

Chapter - 12
Surface Areas and Volumes
Exercise No. 12.1

Multiple Choice Questions:

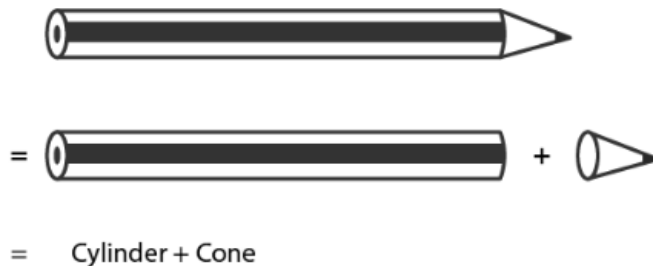
Choose the correct answer from the given four options:

1. A cylindrical pencil sharpened at one edge is the combination of

- (A) a cone and a cylinder
- (B) frustum of a cone and a cylinder
- (C) a hemisphere and a cylinder
- (D) two cylinders.

Solution:

(A) a cone and a cylinder



The Nib of a sharpened pencil = conical shape

And,

The rest of the part of a sharpened pencil = cylindrical

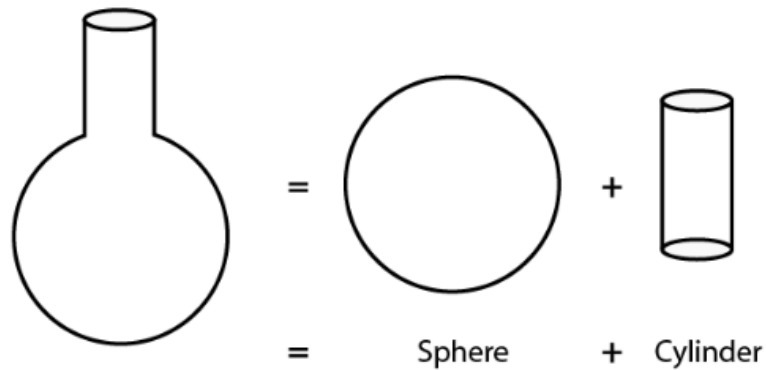
Therefore, a pencil is a combination of cylinder and a cone.

2. A *surahi* is the combination of

- (A) a sphere and a cylinder
- (B) a hemisphere and a cylinder
- (C) two hemispheres
- (D) a cylinder and a cone.

Solution:

(A) a sphere and a cylinder

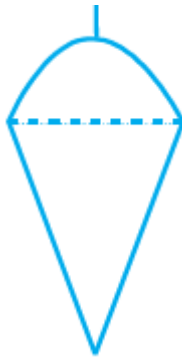


The top part of surahi = cylindrical shape

Bottom part of surahi = spherical shape

Therefore, surahi is a combination of Sphere and a cylinder.

3. A plumblin (sahul) is the combination of



(A) a cone and a cylinder

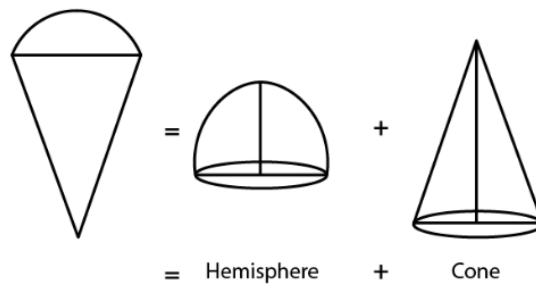
(B) a hemisphere and a cone

(C) frustum of a cone and a cylinder

(D) sphere and cylinder

Solution:

(B) a hemisphere and a cone



The upper part of plumb line = hemispherical,

The bottom part of plumb line = conical

Therefore, it is a combination of hemisphere and cone.

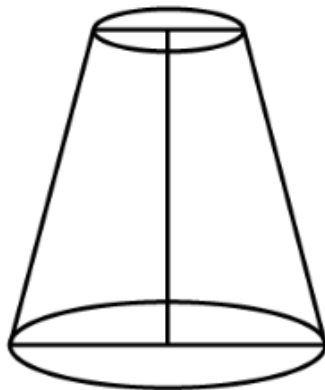
4. The shape of a glass (tumbler) is usually in the form of



- (A) a cone
- (B) frustum of a cone
- (C) a cylinder
- (D) a sphere

Solution:

(B) frustum of a cone



The shape of glass is a frustum or specifically, an inverted frustum.

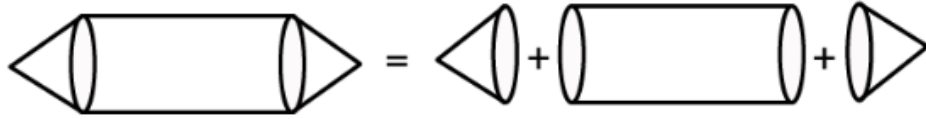
5. The shape of a *gilli*, in the *gilli-danda* game, is a combination of



- (A) two cylinders
- (B) a cone and a cylinder
- (C) two cones and a cylinder
- (D) two cylinders and a cone

Solution:

(C) two cones and a cylinder



The left and right part of a gilli = conical

The central part of a gilli = cylindrical

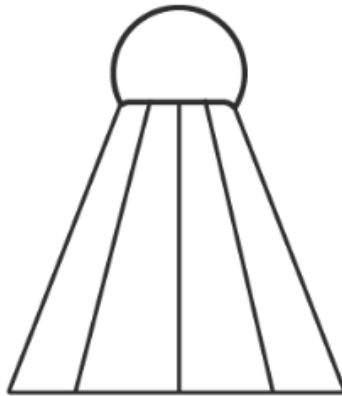
Therefore, it is a combination of a cylinder and two cones.

6. A shuttle cock used for playing badminton has the shape of the combination of

- (A) a cylinder and a sphere**
- (B) a cylinder and a hemisphere**
- (C) a sphere and a cone**
- (D) frustum of a cone and a hemisphere**

Solution:

(D) frustum of a cone and a hemisphere



The cork of a shuttle = hemispherical shapes

The upper part of a shuttle = shape of frustum of a cone.

Therefore, it is a combination of frustum of a cone and a hemisphere.

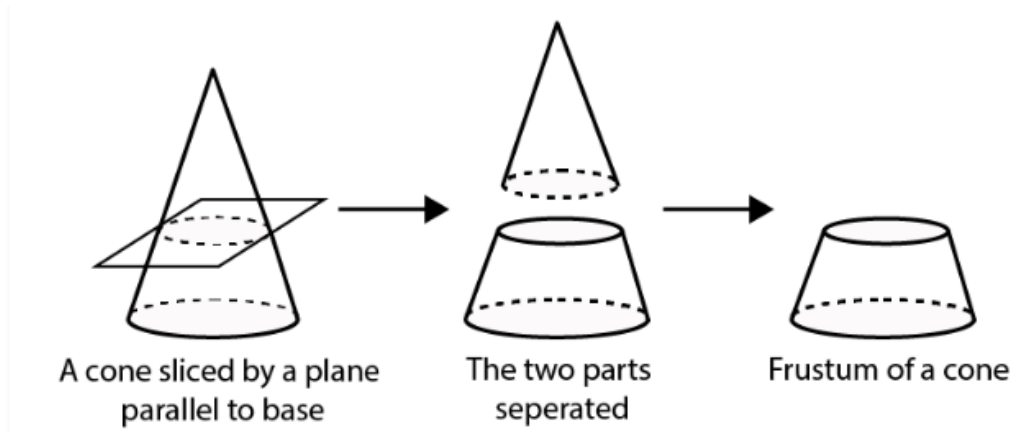
7. A cone is cut through a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is left over on the other side of the plane is called

- (A) a frustum of a cone**

- (B) cone
(C) cylinder
(D) sphere

Solution:

(A) a frustum of a cone



When a cone is divided into two parts by a plane through any point on its axis parallel to its base, the upper and lower parts obtained are cone and a frustum respectively.

