ગુજરાત રાજયના શિક્ષણવિભાગના પત્ર-ક્રમાંક
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STATISTICS

(Part 2)

Standard 12

PLEDGE

India is my country.
All Indians are my brothers and sisters.
I love my country and I am proud of its rich and varied heritage.
I shall always strive to be worthy of it.
I shall respect my parents, teachers and all my elders and treat everyone with courtesy.
I pledge my devotion to my country and its people.
My happiness lies in their well-being and prosperity.

રાજ્ય સરકારની વિનામૂલ્યે યોજના હેઠળનું પુસ્તક

Gujarat State Board of School Textbooks
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PREFACE

Gujarat State Board of School Textbooks has prepared new textbooks as per the new curricula developed by the Gujarat State Secondary and Higher Secondary Education Board and which has been sanctioned by the Education Department of the Government of Gujarat. A panel of experts from Universities/Colleges, Teachers Training Colleges and Schools have put lot of efforts in preparing the manuscript of the subject. It is then reviewed by another panel of experts to suggest changes and filter out the mistakes, if any. The suggestions of the reviewers are considered thoroughly and necessary changes are made in the manuscript. Thus, the Textbook Board takes sufficient care in preparing an error-free manuscript. The Board is vigilant even while printing the textbooks.

The Board expresses the pleasure to publish the Textbook of **Statistics (Part 2)** for **Std. 12** which is a translated version of Gujarati. The Textbook Board is thankful to all those who have helped in preparing this textbook. However, we welcome suggestions to enhance the quality of the textbook.

H. N. Chavda
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Date : 15-02-2017

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FUNDAMENTAL DUTIES

It shall be the duty of every citizen of India : *

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers and wild life, and to have compassion for living creatures;

(h) to develop scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

(k) to provide opportunities for education by the parent, the guardian, to his child, or a ward between the age of 6-14 years as the case may be.

*Constitution of India : Section 51-A

CONTENTS

●

"Statistically, the probability of any one of us being here is so small that the mere fact of our existence should keep us all in a state of contented dazzlement."

– Lewis Thomas

1

Probability

Contents :

1.1 Introduction

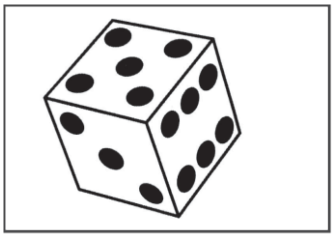
Many events occur in our day-to-day life. We can definitely say for many events that these events will certainly happen. For example, each person taking birth will die, a fruit freely falling from a tree will fall on ground, if the profit per item of a trader is ₹ 10 then he will earn a profit of ₹ 500 by selling 50 items, if a person invests ₹ 1,00,000 in a nationalised bank at an annual interest rate of 7.5 percent then the interest received will be ₹ 7,500, etc. These events are certain but some events are such that we can not be definitely say in advance whether they will happen. For example, getting head on the upper side after tossing a balanced coin, getting number 3 on the upper side of a die when a six faced unbiased die is thrown, the new baby to be born will be a boy, an item produced in a factory is non-defective, what will be the total rainfall in a certain region in the current year, what will be the wheat production in a state in the current year, what will be the result of a cricket match played between teams of two countries, etc. We cannot say with certainty that these events will definitely occur. It is not possible to give precise prediction about the occurrance of such events. We can intuitively get some idea about possibility of happening (or not happening) for these events but there is uncertainty regarding happening (or not happening) of these events. We accept that the occurrence (or non-occurrence) of these events depends upon an unknown element which is called chance. Such events which depend on chance are called random events. Probability is used to numerically express the possibility of these uncertain events. We shall study the theory of probability, the classical definition of probability, its statistical definition and the illustrations showing utility of probability. Now, let us see the explanation of certain terms which are useful to study probability.

1.2 Random Experiment and Sample Space

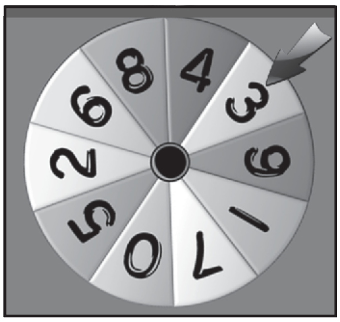
1.2.1 Random Experiment

Let us consider the following experiments :

Experiment 1 : Toss a balanced coin. Any one outcome is obtained out of two posible outcomes (i) Head-H (ii) Tail-T for this experiment. (We assume that the coin does not stand on its edge.) Thus, *'H'* and *'T'* are the only possible outcomes for the experiment of tossing a coin. But which of the outcomes will be obtained among these two outcomes cannot be said with certainty before conducting the experiment.



Experiment 2 : Throw a balanced die with six faces marked with numbers 1, 2, 3, 4, 5, 6 on it. Note the number appearing on its upper face. Any one outcome among the six possible outcomes 1, 2, 3, 4, 5, 6 will be obtained. There are only six possible outcomes 1, 2, 3, 4, 5, 6 for this experiment of throwing a die but which of the six outcomes will be obtained cannot be said with certainty before conducting the experiment.



Experiment 3 : Suppose there is a wheel marked with 10 numbers 0, 1, 2, ... , 9 and a pointer is kept against it. If this wheel is rotated with hand, it will spin and become stable after some time. When the wheel stops, any one of the numbers 0, 1, 2, ... , 9 will appear against the pointer. This number is the winning number. There are total ten possible outcomes 0, 1, 2, ... , 9 for this experiments. But which of the ten numbers will be obtained as a winning number cannot be said with certainty before conducting the experiment.

The experiments 1, 2, 3 shown above are called random experiments. A random experiment is defined as follows. The experiment which can be independently repeated under indentical conditions and all its possible outcomes are known but which of the outcomes will appear cannot be predicted with certainty before conducting the experiment is called a random experiment. The following characteristics of the random experiment can be deduced from its definition :

(1) A random experiment can be independently repeated under almost identical conditions.

(2) All possible outcomes of the random experiment are known but which of the outcomes will appear cannot be predicted before conducting the experiment.

(3) The random experiment results into a certain outcome.

1.2.2 Sample Space

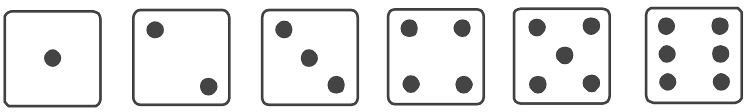
The set of all possible outcomes of a random experiment is called a sample space of that random experiment. The sample space is generally denoted by U or S. The elements of sample space are called sample points.

The sample space of the random experiment in the earlier discussion can be obtained as follows :

Experiment 1 : Toss a balanced coin. There are total two possible outcomes for this random experiment : H and T. Thus, the Sample Space can be written here as $U = \{H, T\}$ or $U = \{T, H\}$.

