reduced from 3500 Å to 290 nm. Find the change in stopping potential $h = 6.625 \times 10^{-34}$ J s. [**Ans. :** 73.42×10^{-2} V]

4. An electric bulb of 100 W converts 3% of electrical energy into light energy. If the wavelength of light emitted is 6625 Å, find the number of photons emitted is 1 s. $h = 6.625 \times 10^{-34}$ J s. [**Ans. :** 10^{19}]

5. When a radiation of wavelength 3000 Å is incident on a metal, stopping potential is found to be 1.85 V and on making radiation of 4000 Å incident on it the stopping potential is found to be 0.82 V. Find (1) Planck's constant (2) Work function of the metal (3) Threshold wavelength of the metal.. [**Ans. :** (1) $h = 6.59 \times 10^{-34}$ J s (2) $\phi_0 = 2.268$ eV (3) $\lambda_0 = 5440$ Å.]

6. Work function of Zn is 3.74 *e*V. If the sphere of Zn is illuminated by the X-rays of wavelength 12 Å, find the maximum potential produced on the sphere.

   $h = 6.25 \times 10^{-34}$ J s. [**Ans. :** 1032.2 V]

7. Find the energy of photon in each of the following :

   (1) Microwaves of wavelength 1.5 cm (2) Red light of wavelength 660 nm

   (3) Radiowaves of frequency 96 MHz (4) X–rays of wavelength 0.17 nm

   [**Ans. :** (1) 8.3×10^{-5} *e*V (2) 1.9 *e*V (3) 4×10^{-7} *e*V (4) 7.3 *ke*V]

8. Human eye can experience minimum 19 photons per second. Light of 560 nm wavelength is required for it. What is the minimum power necessary to excite optic nerves ?

   [**Ans. :** 67.4×10^{-19} W]

9. Power produced by a star is 4×10^{28} W. If the average wavelength of the emitted radiations is considered to be 4500 Å, find the number of photons emitted in 1 s.

[**Ans. :** 9.054×10^{46} photons/s]

10. What should be the ratio of de Broglie wavelenghts of an atom of nitrogen gas at 300 K and 1000 K. Mass of nitrogen atom is 4.7×10^{-26} kg and it is at 1 atm pressure. Consider it as an ideal gas. [**Ans. :** 1.826]

11. Monochromatic light of wavelength 3000 Å is incident normally on a surface of area 4 cm^2. If the intensity of light is 150 $\frac{mW}{m^2}$, find the number of photons being incident on this surface in one second. [**Ans. :** 9.05×10^{13} s^{-1}]

12. A star which can be seen with naked eye from Earth has intensity 1.6×10^{-9} W m^{-2} on Earth. If the corresponding wavelength is 560 nm, and the diameter of the lens of human eye is 2.5×10^{-3} m, find the number of photons entering in our eye in 1 s.

[**Ans. :** 9×10^4 photons/s]

13. Find the velocity at which mass of a proton becomes 1.1 times its rest mass, $m_p = 1.6 \times 10^{-27}$ kg. Also, calculate corresponding temperature. For simplicity, consider a proton as non-interacting ideal-gas particle at 1 atm pressure.

[$c = 3 \times 10^8$ ms^{-1}, $k_B = 1.38 \times 10^{-23}$ SI] [**Ans. :** $v = 0.42$ C, 6.75×10^{11} K]

14. Output power of He-Ne LASER of low energy is 1.00 mW. Wavelength of the light is 632.8 nm. What will be the number of photons emitted per second from this LASER ? $h = 6.25 \times 10^{-34}$ J s. [**Ans. :** 3.18×10^{15} s^{-1}]

• • •

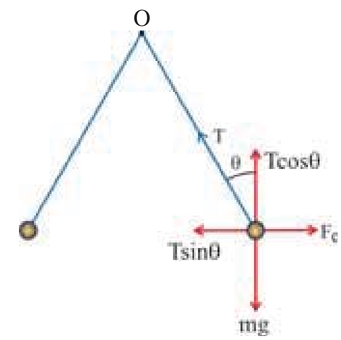
SOLUTIONS

CHAPTER 1

1. Suppose charge on a sphere A and sphere B is q_1 and q_2 respectively. In first case force between two spheres is $F = k\frac{q_1 q_2}{d^2}$. When sphere A is brought in contact with sphere C the charge on sphere A will be, $q_A = \frac{q_1}{2}$ and charge on sphere B will be $q_B = \frac{q_2}{2}$.

Now, force between sphere A and sphere B at distance $\frac{d}{2}$ will be

$$F' = \frac{k(q_1/2)(q_2/2)}{(d/2)^2} = \frac{kq_1q_2}{d^2} = F$$



2. Suppose the density of sphere = ρ and density of kerosene = ρ'. When two spheres are suspended in air forces acting on it are shown in figure. In equilibrium pasition, $F_e = T\sin\theta$ and $mg = T\cos\theta$. From these,

$$\tan\theta = \frac{F_e}{mg} \qquad (1)$$

Now, the sphere is immersed in kerosene, due to buoyant force acting on it, its weghit will be $(m - m')g$ instead of mg and electric force. $F_e = \frac{F_e}{2}$.

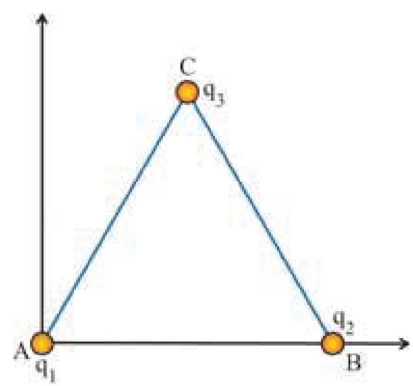
Because dilectric constant of kerosene is, K = 2.

$$\therefore \tan\theta = \frac{F_e/2}{(m-m')g} \qquad (2)$$

Comparing equation (1) and (2), m = 2m'

or ρV = 2(ρ'V)

$\therefore \rho = 2\rho' = 2 \times 800 = 1600 \text{ kg m}^{-3}$



3. Suppose $q_1 = 0.5 \times 10^{-6}$C

$q_2 = -0.25 \times 10^{-6}$C, $q_3 = 0.1 \times 10^{-6}$C

From the figure,

Position vector of q_1 is $\vec{r_1} = (0, 0)$m

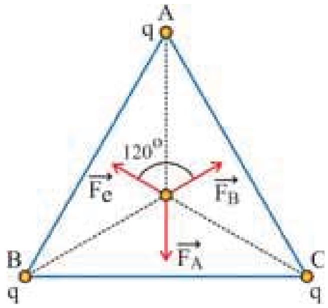
Position vector of q_2 is $\vec{r_2} = (5 \times 10^{-2}, 0)$m

Position vector of q_3 is $\vec{r_3} = (2.5 \times 10^{-2}, 2.5 \times 10^{-2} \times \sqrt{3})$m

Now, force on q_3, $\vec{F_3} = \vec{F_{31}} + \vec{F_{32}}$

$$= kq_3\left[\frac{q_1(\vec{r_3}-\vec{r_1})}{|\vec{r_3}-\vec{r_1}|^3} + \frac{q_2(\vec{r_3}-\vec{r_2})}{|\vec{r_3}-\vec{r_2}|^3}\right]$$

Substituting values in above equation and calculate $\vec{F_3}$.

