SEQUENCE AND SERIES

1. DEFINITION

When the terms of a sequence or series are arranged under a definite rule then they are said to be in a Progression.

2. ARITHMETIC PROGRESSION (A.P.)

Arithmetic Progression is defined as a series in which difference between any two consecutive terms is constant throughout the series. This constant difference is called common difference. If 'a' is the first term and 'd' is the common difference, then an AP can be written as $a + (a + d) + (a + 2d) + (a + 3d) + \dots$

Note: If a,b,c, are in AP \Leftrightarrow 2b = a + c 2.1 General Term of an AP

General term (nth term) of an AP is given by $T_n = a + (n-1) d$

Note:

(i) General term is also denoted by ℓ (last term)
(ii) n (No. of terms) always belongs to set of natural numbers.

(iii) n^{th} term from end is given by = m - (n- 1) d or = (m - n + 1) th term from

beginning where m is total no. of terms.

2.2 Sum of n terms of an A.P. The sum of first n terms of an A.P. is given by

$$S_n = \frac{n}{2} [2a + (n-1) d]$$
 or
 $S_n = \frac{n}{2} [a + T_n]$

Note:

(i) If sum of n terms S_n is given then general term $T_n = S_n - S_{n-1}$ where S_{n-1} is sum of (n-1) terms of A.P.

(ii) If for two A.P.'s

$$\frac{T_n}{T'_n} = \frac{An+B}{Cn+D}$$

then
$$\frac{S_n}{S'_n} = \frac{A \frac{D+1}{2} + B}{C \frac{D+1}{2} + D}$$

3. SUPPOSITION OF TERMS IN A.P.
(i) When no. of terms be odd then we take three terms as: a - d, a, a+ d five terms are as :
a - 2d, a - d, a, a+ d, a + 2d Here we take middle term as 'a' and common difference as 'd'.

4. SOME PROPERTIES OF AN A.P.

(i) If each term of a given A.P. be increased, decreased, multiplied or divided by some non zero constant number then resulting series thus obtained will also be in A.P.

(ii) In an A.P., the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last term.

(iii) Any term of an AP (except the first term) is equal to the half of the sum of terms equidistant from the term Page # 28

