Time allowed: 3 hours   Maximum Marks: 100

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

Section A

1. State whether the functions $\dfrac{a^x + a^{-x}}{2}$ odd or even

2. Prove that $f(x) + f(-x)$ is an even function

3. Write the condition so that the line $y = mx + c$ is a tangent to the circle $x^2 + y^2 = a^2$

4. What is the period of the function $f(x) = 2\sin(3x + 5)$

5. If $A = \{y = \sin x, 0 \le x < \dfrac{\pi}{4}\}\ and\ B = \{y = \cos x, 0 \le x \le \dfrac{\pi}{4}\}$ Then what is $(A \cap B)$

6. Are the functions $f(x) = \dfrac{x}{x^2}\ and\ g(x) = \dfrac{1}{x}$ identical

Section B

7. If $f(x-1) = x^2 - 2x + 1$ Find $f(x)$

8. Find the constant term in the expansion $(x + \dfrac{1}{x})^{12}$

9. Given that $f(x) = x^2 - 2x = 3$ Find all roots of the equation when $(1). f(x) = f(-1)\ and\ (2)\ f(x) = 0$

10. Prove by mathematical induction that for all positive integral values of n,

$$\frac{{}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots + {}^nC_n}{2} \le n!$$

11. Find the domain and range of the function $f(x) = \dfrac{1}{3 - \sin 3x}$

12. Find the limit $\lim\limits_{x \to \infty} \dfrac{a + b^x}{a - b^x}$

13. Solve $\sin 2\theta + \cos 2\theta + \sin\theta + \cos\theta + 1 = 20$, $0 \le x \le 2\pi$

14. Find the inverse of the function $f(x) = 2x - 1$

15. Find the equation of a parabola whose axis is parallel to the y axis and which passes through the points $(0,1),(1,0),(2,0)$

16. Find $\sqrt{2-3i}$

17. Find the maximum value of $a\cos x + b\sin x$

18. Prove that AM of a, b is greater than their GM.

19. A box contains coins and beads. 30% of the objects in the box are beads. Coins are either silver or gold. 25 % of the coins are silver. One item is randomly selected what is the probability that it is a gold coin

Section C

20. The sum of all numbers that can be formed with the digits 1,3,4,7 taken all at a time

21. A problem is given to three students whose chance of solving it are $\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}$ What is the probability that the problem gets solved.

22. There are 300 boys who represent a team in sports in summer and in winter. In summer 60 plays cricket and the remainder play tennis. In winter they must play foot ball or Kabadi but not both. 56% of the Kabadi players play cricket in summer and 30% of the cricket players play Kabadi in winter. How many boys play tennis and foot ball ?

23. Differentiate $\log_e x$ from the first principle

24. Find the sum of n terms of the series $\dfrac{1}{1.2} + \dfrac{1}{2.3} + \dfrac{1}{3.4} + \dfrac{1}{4.5} + \ldots\ldots + \dfrac{1}{n.(n+1)}$

25. Find the equation of a circle whose diameter is on the line $y - 2x + 3 = 0$ and passes through two points $(-1,0),(2,1)$

26. Calculate the mean deviation from the median for the following data

x_i	10	15	20	25	30	35	40	45
f_i	9	5	8	5	5	8	3	9