4. 

As shown in figure, the net force on charge $2q$ is $\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C$ and $|\vec{F}_A| = |\vec{F}_B| = |\vec{F}_C|$ because of equilateral triangle. All three forces are at 120° respectively. Vector addition of $\vec{F}_A$, $\vec{F}_B$ and $\vec{F}_C$ using triangular method forms close loop. So, the resultant force is zero.

5. Torque on dipole, $\tau = PE\sin\theta = PE\theta$ ($\because$ θ is small)

Torque is in clockwise direction, therefore $\tau = -PE\theta$

Now, $\tau = I\alpha$ and $\alpha = -\omega^2\theta$

$$\omega = \sqrt{\frac{PE}{I}} \quad \therefore f = \frac{1}{2\pi}\sqrt{\frac{PE}{I}}$$

6. Suppose an electron is thrown from the distance r from surface with 150 eV energy. Work done on electron against the force,

$$W = \vec{F}\cdot\vec{r} = (-eE)(r) = \left(\frac{-e\sigma}{2\varepsilon_0}\right)(r)$$

Now substitue the values in this equation and find the value of r.

7. Suppose charge on two spheres is q, and q_2 respectively.

In the first case According to Coulomb's Law.

$$0.108 = 9 \times 10^9 \frac{q_1q_2}{(0.5)^2}$$

$$\therefore\ q_1q_2 = 3 \times 10^{-6} \qquad (1)$$

Now, when both the spheres are bring in contact, the charge on both spheres will be is $\frac{q_1 - q_2}{2}$. In this case, force between them, $0.036 = \frac{9\times10^9\left(\frac{q_1-q_2}{2}\right)^2}{(0.5)^2}$

$$\therefore\ q_1 - q_2 = 2 \times 10^{-6} \qquad (2)$$

Equating equation (1) and (2)

$q_1 = 3 \times 10^{-6}$C and $q_2 = 1 \times 10^{-6}$C

8. Acceleration of $2q$ charge, $a_1 = \frac{F_1}{m} = \frac{2qE}{m}$

Velocity of charge after t time, $v_1 = a_1t = \frac{2qE}{m}t$

So, Kinetic energy $K_1 = \frac{1}{2}mv_1^{\,2} = \frac{2q^2E^2}{m}t^2 \qquad (1)$

Similarly, calculating kinetic energy of charge q will be

$$K_2 = \frac{q^2E^2}{4m}t^2 \qquad (2)$$

From, equation (1) and (2)

$$\frac{K_1}{K_2} = \frac{8}{1}$$

9. Two forces are acting on a simple pendulum (1) Electric force $q\vec{E}$ (2) Gravitational force $m\vec{g}$.

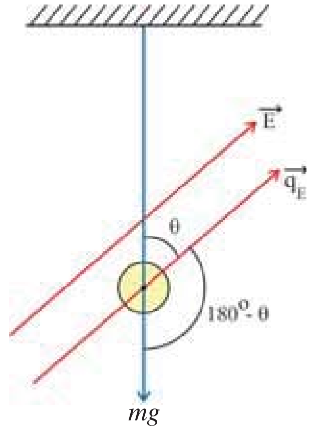
Resultant force, $\vec{F} = m\vec{g} + q\vec{E}$

$\therefore \ |\vec{F}| = \sqrt{(mg)^2 + (qE)^2 + 2(mg)(qE)cos(180^o - \theta)}$

Taking effective acceleration g_e of sphere,

$mg_e = \sqrt{(mg)^2 + (qE)^2 - 2(mg)(qE)cos\theta}$

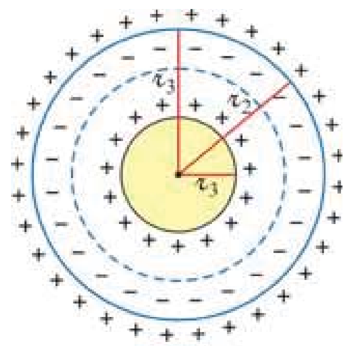
$$\therefore \ g_e = \left(g^2 + \frac{q^2E^2}{m^2} - \frac{2gqE}{m}cos\theta\right)^{\frac{1}{2}}$$



Periodic time of pendulum $T = 2\pi\sqrt{\frac{l}{g_e}}$. Substitute the value of g_e in it.

10. Due to charge q on a sphere of radius 1 cm, $+q$ charge is induced on outer region of sphere having radius r_3 = 5 cm and $-q$ charge is induced in inner region. Now, draw the spherical Gaussian surface of radius r_2 = 2 cm. Applying Gauss's Law,

$$\int_s \vec{E} \cdot d\vec{a} = \frac{\Sigma q}{\varepsilon_0}$$



$$E(4\pi r_2^2) = \frac{q}{\varepsilon_0}$$

$$\therefore \ E = \frac{q}{4\pi\varepsilon_0 r_2^2}$$

Calculate the value of E by substituting the values in above equation.

11. Solved according to illustration 15.

12. When charge q is established on a particle and if electric force qE acting vertically downward on it and gravitational force mg acting on it will be same then particle will be in equalibrium.

$qE = mg$

$$\therefore \ q = \frac{mg}{E} = \frac{mg}{\sigma/\varepsilon_0}$$

Calculate q by substituting the values in above equation.