8. A hollow cube of internal edge 22cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then the number of marbles that the cube can accommodate is

- (A) 142296
(B) 142396
(C) 142496
(D) 142596

Solution:

(A) 142296

According to the question,

$$\text{Volume of cube} = 22^3 = 10648 \text{ cm}^3$$

$$\begin{aligned} \text{Volume of cube that remains unfilled} &= \frac{1}{8} \times 10648 \\ &= 1331 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume occupied by spherical marbles} &= 10648 - 1331 \\ &= 9317 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Radius of the spherical marble} &= 0.5/2 \\ &= 0.25 \text{ cm} \\ &= 1/4 \text{ cm} \end{aligned}$$

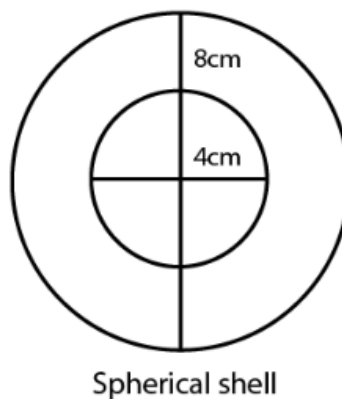
$$\begin{aligned}\text{Volume of 1 spherical marble} &= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{1}{4}\right)^3 \\ &= \frac{11}{168} \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Numbers of spherical marbles, } n &= 9317 \times \left(\frac{11}{168}\right) \\ &= 142296\end{aligned}$$

9. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recast into the form a cone of base diameter 8cm. The height of the cone is

- (A) 12cm
- (B) 14cm
- (C) 15cm
- (D) 18cm

Solution:



Volume of spherical shell = Volume of cone recast by melting

For Spherical Shell,

Internal diameter,

$$d_1 = 4 \text{ cm}$$

Internal radius,

$$r_1 = 2 \text{ cm}$$

External diameter,

$$d_2 = 8 \text{ cm}$$

External radius,

$$r_2 = 4 \text{ cm}$$

Now,

As volume of spherical shell = $\frac{4}{3} \pi (r_2^3 - r_1^3)$

Where r_1 and r_2 are internal and external radii respectively.

$$\begin{aligned}\text{Volume of given shell} &= \frac{4}{3} \pi (4^3 - 2^3) \\ &= \frac{4}{3} \pi (56) \\ &= \left(\frac{224}{3} \right) \pi\end{aligned}$$

We know that,

$$\text{Volume of cone} = \frac{224}{3} \pi \text{ cm}^3$$

For cone,

Base diameter = 8 cm

Base radius, $r = 4$ cm

Let Height of cone = 'h'.

We know,

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h,$$

Where r = Base radius and h = height of cone,

$$\begin{aligned}\text{Volume of given cone} &= \left(\frac{1}{3} \right) \pi 4^2 h \\ \frac{224}{3} \pi &= 16\pi h / 3 \\ 16h &= 224 \\ h &= 14 \text{ cm}\end{aligned}$$

So, Height of cone is 14 cm.

10. A solid piece of iron in the form of a cuboid of dimensions 49cm × 33cm × 24cm, is molded to form a solid sphere. The radius of the sphere is

(A) 21cm

(B) 23cm

(C) 25cm

(D) 19cm

Solution:

As we know,

Volume of cuboid = lbh

Where,

l = length,

b = breadth and

h = height

For given cuboid,

Length,

l = 49 cm

Breadth,

b = 33 cm

Height,

h = 24 cm

Volume of cube = $49 \times (33) \times (24) \text{ cm}^3$

Now,

Let the radius of cube be r.

As volume of sphere = $\frac{4}{3} \pi r^3$ Where r = radius of sphere

Also,

Volume of cuboid = volume of sphere molded

$$49(33)(24) = \frac{4}{3} \pi r^3$$

$$\pi r^3 = 29106$$

$$r^3 = 29106 \times \frac{22}{7}$$

$$r^3 = 9261$$
$$= 21 \text{ cm}$$

Hence, radius of sphere is 21 cm

11. A mason constructs a wall of dimensions 270cm × 300cm × 350cm with the bricks each of size 22.5cm × 11.25cm × 8.75cm and it is assumed that $\frac{1}{8}$ space is covered by the mortar. Then the number of bricks used to construct the wall is

(A) 11100

(B) 11200

(C) 11000

(D) 11300

Solution:

(B) 11200

$$\begin{aligned}\text{Volume of wall} &= 270\text{cm} \times 300\text{cm} \times 350\text{cm} \\ &= 28350000 \text{ cm}^3\end{aligned}$$

$\frac{1}{8}$ space is covered by the mortar,

$$\begin{aligned}\text{Remaining space of wall} &= \text{Volume of wall} - \text{volume of mortar} \\ &= 28350000 - 28350000 \times \frac{1}{8} \\ &= 24806250\end{aligned}$$

Now,

$$\begin{aligned}\text{Volume of 1 brick} &= 22.5\text{cm} \times 11.25\text{cm} \times 8.75\text{cm} \\ &= 2214.844 \text{ cm}^3\end{aligned}$$

So,

Required no. of bricks,

$$\begin{aligned}&= \frac{24806250}{2214.844} \\ &= 11200\end{aligned}$$

12. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is

(A) 4 cm

(B) 3 cm

(C) 2 cm

(D) 6 cm

Solution:

(c)

Given,

Diameter of the cylinder = 2 cm

Radius = 1 cm and

Height of the cylinder = 16 cm

[diameter = 2 x radius]

$$\begin{aligned}\text{Volume of the cylinder} &= \pi \times (1)^2 \times 16 \\ &= 16 \pi \text{ cm}^3\end{aligned}$$

According to question,

Volume of 12 solid spheres = Volume of cylinder

$$\begin{aligned}12 \times \frac{4}{3} \pi r^3 &= 16 \pi \\ r &= 1\end{aligned}$$

Diameter of each sphere, $d=2r$

$$= 2 \times 1$$

$$= 2 \text{ cm}$$

Hence, the required diameter of each sphere is 2 cm.

13. The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm, respectively. The curved surface area of the bucket is

- (A) 4950 cm^2
- (B) 4951 cm^2
- (C) 4952 cm^2
- (D) 4953 cm^2

Solution:

(a)

Given,

The radius of the top of the bucket,

$$R = 28 \text{ cm}$$

The radius of the bottom of the bucket,

$$r = 7 \text{ cm}$$

Slant height of the bucket,

$$l = 45 \text{ cm}$$

Since, bucket is in the form of frustum of a cone.

$$\begin{aligned}\text{Curved surface area of the bucket} &= \pi l (R + r) \\ &= \pi \times 45 (28 + 7) \\ &= 4750 \text{ cm}^2\end{aligned}$$

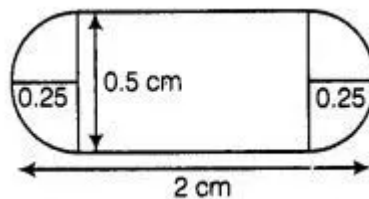
14. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of entire capsule is 2 cm. The capacity of the capsule is

- (A) 0.36 cm^3
- (B) 0.35 cm^3
- (C) 0.34 cm^3
- (D) 0.33 cm^3

Solution:

(A) 0.36 cm^3

Given, diameter of cylinder = Diameter of hemisphere = 0.5 cm



Radius = 0.25 cm

Total length of capsule = 2 cm

So,

Length of cylindrical part of capsule = $2 - (0.25 + 0.25)$

$$= 1.5 \text{ cm}$$

So,

$$\begin{aligned}\text{Capacity of capsule} &= \text{Volume of cylindrical part} + 2 \times \text{volume of hemisphere} \\ &= \pi r^2 h + 2/3 \pi r^3 \\ &= 0.36 \text{ cm}^3\end{aligned}$$

15. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is

- (A) $4\pi r^2$
- (B) $6\pi r^2$
- (C) $3\pi r^2$
- (d) $8\pi r^2$

Solution:

(A) $4\pi r^2$

Because curved surface area of a hemisphere is $2\pi r^2$ and here, we join two solid hemispheres along their bases of radius r , from which we get a solid sphere.

$$\begin{aligned}\text{Hence, the curved surface area of new solid} &= 2\pi r^2 + 2\pi r^2 \\ &= 4\pi r^2\end{aligned}$$

16. A right circular cylinder of radius r cm and height h cm ($h > 2r$) just encloses a sphere of diameter

- (A) r cm
- (B) $2r$ cm
- (C) h cm
- (D) $2h$ cm

Solution:

(B) $2r$ cm

Because the sphere encloses in the cylinder, therefore the diameter of sphere is equal to diameter of cylinder which is $2r$ cm.

17. During conversion of a solid from one shape to another, the volume of the new shape will

- (A) increase
- (B) decrease
- (C) remain unaltered
- (D) be doubled

Solution:

(C) remain unaltered

During conversion of a solid from one shape to another, the volume of the new shape will remain unaltered.

18. The diameters of the two circular ends of the bucket are 44 cm and 24 cm. The height of the bucket is 35 cm. The capacity of the bucket is

- (A) 32.7 litres
- (B) 33.7 litres
- (C) 34.7 litres
- (D) 31.7 litres

Solution:

(A) 32.7 litres

Given,

Diameter of one end of the bucket = 44cm

Radius(R) = 22cm

Diameter of other end of the bucket = 24cm

Radius(r) = 12cm

h= 35cm

Capacity of bucket = Volume of frustum of cone

$$= \frac{1}{3} \pi h (R^2 + r^2 + rR)$$

Putting value of R and r, we get,

Capacity of bucket = 32.7 l

19. In a right circular cone, the cross-section made by a plane parallel to the base is a

- (A) circle
- (B) frustum of a cone
- (C) sphere
- (D) hemisphere

Solution:

(B)

We know that, if a cone is cut by a plane parallel to the base of the cone, then the portion between the plane and base is called the frustum of the cone.

20. Volumes of two spheres are in the ratio 64:27. The ratio of their surface areas is

- (A) 3 : 4
(B) 4 : 3
(C) 9 : 16
(D) 16 : 9

Solution:

(D) 16 : 9

Let the radii of the two spheres are r_1 and r_2 , respectively.