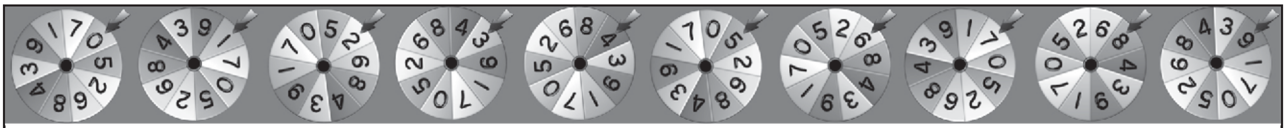
Head - H   Tail - T

Experiment 2 : Throw a balanced die with six faces with numbers 1, 2, 3, 4, 5, 6 on it. There are total six possible outcomes for this random experiment : 1, 2, 3, 4, 5, 6. Thus, the sample space is $U = \{1, 2, 3, 4, 5, 6\}$.



1 2 3 4 5 6

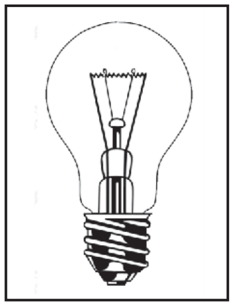
Experiment 3 : To decide the winning number by rotating a wheel marked with numbers 0, 1, 2, ... , 9. There are total ten possible outcomes for this random experiment. Thus, the sample space $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.



0 1 2 3 4 5 6 7 8 9

Finite Sample Space : If the total number of possible outcomes in the sample space is finite then it is called a finite sample space. For example, the sample spaces of all the three random experiments given above are finite sample spaces.

Infinite Sample Space : If the total number of possible outcomes in the sample space of a random experiment is infinite then it is called an infinite sample space. For example, if the life of electric bulbs (L) from a production is recorded in hours then it is a real number. The value of L will be 0 or more. Thus, there will be infinite possible outcomes for an experiment of measuring life of bulbs. The sample space will be $U = \{L \mid L \geq 0,\ L \in R\}$. If the maximum life of electric bulbs is assumed to be 700 hours, the sample space will be $U = \{L \mid 0 \leq L \leq 700;\ L \in R\}$ which is an Infinite sample space.



Now we shall see some more illustrations of sample space of a random experiment.

Illustration 1 : Two balanced coins are tossed simultaneously. Write the sample space of this random experiment.

We shall consider any one of the two coins here as the first coin and the other as the second coin. The outcome of this experiment will be as shown in the following diagram.

If we denote the head as H and the tail as T, the sample space will be as follows :

$U = \{HH,\ HT,\ TH,\ TT\}$

Any one of the outcomes out of H and T can be obtained on the first coin. Thus, this action can be done in two ways and the other coin can also show one of the outcomes H and T which can also be done in two ways. According to the fundamental principle of counting for multiplication, the total number of outcomes will be $2 \times 2 = 2^2 = 4$. It should be noted here that the sample space for the experiment of tossing one balanced coin two times will also be the same as above.

Illustration 2 : Two balanced dice are thrown where each die has numbers 1 to 6 on the six sides. Write the sample space of this experiment.

We shall consider any one die as the first die and the other will be called the second die. The number on the first die will be shown as i and the number on the second die will be shown as j. The following sample space will be obtained by denoting the pair of numbers on the two dice as $(i,\ j)$ where $i,\ j = 1, 2, 3, 4, 5, 6$.

$$U = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

OR

$$U = \{(i,\ j);\ i, j = 1, 2, 3, 4, 5, 6\}$$

Any one of the integers 1 to 6 can be shown on the upper side of the first die which can occur in 6 ways and the second die can also show one of the integers among 1 to 6 which will also occur in 6 ways. The total number of outcomes will be $6 \times 6 = 6^2 = 36$ according to fundamental principle of counting for multiplication. Similarly, the sample space for the random experiment of throwing three balanced dice simultaneously will have $6^3 = 216$ total outcomes.

Illustration 3 : Write the sample space of the random experiment of finding the number of defective items while testing the quality of 1000 items produced in a factory.

If the defective items are found among 1000 items produced in the factory then the number of defective items in the production can be 0, 1, 2, ... , 1000. Thus, the sample sapce will be as follows :

$$U = \{0, 1, 2, ..., 1000\}$$

Illustration 4 : Write the sample space of random experiment of randomly selecting three numbers from the first four natural numbers.

If three numbers are selected simultaneously from the first four natural numbers 1, 2, 3, 4 then those three numbers can be $(1, 2, 3)$, $(1, 2, 4)$, $(1, 3, 4)$ or $(2, 3, 4)$. Thus, the sample space of the random experiment will be as follows :

$$U = \{(1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4)\}$$

3 numbers are to be selected here from the 4 numbers which has ${}^4C_3 = 4$ combinations. Thus, the total number of outcomes for this random experiment is 4.

Illustration 5 : Write the sample space of a random experiment of randomly selecting any one number from the natural numbers.

The natural numbers are 1, 2, 3, ... If one number is randomly selected from these numbers then the sample space will be as follows :

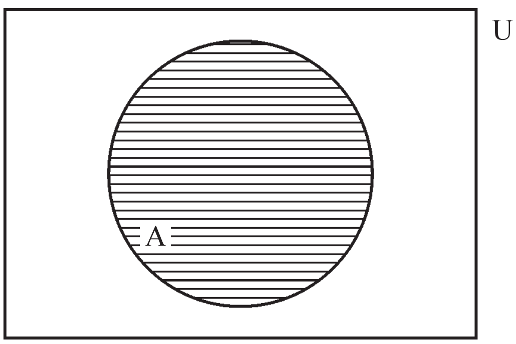
$$U = \{1, 2, 3, 4, ...\}$$

It should be noted here that this is an infinite sample space.

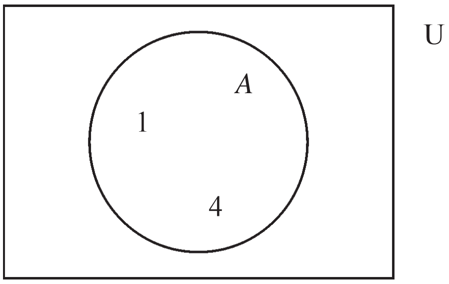
1.3 Events : Certain Event, Impossible Event, Special Events

We will study the different types of events by first understanding the meaning of an event.

(1) Event : A subset of the sample space of a random experiment is called an event. The events are generally denoted by letters *A, B, C,* ... or as A_1, A_2, A_3, ... The set formed by the sample points showing favourable outcomes of an event *A* will be a subset of the sample space *U*. Thus, any event *A* associated with the random experiment is the subset of sample space *U*. This is denoted as $A \subset U$.



For example, the sample space of a random experiment of throwing a balanced die is $U = \{1, 2, 3, 4, 5, 6\}$. If the event of obtaining a complete square as a number on the upper side of the die is denoted by A then event $A = \{1, 4\}$.



Now, we shall show that an event is a subset of the sample space by taking a few examples of events in the random experiment of throwing two balanced dice.

- A_1 = the sum of numbers on the dice is 6.

  $\therefore A_1 = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$

- A_2 = the numbers on the dice are same.

  $\therefore A_2 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

- A_3 = the sum of numbers on the dice is more than 9.

  $\therefore A_3 = \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

All these subsets are called events.