13. Force between the electron and the proton is,

$$F = k\frac{q_1q_2}{r^2} = \frac{9\times10^9(1.6\times10^{-19})^2}{(0.53\times10^{-10})^2} = 8.2 \times 10^{-8}N$$

The force F on a revolving electron will be equal to centripetal force.

$$\frac{mv^2}{r} = F.$$

Now, putting $v = r\omega$,

$mr\omega^2 = F$

$$\therefore \ \text{Radial acceleration } r\omega^2 = \frac{F}{m} = \frac{8.2\times10^{-8}}{9.1\times10^{-31}} = 9 \times 10^{22} \ m/s^2$$

$$\therefore \ \omega = \sqrt{\frac{8.2\times10^{22}}{0.53\times10^{-10}}} = 3.9 \times 10^{16} \ rad/s.$$

CHAPTER 2

1. **Figure**

$q_1 = 2\ \text{C}$ $\qquad q_2 = -3\ \text{C}$

(i) $V = \dfrac{Kq_1}{x_1} + \dfrac{Kq_2}{(100 - x_1)} = 0$ $\qquad$ (ii) $V = \dfrac{Kq_1}{x_2} + \dfrac{Kq_2}{(100 + x_2)} = 0$

Hence find x_1. $\qquad$ Hence find x_2.

(Take $q_1 = 2$ C and $q_2 = -3$ C)

2. When both the spheres are joined by a conducting wire; the charges distribute in such a way that their potentials become equal.

$\therefore\ \dfrac{KQ_a}{a} = \dfrac{KQ_b}{a}$. But $Q_b = Q - Q_a$. Hence find Q_a. Similarly find Q_b.

3. $E_x = \dfrac{-\partial V}{\partial x} = -(4xy - 4z^4)$, $E_y = \dfrac{-\partial V}{\partial y} = -(2x^2 + 9y^2z)$, $E_z = \dfrac{-\partial V}{\partial z} = -(3y^3 - 16z^3x)$

For point (1, 1, 1) put $x = 1$, $y = 1$, $z = 1$ to find E_x, E_y, E_z. Hence find $\vec{E}$.

4. Find r from $V = \dfrac{Kq}{r}$.

If the radius of big drop is r', $\frac{4}{3}\pi r'^3 = (8)(\frac{4}{3}\pi r^3)$ $\therefore$ $r' = 2r$

Now, find $V' = \dfrac{Kq'}{r'}$ where, $q' = 8q$.

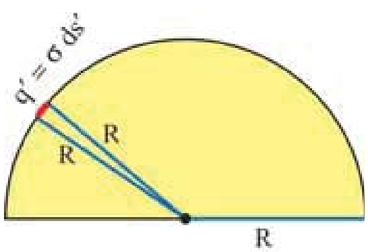
5. When Q = 0, the potential = 0, When Q = Q, the potential = $V = \dfrac{KQ}{R}$.

$\therefore$ Average potential $= \dfrac{O+V}{2} = \dfrac{V}{2}$.

Now, find potential energy = (average potential) (charge)

6. Electric charge in small surface element $= \sigma ds'$. Potential due to it at 0

$dV' = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\sigma ds'}{R}$.



$\therefore$ Total potential $V = \int dV' = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\sigma}{R}\int ds'$

$\displaystyle\int_{\text{on semi-sphere}} ds' = 2\pi R^2$. Hence find V.

7. Potential on the sphere $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\sigma(4\pi R^2)}{R}$$

$$= \frac{\sigma R}{\varepsilon_0}$$

Hence find V_A, V_B, V_C and $V = V_A + V_B + V_C$

8. $\dfrac{C_1}{C_2} = \dfrac{C_3}{C_4}$ $\therefore$ Potential difference for $C_5 = 0$

$\therefore$ It is not in action (not effective)

$\therefore \frac{1}{C'} = \frac{1}{1} + \frac{1}{3} = \frac{4}{3}$. find C'.

$\frac{1}{C''} = \frac{1}{2} + \frac{1}{6} = \frac{8}{12}$ find C''.

Find C = C' + C''.

9. Initially the charge on capacitor = Q.

Find initial energy $U = \frac{1}{2}CV^2$. It can also be written as $= \frac{Q^2}{2C}$.

When it is joined with other uncharged capacitor, the charge on each one will be $= Q' = \frac{Q}{2}$.

Now find energy of each one $U' = \frac{Q'}{2C}$

Find total energy = U' + U' = 2U'

10. If capacitance on path MNOP is C',

$\frac{1}{C'} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \therefore C' = \frac{10}{3} pF.$

C' and C_4 are in parallel. ∴ Their equivalent capacitance $C'' = \frac{10}{3} + 10 = \frac{40}{3} pF.$

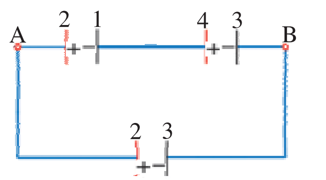
Find charge coming from battery Q'' = C''V

Find $Q_4 = C_4V$. Find equal charge on

C_1, C_2, C_3 as = Q'' − Q_4.

11. If capacitance on the path B C_2 C_3 D, is C', $\frac{1}{C'} = \frac{1}{C_2} + \frac{1}{C_3}$. Find C'. Now between A and C, C_1, C' and C_4 are in parallel. ∴ $C'' = C_1 + C' + C_4$.

12. The equivalent connections are as shown here.



C_{21} and C_{43} are in series. Their equivalent $C' = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)$

With this combination C_{23} is in parallel.

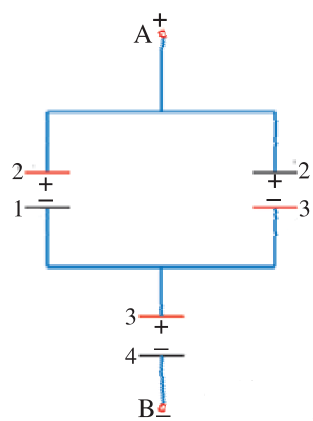
find $C_{AB} = C' + C_{23}$.

13. C_{21} and C_{23} are in parallel

$\therefore$ Their equivalent $C' = 2\left(\frac{\varepsilon_0 A}{d}\right)$

C' and C_{34} are in series.

Use $\frac{1}{C_{AB}} = \frac{1}{C'} + \frac{1}{C_{34}}$ to find C_{AB}.



CHAPER 3

1. If the number of electrons striking the screen per second = n

Using the equation of current, $I = \frac{Q}{t} = \frac{ne}{t}$, calculate n.

Electric charge striking the screen in t = 1 minute = 60 s can be calculated from $Q = It$.

2. The speed of electron in a circular orbit $v = \frac{2\pi r}{T} = 2\pi rf$

Substituting given values of v and r, calculate f.
Now, apply the equation $I = ef$.