Therefore,

Ratio of volume of spheres:

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\frac{r_1}{r_2} = \frac{4}{3}$$

$$\begin{aligned}\text{Now, Ratio of their surface area} &= \frac{4\pi r_1^2}{4\pi r_2^2} \\ &= \left(\frac{r_1}{r_2}\right)^2 \\ &= \left(\frac{4}{3}\right)^2 \\ &= 16:9\end{aligned}$$

Exercise No. 12.2

Short Answer Questions with Reasoning:

Write 'True' or 'False' and justify your answer in the following:

1. Two identical solid hemispheres of equal base radius r cm are stuck together along their bases. The total surface area of the combination is $6\pi r^2$.

Solution:

False

Explanation:

When two hemispheres are joined together along their bases, a sphere of same base radius is formed.

Curved Surface Area of a sphere = $4\pi r^2$.

2. A solid cylinder of radius r and height h is placed over other cylinder of same height and radius. The total surface area of the shape so formed is $4\pi rh + 4\pi r^2$.

Solution:

False

Explanation:

When one cylinder is placed over another, the base of first cylinder and top of other cylinder will not be covered in total surface area.

We know that,

Total surface area of cylinder = $2\pi rh + 2\pi r^2$

(Where r = base radius and h = height)

Total surface area of shape formed = 2(Total surface of single cylinder) – 2(Area of base of cylinder)

$$= 2(2\pi rh + 2\pi r^2) - 2(\pi r^2)$$

$$= 4\pi rh + 2\pi r^2$$

3. A solid cone of radius r and height h is placed over a solid cylinder having same base radius and height as that of a cone. The total surface area of the combined solid is $\pi r[\sqrt{r^2 + h^2} + 3r + 2h]$.

Solution:

False

Explanation:

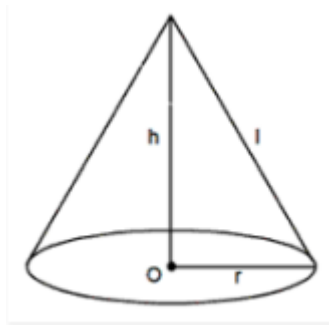
When a solid cone is placed over a solid cylinder of same base radius, the base of cone and top of the cylinder will not be covered in total surface area.

Since the height of cone and cylinder is same,

We get,

Total surface area of cone = $\pi r l + \pi r^2$, (where r = base radius and l = slant height)

Total surface area of shape formed = Total surface area of cone + Total Surface area of cylinder – 2(Area of base)



Total surface area of cylinder = $2\pi r h + 2\pi r^2$

(Where r = base radius and h = height)

Total surface area of cylinder = $\pi r(r + l) + (2\pi r h + 2\pi r^2) - 2(\pi r^2)$

$$= \pi r^2 + \pi r l + 2\pi r h + 2\pi r^2 - 2\pi r^2$$

$$= \pi r(r + l + 2h)$$

$$= \pi r [\sqrt{r^2 + h^2} + r + 2h]$$

4. A solid ball is exactly fitted inside the cubical box of side a . The volume of the ball is $\frac{4}{3}\pi a^3$.

Solution:

False

Explanation:

Let the radius of sphere = r

When a solid ball is exactly fitted inside the cubical box of side a ,

We get,

Diameter of ball = Edge length of cube

$$2r = a$$

Radius,

$$r = \frac{a}{2}$$

We also know that,

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\begin{aligned} \text{Volume of ball} &= \frac{4}{3} \pi \left(\frac{a}{2}\right)^3 \\ &= \frac{4}{3} \pi \left(\frac{a^3}{8}\right) \\ &= \frac{1}{6} \pi a^3 \end{aligned}$$

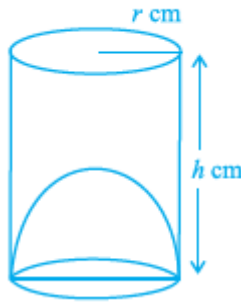
5. The volume of the frustum of a cone is $\frac{1}{3} \pi h [r_1^2 + r_2^2 + r_1 r_2]$, where h is vertical height of the frustum and r_1, r_2 are the radii of the ends.

Solution:

False

As volume of the frustum of a cone is $\frac{1}{3} \pi h [r_1^2 + r_2^2 + r_1 r_2]$

6. The capacity of a cylindrical vessel with a hemispherical portion raised upward at the bottom as shown in the fig. is $\frac{\pi r^2}{3} [3h - 2r]$.



Solution:

True

We know that, capacity of cylindrical vessel = $\pi r^2 h \text{ cm}^3$

Capacity of hemisphere = $\frac{2}{3} \pi r^3$

$$\begin{aligned} \text{Capacity of cylindrical vessel} &= \pi r^2 h - \frac{2}{3} \pi r^3 \\ &= \frac{\pi r^2}{3} [3h - 2r] \end{aligned}$$

7. The curved surface area of a frustum of a cone is $\pi l(r_1 + r_2)$, where $l = \sqrt{h^2 + (r_1 - r_2)^2}$, r_1 and r_2 are the radii of the two ends of the frustum and h is the vertical height.

Solution:

False

We know that,

Curved surface area of a frustum of a cone is $\pi l(r_1 + r_2)$,

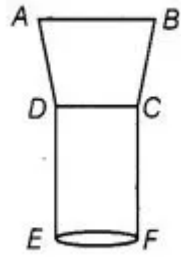
Where, $l = \sqrt{h^2 + (r_1 - r_2)^2}$, r_1 and r_2 are the radii of the two ends of the frustum and h is the vertical height.

8. An open metallic bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet. The surface area of the metallic sheet used is equal to curved surface area of frustum of a cone + area of circular base + curved surface area of cylinder.

Solution:

True

Because the resulting figure is:



Here, ABCD is a frustum of a cone and CDEF is a hollow cylinder.

Exercise No. 12.3

Short Answer Questions:

1. Three metallic solid cubes whose edges are 3 cm, 4 cm and 5 cm are melted and formed into a single cube. Find the edge of the cube so formed.

Solution:

We know that,

Volume of cube = a^3 ,

According to the question,

Side of first cube,

$$a_1 = 3 \text{ cm}$$

Side of second cube,

$$a_2 = 4 \text{ cm}$$

Side of third cube,

$$a_3 = 5 \text{ cm}$$

Let us assume that the side of cube recast from melting these cubes = a

We know that the total volume of the 3 cubes will be the same as the volume of the newly formed cube,

Volume of new cube = (volume of 1st + 2nd + 3rd cube)

$$a^3 = a_1^3 + a_2^3 + a_3^3$$

$$a^3 = (3)^3 + (4)^3 + (5)^3$$

$$a^3 = 27 + 64 + 125$$

$$= 216$$

$$a = 6 \text{ cm}$$

Therefore, side of cube so formed is 6 cm.

2. How many shots each having diameter 3 cm can be made from a cuboidal lead solid of dimensions 9cm × 11cm × 12cm?

Solution:

Volume of cuboid = lbh ,

Where, l = length, b = breadth and h = height

Cuboidal lead:

Length, $l = 9 \text{ cm}$

Breadth, $b = 11 \text{ cm}$

Height, $h = 12$ cm

$$\begin{aligned}\text{Volume of lead} &= 9(11)(12) \\ &= 1188 \text{ cm}^3\end{aligned}$$

Volume of sphere $= \frac{4}{3}\pi r^3$, where r = radius of sphere

Spherical shots,

Diameter = 3 cm

Radius, $r = 1.5$ cm

$$\begin{aligned}\text{Volume of one shot} &= \frac{4}{3} \times \frac{22}{7} \times (1.5)^3 \\ &= \frac{99}{7} \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{No. of shots can be made} &= \frac{\text{Volume of lead}}{\text{Volume of one shot}} \\ &= \frac{1188}{\frac{99}{7}} \\ &= 84\end{aligned}$$

Hence, the number of bullets that can be made from lead = 84.

3. A bucket is in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm, respectively. Find the height of the bucket.

Solution:

According to the question,

The bucket is in the form of frustum of a cone.

We know that,

$$\text{Volume of frustum of a cone} = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1r_2),$$

Where, h = height, r_1 and r_2 are the radii ($r_1 > r_2$)

For bucket,

$$\begin{aligned}\text{Volume of bucket} &= 28.490 \text{ L} \\ 1 \text{ L} &= 1000 \text{ cm}^3\end{aligned}$$

$$\text{Volume of bucket} = 28490 \text{ cm}^3$$

Radius of top, $r_1 = 28$ cm

Radius of bottom, $r_2 = 21$ cm

Let the height = h .

Substituting these values in the equation to find the volume of bucket,

$$\text{Volume of bucket} = \frac{1}{3} \pi h [28^2 + 21^2 + 28(21)]$$

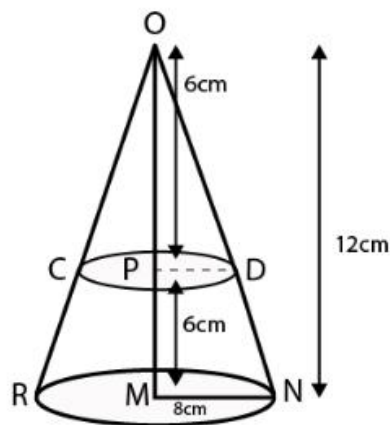
$$28490 = \frac{1}{3} \times \frac{22}{7} \times h (784 + 441 + 588)$$

$$= \frac{22}{7} \times h \times 1813$$

$$h = 15$$

4. A cone of radius 8 cm and height 12 cm is divided into two parts by a plane through the mid-point of its axis parallel to its base. Find the ratio of the volumes of two parts.