SOLVED PROBLEMS Write first five terms of the sequence given by \Leftrightarrow 5n = 110 \Leftrightarrow n = 22. Ex.1 Hence, 22nd term of the given AP is 109. the rule $a_n = (2n + 1)$ and obtain the corresponding series. How many terms are there in the AP Ex.4 We have, $a_n = (2n + 1)$(i) Sol. 7, 13, 19,...., 205 ? Putting $n = 1, 2, 3, 4, 5, \dots$ successively in Sol. Let the given AP contain n terms. Then, (i), we get : a = 7, d = (13 - 7) = 6 and $a_n = 205$. $a_1 = (2 \times 1 + 1) = 3; a_2 = (2 \times 2 + 1) = 5;$ Now, $a_n = 205 \Leftrightarrow a + (n - 1) d = 205$ $a_2 = (2 \times 3 + 1) = 7;$ \Leftrightarrow 7 + (n - 1) × 6 = 205 $a_4 = (2 \times 4 + 1) = 9$ and $a_5 = (2 \times 5 + 1) = 11$. [:: a = 7, d = 6]Hence, the required sequence is $\Leftrightarrow (n-1) = \frac{198}{6} = 33 \Leftrightarrow n = 34.$ 3, 5, 7, 9, 11, Hence, the given AP contains 34 terms. The corresponding series is 3 + 5 + 7 + 9 + 11 + Ex.5 Is 301 a term of the AP 5, 11, 17, 23, ..? Ex.2 Write first four terms of the sequence given Sol. If possible, let the nth term of given AP be 301. by $a_n = \frac{1}{6}$ (2n - 3) and obtain the Then, a = 5, d = (11 - 5) = 6 and $a_n = 301$ corresponding series. Now, $a_n = 301$ We have, $a_n = \frac{1}{6}(2n - 3)$. \Leftrightarrow a + (n - 1) d = 301 Sol. \Leftrightarrow 5 + (n - 1) × 6 = 301 Putting $n = 1, 2, 3, 4, \dots$ successively in (i), we get : [::a = 5, d = 6] $a_1 = \frac{1}{6}(2 \times 1 - 3) = \frac{-1}{6}; a_2 = \frac{1}{6}(2 \times 2 - 3) = \frac{1}{6};$ $\Leftrightarrow 6(n-1) = 296$ \Leftrightarrow (n - 1) = $\frac{148}{3}$ $a_3 = \frac{1}{6}(2 \times 3 - 3) = \frac{3}{6} = \frac{1}{2}; a_4 = \frac{1}{6}(2 \times 4 - 3) = \frac{5}{6};$ \Leftrightarrow n = $\left(\frac{148}{3} + 1\right) = \frac{151}{3}$ Hence, the required sequence is $\frac{-1}{6}$, $\frac{1}{6}$, $\frac{1}{2}$, $\frac{5}{6}$, $\Leftrightarrow n = \left(\frac{148}{3} + 1\right) = \frac{151}{3} = 53\frac{1}{3}$ The corresponding series is $-\frac{1}{6}+\frac{1}{6}+\frac{1}{2}+\frac{5}{6}+\ldots$ But, $53\frac{1}{2} \notin N$. Hence, 301 is not a term of the given AP. Which term of the AP 4,9, 14, 19, is 109? Ex.3 Ex.6 Find three numbers in AP whose sum is 24 and Here a = 4 and d = (9 - 4) = 5. Sol. whose product is 440. Let the nth term of the given AP be 109. Then Sol. Let the required numbers be (a - d), $a_{n} = 109.$ a and (a + d). Then, \therefore $a_n = 109 \Leftrightarrow a + (n-1) d = 109$ \Leftrightarrow 4 + (n - 1) × 5 = 109 (a - d) + a + (a + d) = 24[:: a = 4, d = 5] \Leftrightarrow 3a = 24 \Leftrightarrow a = 8.

	Thus, the numbers are		++ 100)- (10 + 20 ++ 100)				
	(8 – d), 8 and (8 + d).		$= \frac{50}{2} [2 \times 2 + (50 - 1) \times 2] + \frac{20}{2} [2 \times 5 + (20 - 1) \times 10] - \frac{10}{2} [2 \times 10 + (10 - 1) \times 10]$				
	Now, $(8 - d) \times 8 \times (8 + d) = 440$						
	$\Leftrightarrow (8-d) (8+d) = 55$						
	$\Leftrightarrow (64 - d^2) = 55 \Leftrightarrow d^2 = 9$		= 50 [2+49] + 10 [10 + 95] - 5 [20 + 90]				
	$\Leftrightarrow d = \pm 3.$		= 51 × 50 + 105 × 10 - 110 × 5 = 3050				
	Hence, the required numbers are	Ex.9	If a^2 , b^2 , c^2 are in A.P. then				
	5, 8, 11 or 11, 8, 3		$\frac{1}{b+c}$, $\frac{1}{c+a}$, $\frac{1}{a+d}$ are in-				
Ex.7 Sol.	If for an A.P. $T_3 = 18$ and $T_7 = 30$ then S_{17} is equal to- (A) 612 (B) 622 (C) 306 (D) None of these Let first term = a, common difference = d	Sol.	(A) A.P. (B) G.P. (C) H.P. (D) None of these a^2 , b^2 , c^2 are in A.P. a^2 + ab + bc + ca, b^2 + bc + ca + ab, c^2 + ca + ab + bc are also in A.P. [adding ab + bc + ca]				
	Then $T_3 = a + 2d = 18$ and $T_7 = a + 6d = 30$ Solving these, $a = 12$, $d = 3$ ∴ $S_{17} = \frac{17}{2} [2a + (17-1)d]$ $= \frac{17}{2} [24 + 16 \times 3] = 612$		or $(a+c) (a+b)$, $(b+c) (a+b)$, $(c+a) (b+c)$ are also in A.P. or $\frac{1}{b+c}$, $\frac{1}{c+a}$, $\frac{1}{a+d}$ are in A.P. [dividing by $(a + b) (b + c) (c + a)$]				
Ex.8 Sol.	The sum of integers in between 1 and 100 which are divisible by 2 or 5 is- (A) 3100 (B) 3600 (C) 3050 (D) 3500 Required sum = (sum of integers divisible by 2) + (sum of integers divisible by 5) – (sum of						
	integers divisible by 2 and 5) = $(2 + 4 + 6 + + 100) + (5 + 10 + 15)$						