3. (i) Calculate the potential difference across the ends of a wire from equation,

$$V = IR = I\left(\rho\frac{l}{A}\right)$$

(ii) From equation $I = Av_d ne$, drift velocity $v_d = \frac{I}{Ane}$

where, number density of electron $n = \frac{dN_A}{M}$

4. Area of cross-section of a semiconductor
$A = bh$
$A = (4 \times 10^{-3})(25 \times 10^{-5}) = 10^{-6} m^2$

Find the current density from $J = \frac{I}{A}$

Now, using equations $J = ne\, v_d$ and $v_d = \frac{l}{t}$ calculate time t.

5. New length of a wire $l' = l + 10\ \%$ of l

$$l' = 1 + 0.1\ l = 1.1\ l$$

$$\frac{l'}{l} = 1.1$$

Initially, $R = \rho\frac{l}{A}$

After stretching the wire, $R' = \rho\frac{l'}{A'}$

volume of the wire is constant.

$\therefore\ Al = A'l' \Rightarrow \frac{A}{A'} = \frac{l'}{l} = 1.1$

Now, $\frac{R'}{R} = \frac{l'}{l}\cdot\frac{A}{A'} = \left(\frac{l'}{l}\right)^2 = 1.21$

Percentage increase in resistance $= \frac{R'-R}{R} \times 100 = (1.21 - 1) \times 100 = 21\%$

6. Let the length of P part of wire = l,
and Q part of wire = $(1 - l)$
If the resistances of P, Q and R part of wires be R_P, P_Q and R_R respectively then,

$$R_P = \rho.\frac{l}{A},\ R_R = \frac{\rho(2l)}{A/2}\ 4\frac{\rho l}{A} = 4R_P$$

and $R_Q = \frac{\rho(1-l)}{A}$

given that, $R_R = R_Q$

$4 \cdot \frac{\rho l}{A} = \frac{\rho(1-l)}{A}$

$\Rightarrow l = \frac{1}{5}$ m

7. Given that, $R_{Al} = R_{CU}$

$\rho_1 \cdot \frac{l_1}{A_1} = \rho_2 \cdot \frac{l_2}{A_2}$

$\Rightarrow \frac{\rho_2}{\rho_1} = \frac{A_2}{A_1}$

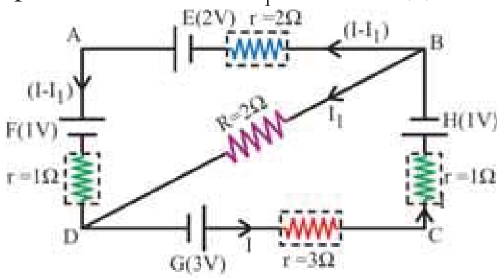
Now, mass of Al wire $m_{Al} = A_1 l_1 d_1$ and mass of C_{Cu} wire $m_{Cu} = A_2 l_2 d_2$

Their ratio gives, $m_{Cu} = 2.15\ m_{Al}$

$\therefore m_{Al} < m_{Cu}$

8. (i) Applying Kirchhoff's second rule to the closed loop BADB, $-3I + 5I_1 + 1 = 0$ (1)

(ii) For a closed loop DCBD, $-2I - I_1 + 1 = 0$ (2)

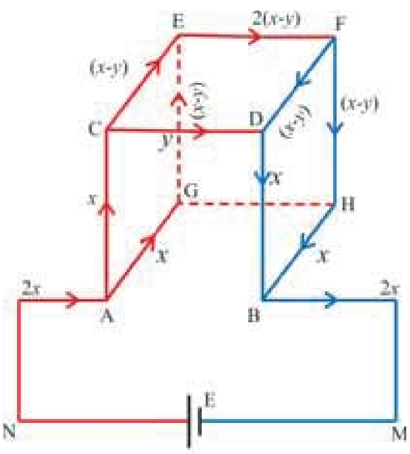


Solving equation (1) and (2) we get, $I_1 = \frac{1}{13}$ A

and $V_{BD} = I_1 R = \left(\frac{1}{13}\right)(2) = \frac{2}{13}$ V

9. (i) Applying Kirchhoff's second rule to the closed loop ACDBMNA, $r(2x + y) = \varepsilon$ (1)

(ii) Similarly for a closed loop ACEFDBMNA, $2r(3x - 2y) = \varepsilon$ (2)



From equation (1) and (2), $y = \frac{4}{5}x$ (3)

From equation (1) and (3), $\varepsilon = \left(\frac{14}{5}x\right)r$ (4)

If the effective resistance between A and B is r' then,

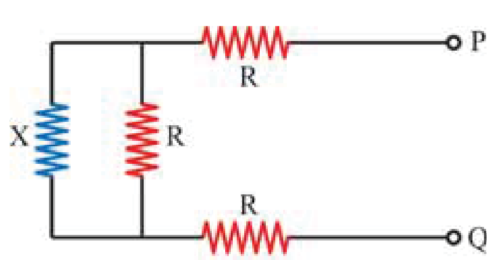
$\varepsilon = 2xr'$ (5)

comparing equation (4) and (5)

$r' = \frac{7}{5}r$

10. Let the required equivalent resistance be X. The network is infinite. Therefore adding one more stage to the network does not affect the value of X.

The equivalent resistance of the above circuit should be equal to X.



$\frac{XR}{X+R} + 2R = X$

$\therefore X^2 - 2RX - 2R^2 = 0$

Solving the above equation by the method of quadratic equation, we get,

$X = R(1 + \sqrt{3})$

11. Length of the potentiometer wire $L_1 = 200$ cm

$\Rightarrow$ null point length $l_1 = 80$ cm

$$\therefore\ \varepsilon = \sigma\, l_1 = \left(\frac{IR}{L_1}\right) l_1 \qquad (1)$$

Length of potentiometer wire $L_2 = 300$ cm

$\Rightarrow$ null point length $l_2 = ?$

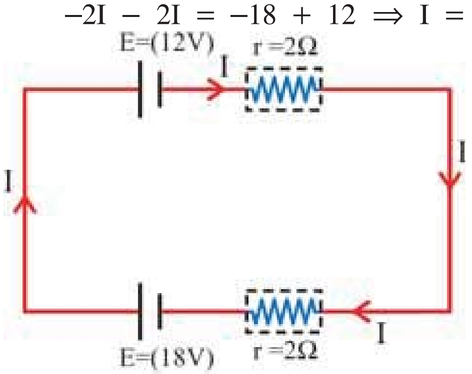
$$\varepsilon = \sigma l_2 = \left(\frac{IR}{L_2}\right) l_2 \qquad (2)$$

Comparing equation (1) and (2),

$l_2 = 120$ cm

12. (1) Applying Kirchhoff's second rule to the closed circuit shown in Figure,

$-2I - 2I = -18 + 12 \Rightarrow I = 1.5A$



(2) Calculate electrical power in Battery using equation $P = \varepsilon I$

(3) Terminal voltage of a 18 V battery

$V = \varepsilon - Ir \Rightarrow V = 15$ V

Terminal voltage of a 12 V battery

$V = \varepsilon + Ir \Rightarrow V = 15$ V ($\because$ The battery of 12 V is being charged.)