Solution:



According to the question,
Height of cone = OM = 12 cm

The cone is divided from mid-point.
Hence, let the mid-point of cone = P

$$OP = PM = 6 \text{ cm}$$

From $\triangle OPD$ and $\triangle OMN$

$$\angle POD = \angle POM \quad [\text{Common}]$$

$$\angle OPD = \angle OMN \quad [\text{Both } 90^\circ]$$

Hence, by the Angle-Angle similarity criterion

We have,

$$\triangle OPD \sim \triangle OMN$$

and

Similar triangles have corresponding sides in equal ratio,

So,

$$\frac{PD}{MN} = \frac{OP}{OM}$$

$$\frac{PD}{8} = \frac{6}{12}$$

$$PD = 4\text{cm}$$

[MN = 8 cm = radius of base of cone]

For First part i.e. cone

Base Radius,

$$r = PD = 4\text{ cm}$$

Height,

$$h = OP = 6\text{ cm}$$

We know that,

Volume of cone for radius r and height h,

$$V = \frac{1}{3} \pi r^2 h$$

$$\begin{aligned} \text{Volume of first part} &= \frac{1}{3} \pi (4)^2 (6) \\ &= 32\pi \end{aligned}$$

For second part, Frustum,

Bottom radius,

$$r_1 = MN = 8\text{ cm}$$

Top Radius,

$$r_2 = PD = 4\text{ cm}$$

Height,

$$h = PM = 6\text{ cm}$$

We know that,

$$\text{Volume of frustum of a cone} = \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2),$$

Where, h = height, r_1 and r_2 are radii, ($r_1 > r_2$)

$$\begin{aligned} \text{Volume of second part} &= \frac{1}{3} \pi (6) [8^2 + 4^2 + 8(4)] \\ &= 2\pi (112) \\ &= 224\pi \end{aligned}$$

Therefore, we get the ratio,

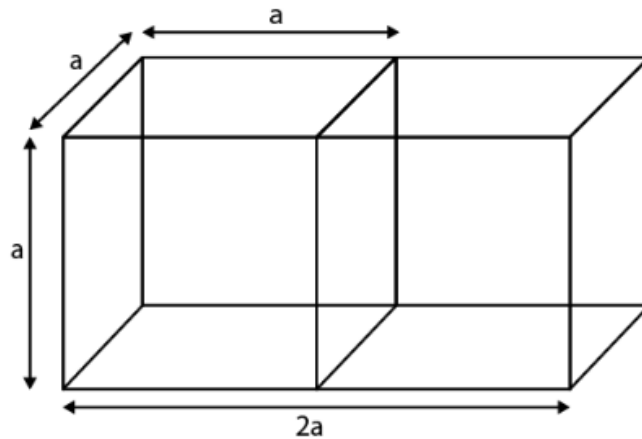
Volume of first part: Volume of second part

$$32\pi : 224\pi$$

$$1 : 7$$

5. Two identical cubes each of volume 64 cm^3 are joined together end to end. What is the surface area of the resulting cuboid?

Solution:



Let the side of one cube = a

Surface area of resulting cuboid = $2(\text{Total surface area of a cube}) - 2(\text{area of single surface})$

We know that,

Total surface area of cube = $6a^2$, Where a = side of cube

$$\begin{aligned}\text{Surface area of resulting cuboid} &= 2(6a^2) - 2(a^2) \\ &= 10a^2\end{aligned}$$

Also,

According to the question,

Volume of cube = 64 cm^3

Volume of cube = a^3

$$64 = a^3$$

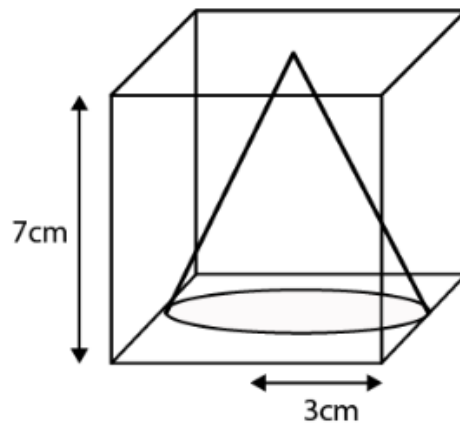
$$a = 4 \text{ cm}$$

Therefore,

$$\begin{aligned}\text{Surface area of resulting cuboid} &= 10a^2 \\ &= 10(4)^2 \\ &= 160 \text{ cm}^2\end{aligned}$$

6. From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of the remaining solid.

Solution:



From the figure, we get,

Volume of remaining solid = volume of cube – volume of cone

For Cube

Side, $a = 7$ cm

We know that,

Volume of cube = a^3 , where a = side of cube

$$\begin{aligned}\text{Volume of cube} &= (7)^3 \\ &= 343 \text{ cm}^3\end{aligned}$$

For cone,

Radius, $r = 3$ cm

Height, $h = 7$ cm

$$\begin{aligned}\text{Volume of cone} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi (3)^2 7 \\ &= 3 \times (22/7) \times 7 \\ &= 66 \text{ cm}^3\end{aligned}$$

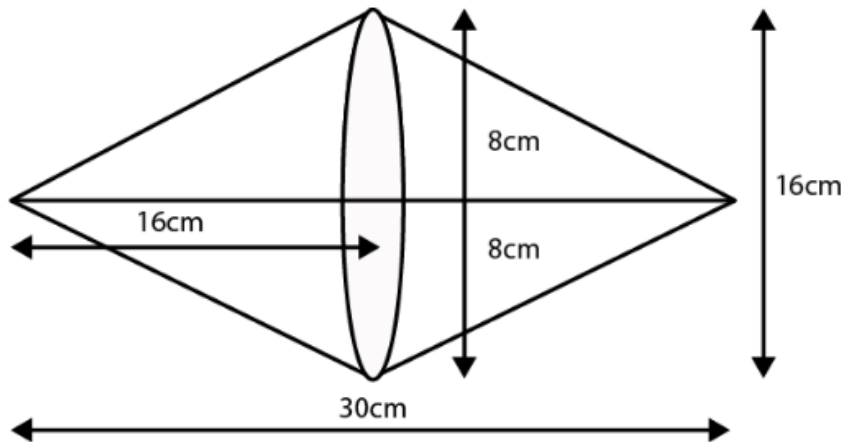
$$\begin{aligned}\text{Volume of remaining solid} &= \text{volume of cube} - \text{volume of cone} \\ &= 343 - 66 \\ &= 277 \text{ cm}^3\end{aligned}$$

7. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.

Solution:

According to the question,

We get the figure given below,



Total surface area of shape formed = Curved area of first cone + Curved surface area of second cone

Since, both cones are identical,

Total surface area of shape formed = Curved area of first cone + Curved surface area of second cone

$$= 2(\text{Surface area of cone})$$

We also know that,

Surface area of cone = πrl , where r = radius and l = slant height

The total Surface area of shape so formed = $2\pi rl$

Given in the question that,

Radius, $r = 8$ cm

Height, $h = 15$ cm

Therefore,

$$\begin{aligned} \text{Area} &= 2(3.14)(8)(15) \\ &= 753.6 \text{ cm}^2 \end{aligned}$$

8. Two solid cones A and B are placed in a cylindrical tube as shown in the Fig. The ratio of their capacities are 2:1. Find the heights and capacities of cones.

Also, find the volume of the remaining portion of the cylinder.

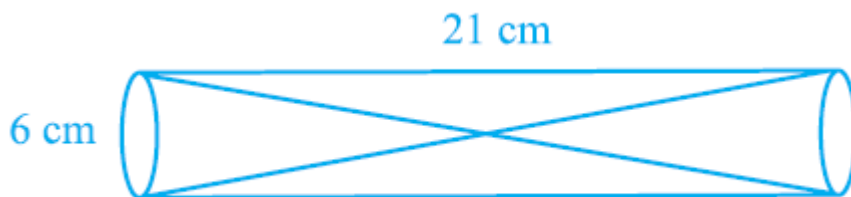
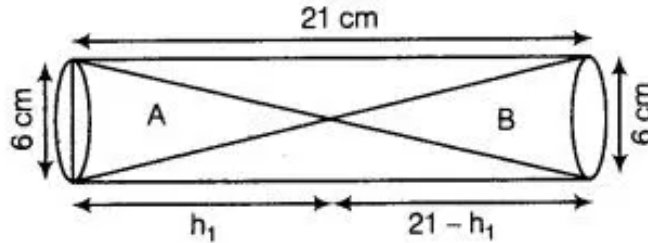


Fig. 12.9

Solution:

Let volume of cone A be $2V$ and volume of cone B be V .