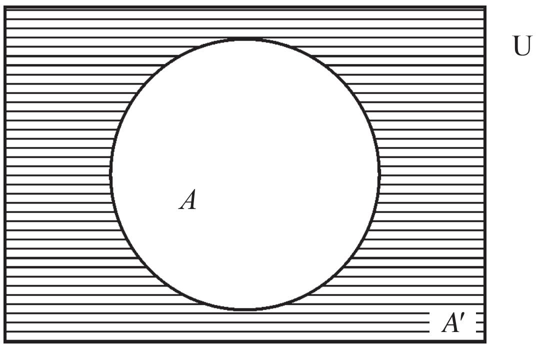
(2) Impossible Event : The special subset ϕ or $\{\ \}$ of the sample space of a random experiment is called an impossible event. Impossible event is an event which never occurs. It is denoted by ϕ or $\{\ \}$.

For example, the event of getting both head (*H*) and tail (*T*) on a balanced coin is an impossible event.

(3) Certain Event : The special subset U of the sample space of random experiment is called a certain event. The certain event is an event which always occurs. It is denoted by U.

For example, the day next to Saturday is Sunday, the number on the upper side die when a balanced die is thrown is less than 7, etc. are certain events.

(4) Complementary Event : Suppose U is a finite sample space and A is one of its events. The set of all the outcomes or elements of U which are not in the event A is called as complementary event of A. The complementary event of event A is denoted by A', $\overline{A}$, A^c. We will use the notation A' for complementary event of A.



A' = Complementary event of event A.

= Non-occurrence of event A.

= $U - A$.

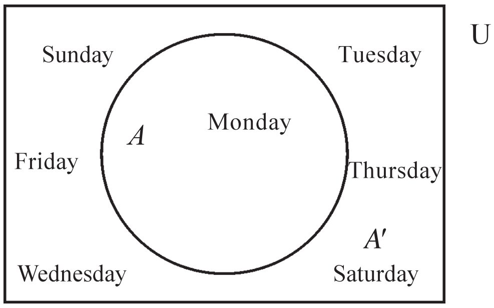
For example, the sample space of the random experiment of finding the day when a cargo ship will reach port Y after leaving from port X will be as follows.

U = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

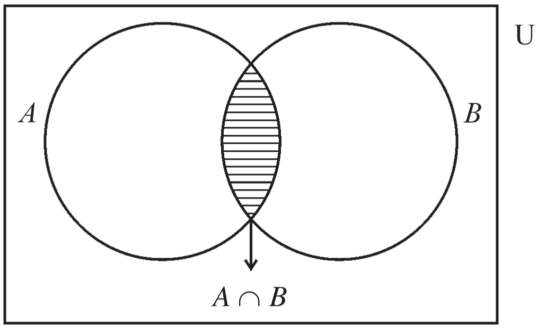
Suppose A denotes that this ship reaches port Y on Monday. Then the set of days except Monday will be the set of outcomes of event A'.

A = {Monday}

$A' = U - A$ = {Sunday, Tuesday, Wednesday, Thursday, Friday, Saturday}



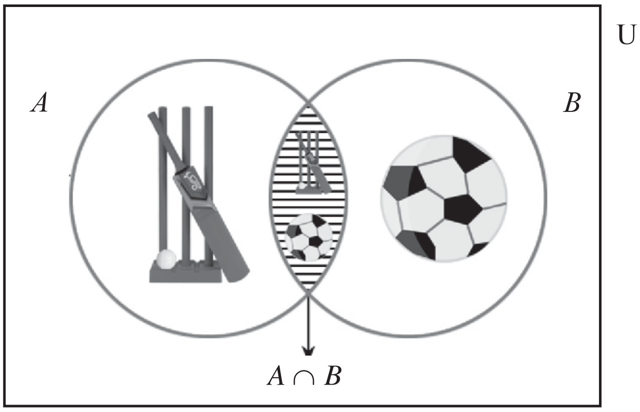
(5) Intersection of Events : Suppose A and B are two events of a finite sample space U. The event where events A and B occur simultaneously is called the intersection of two events A and B. It is denoted by $A \cap B$.



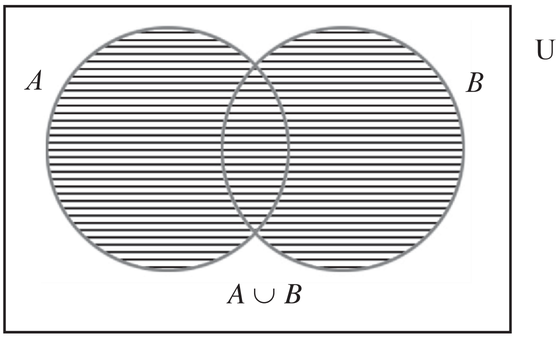
$A \cap B$ = Intersection of two events A and B

= Simultaneous occurrence of events A and B

For example, some of the students studying in a class of a school are the members of school cricket team and some students are members of school football team. Let us denote the event that a student is a member of cricket team by event A and the event that a student is a member of football team by B. If one student is randomly selected from this class then the event that the student is a member of school cricket and football team is called $A \cap B$, the intersection of events A and B.



(6) Union of Events : Suppose A and B are any two events of a finite sample space U. The event where the event A occurs or the event B occurs or both the events A and B occur is called the union of events A and B. It is denoted by $A \cup B$.

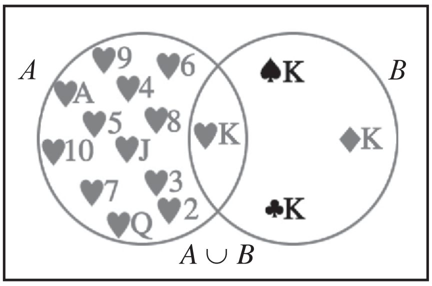


$A \cup B$ = Union of events A and B

= Event A occurs or event B occurs or both events A and B occur together

= At least one of the event A and B occurs.

For example, the event $A \cup B$ of getting heart card (say event A) or a king (say event B) when a card is drawn randomly from a pack of 52 cards will be as follows :



$$A = \{H_A, H_2, H_3, H_4, H_5, H_6, H_7, H_8, H_9, H_{10}, H_J, H_Q, H_K\}$$

$$B = \{S_K, D_K, C_K, H_K\}$$

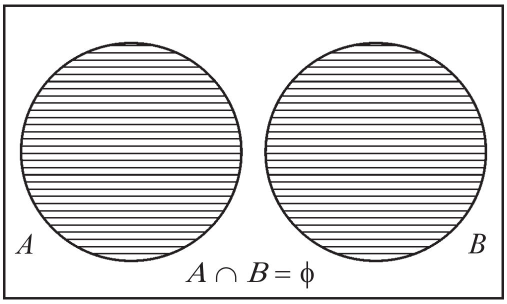
$$A \cup B = \{H_A, H_2, H_3, H_4, H_5, H_6, H_7, H_8, H_9, H_{10}, H_J, H_Q, H_K, S_K, D_K, C_K\}$$

Thus, the occurrence of $A \cup B$ is selecting any one of these 16 cards.