(4) Calculate power consumed in battery from equation $I^2 r$.

13. V and H is same for both the coils

For coil-1, $H = \frac{I_1^2 R_1 t_1}{J} = \left(\frac{V^2}{R_1^2}\right)\frac{R_1 t_1}{J} = \frac{V^2 t_1}{R_1 J}$

$$\Rightarrow \frac{1}{R_1} = \frac{JH}{V^2 t_1} \qquad (1)$$

$$\text{For coil-2, } \frac{1}{R_2} = \frac{JH}{V^2 t_2} \qquad (2)$$

when two coils are connected in parallel,

$$\frac{1}{R} = \frac{JH}{V^2 t} \qquad (3)$$

where R = equivalent resistance of a parallel connection

From equations (1), (2) and (3)

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{JH}{V^2 t} = \frac{JH}{V^2 t_1} + \frac{JH}{V^2 t_2}$$

$$\therefore \frac{I}{t} = \frac{I}{t_1} + \frac{I}{t_2} \Rightarrow t = 3.43 \text{ minute}$$

14. Heat required to melt the fusewire

$H = mc\Delta\theta$

$= (Ald)\, c\Delta\theta \qquad (1)$

where, d = density of fusewire

c = specific heat of fusewire

m = Ald = mass of fusewire

$\Delta\theta$ = Increase in temperature required to melt the fusewire

Heat produced in the fusewire by passing current I through it for time t,

$$H = \frac{I^2Rt}{J} \qquad (2)$$

For melting of a fusewire,

$$\frac{I^2Rt}{J} = Aldc\Delta\theta$$

$$I^2 \left(\rho\frac{l}{A}\right)\frac{t}{J} = Aldc\Delta\theta$$

$$\therefore\ I^2\rho t = JA^2dc\Delta\theta$$

$$\therefore\ t = \frac{JA^2dc\Delta\theta}{I^2\rho} \qquad (3)$$

From this equation both the fuse wire will melt in the same time for the same value of current flowing through them.

($\because$ J, A, d, C, $\Delta\theta$, ρ are constant for both the fusewires)

15. Using $P = \frac{V^2}{R}$, $R_A = 302.5\ \Omega$ and $R_B = 121\ \Omega$

Similarly from equation P = VI,

$I_A = 0.3636$ A and $I_B = 0.9091$ A

Same current will flow through each of the bulb when two bulbs are connected in series with a supply of 220 V.

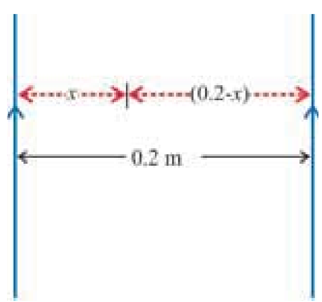
$$\therefore\ I = \frac{V}{R_A + R_B} = \frac{220}{302.5+121} = 0.519\ A$$

Here $I > I_A$, therefore A bulb will fuse.

CHAPTER 4

1. $\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(0.2-x)}$ (in opposite directions)

Hence find x.



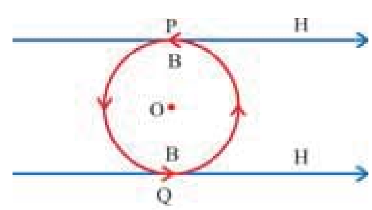
2. At P, B = H (in opposite directions)

$$\frac{\mu_0 I}{2\pi y} = H$$

Hence find I.

At Q, B = H (in the same direction)

$\therefore$ Total magnetic field = B + H = 2B = 2H.



3. $S = \frac{GI_G}{I - I_G}$ I = 100 unit and I_G = 2 unit, find S.

4. Find velocity v from $qV = \frac{1}{2}Mv^2$ ...... V = voltage. Put it in $\frac{Mv^2}{R} = Bqv$ and then

Make R the subject, to get $R = \left(\frac{2MV}{q}\right)^{\frac{1}{2}} \times \frac{1}{B}$. V, q, B are same for both particles.

$\therefore \frac{M_1}{M_2} = \left(\frac{R_1}{R_2}\right)^2$

5. $\vec{\tau} = \vec{\mu} \times \vec{B} = NI\vec{A} \times \vec{B}$ gives $\tau_{max} = NIAB$,

Length of wire $L = N(2\pi R) \Rightarrow R = \frac{L}{2\pi N}$; $A = \pi R^2 = \pi\frac{L^2}{4\pi^2 N^2}$, Put A in τ_{max}.

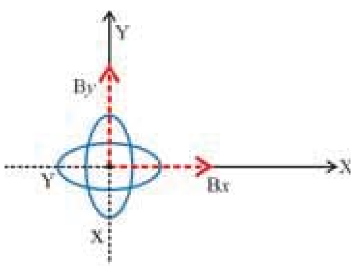
6. $\frac{mv^2}{r} = Bqv$ gives $r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2mE}}{Bq}$ $\therefore E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $\therefore p = \sqrt{2mE}$

$\frac{r_d}{r_p} = \frac{p_d}{Bq_d} \times \frac{Bq_p}{p_p}$, ... $(q_p = q_d)$,

$\frac{p_d}{p_p} = \frac{\sqrt{2m_d E}}{\sqrt{2m_p E}}$ (But $m_d = 2m_p$; E is same)

$\therefore \frac{r_d}{r_p} = \sqrt{2}$

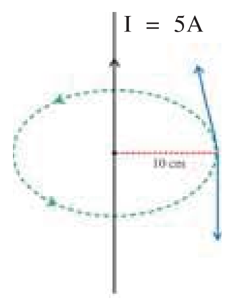
7. $k\phi = NABI$; Take $\phi = \left(36\times\frac{\pi}{180}\right)$rad, $k = \frac{B_I N}{\phi}$. Find k.

8. 

$B_x = \frac{\mu_0 I_x}{2a}$ and $B_y = \frac{\mu_0 I_y}{2a}$.

Resultant magnetic field is $B' = \sqrt{B_x^2 + B_y^2}$.

9. $\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{y}$ Find $\frac{F}{l}$.



10. $B = \frac{\mu_0 I}{2\pi y}$. The velocity of electron is perpendicular to this B.