Again, let height of the cone A = h_1 cm, then height of cone B = $(21 - h_1)$ cm



Diameter of cone = 6 cm

Radius of cone = 3cm

And,

Volume of cone A = $2V$

$$= \frac{4}{3} \times \frac{22}{7} \times (3)^2 \times h_1$$

$$= \frac{3}{2} h_1 \pi$$

Volume of cone B = V

$$= \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi r^2 h$$

$$= \frac{4}{3} \times \frac{22}{7} \times (3)^2 \times (21 - h_1)$$

$$= 3(21 - h_1) \pi$$

According to question,

$$\frac{3}{2} h_1 \pi = 3(21 - h_1) \pi$$

$$h_1 = 14 \text{ cm}$$

So,

$$\text{Height of cone B} = 21 - h_1$$

$$= 7 \text{ cm}$$

$$\text{Volume of cone A} = \frac{3}{2} h_1 \pi$$

$$= 132 \text{ cm}^3$$

$$\text{Volume of cone B} = 3(21 - h_1) \pi$$

$$= 66 \text{ cm}^3$$

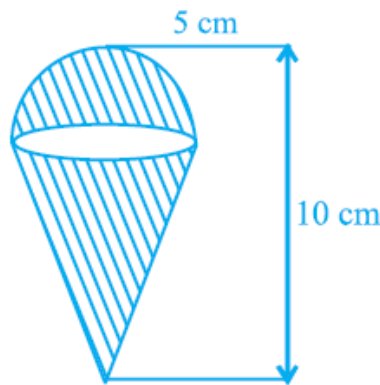
$$\text{Volume of cylinder} = \pi r^2 h$$

$$= 594 \text{ cm}^3$$

Volume of remaining portion = Volume of cylinder – (Volume of cone A+ Volume of cone B)

$$= 594 - (132 + 66) \\ = 396 \text{ cm}^3$$

9. An ice cream cone full of ice cream having radius 5 cm and height 10 cm as shown in the Fig.12.10. Calculate the volume of ice cream, provided that its $\frac{1}{6}$ part is left unfilled with ice cream.



Solution:

Given, ice-cream cone is the combination of a hemisphere and a cone.

Also, radius of hemisphere = 5 cm

Volume of hemisphere,

$$= \frac{2}{3} \pi r^3 \\ = \frac{4}{3} \times \frac{22}{7} \times (5)^3 \\ = \frac{5500}{21} \\ = 261.90 \text{ cm}^3$$

Now,

Radius of cone = 5

Height of cone = 5

Volume of cone,

$$= \frac{1}{3} \pi r^2 h \\ = \frac{4}{3} \times \frac{22}{7} \times (5)^2 \times 5 \\ = 130.95 \text{ cm}^3$$

Total volume of ice-cream cone = 261.90 + 130.95

As,

1/6 part is left unfilled,

$$\begin{aligned}\text{Required volume of ice-cream} &= 392.85 - 392.85 \times 1/6 \\ &= 327.4 \text{ cm}^3\end{aligned}$$

10. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm containing some water. Find the number of marbles that should be dropped into the beaker so that the water level rises by 5.6 cm.

Solution:

Given,

Diameter of a marble = 1.4 cm

Radius = 0.7

$$\begin{aligned}\text{Volume of spherical marble} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times (0.7)^3 \\ &= \frac{1.372}{3} \pi \text{ cm}^3\end{aligned}$$

Diameter of beaker = 7cm

Radius = 3.5 cm

Height of water raised = 5.6 cm

Volume of water raised in beaker,

$$\begin{aligned}&= \pi (3.5)^2 \times 5.6 \\ &= 68.6 \pi\end{aligned}$$

$$\begin{aligned}\text{Required no. of marbles} &= \frac{\text{Volume of water raised in beaker}}{\text{Volume of marble}} \\ &= 150\end{aligned}$$

11. How many spherical lead shots each of diameter 4.2 cm can be obtained from a solid rectangular lead piece with dimensions 66 cm, 42 cm and 21 cm.

Solution:

Given that, lots of spherical lead shots made out of a solid cube of lead.

$$\text{Number of spherical lead shots} = \frac{\text{Volume of solid rectangular lead piece}}{\text{Volume of spherical lead shot}}$$

Diameter of a spherical lead shot = 4.2cm

Radius = 2.1cm

Now,

$$\text{Volume of solid rectangular lead piece} = 46 \times 42 \times 21$$

$$\begin{aligned}
 \text{Volume of spherical lead shot} &= \frac{4}{3} \pi r^3 \\
 &= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 \\
 &= \frac{4 \times 22 \times 2.1 \times 2.1 \times 2.1}{21}
 \end{aligned}$$

$$\begin{aligned}
 \text{Number of spherical lead shots} &= \frac{\text{Volume of solid rectangular lead piece}}{\text{Volume of spherical lead shot}} \\
 &= 1500
 \end{aligned}$$

Hence, the required number of spherical lead shots is 1500.

12. How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge measures 44 cm.

Solution:

Given that, lots of spherical lead shots made out of a solid cube of lead.

$$\text{Number of spherical lead shots} = \frac{\text{Volume of solid cube of lead}}{\text{Volume of spherical lead shot}}$$

Diameter of a spherical lead shot = 4cm
 Radius = 2cm

Now,

$$\text{Volume of solid cube} = 44 \times 44 \times 44$$

$$\begin{aligned}
 \text{Volume of spherical lead shot} &= \frac{4}{3} \pi r^3 \\
 &= \frac{4}{3} \times \frac{22}{7} \times 2^3 \\
 &= \frac{4 \times 22 \times 8}{21}
 \end{aligned}$$

$$\begin{aligned}
 \text{Number of spherical lead shots} &= \frac{\text{Volume of solid cube of lead}}{\text{Volume of spherical lead shot}} \\
 &= 2541
 \end{aligned}$$

Hence, the required number of spherical lead shots is 2541.

13. A wall 24 m long, 0.4 m thick and 6 m high is constructed with the bricks each of dimensions 25 cm × 16 cm × 10 cm. If the mortar occupies

$\frac{1}{10}$ th of the volume of the wall, then find the number of bricks used in constructing the wall.

Solution:

Given that, a wall is constructed with the help of bricks and mortar.

$$\text{Number of bricks} = \frac{\text{Volume of wall} - \frac{1}{10} \text{Volume of wall}}{\text{Volume of a brick}}$$

Given,

Length of wall = 24

Breadth of wall = 0.4

Height of wall = 6

Now,

Volume of wall constructed with bricks = $24 \times 0.4 \times 6 \text{ m}^3$

So,

$$\frac{1}{10} \text{Volume of wall} = \frac{1}{10} \times 24 \times 0.4 \times 6$$

Also,

Length of brick = 25 cm

Breadth of brick = 16cm

Height of brick = 10cm

Volume of bricks = $0.25 \times 0.16 \times 0.10 \text{ m}^3$

So,

$$\begin{aligned} \text{Number of bricks} &= \frac{\text{Volume of wall} - \frac{1}{10} \text{Volume of wall}}{\text{Volume of a brick}} \\ &= \frac{24 \times 0.4 \times 6 - \frac{1}{10}(24 \times 0.4 \times 6)}{0.25 \times 0.16 \times 0.10} \\ &= 12960 \end{aligned}$$

14. Find the number of metallic circular disc with 1.5 cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Solution:

Given that, lots of metallic circular disc to be melted to form a right circular cylinder.

Here, a circular disc work as a circular cylinder.

Base diameter of metallic circular disc = 1.5 cm

$$\text{Radius} = \frac{1.5}{2}$$

Height = 0.2cm

Volume = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{1.5}{2} \times \frac{1.5}{2} \times 0.2$$

Now,

$$\text{Radius of right circular cylinder} = \frac{4.5}{2}$$

$$\text{Height of right circular cylinder} = 10 \text{ cm}$$

$$\begin{aligned} \text{Volume of right circular cylinder} &= \pi r^2 h \\ &= \frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2} \times 10 \end{aligned}$$

Number of metallic disc:

$$\begin{aligned} &= \frac{\text{Volume of right circular cylinder}}{\text{Volume of metallic disc}} \\ &= 450 \end{aligned}$$

Hence, the required number of metallic circular disc is 450.

Exercise No. 12.4

Long Answer Questions:

1. A solid metallic hemisphere of radius 8 cm is melted and recasted into a right circular cone of base radius 6 cm. Determine the height of the cone.

Solution:

For hemisphere,
Radius, $r = 8$ cm

We know that,

Volume of hemisphere = $\frac{2}{3} \pi r^3$, where, r = radius of hemisphere

$$\begin{aligned}\text{Volume of given hemisphere} &= \frac{2}{3} \times \pi \times 8^3 \\ &= (1024/3) \pi \text{ cm}^3\end{aligned}$$

Now,

For the cone that is recast from a hemisphere,
Base radius, $r = 6$ cm

We also know that,

Volume of cone = $\frac{1}{3} \pi r^2 h$, where, r is base radius and h is the height of the cone.

$$\begin{aligned}\text{Volume of cone} &= \frac{1}{3} \pi (6)^2 h \\ &= 12\pi h\end{aligned}$$

According to the question, we know that,

The volume remains same, when a body is reformed to another body

Volume of cylinder = Volume of cone

$$12\pi h = 1024\pi / 3$$

$$h = 28.44 \text{ cm}$$

2. A rectangular water tank of base $11 \text{ m} \times 6 \text{ m}$ contains water upto a height of 5 m. If the water in the tank is transferred to a cylindrical tank of radius 3.5 m, find the height of the water level in the tank.