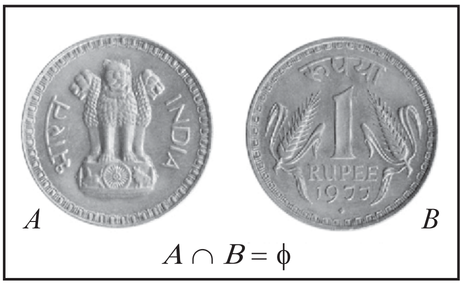
The suits and types of cards are shown as follows :

Spade - S	Diamond - D	Club - C	Heart - H
Ace - A	King - K	Queen - Q	Jack - J

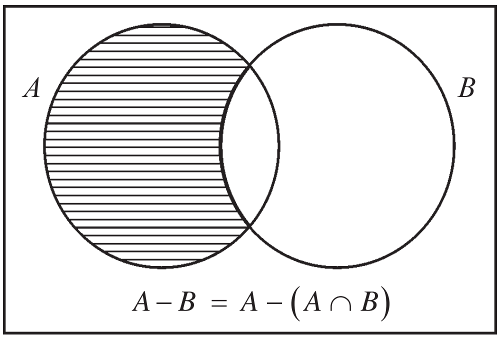
(7) Mutually Exclusive Events : Suppose A and B are any two events of a finite sample space U. Events A and B do not occur together which means $A \cap B = \phi$ or in other words, event B does not occur when event A occurs and event A does not occur when event B occurs then the events A and B are called mutually exclusive events.



For example, toss a balanced coin. Denote the outcome H on the coin as event A and the outcome T on the coin as B. We get $A = \{H\}$ and $B = \{T\}$. It is clear that $A \cap B = \phi$ because when we get H in a trial, it is not possible to get the outcome T in the same trial and vice versa, when we get T in a trial, it is not possible to get the outcome H in the same trial in the random experiment of tossing a balanced coin. Thus, these two events cannot occur simultaneously.



(8) Difference Event : Suppose A and B are any two events of a finite sample space U. The set of elements or outcomes where event A happens but event B does not happen is called the difference of events A and B. It is denoted by $A - B$. It is clear from the venn diagram given here that



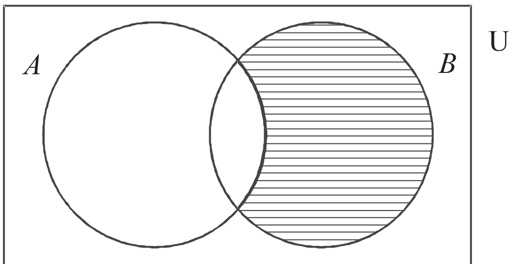
$$A - B = A \cap B' = A - (A \cap B) = (A \cup B) - B$$

$A - B$ = Difference of events A and B

= Event A happens but event B does not happen

= Only A happens out of events A and B.

Similarly, for two events A and B of a finite sample space U, the set of elements or outcomes where B happens but A does not happen is called as the difference of event B and event A. It is denoted by $B - A$. It is clear from the venn diagram given here that,



$B - A = B - (A \cap B)$

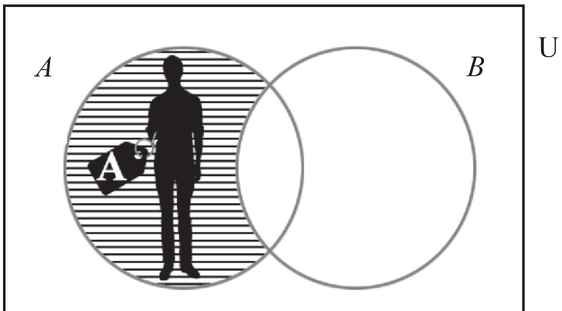
$$B - A \;=\; A' \cap B \;=\; B - (A \cap B) \;=\; (A \cup B) - A$$

$B - A$ = Difference of events B and A

= Event B happens but event A does not happen

= Only event B happens out of events A and B

For example, two employees A and B among the employees working in an office are friends. Denote the presence of employee A in the office as event A and the presence of employee B in the office as event B. On a ctertain day, if it is said that only the employee A is present in the office out of employees A and B then it is clear that among two employees A and B, employee A is present but employee B is not present. Thus, it is called the difference of two events $A - B$ for events A and event B. Here,

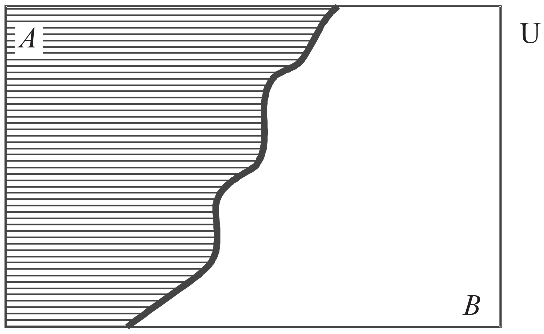


$A - B = A - (A \cap B)$

$A - B$ = Only employee A is present in the office among the employees A and B

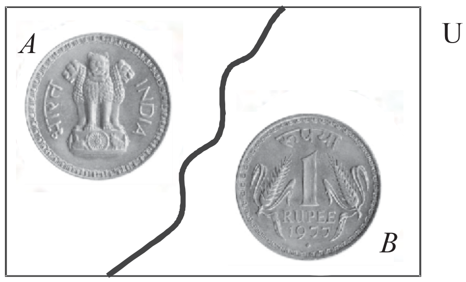
$B - A$ = Only employee B is present in the office among the employees A and B

(9) Exhaustive Events : If the group of favourable outcomes of events of random experiment is the sample space then the events are called exhaustive events. Suppose A and B are any two events of a sample space U. The events A and B are called the exhaustive events if the union $A \cup B$ of the two events A and B is the sample space U, that is $A \cup B = U$.



For example, denote the outcome H as event A and the outcome T as event B when a balanced coin is tossed. It is clear in this case that $A = \{H\}$, $B = \{T\}$ and $A \cup B = \{H, T\} = U$.

$\therefore$ A and B are exhaustive events.



(10) Mutually Exclusive and Exhaustive Events : Suppose A and B are two events of a finite sample space U. These two events A and B are called the mutually exclusive and exhaustive events if $A \cap B = \phi$ and $A \cup B = U$. It should be noted here that all the mutually exclusive events need not be exhaustive events and similarly, all the exhaustive events need not be the mutually exclusive events.

For example, consider the sample space $U = \{1, 2, 3, 4, 5, 6\}$ of the experiment of throwing a balanced die. Let the event A = getting odd number on the die $= \{1, 3, 5\}$ and event B = getting even number on the die $= \{2, 4, 6\}$. It is clear that $A \cap B = \phi$ and $A \cup B = U$. Thus, the events A and B are mutually exclusive and exhaustive.

(11) Elementary Events : The events formed by all the subsets of single elements of the sample space U of a random experiment are called the elementary events. The elementary events are mutually exclusive and exhaustive.