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	EXERCISE												
Q.1	Write first	t 4 terr	4 terms in each of the sequences: $(2p - 2)$				Q.14	Find the sum of 100 terms of the AP 0.7, C					
Q.2	(i) $a_n = 5$ Find 23^{rd}	ōn + 2 term d	(ii) a of the AP	$a_n = \frac{(21)}{7, 5, 3}$	4 8, 1,		Q15	Find the sum of the series 101 + 99 + 97 + 47.					
Q.3	Which ter	m of th	ne AP 5, 8	3, 11, ⁻	14,Is 32	20?	Q.16	How many terms of the AP 26, 21, 16, 11, .					
Q.4	How man 16, , 4	ow many terms are there in the AP 10, 13, 6, , 43 ?						Find the sum of 32 terms of an AP whose third term is 1 and the 6^{th} term is -11					
Q.5	Is 302 a t	erm of	the AP 3	8, 8, 13	8, 18, 7	?	0.10						
Q.6	The 5 th ar	nd 13 th Ny Fin	terms of	an AP	AP are 5 and -3		Q. 18	AP 25, 22	m of certa 2, 19, i	s 115, f	ind the last term.		
	term.					Q.19	The sum of n term of two arithmetic progressions are in the ratio $(3n + 8)$: $(7n - 3n)$						
Q.7	How many two-digit numbers are divisible by							15). Find the ratio of their 12^{th}			12^{th} terms.		
Q.8	Find the 15 th term from the end of the AP 3, 5, 7, 9,, 201.						Q.20	A man repays a loan of Rs 3250 by paying Rs 20 in the first month and then increasing it by Rs 15 every month. How long will it take him to					
Q.9	If 7 times the 7^{th} term of an AP is equal to 11 times its 11^{th} term, show that its 18^{th} term is 0.						Q.21	A man saved Rs 16500 in 10 years. In each year after the first year he saved Rs 100 more than he did in the preceding year. How much did he save in the first year? A sum of Rs 62400 is paid off in 30 instalments such that each instalment is Rs 100 more than the preceding instalment. Calculate the first instalment.					
Q.10	Three numbers are in AP. If their sum is 27 and the product 648, find the numbers.												
Q.11	The sum of three consecutive terms of an AP is 21 and the sum of the squares of these terms is 165. Find these terms.						Q.22						
Q.12	Find the sum of 24 terms of the AP 1, 3, 5, 7,							The digits of a three-digit number are ir					
Q.13	Find the sum of 10 terms of the AP $6,5\frac{1}{3},4\frac{2}{3},\dots$							and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.					
ANSWER KEY													
1.	(i) 7, 12,	17, 22	(ii) $\frac{-1}{4}$,	$\frac{1}{4}$, $\frac{3}{4}$,	<u>5</u> 4	2.	-37	3.	106 th	4.	12		
5.	No	6.	9, 8, 7,	6, and	a ₁₆ =-6	7.	13	8.	173	10.	6, 9, 12		
11.	4, 7, 10	12.	576	13.	30	14.	119.	5 15.	2072	16.	11		
17.	-1696	18.	x = −2	19.	7:16	20.	20 months 21. Rs 12				Rs 1200		
22.	Rs. 630	23.	852										