Solution:

Volume of water in tank = volume of cuboidal tank up to a height of 5 m

According to the question,

For cuboidal tank

Length, $l = 11$ m

Breadth, $b = 6$ m

Height, $h = 5$ m

We know that the equation to find the volume of the tank,

Volume of tank = lbh , where, l , b and h are the length, breadth and height of tank respectively

$$\text{Volume of water} = 11(6)(5) = 330 \text{ m}^3$$

We also know that,

Base radius of cylindrical tank, $r = 3.5$ m

Let the height till which the cylindrical tank is filled = h m

Hence, using the formula,

$$\text{Volume of a cylinder} = \pi r^2 h,$$

where r is base radius and h is the height of cylinder

$$\text{Volume of water in cylindrical tank} = \pi(3.5)^2 h$$

$$330 \text{ m}^3 = \frac{22}{7} \times 3.5 \times 3.5 \times h$$

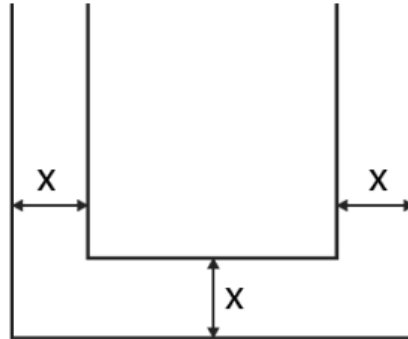
$$h = 8.57 \text{ m}$$

Hence, the height till which the cylindrical tank is filled = 8.57 m

3. How many cubic centimetres of iron is required to construct an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm provided the thickness of the iron is 1.5 cm. If one cubic cm of iron weighs 7.5 g, find the weight of the box.

Solution:

Let the length (l), breadth (b), and height (h) be the external dimension of an open box and thickness be x.



The volume of metal used in box = Volume of external box – Volume of internal box

Consider external box,

Length, $l = 36$ cm

Breadth, $b = 25$ cm

Height, $h = 16.5$ cm

We know that the equation of the volume of cuboid is given by,

Volume of cuboid = lbh , where, l , b and h are the length, breadth and height of tank respectively

$$\begin{aligned}\text{Volume of external box} &= 36(25)(16.5) \\ &= 14850 \text{ cm}^3\end{aligned}$$

Since the box is open from top,

Consider internal box,

The thickness of two sides is reduced as follows,

$$\begin{aligned}\text{Length, } l' &= \text{Length of external box} - 2(\text{thickness of box}) \\ &= 36 - 2(1.5) \\ &= 33 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Breadth, } b' &= \text{Breadth of external box} - 2(\text{thickness of box}) \\ &= 25 - 2(1.5) \\ &= 22 \text{ cm}\end{aligned}$$

Height,

$$\begin{aligned}h' &= \text{Height of external box} - \text{thickness of box} \\ &= 16.5 - 1.5 \\ &= 15 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Volume of internal box} &= 33(22)(15) \\ &= 10890\end{aligned}$$

$$\begin{aligned}\text{Volume of metal in box} &= 14850 - 10890 \\ &= 3960 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}1 \text{ cm}^3 &\text{ weighs } 7.5 \text{ g.} \\ 3960 \text{ cm}^3 &\text{ weighs } 3960(7.5) = 29,700 \text{ g}\end{aligned}$$

Therefore, the weight of box is 29,700 g i.e. 29.7 kg

4. The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up on writing 3300 words on an average. How many words can be written in a bottle of ink containing one fifth of a litre?

Solution:

Let us first calculate the volume of barrel of pen that is of cylindrical shape
Consider barrel,

$$\begin{aligned}\text{Since } 1\text{cm} &= 10 \text{ mm} \\ \text{Base diameter} &= 5 \text{ mm} \\ &= 0.5 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Base radius, } r &= 0.25 \text{ cm} \\ \text{Height, } h &= 7 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{We know that,} \\ \text{Volume of a cylinder} &= \pi r^2 h\end{aligned}$$

$$\begin{aligned}\text{Volume of barrel} &= \pi (0.25)^2 7 \\ \text{Volume of barrel} &= \frac{22}{7} \times 0.25 \times 0.25 \times 7 \\ &= 1.375 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Hence, according to the question,} \\ 1.375 \text{ cm}^3 \text{ of ink can write } &3300 \text{ words} \\ \text{No of words that can be written by } 1 \text{ cm}^3 \text{ of ink} &= 3300/1.375 \\ &= 2400 \text{ words}\end{aligned}$$

$$\begin{aligned}1/5\text{th of a liter} &= 0.2 \text{ L} \\ \text{We know that,} \\ 1 \text{ L} &= 1000 \text{ cm}^3 \\ 0.2 \text{ L} &= 200 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{So, no of words that can be written by } 200 \text{ cm}^3 &= 2400(200) \\ &= 480000 \text{ words}\end{aligned}$$

Therefore, 1/5th of a liter ink can write 480000 words.

5. Water flows at the rate of 10m/minute through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24 cm?

Solution:

Let the time taken by pipe to fill vessel = t minutes

Since water flows 10 m in 1 minute, it will flow 10t meters in t minutes.

According to the question,

Volume of conical vessel = Volume of water that passes through pipe in t minutes

Consider conical pipe,

Base Diameter = 40 cm

Base radius, r = 20 cm

Height, h = 24 cm

We know that the volume of cone = $\frac{1}{3}\pi r^2 h$

$$\begin{aligned}\text{Volume of conical vessel} &= \frac{1}{3}\pi(20)^2(24) \\ &= 3200\pi \text{ cm}^3\end{aligned}$$

Consider cylindrical pipe

Base diameter = 5 mm

= 0.5 cm

Base radius,

r = 0.25 cm

Water covers 10t m distance in pipe,

Hence, we get,

Height,

h = 10t m = 1000t cm

We also know that,

Volume of a cylinder = $\pi r^2 h$

$$\begin{aligned}\text{Volume of water passed in pipe} &= \pi(0.25)^2(1000t) \\ &= 62.5t\pi \text{ cm}^3\end{aligned}$$

So, we have

$$62.5t\pi = 3200$$

$$62.5t = 3200$$

$$t = 51.2 \text{ minutes}$$

We know that,

$$\begin{aligned} 0.2 \text{ minutes} &= 0.2(60) \text{ seconds} \\ &= 12 \text{ seconds} \end{aligned}$$

Therefore,

$$t = 51 \text{ minutes } 12 \text{ seconds}$$

6. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

Solution:

According to the question,

Consider conical heap,

Base Diameter = 9 cm

So,

Base radius, $r = 4.5$ cm

Height, $h = 3.5$ cm

We know that,

Slant height,

$$\begin{aligned} l &= \sqrt{h^2 + r^2} \\ &= \sqrt{4.5^2 + 3.5^2} \\ &= 5.7 \text{ cm} \end{aligned}$$

$$\text{The equation of volume of cone} = \frac{1}{3} \pi r^2 h$$

We know that,

Volume of rice = Volume of conical heap

$$\begin{aligned} \text{Volume of rice} &= \frac{1}{3} \pi (4.5)^2 (3.5) \\ &= 74.25 \text{ cm}^3 \end{aligned}$$

We also know that,

Canvas requires to just cover heap = Curved surface area of conical heap

And curved surface area of a cone = $\pi r l$

$$\text{Therefore, the canvas required} = \pi (4.5)(5.7) = 80.61 \text{ cm}^2 \text{ [appx]}$$

7. A factory manufactures 120000 pencils daily. The pencils are cylindrical in shape each of length 25 cm and circumference of base as 1.5 cm.

Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at Rs 0.05 per dm².

Solution:

The shape of pencil = cylinder.

Let the radius of base = r cm

Circumference of base = 1.5 cm

Circumference of circle is $2\pi r = 1.5$ cm

$$r = \frac{1.5}{2\pi} \text{ cm}$$

According to the question,

Height, h = 25 cm

We know that,

Curved surface area of cylinder = $2\pi rh$

$$\begin{aligned}\text{Curved surface area of pencil} &= 2\pi \left(\frac{1.5}{2\pi}\right) 25 \\ &= 37.5 \text{ cm}^2\end{aligned}$$

$$1 \text{ cm} = 0.1 \text{ dm}$$

$$1 \text{ cm}^2 = 0.01 \text{ dm}^2$$

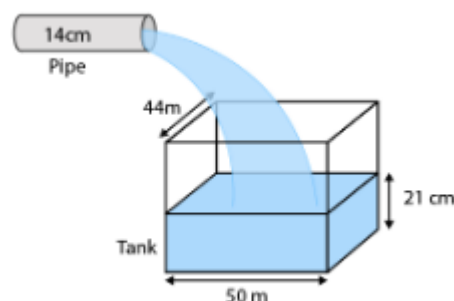
$$37.5 \text{ cm}^2 = 0.375 \text{ dm}^2$$

Cost for colouring 1 dm² = Rs. 0.05

Cost for colouring 0.375 dm² (i.e. 1 pencil) = Rs. 0.01875

Cost for colouring 120000 pencils = 120000×0.01875
= Rs. 2250

8. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm?