For example, consider the sample space $U = \{H, T\}$ for the random experiment of tossing a balanced coin. The events $A = \{H\}$ and $B = \{T\}$ having single elements are the elementary events. Since $A \cap B = \phi$ and $A \cup B = U$ in this case, it can be said that the elementary events are mutually exclusive and exhaustive.

Illustration 6 : There are 3 yellow and 2 pink flowers in a basket. One flower is randomly selected from this basket. Denote the selection of yellow flower as an event *A* and the selection of pink flower as the event *B*. Find the sets representing the following events and answer the given questions.

(1) U **(2)** A **(3)** B **(4)** A' **(5)** B' **(6)** $A \cap B$ **(7)** $A \cup B$ **(8)** $A \cap B'$ **(9)** $A' \cap B$

(10) State the elementary events of the sample space for this random experiment.

(11) Can it be said that the events *A* and *B* are mutually exclusive events ? Give reason.

(12) Can it be said that the events *A* and *B* are exhaustive events ? Give reason.

We will denote the 3 yellow flowers in the basket as Y_1, Y_2, Y_3 and the 2 pink flowers as P_1, P_2. The sets representing the required events will be as follows :

(1) $U = \{Y_1, Y_2, Y_3, P_1, P_2\}$

(2) $A = \{Y_1, Y_2, Y_3\}$

(3) $B = \{P_1, P_2\}$

(4) $A' = U - A = \{Y_1, Y_2, Y_3, P_1, P_2\} - \{Y_1, Y_2, Y_3\}$

$= \{P_1, P_2\}$

(5) $B' = U - B = \{Y_1, Y_2, Y_3, P_1, P_2\} - \{P_1, P_2\}$

$= \{Y_1, Y_2, Y_3\}$

(6) $A \cap B = \{Y_1, Y_2, Y_3\} \cap \{P_1, P_2\}$

$= \phi$

(7) $A \cup B = \{Y_1, Y_2, Y_3\} \cup \{P_1, P_2\}$

$= \{Y_1, Y_2, Y_3, P_1, P_2\}$

(8) $A \cap B' = \{Y_1, Y_2, Y_3\} \cap \{Y_1, Y_2, Y_3\}$

$= \{Y_1, Y_2, Y_3\}$

OR

$A \cap B' = A - (A \cap B)$

$= \{Y_1, Y_2, Y_3\} - \phi$

$= \{Y_1, Y_2, Y_3\}$

(9) $A' \cap B = \{P_1, P_2\} \cap \{P_1, P_2\}$

$= \{P_1, P_2\}$

OR

$A' \cap B = B - (A \cap B)$

$= \{P_1, P_2\} - \phi$

$= \{P_1, P_2\}$

(10) The elementary events are the subsets with one element. If we denote the different elementary events as $E_1, E_2, E_3, ...$ then

$E_1 = \{Y_1\}, \ E_2 = \{Y_2\}, \ E_3 = \{Y_3\}, \ E_4 = \{P_1\}, \ E_5 = \{P_2\}$

(11) The events A and B can be called mutually exclusive events because according to the definition of mutually exclusive events, the events A and B are called the mutually exclusive events if $A \cap B = \phi$. It can be seen from the answer to the question 6 that $A \cap B = \phi$.

(12) The events A and B can be called exhaustive events because according to the definition of exhaustive events, the A and B are called the exhaustive events if $A \cup B = U$. It can be seen from the answer to the question 7 that $A \cup B = U$.

Illustration 7 : The events A and B of a random experiment are as follows :

$$\mathbf{A = \{1, 2, 3, 4\}, \quad B = \{-1, 0, 1\}}$$

If the sample space $U = A \cup B$ then find the sets showing the following events.

(1) B' (2) $A' \cap B$ (3) $A - B$

Here, $A = \{1, 2, 3, 4\}$

$B = \{-1, 0, 1\}$

$U = A \cup B = \{1, 2, 3, 4\} \cup \{-1, 0, 1\}$

$= \{-1, 0, 1, 2, 3, 4\}$

(1) $B' = U - B$

$= \{-1, 0, 1, 2, 3, 4\} - \{-1, 0, 1\}$

$= \{2, 3, 4\}$

(2) $A' \cap B = B - (A \cap B)$

First we find $A \cap B$,

$A \cap B = \{1, 2, 3, 4\} \cap \{-1, 0, 1\}$
$= \{1\}$

Alternate Method :

$A' = U - A = \{-1, 0, 1, 2, 3, 4\} - \{1, 2, 3, 4\} = \{-1, 0\}$

$\therefore A' \cap B = \{-1, 0\} \cap \{-1, 0, 1\} = \{-1, 0\}$

Now, $A' \cap B = B - (A \cap B)$

$= \{-1, 0, 1\} - \{1\}$

$= \{-1, 0\}$

(3) $A - B = \{1, 2, 3, 4\} - \{-1, 0, 1\}$
$= \{2, 3, 4\}$

Illustration 8 : One number is randomly selected from the first 50 natural numbers. Find the sets showing the following events.

(1) The number selected is a multiple of 5 or 7.

(2) The number selected is a multiple of both 5 and 7.

(3) The number selected is a multiple of 5 but not a multiple of 7.

(4) The number selected is only a multiple of 7 out of 5 and 7.

If one number is selected from the first 50 natural numbers then the group of all possible outcomes of this experiment, which is the sample space U, is as follows :

$U = \{1, 2, 3, ..., 50\}$

Event A = Selected number is a multiple of 5

$= \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$

Event B = Selected number is a multiple of 7

$= \{7, 14, 21, 28, 35, 42, 49\}$

Now, the required events are as follows :

(1) The event of selecting a number which is a multiple of 5 or 7 = $A \cup B$

$\therefore A \cup B = \{5, 7, 10, 14, 15, 20, 21, 25, 28, 30, 35, 40, 42, 45, 49, 50\}$

(2) The event of selecting a number which is a multiple of both 5 and 7 = $A \cap B$

$\therefore A \cap B = \{35\}$

(3) The event of selecting a number which is a multiple of 5 but not of 7 = $A \cap B'$

$$\therefore \; A \cap B' = A - (A \cap B)$$
$$= \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\} - \{35\}$$
$$= \{5, 10, 15, 20, 25, 30, 40, 45, 50\}$$

(4) The event of selecting a number which is only a multiple of 7 out of 5 and 7 = $A' \cap B$

$$\therefore \; A' \cap B = B - (A \cap B)$$
$$= \{7, 14, 21, 28, 35, 42, 49\} - \{35\}$$
$$= \{7, 14, 21, 28, 42, 49\}$$

Illustration 9 : The events A_1 and A_2 of a random experiment are defined as follows. Find the sets showing union event $A_1 \cup A_2$ and intersection event $A_1 \cap A_2$.

$A_1 = \{x \mid x = -1, 0, 1\}$, $\quad A_2 = \{x \mid x = 1, 2, 3\}$

It is given that $A_1 = \{-1, 1, 0\}$ and $A_2 = \{1, 2, 3\}$.

Union of events $A_1 \cup A_2 = \{-1, 0, 1, 2, 3\}$

Intersection of events $A_1 \cap A_2 = \{1\}$

Illustration 10 : A factory produces screws of different lengths. The length (in cm) of screw is denoted by x. The events A_1 and A_2 are defined as follows in the experiment of finding the length of selected screws. Find the events showing union event $A_1 \cup A_2$ and intersection event $A_1 \cap A_2$.