Use $F = Bqv\sin\theta = Bev$

11. Use formula obtained in illustration-1 $B = \frac{\mu_0 I\theta}{2\pi R}$; θ in radian $= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$, $I = 6A$ and $R = 0.02$. Find B.

CHAPTER 5

1. Average radius $r = \frac{r_1 + r_2}{2}$, $n = \frac{N}{2\pi r}$

Now $H = nI_f$, Hence use $\mu_r = \frac{B}{\mu_0 H}$

2. $m_{atom} = 1.5 \times 10^{-23}$ A m^2

$\therefore$ $m_{net} = m_{atom} \times$ number of atoms per unit volume

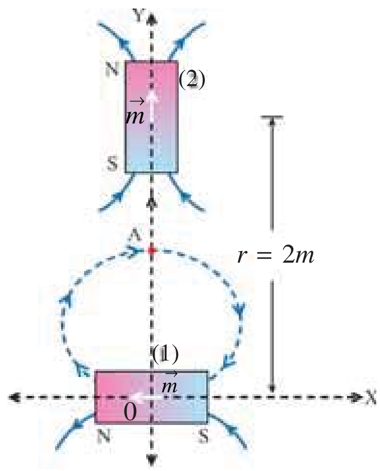
Hence use $M_{max} = \dfrac{m_{net}}{V}$ (1)

Thermal energy of the atom of gas $= \dfrac{3}{2}kT$ (2)

Maximum potential energy of atom $= m_{net}$ B (3)

Find the ratio $\left(\dfrac{\frac{3}{2}kT}{m_{net}B}\right)$ and give answer.

3. For magnet (1) the equatorial magnetic field at A (1 m from its centre) is $B_1 = \dfrac{\mu_0}{4\pi}\dfrac{m}{r^3}\hat{i}$



For magnet (2) the axial magnetic field at A (1 m from its centre) is

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2m}{r^3}\hat{j}$$

Hence resultant magnetic field at A is

$$B = \sqrt{B_1^2 + B_2^2}$$

4. Axial magnetic field of magnet is

$$B(z) = \frac{\mu_0}{4\pi}\frac{2m}{z^3} = \frac{\mu_0}{4\pi}\frac{2(l\,p_b)}{z^3}$$

l = length of magnet, p_b = pole strength

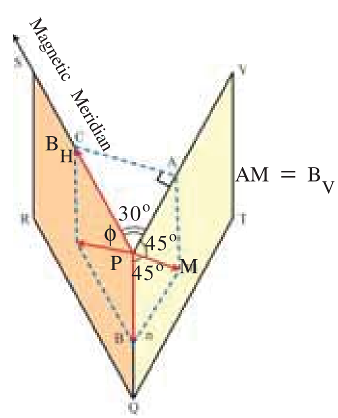
Hence calculate force on magnetic pole as $F = p_b\,B(z)$

5. Work done for rotating magnet of magnetic dipole moment m by angle θ is

$$W(\theta) = \int_0^\theta mB\sin\theta\,d\theta = \left[-mB\cos\theta\right]_0^\theta$$

Now $W(90^o) = nW(60^o) \Rightarrow n = \dfrac{W(90^o)}{W(60^o)}$

6. Magnetic moment of magnet is shown by $\vec{PM}$ making angle of 45^o in PQTV plane. The plane PQTV makes an angle of 30^o with magnetic meridian plane represented by PQRS.



For rectangular triangle PAC, $PA = B_H\cos 30^o$.

Hence in plane PQTV, for rectangular triangle

$$PAm\ \tan 45^o = \frac{AM}{PA}$$

Now calculate B_v in terms of B_H.

In plane PQRS, $\tan\phi = \dfrac{B_V}{B_H}$

7. Use $m = \frac{1}{2}evr$, and $L = m_e vr$

8. (a) Use $B(x) = \frac{\mu_0}{4\pi}\frac{2m}{x^3}$ to calculate m.

(b) Using the value of m from (a), calculate $B(y) = \frac{\mu_0}{4\pi}\frac{m}{y^3}$

9. Volume of cylindrical rod $V = \pi r^2 l$

Then use, $m_{net} = M \times V$

10. Number of electrons (n_e) = number of ions (n_i)

Average kinetic energy of electron = K_e

Average kinetic energy of ion = K_i

Total kinetic energy of the gas is $K = (n_e \times K_e) + (n_i \times K_i)$

When the gas is completely magnetized, the resultant magnetic moment of gas is equal to its magnetization M or M_{max},

When $U = \vec{M} \cdot \vec{B} = K$, $(\vec{M} = n_e\vec{m} + n_i\vec{m} = 2n_e\vec{m})$

$\therefore$ $K = MB\cos\theta$, Calculate M when $\theta = 0°$

11. Length of solenoid = l, Number of turns per unit length = n

Hence, magnetic moment of solenoid $m = NIA = nlIA$

Also pole strength of solenoid $p_S = \frac{m}{l}$

CHAPTER 6

1. Using Gauss' formula for mirror $\frac{2}{R} = \frac{1}{u} + \frac{1}{v}$, obtain $v = \frac{u \cdot R}{2u - R}$

Differentiate it with respect to time, i.e. obtain $\frac{dv}{dt}$ and simplify.

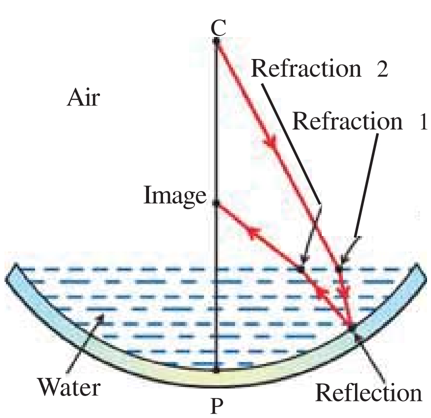
Let $\frac{dv}{dt} = v_i$ = velocity of image

and $\frac{du}{dt} = v_0$ = velocity of object.

2. Use $m = \frac{-v}{u} = \frac{h'}{h}$ and $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ with proper sign convention.

[**Ans. :** 37.5 cm]

3. This is the combination of plano-convex lens formed by water and a concave mirror. Focal length of a lens,



$$\frac{1}{f_1} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For plano-convex lens, $R_1 = \infty$, $R_2 = -R$ (say), n is the refractive index of the material forming lens, here water.

$$\therefore\ f_1 = \frac{R}{(n-1)} \qquad (1)$$

If focal length of a mirror is f_2 then effective focal length of this combination

$$\frac{1}{f'} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

where f_3 focal length corresponding to emerging ray, which is from denser (water) to rarer (air). f_2 is the focal length of the mirror.