Solution:

Let the time taken by pipe to fill pond = t hours

Water flows 15 km in 1 hour, so, it will flow 15t meters in t hours.

We know that,

Volume of cuboidal pond up to height 21 cm = Volume of water that passes through pipe in “t” hours

Considering cuboidal pond,

Length, $l = 50$ m

Breadth, $b = 44$ m

Height, $h = 21$ cm
 $= 0.21$ m

We know that,

Volume of tank = lbh

$$\begin{aligned}\text{Volume of water} &= 50(44)(0.21) \\ &= 462 \text{ m}^3\end{aligned}$$

Considering cylindrical pipe

Base diameter = 14 cm

Base radius, $r = 7$ cm
 $= 0.07$ m

Height, $h = 15t$ km
 $= 15000t$ m

We also know that,

Volume of a cylinder = $\pi r^2 h$

$$\begin{aligned}\text{Volume of water passed in pipe} &= \pi(0.07)^2(15000t) \\ &= \frac{22}{7} \times 0.07 \times 0.07 \times 15000t \\ &= 231t \text{ cm}^3\end{aligned}$$

So, we have

$$231t = 462$$

$$t = 2 \text{ hours}$$

Time required to fill tank up to a height of 25 cm is 2 hours.

9. A solid iron cuboidal block of dimensions $4.4 \text{ m} \times 2.6 \text{ m} \times 1 \text{ m}$ is recast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

Solution:

Considering cuboidal block

Length, $l = 4$ m

Breadth, $b = 2.6$ m

Height, $h = 1$ m

We know that,

Volume of tank = lbh

$$\begin{aligned}\text{Volume of cuboid} &= 4.4(2.6)(1) \\ &= 11.44 \text{ m}^3\end{aligned}$$

We know that,

The volume remains same when a body is recast to another body.

According to the question,

$$\text{Volume of cylindrical pipe} = 11.44 \text{ m}^3$$

Considering pipe or the hollow cylinder

Internal radius,

$$\begin{aligned}r^2 &= 30 \text{ cm} \\ &= 0.3 \text{ m}\end{aligned}$$

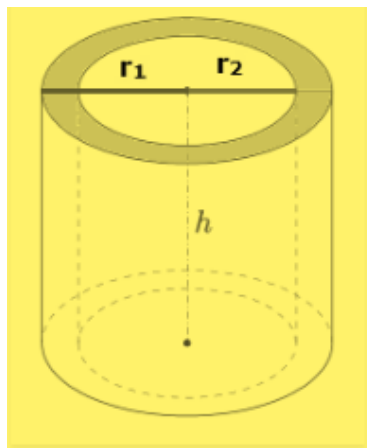
Thickness = 5 cm

$$\begin{aligned}\text{External radius, } r_1 &= \text{Internal radius} + \text{thickness} \\ &= 30 + 5 \\ &= 35 \text{ cm} \\ &= 0.35 \text{ m}\end{aligned}$$

Let the length of pipe = h

We know that,

$$\text{Volume of a hollow cylinder} = \pi h(r_1^2 - r_2^2)$$



Hence,

$$\text{Volume of pipe} = \pi h((0.35)^2 - (0.3)^2)$$

So, the length of pipe is 112 m.

10. 500 persons are taking a dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04m^3 ?

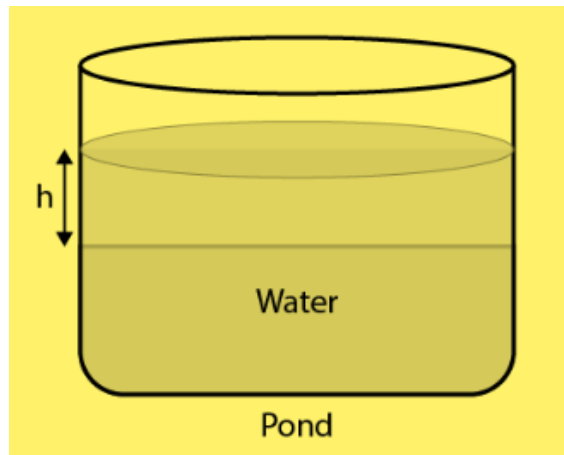
Solution:

According to the question,

Average displacement by a person = 0.04 m^3

Average displacement by 500 persons = 500×0.04
 $= 20\text{ m}^3$

Hence, the volume of water raised in pond = 20 m^3



It is also given that,

Length of pond, $l = 80\text{ m}$

Breadth of pond, $b = 50\text{ m}$

Height = h

Volume of water raised in pond = $80(50)(h)$

$20\text{ m}^3 = 4000h$

$h = 0.005\text{ m}$

$= 0.5\text{ cm}$

Therefore, raise in the height of water = 0.5 cm .

11. 16 glass spheres each of radius 2 cm are packed into a cuboidal box of internal dimensions $16\text{ cm} \times 8\text{ cm} \times 8\text{ cm}$ and then the box is filled with water. Find the volume of water filled in the box.

Solution:

Given,

Dimensions of the cuboidal = $16\text{ cm} \times 8\text{ cm} \times 8\text{ cm}$

Volume of the cuboidal = $16 \times 8 \times 8 = 1024\text{ cm}^3$

Also, given radius of one glass sphere = 2 cm

Volume of one glass sphere,

$$\begin{aligned}
&= \frac{4}{3} \pi r^3 \\
&= \frac{4}{3} \pi (2)^3 \\
&= 33.523 \text{ cm}^3
\end{aligned}$$

$$\begin{aligned}
\text{Volume of 16 glass sphere} &= 16 \times 33.523 \\
&= 536.37 \text{ cm}^3
\end{aligned}$$

$$\begin{aligned}
\text{Required volume of water} &= \text{Volume of the cuboidal} - \text{Volume of 16 glass sphere} \\
&= 1024 - 536.37 \\
&= 487.6 \text{ cm}^3
\end{aligned}$$

12. A milk container of height 16 cm is made of metal sheet in the form of a frustum of a cone with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk at the rate of Rs. 22 per litre which the container can hold.

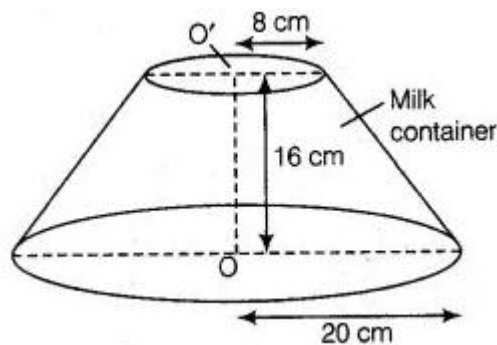
Solution:

Given,

Height of milk container (h) = 16 cm,

Radius of lower end of milk container (r) = 8 cm

Radius of upper end of milk container (R) = 20 cm



Volume of milk container made of metal sheet in form of frustum of a cone,

$$\begin{aligned}
&= \frac{\pi h}{3} (R^2 + r^2 + Rr) \\
&= \frac{\pi \times 16}{3} (20^2 + 8^2 + 20 \times 8) \\
&= 10.45942 \text{ L}
\end{aligned}$$

Cost of milk per litre = Rs.22

For,

$$\begin{aligned}
10.45942 \text{ L} &= 22 \times 10.45942 \\
&= \text{Rs. } 230.12
\end{aligned}$$

Hence, the required cost of milk is ₹ 230.12.

13. A cylindrical bucket of height 32 cm and base radius 18 cm is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Solution:

Given,

Radius of the base of the bucket = 18 cm

Height of the bucket = 32 cm

So,

$$\begin{aligned}\text{Volume of the sand in cylindrical bucket} &= \pi r^2 h \\ &= \pi (18)^2 \times 32 \\ &= 10368 \pi\end{aligned}$$

Also,

Height of the conical heap (h) = 24 cm

Let radius of heap be r cm.

So,

$$\text{Volume of sand in heap (cone)} = \frac{1}{3} \pi r^2 h$$

According to the question,

Volume of the sand in cylindrical bucket = Volume of the sand in conical heap

$$10368 \pi = \frac{1}{3} \pi r^2 \times 24$$

On solving,

$$r = 36 \text{ cm}$$

Let, the slant height of heap be l

$$\begin{aligned}l^2 &= h^2 + r^2 \\ &= 24^2 + 36^2 \\ &= 43.267 \text{ cm}\end{aligned}$$

Hence, radius of conical heap of sand = 36 cm

Slant height of conical heap = 43.267 cm

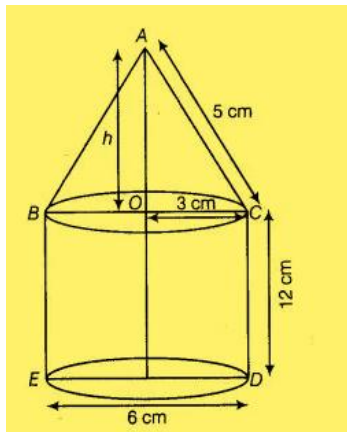
14. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, find the total surface area and volume of the rocket

[Use $\pi = 3.14$].