$A_1 = \{x \mid 0 < x < 1\}$, $\quad A_2 = \{x \mid \frac{1}{2} \le x < 2\}$

If is given that $A_1 = \{x \mid 0 < x < 1\}$ and $A_2 = \{x \mid \frac{1}{2} \le x < 2\}$.

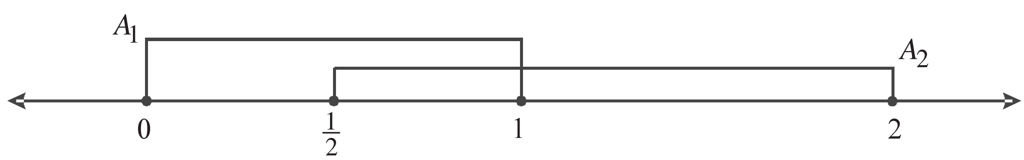
Union of events $A_1 \cup A_2 = \{x \mid 0 < x < 2\}$

$= (0, 2)$ (interval form)

Intersection of events $A_1 \cap A_2 = \{x \mid \frac{1}{2} \le x < 1\}$

$= \left[\frac{1}{2}, 1\right)$ (interval form)

See the following diagram carefully for better explanation of $A_1 \cup A_2$ and $A_1 \cap A_2$.



Exercise 1.1

1. State the sample space for the following random experiments :

(1) A balanced die is thrown three times.

(2) A balanced die with six sides and a balanced coin are tossed together.

(3) Two persons are to be selected from five persons *a, b, c, d, e*.

2. Write the sample space for the marks (in integers) scored by a student appearing for an examination of 100 marks and state the number of sample points in it.

3. Write the sample space for randomly selecting one minister and one deputy minister from four persons.

4. A balanced die in thrown in a random experiment till the first head is obtained. The experiment is terminated with a trial of first head. Write the sample space of this experiment and state whether it is finite or infinite.

5. Write the sample space for the experiment of randomly selecting three numbers from the first five natural numbers.

6. The sample space of a random experiment of selecting a number is $U = \{1, 2, 3, ..., 20\}$.

Write the sets showing the following events :

(1) The selected number is odd number

(2) The selected number is divisible by 3

(3) The selected number is divisible by 2 or 3.

7. One family is selected from the families having two children. The sex (male or female) of the children from this family is noted. State the sample space of this experiment and write the sets showing the following events :

(1) Event A_1 = One child is a female

(2) Event A_2 = At least one child is a female.

8. Two six faced balanced dice are thrown simultaneously. State the sample space of this random experiment and hence write the sets showing the following events :

(1) Event A_1 = The sum of numbers on the dice is 7

(2) Event A_2 = The sum of numbers on the dice is less than 4

(3) Event A_3 = The sum of numbers on the dice is divisible by 3

(4) Event A_4 = The sum of numbers on the dice is more than 12.

9. Two numbers are selected at random from the first five natural numbers. The sum of two selected numbers is at least 6 is denoted by event A and the sum of two selected numbers is even is denoted by event B. Write the sets showing the following events and answer the given questions :

(1) U (2) A (3) B (4) $A \cup B$ (5) $A \cap B$ (6) A' (7) $A - B$ (8) $A' \cap B$

(9) Can it be said that the events A and B are mutually exclusive ? Give reason.

(10) State the number of sample points in the sample space of this random experiment.

10. Three female employees and two male employees are working in an office. One employee is selected from the employees of this office for training. The event that the employee selected for the training is a female is denoted by A and the event that this employee is a male is denoted by B. Find the sets showing the following events and answer the given questions :

(1) U (2) A (3) B (4) $A \cup B$ (5) $A \cap B$ (6) $A' \cap B$

(7) Can it be said that the events A and B are mutually exclusive ? Give reason.

(8) Can it be said that the events A and B are exhaustive ? Give reason.

11. One card is randomly drawn from a pack of 52 cards. If drawing a spade card is denoted by event A and drawing a card from ace to ten (non-face card) is denoted by B then write the sets showing the following events :

(1) U (2) A (3) B (4) $A \cup B$ (5) $A \cap B$ (6) B'

12. The events A_1 and A_2 of a random experiment are as follows. Find the sets showing the union event $A_1 \cup A_2$ and intersection event $A_1 \cap A_2$.

$$A_1 = \{x \mid 0 < x < 5\}, \qquad A_2 = \{x \mid -1 < x < 3, x \text{ is an integer}\}$$

13. The events A_1 and A_2 of a random experiment are as follows. Find the sets showing the union event $A_1 \cup A_2$ and intersection event $A_1 \cap A_2$.

$$A_1 = \{x \mid 2 \le x < 6,\ x \in N\}, \qquad A_2 = \{x \mid 3 < x < 9,\ x \in N\}$$

14. The sample space U of a random experiment and its event A are defined as follows. Find the complementary event A' of A.

$$U = \{x \mid x = 0, 1, 2, \ldots.., 10\}, \qquad A = \{x \mid x = 2, 4, 6\}$$

15. The sample space U of a random experiment and its event A are defined as follows. Find the complementary event A' of A.

$$U = \{x \mid 0 < x < 1\}, \qquad A = \{x \mid \tfrac{1}{2} \le x < 1\}$$

*

After getting aquainted with the random experiment, sample space and different events, we shall now study the probability. We shall begin with the definition of probability.

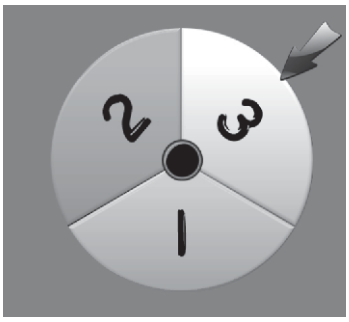
1.4 Mathematical Definition of Probability

To understand the mathematical definition of probability, we shall first understand the two important terms namely equiprobable events and favourable outcomes.

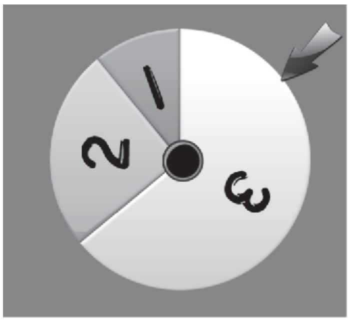
Equiprobable Events : If there is no apparent reason to believe that out of one or more events of a random experiment, any one event is more or less likely to occur than the other events then the events are called as equiprobable events.

For example, a manufacturer of a certain item has two machines M_1 and M_2 in his factory for the production of items. Both the machines produce the same number of items during a day. The lots are made of the produced goods by properly mixing the items produced on both the machines during the day. An item randomly selected from such a lot is made on machine M_1 or machine M_2 are the elementary events which are equiprobable.