Here, $f_2 \rightarrow -f_2$ and $f_3 \rightarrow f_1$

$$\therefore \frac{1}{f'} = \frac{2}{f_1} - \frac{1}{f_2}$$

$$= \frac{2}{\left(\frac{R}{(n-1)}\right)} - \frac{1}{\left(\frac{2}{R}\right)} \quad (\because f = \frac{2}{R} \text{ for mirror and using equation (1))}$$

$$\therefore f' = \frac{R}{2(n-2)}$$

$\therefore$ effective radius of curvature,

$$R' = 2f' = \frac{R}{2(n-2)}$$

Since $n = 1.33 \Rightarrow |R'| > |R|$

i.e. image will form between C' and pole.

5. Since Snell's law is applicable to all points, first apply it to the point of incident.

i.e., $n_1 \cdot sin\theta_1$ = const, A (say)

Let at a distance y in the medium, refracted ray is horizontal (i.e. $\theta_2 = 90°$). Then again using Snell's law at this point,

$n_2 \cdot sin\theta_2$ = A;

Where $n_2 = (1.5 - 0.25y)$. This gives value of y. [**Ans. :** $y = 3$m]

6. For large incidence angle, lateral shift,

$$x = \frac{t \cdot sin(\theta_1 - \theta_2)}{cos\,\theta_2}, \text{ where } \theta_1 = 53°$$

Find θ_2 using Snell's law. t is given. [**Ans. :** $x = 9$ mm]

7. Use $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ to obtain $m = \frac{f}{u-f}$

(1) When $m = 4$, obtain object distances $u_1 = \frac{3}{4}f$.

(2) Now, on displacing object by 3 cm away from the mirror,

$u_2 = u_1 + 3$ (in cm)

Now, calculate f. [**Ans. :** $|f| = 36$ cm]

8. For optical fibre, requirement for total internal reflection is $(90° - \theta_f) > C$

$\therefore sin(90° - \theta_f) > sin C$

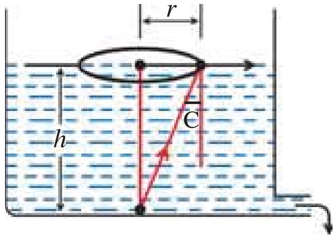
$\therefore cos\theta_f > \frac{1}{n}$ ($\because$ for air-medium interface $sin C = \frac{1}{n}$)

Now apply Snell's law to find maximum incident angle. [**Ans. :** $\frac{\pi}{2}$]

9. Apply Snell's law at the point of incidence to the river. Then use simple trigonometry.

[**Ans. :** length of a shadow = 3.44 m]

10. $\sin C = \frac{1}{n}$ gives C. From the Figure, $\tan C = \frac{r}{h}$ find h.



[**Ans. :** 1.33 cm]

11. For image due to surface on right,

$$\frac{-n_1}{u} + \frac{n_2}{v} = (n_2 - n_1) \times \frac{1}{R_1} \quad (1)$$

Where $n_1 = 1$, $n_2 = 1.5$, $R_1 = -R$, $u = -\frac{R}{2}$

$$\therefore \; v = \frac{-3R}{5} \quad (2)$$

The image due to this surface is the object for the second. For surface on the left, $n_1 = 1$, $n_2 = |1.5 \; R_2 = +R|$.

From equation (2), image of right-surface is at a distance $\left(R - \frac{3}{5}R\right)$ from the centre towards the right-surface. Therefore, u for left-surface is $\frac{3}{2}$R. Using equation (1), image distance due to left-surface from the centre is $\frac{2R}{7}$.

$\therefore$ distance between two images due to both surfaces is $\frac{2}{5}R - \frac{2}{7}R = 0.114R$

14. For plano-convex lens,

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = (n - 1)\cdot\left(\frac{1}{R_1} - \frac{1}{\infty}\right)$$

$\therefore$ $R_1 = 10$ cm, $f = 20$ cm, $n = 1.5$

This lens would have given image at I'. But back plane reflecting surface gives image at I''. This I'' image is the virtual object for the curved surface. Using

$$\frac{-n_1}{u} + \frac{n_2}{v} = \left(\frac{n_2 - n_1}{R}\right)$$ formula for incident

rays : $u \to -\infty$, $v' = ?$, $n_2 = 1.5$, $n_1 = 1$.

$\therefore$ $v' = 30$ cm.

For emerging ray (second refraction),

$n_1 = 1.5$, $n_2 = 1$, R = −10 cm, $u = +30$ cm, $v = ?$

$v = 10$ cm.

Since object was at infinite, final image distance (v) gives the focal length of the system.

15. From the similarity of ΔBH_1H_2 and ΔF_1OH_2,

$$\frac{OF_1}{OA} = \frac{OH_2}{H_1H_2}$$

$$\therefore \frac{-f_1}{-u} = \frac{-h'}{(-h'+h)} \quad \text{(sign convention)}$$

Similarly, for $\Delta B'H_1H_2$ and ΔF_2OH_1,

$$\frac{f_2}{v} = \frac{h}{(-h'+h)}$$

Adding these equations,

$$\frac{f_1}{u} + \frac{f_2}{v} = \frac{-h'+h}{(-h'+h)} = 1 \quad (1)$$

$$\therefore |m| = \frac{v}{u} = \frac{(v-f_2)}{f_1}$$

From equation (1) for special case, $f_2 = -f_1 = f$

$$\frac{-f}{u} + \frac{f}{v} = 1$$

$$\therefore \frac{-1}{u} + \frac{1}{v} = \frac{1}{f}.$$ This is the Gauss' formula.