Solution:

Since, rocket is the combination of a right circular cylinder and a cone.

Given, diameter of the cylinder = 6 cm



Radius of cylinder = 3cm

Height = 12cm

Volume of cylinder ,

$$\begin{aligned}
 &= \pi r^2 h \\
 &= 3.14 \times 3^2 \times 12 \\
 &= 339.12 \text{ cm}^3
 \end{aligned}$$

$$\begin{aligned}
 \text{Curved surface area of cylinder} &= 2 \times 3.14 \times 3 \times 12 \\
 &= 226.08
 \end{aligned}$$

Now,

Height of cone = 4 cm

Radius of cone = 3 cm

So,

Volume of cone

$$\begin{aligned}
 &= \frac{1}{3} \pi r^2 h \\
 &= \frac{1}{3} \pi \times 3^2 \times 4 \\
 &= 37.68 \text{ cm}^3
 \end{aligned}$$

Also,

$$\begin{aligned}
 \text{C.S.A.} &= \pi r l \\
 &= \pi \times 3 \times 5 \\
 &= 47.1
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 \text{Total volume of rocket} &= 339.12 + 37.68 \\
 &= 376.8 \text{ cm}^3
 \end{aligned}$$

Now,

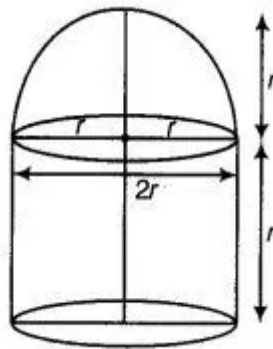
$$\begin{aligned}
 \text{Total surface area of rocket} &= \text{CSA of cone} + \text{CSA of cylinder} + \text{Area of base of cylinder} \\
 &= 47.1 + 226.08 + 28.26 \\
 &= 301.44 \text{ cm}^2
 \end{aligned}$$

15. A building is in the form a cylinder surmounted by a hemispherical vaulted dome and contains

$41\frac{19}{21}$ m³ of air. If the internal diameter of dome is equal to its total height above the floor, find the height of the building?

Solution:

Let total height of the building = Internal diameter of the dome
= $2r$ m



Radius of building = r

Height of cylinder = $2r - r$
= r m

So,

Volume of cylinder = $\pi r^2 \times r$

Also,

Volume of hemispherical dome = $\frac{2}{3} \pi r^3$

Therefore,

Total volume of building = Volume of cylinder + Volume of hemispherical dome

$$= \pi r^3 + \frac{2}{3} \pi r^3$$

$$= \frac{5}{3} \pi r^3$$

According to question,

Volume of building = Volume of air

$$\frac{5}{3} \pi r^3 = 41\frac{19}{21} \text{ m}^3$$

On solving,

$r = 2$ m

Also,

Height = $2r$

= 2×2

= 4 m

16. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped bottles each of radius 1.5 cm and height 4 cm. How many bottles are needed to empty the bowl?

Solution:

Given,

Radius of hemispherical bowl, $r = 9$ cm

Radius of cylindrical bottles,

$R = 1.5$ cm and

$h = 4$ cm

$$\begin{aligned}\text{Number of required cylindrical bottles} &= \frac{\text{Volume of hemispherical bowl}}{\text{Volume of one cylindrical bottle}} \\ &= \frac{\frac{2}{3}\pi(9)^3}{\pi(1.5)^2 \times 4} \\ &= 54\end{aligned}$$

17. A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is equal to the radius of the cone.

Solution:

(i) Whenever we placed a solid right circular cone in a right circular cylinder with full of water, then volume of a solid right circular cone is equal to the volume of water failed from the cylinder.

(ii) Total volume of water in a cylinder is equal to the volume of the cylinder.

(iii) Volume of water left in the cylinder = Volume of the right circular cylinder – volume of a right circular cone.

Now, given that

Height of a right circular cone = 120 cm

Radius of a right circular cone = 60 cm

Volume of right circular cone,

$$\begin{aligned}&= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi (60)^2 \times 120 \\ &= 144000\pi\end{aligned}$$

$$\begin{aligned}\text{Volume of right circular cone} &= \text{Volume of water filled from the cylinder} \\ &= 144000\pi\end{aligned}$$

Now,

Height of right circular cylinder = 180 cm

Radius = 60 cm

$$\begin{aligned}\text{Volume of right circular cylinder} &= \pi r^2 h \\ &= \pi (60)^2 \times 180 \\ &= 648000\pi\end{aligned}$$

Hence, the required volume of water left in the cylinder is 1.584 m³.

18. Water flows through a cylindrical pipe, whose inner radius is 1 cm, at the rate of 80 cm/sec in an empty cylindrical tank, the radius of whose base is 40 cm. What is the rise of water level in tank in half an hour?

Solution:

Given,

Radius of tank, $r_1 = 40$ cm

Let height of water level in tank in half an hour = 1 cm.

Also,

Internal radius of cylindrical pipe, $r_2 = 1$ cm

Speed of water = 80 cm/s (in 1 water flow = 80 cm)

In 30 (min) water flow = $80 \times 60 \times 30 = 144000$ cm

According to the question,

Volume of water in cylindrical tank = Volume of water flow from circular pipe

$$\pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$40^2 \times h_1 = 1^2 \times 144000$$

$$h_1 = 90 \text{ cm}$$

Hence, the level of water in cylindrical tank rises 90 cm in half an hour.

19. The rain water from a roof of dimensions 22 m 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the rain water collected from the roof just fill the cylindrical vessel, then find the rainfall in cm.

Solution:

Given,

Length of roof = 22 m and

Breadth of roof = 20 m

Let the rainfall be a cm.

$$\begin{aligned}\text{Volume of water on roof} &= 22 \times 20 \times \frac{a}{100} \\ &= \frac{22a}{5}\end{aligned}$$

Now,

$$\begin{aligned}\text{Volume of cylindrical vessel} &= \frac{22}{7} \times 1 \times 1 \times \frac{7}{2} \\ &= 11 \text{ m}^3\end{aligned}$$

According to question,

Volume of water on roof = Volume of cylindrical vessel

$$\frac{22a}{5} = 11$$

$$A = 2.5 \text{ cm}$$

Hence, the rainfall is 2.5 cm

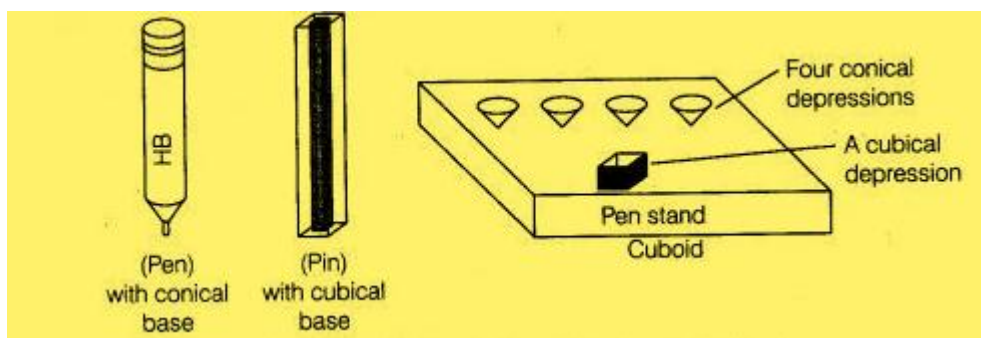
20. A pen stand made of wood is in the shape of a cuboid with four conical depressions and a cubical depression to hold the pens and pins, respectively. The dimension of the cuboid are 10 cm, 5 cm and 4 cm. The radius of each of the conical expressions is 0.5 cm and the depth is 2.1 cm. The edge of the cubical depression is 3 cm. Find the volume of the wood in the entire stand.

Solution:

Given that, length of cuboid pen stand (l) = 10 cm

Breadth of cuboid pen stand (b) = 5 cm

Height of cuboid pen stand (h) = 4 cm



$$\begin{aligned}\text{Volume of cuboidal pen stand} &= 10 \times 5 \times 4 \\ &= 200 \text{ cm}^3\end{aligned}$$

Radius of conical depression = 0.5 cm

Depth = 2.1 cm

Volume of conical depression,

$$\begin{aligned}
&= \frac{1}{3} \pi r^2 h \\
&= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 2.1 \\
&= 0.55 \text{ cm}^3
\end{aligned}$$

So,

Volume of four conical depression = 4×0.55

$$= \frac{11}{5} \text{ cm}^3$$

Now,

Edge of cube = 3 cm

Volume of cube = 27 cm^3

Therefore,

Volume of wood in entire pen stand = Volume of cuboidal pen stand - Volume of four conical depression - Volume of cube

$$\begin{aligned}
&= 200 - \frac{11}{5} - 27 \\
&= 170.8 \text{ cm}^3
\end{aligned}$$

So, the required volume of the wood in the entire stand is 170.8 cm^3 .