Similarly, the wheels A and B marked with numbers 1, 2, 3 as shown in the following pictures are rotated by hand. The number against the pointer is noted down when the wheels stop rotating after some time. It is clear from the picture that all the three numbers on wheel A will come against the pointer are equiprobable events. But the numbers 1, 2 and 3 coming against the pointer for wheel B are not equiprobable events.



Wheel A



Wheel B

Favourable Outcomes : If some outcomes out of all the elementary outcomes in the sample space of random experiment indicate the occurrence of a certain event A then these outcomes are called the favourable outcomes of the event A. For example, a card is drawn from a pack of 52 cards. If event A denotes that the card drawn is a face card then the set of favourable outcomes is as follows :

$$A = \left\{S_K, D_K, C_K, H_K, S_Q, D_Q, C_Q, H_Q, S_J, D_J, C_J, H_J\right\}$$

Thus, 12 outcomes are favourable for event A.

Mathematical Definition of probability : Suppose there are total n outcomes in the finite sample space of a random experiment which are mutually exclusive, exhaustive and equiprobable. If m outcomes among them are favourable for an event A then the probability of the event A is $\frac{m}{n}$. The probability of event A is denoted by $P(A)$.

$$P(A) = \text{Probability of event } A$$

$$= \frac{\text{Favourable outcomes of event } A}{\text{Total number of mutually exclusive, exhaustive and equi-probable outcomes of sample space}}$$

$$= \frac{m}{n}$$

Both the numbers $m(\geq 0)$ and $n(> 0)$ are integers and $m \leq n$. It should be noted here that n can not be zero and infinity. The mathematical definition of probability is also called the classical definition.

The assumptions of the mathematical definition are as follows :

(1) The number of outcomes in the sample space of the random experiment is finite.

(2) The number of outcomes in the sample space of the random experiment is known.

(3) The outcomes in the sample space of the random experiment are equi-probable.

We will accept some of the following important results about probability without proof :

(1) The range for the value of probability $P(A)$ for any event A in the sample space U is 0 to 1. Thus, $0 \leq P(A) \leq 1$.

(2) The probability of an impossible event is zero. Earlier we have denoted an impossible event by ϕ. Hence, $P(\phi) = 0$.

(3) The probability of certain event is always 1. Earlier we have denoted a certain event by U. Hence, $P(U) = 1$.

(4) The probability of complementary event A' of event A in the sample space U is $P(A') = 1 - P(A)$.

(5) If $A \subset B$ for two events A and B in the sample space of a random experiment then

- $P(A) \leq P(B)$
- $P(B - A) = P(B) - P(A)$

(6) For two events A and B in the sample space of a random experiment,

- $P(A \cap B) \leq P(A)$ $[\because A \cap B \subset A]$
- $P(A \cap B) \leq P(B)$ $[\because A \cap B \subset B]$
- $P(A) \leq P(A \cup B)$ $[\because A \subset A \cup B]$
- $P(B) \leq P(A \cup B)$ $[\because B \subset A \cup B]$
- $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$
- $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$
- $P(A - B) = P(A \cap B') = P(A) - P(A \cap B)$
- $P(B - A) = P(A' \cap B) = P(B) - P(A \cap B)$
- $0 \leq P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$

We shall now consider illustrations of finding probability of different events using the mathematical definition.

Illustration 11 : If two balanced coins are tossed, then find the probability of (1) getting one head and one tail and (2) getting at least one head.

The sample space for the random experiment of tossing two balanced coins is as follows :

$U = \{HH, HT, TH, TT\}$

$\therefore$ No. of mutually exclusive, exhaustive and equi-probable outcomes $n = 4$.

(1) If A denotes the event of getting one head H and one tail T then HT and TH are two favourable outcomes of event A. Thus, $m = 2$.

From the mathematical definition of probability,

$$P(A) = \frac{m}{n}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

(2) If B denotes the event of getting at least one head then HT, TH, HH are the favourable outcomes of event B. Hence, the number of favourable outcomes $m = 3$ for even B.

From the mathematical definition of probability,

$$P(B) = \frac{m}{n}$$

$$= \frac{3}{4}$$

Required probability $= \frac{3}{4}$

Illustration 12 : Two balanced dice marked with numbers 1 to 6 are thrown simultaneously. Find the probability that (1) sum of numbers on both the dice is 7 (2) sum of numbers on both the dice is more than 10 (3) sum of number on both the dice is at the most 4 (4) both the dice show same numbers (5) sum of numbers on both the dice is 1 (6) sum of numbers on both the dice is 12 or less.

The sample space for throwing two balanced dice simultaneously is as follows :

$U = \{(i, j) ; i, i, j = 1, 2, 3, 4, 5, 6\}$

$\therefore$ Total number of outcomes $n = 36$.

(1) If A_1 denotes that sum of the numbers on the dice is 7 then there are total 6 outcomes $(1, 6)$, $(2, 5)$, $(3, 4)$, $(4, 3)$, $(5, 2)$, $(6, 1)$ favourable for this event A_1. Thus, the number of favourable outcomes $m = 6$ for event A_1. Probability of event A_1

$P(A_1) = \frac{m}{n}$

$= \frac{6}{36}$

$= \frac{1}{6}$

$\therefore$ Required probability $= \frac{1}{6}$

(2) If A_2 denotes the event that the sum of numbers on two dice is more than 10 then $(5, 6)$, $(6, 5)$, $(6, 6)$ are the favourable outcomes of event A_2. Thus, the number of favourable outcomes $m = 3$ for even A_2. Probability of A_2

$P(A_2) = \frac{m}{n}$

$= \frac{3}{36}$

$= \frac{1}{12}$

Required probability $= \frac{1}{12}$

(3) If A_3 denotes the event that the sum of numbers on two dice is at the most 4 then total 6 outcomes $(1, 1)$, $(1, 2)$, $(1, 3)$, $(2, 1)$, $(2, 2)$, $(3, 1)$ are favourable outcomes of event B. Thus, the number of favourable outcomes $m = 6$ for event A_3. Probability of event A_3

$P(A_3) = \frac{m}{n}$

$= \frac{6}{36}$

$= \frac{1}{6}$

Required probability $= \frac{1}{6}$

(4) Event A_4 = both the dice show the same numbers.

$\therefore$ Total 6 outcomes $(1, 1)$, $(2, 2)$, $(3, 3)$, $(4, 4)$, $(5, 5)$, $(6, 6)$ are favourable for the event A_4. Thus, the number of favourable outcomes $m = 6$ for event A_4. Probability of event A_4

$P(A_4) = \frac{m}{n}$

$= \frac{6}{36}$

$= \frac{1}{6}$

Required probability $= \frac{1}{6}$

(5) Let A_5 be the event that the sum of numbers on two dice is 1. It is obvious that not a single outcome in the sample space is favourable for A_5. Hence, the number of favourable outcomes $m = 0$ for event A_5. Probability of event A_5

$$P(A_5) = \frac{m}{n}$$

$$= \frac{0}{36}$$

$$= 0$$

Required probability = 0

(The probability of impossible event is always 0.)