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CHAPTER 7

1. Minimum Energy, $W = Fd = (eE)(d)$

$$= e\left(\frac{\sigma}{\varepsilon_0}\right)d$$

$$= 29.83 \ eV$$

Minimum Energy $= \frac{1}{2}mV^2 + W$

$$= (hf - \phi_0) + W$$

$$= \frac{hc}{\lambda} - \phi_0 + W$$

$$= 30.63 \ eV$$

2. (1) Threshold Frequency, $f_0 = \frac{c}{\lambda_0} = 1.098 \times 10^{15}\text{Hz} \approx 1.1 \times 10^{15}$ Hz

(2) Work Function $\phi_0 = hf_0 = 4.54 \ eV$

(3) *max* K.E., $\frac{1}{2}mV^2_{max} = hf - hf_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

$$= 2.35 \ eV$$

(4) Stopping Potential $V_0 = (\frac{1}{2}mV^2{}_{max})$

$= 2.35\ eV$

(5) $\frac{1}{2}mV_{max}{}^2 = eV_0$

$\therefore V_{max} = \sqrt{\frac{2eV_0}{m}}$ and $V_{min} = 0$ m/s

3. $eV_0 = \frac{hc}{\lambda} - \phi_0$

$\therefore eV_0 = \frac{hc}{\lambda_1} - \phi_0$ (1) and $\frac{hc}{\lambda_2} - \phi_0$ (2)

$\therefore$ Subtract (1) from (2)

Change in stopping potential, $V_{0_2} - V_{0_1} = \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$

4. $p = \frac{3}{100} \times = 100 = 3$ J/s

$p = \frac{E}{t} = \frac{nhf}{t} \Rightarrow n = \frac{p\lambda t}{hc}$

5. $eV_0 = \frac{hc}{\lambda} - \phi_0$

$\therefore eV_{0_1} = \frac{hc}{\lambda_1} - \phi_0$ (1) and $eV_{0_2} = \frac{hc}{\lambda_2} - \phi_0$

$\therefore e(V_{0_1} - V_{0_2}) = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$

Now, subject the formula for h and calculate if.
From equation (1)

$\phi_0 = \frac{hc}{\lambda_1} - V_{0_1}e = ...$.

$\phi_0 = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{hc}{\phi_0} = ...$.

6. $Ve = \frac{hc}{\lambda} - \phi_0$

$\therefore V = \frac{hc}{\lambda e} - \frac{\phi_0}{e} = ...$.

7. For (1) and (2)

Energy of photon $E = \frac{hc}{\lambda}$

for (3) and (4) $E = hf$

8. $p = \frac{E}{t} = \frac{nhf}{t} = \frac{nhc}{t\lambda} = ...$.

9. $p = \frac{E}{t} = \frac{nhf}{t} = \frac{nhc}{t \cdot \lambda} \Rightarrow$ no. of photon emitted per second is $n = \frac{p\lambda t}{hc} = ...$.

10. $\frac{1}{2}mv^2 = \frac{3}{2}k_B T$

$\therefore \; m^2v^2 = 3k_B Tm$

$\therefore \; p = \sqrt{3k_B Tm}$

$\lambda = \frac{h}{p}$

$\therefore \; \lambda \; \alpha \; \frac{1}{\sqrt{T}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}} = ...$.

11. $I = \frac{E}{At} = \frac{nhf}{At} = \frac{nhc}{At\lambda}$

$\Rightarrow n = \frac{IAT\lambda}{hc} = ...$.

12. $I \; \frac{E}{At} = \frac{nhf}{At} = \frac{nhc}{At\lambda}$

$\Rightarrow n = \frac{I\lambda At}{hc}$

13. $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ and $m = 1.1 \; m_0$ and calculate V.

$\frac{1}{2}mv^2 = \frac{3}{2}k_B T \Rightarrow T = \frac{mv^2}{3k_B} = ...$.

14. $p = \frac{E}{t} = \frac{nhf}{t} = \frac{nhc}{\lambda t} \Rightarrow$ no. of photons emitted per second is, $n = \frac{p\lambda t}{hc} = ...$.

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LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
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24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
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36	5563	5575	5587	5566	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
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39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6345	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7769	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9843	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

AntilogarithmS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1289	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

AntilogarithmS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
82	6607	6622	6637	6653	6668	6683	6699	6715	6730	6745	2	3	5	6	8	9	11	12	14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	12	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

NATURAL SINES

Degree	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	Mean Deifferences 1'	2'	3'	4'	5'
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	.0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	.0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	14
5	.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	.1045	1063	1080	1197	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	.1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	.1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	.1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	11	14
11	.1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	.2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	3	6	9	11	14
13	.2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	.2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	.2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	.2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	.3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	.3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	.3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	.3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	.3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	.4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	.4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	.4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	.4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	.4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	.5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	.5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	.5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	.5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	9	12
36	.5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	.6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	.6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	.6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	.6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	.6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	.6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	.6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

Degree	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	Mean Deifferences 1'	2'	3'	4'	5'
45	.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	.7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	.7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	.7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	.7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	.7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	.7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	.7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	.8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	.8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	.8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	.8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	.8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	.8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	.8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	.8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	.8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	.9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	.9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	.9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	.9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	.9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	.9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	.9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	.9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	.9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	.9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	.9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	.9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	.9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	.9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	.9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	.9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	.9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	.9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	.9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	.9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0

NATURAL TANGENTS

Degree	0 0.0°	6 0.1°	12 0.2°	18 0.3°	24 0.4°	30 0.5°	36 0.6°	42 0.7°	48 0.8°	54 0.9°	Main Deifferences 1	 2	 3	 4	 5
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	.0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	.0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	.1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	.1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	.1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	.1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	.1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	.2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	.2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	.2493	2512	2530	2549	2568	2686	2605	2623	2642	2661	3	6	9	12	16
15	.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	.2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	.3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	.3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	.3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	.3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	.4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	.4245	4265	8286	4307	4327	4748	4369	4390	4411	4431	3	7	10	14	17
24	.4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	.4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	.5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	.5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	.5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	.6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	.6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	.6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	.6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	.7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	.7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	.7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	.8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	.8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	.9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	.9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degree	0 0.0°	6 0.1°	12 0.2°	18 0.3°	24 0.4°	30 0.5°	36 0.6°	42 0.7°	48 0.8°	54 0.9°	Main Deifferences 1	 2	 3	 4	 5
45	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1.0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1.0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1.1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1.2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1.3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1.4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1.5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1.6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1.7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1.8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1.8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1.9626	9711	9797	9883	9970	2.0057	2.0145	2.0233	2.0323	2.413	15	29	44	58	73
64	2.0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2.1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2.2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2.3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2.4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2.6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2.7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2.9042	9208	9375	9544	9714	9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3.0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3.2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3.4824	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3.7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4.0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4.3315	3662	4015	4374	4737	5107	5483	5864	6252	6646	Mean Differences no longer sufficiently accurate				
78	4.7046	7453	7867	8288	8716	9152	9594	5.0045	5.0504	5.0970					
79	5.1446	1929	2422	2924	3465	3955	4486	5026	5578	6140					
80	5.6713	7297	7894	8502	9124	9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	3859	4596	5350	6122	6912	7720	8548	9395	7.0264					
82	7.1154	2066	3002	3962	4997	5958	6996	8062	9158	8.0285					
83	8.1443	2636	3863	5126	6427	7769	9152	9.0579	9.2052	9.3572					
84	9.514	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					