(6) Let A_6 be the event that the sum of numbers on two dice is 12 or less. It is obvious that all the outcomes in the sample space are favourable for event A_6. Hence, the number of favourable outcomes $m = 36$ for the event A_6. Probability of event A_6

$$P(A_6) = \frac{m}{n}$$

$$= \frac{36}{36}$$

$$= 1$$

Required probability = 1

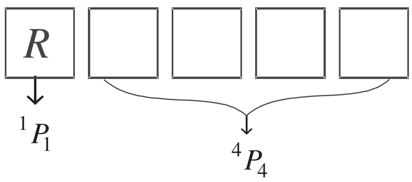
(The probability of certain event is always 1.)

Illustration 13 : Find the probability of getting *R* in the first place in all possible arrangements of each and every letter of the word *RUTVA*.

There are 5 letters *R, U, T, V, A* in the word *RUTVA*. These five letters can be arranged in $^5P_5 = 5! = 120$ different ways. Thus, total number of outcomes $n = 120$.

Event of getting *R* in the first place of the arrangement = *A*.

The favourable outcomes of event *A* are obtained as follows :



R can be arranged in the first place in $^1P_1$ ways and the remaining four letters *U, T, V, A* in the rest of the four places can be arranged in $^4P_4$ ways. According to the fundamental principle of multiplication, there will be $^1P_1 \times {}^4P_4$ arrangements of getting *R* in the first place. Hence, the number of favourable outcomes for event *A* will be

$$m = {}^1P_1 \times {}^4P_4 = 1! \times 4! = 1 \times 24 = 24.$$

Probability of event *A* $P(A) = \frac{m}{n}$

$$= \frac{24}{120}$$

$$= \frac{1}{5}$$

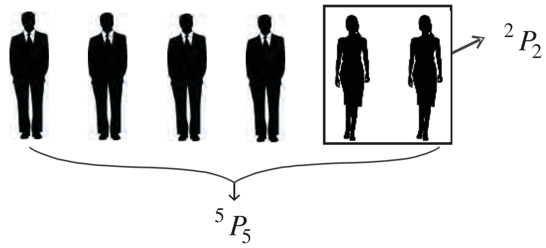
Required probability $= \frac{1}{5}$

Illustration 14 : Four male employees and two female employees working in a government department are sent one by one in turns to the training centre for training. Find the probability that the two female employees go successively for the training.

Total 6 persons, 4 males and 2 females can be sent for training at the training centre one by one in ${}^6P_6 = 6! = 720$ ways. Thus, total number of outcomes will be $n = 720$.

Event of two female employees go successively for training = A.

The favourable outcomes of event A can be obtained as follows :



Considering the two female employees going successively for the training as one person, total 5 persons can be arranged in ${}^5P_5$ ways and two female employees can be arranged among themselves in ${}^2P_2$ ways in each of these arrangements.

Thus, the number favourable outcomes of event A is m

$$= {}^5P_5 \times {}^2P_2$$
$$= 5! \times 2!$$
$$= 120 \times 2$$
$$= 240$$

Probability of event A $P(A) = \frac{m}{n}$

$$= \frac{240}{720}$$

$$= \frac{1}{3}$$

Required probability = $\frac{1}{3}$

Illustration 15 : Find the probability of having 53 Thursdays in a leap year.

There are 366 days in a leap year where we have 52 complete weeks (52×7=364 days) and 2 additional days. Each day appears once in each week and thus each day will appear 52 times in 52 weeks. Now, the additional 2 days can be as follows which gives the sample space for this experiment.

U = {Sunday-Monday, Monday-Tuesday, Tuesday-Wednesday,
Wednesday-Thursday, Thursday-Friday, Friday-Saturday, Saturday-Sunday}

Thus, total number of outcomes will be $n = 7$.

Event A = leap year has 53 Thursdays.

Wednesday-Thursday and Thursday-Friday are the 2 favourable outcomes of event A from the above 7 outcomes. Thus, $m = 2$.

Probability of event A $\quad P(A) = \frac{m}{n}$

$= \frac{2}{7}$

Required probability $= \frac{2}{7}$

Illustration 16 : There are 2 officers, 3 clerks and 2 peons among the 7 employees working in the cash department of a bank. A committee is formed by randomly selecting two employees from the employees of this department. Find the probability that there are

(1) two peons

(2) two clerks

(3) One officer and one clerk among the two employees selected in the committee.

There are 7 employees working in the cash department of the bank. If the employees are randomly selected from them then the total number of mutually exclusive, exhaustive and equi-probable outcomes will be $n = {}^7C_2 = \frac{7 \times 6}{2 \times 1} = 21$.

(1) Event of selecting two peons = A

Selecting 2 peons from the 2 peons and not selecting any employee from the remaining 5 employees will be the favourable outcomes of event A.

The number of such outcomes will be $m = {}^2C_2 \times {}^5C_0 = 1 \times 1 = 1$.

Probability of event A $\quad P(A) = \frac{m}{n}$

$= \frac{1}{21}$

Required probability $= \frac{1}{21}$

(2) Event of selecting two clerks = B

Selecting 2 clerks from the 3 clerks and not selecting any employee from the remaining four employees will be the favourable outcomes of event B.

The number of such outcomes will be $m = {}^3C_2 \times {}^4C_0 = 3 \times 1 = 3$.

Probability of event B $\quad P(B) = \frac{m}{n}$

$= \frac{3}{21}$

$= \frac{1}{7}$

Required probability $= \frac{1}{7}$

(3) Event of selecting one officer and one clerk = C

Selecting 1 officer from 2 officers, one clerk from three clerks and not selecting any peon from two peons will be the favourable outcomes of event C.

The number of such outcomes will be $m = {}^{2}C_1 \times {}^{3}C_1 \times {}^{2}C_0 = 2 \times 3 \times 1 = 6$.

Probability of event C $P(C) = \frac{m}{n}$

$= \frac{6}{21}$

$= \frac{2}{7}$

Required probability $= \frac{2}{7}$

Illustration 17 : A box contains 20 items and 10% of them are defective. Three items are randomly selected from this box. Find the probability that,

(1) two items are defective

(2) two items are non-defective

(3) all three items are non-defective among the three selected items.

There are 20 items wherein 10% that is 20 × 10 % = 2 items are defective and the rest 18 are non-defective. 3 items are selected from this box of 20 items at random. Hence, the total number of outcomes in the sample space will be $n = {}^{20}C_3 = \frac{20 \times 19 \times 18}{3 \times 2} = 1140$.

(1) Event of getting two defective items among three selected items = A

Selecting 2 items from 2 defective items and selecting 1 item from the 18 non-defective items will be the favourable outcomes for the event A.

The number of such outcomes $m = {}^{2}C_2 \times {}^{18}C_1 = 1 \times 18 = 18$.

Probability of event A $P(A) = \frac{m}{n}$

$= \frac{18}{1140}$

$= \frac{3}{190}$

Required probability $= \frac{3}{190}$

(2) Event of getting two non-defective items among three selected items = B

Selecting 2 items from 18 non-defective items and selecting one item from 2 defective items will be the favourable outcomes of the event B.

The number of such outcomes $m = {}^{18}C_2 \times {}^{2}C_1 = 153 \times 2 = 306$.