Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper -3

with Solution

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. Two circles touch each other externally at P. AB is a common tangent to the circle touching them at A and B. The value of ∠APB is **[1]**

a) 30^o b) 45^o

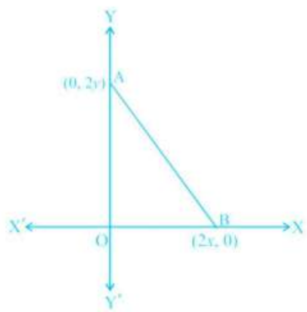
c) 90^o d) 60^o

2. The perimeter of the triangle formed by the points (0, 0), (1, 0) and (0, 1) is **[1]**

a) $2+\sqrt{2}$ b) 3

c) $\sqrt{2}+1$ d) $1\pm\sqrt{2}$

3. The coordinates of the point which is equidistant from the three vertices of a ΔAOB as shown in the figure is **[1]**



a) $\left(\frac{x}{2}, \frac{y}{2}\right)$ b) (y, x)

c) (0, 0) d) (x, y)

4. Two coins are tossed together. The probability of getting at most one head is **[1]**

a) $\frac{2}{4}$ b)

c) $\frac{1}{4}$ d) $\frac{3}{4}$

5. If R(5, 6) is the midpoint of the line segment AB joining the points A(6, 5) and B(4, y) they y equals **[1]**

a) 7 b) 12

c) 5 d) 6

6. The graphs of the equations $2x + 3y - 2 = 0$ and $x - 2y - 8 = 0$ are two lines which are **[1]**

a) perpendicular to each other b) parallel

c) intersecting exactly at one point d) coincident

7. How many bricks each measuring (25 cm × 11.25 cm × 6 cm) will be required to construct a wall (8 m × 6 m × 22.5 cm)? **[1]**

a) 7200 b) 4800

c) 8000 d) 6400

8. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and these values are equally likely outcomes. The probability that it will point at a number greater than 5 is **[1]**

a) $\frac{1}{2}$ b) $\frac{1}{4}$

c) $\frac{1}{5}$ d) $\frac{1}{3}$

9. A bag contains 8 red, 2 black and 5 white balls. One ball is drawn at random. What is the probability that the ball drawn is not black? **[1]**

a) $\frac{13}{15}$ b) $\frac{1}{3}$

c) $\frac{8}{15}$ d) $\frac{2}{15}$

10. If one root of $5x^2 + 13x + k = 0$ be the reciprocal of the other root then the value of k is **[1]**

a) 0 b) 5

c) 1 d) 2

11. If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then **[1]**

a) $-3 < b < 3$ b) $b > 2$

c) $-2 < b < 2$ d) $b < -2$

12. The difference of a rational and an irrational number is always **[1]**

a) a rational number

b) an irrational number

c) an integer

d) None of these

13. If the distance between the points (2, -2) and (-1, x) is 5, one of the values of x is **[1]**

a) -2

b) -1

c) 1

d) 2

14. $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ **[1]**

a) $\tan 90^o$

b) 1

c) $\sin 45^o$

d) 0

15. The length of the shadow of a 20 m tall pole on the ground when the sun's elevation is 45° is **[1]**

a) 20 m

b) $20\sqrt{2}$ m

c) 40 m

d) $20\sqrt{3}$ m

16. If two numbers do not have common factor (other than 1), then they are called **[1]**

a) prime numbers

b) co-prime numbers

c) composite numbers

d) twin primes

17. Consider the frequency distribution of the heights of 60 students of a class: **[1]**

Height (in cm)	No. of Students	Cumulative Frequency
150-155	16	16
155-160	12	28
160-165	9	37
165-170	7	44
170-175	10	54
175-180	6	60

The sum of the lower limit of the modal class and the upper limit of the median class is

a) 320

b) 315

c) 330

d) 310

18. The sum of the numerator and denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to $\frac{1}{3}$. The fraction is **[1]**

a) $\frac{-7}{11}$

b) $\frac{5}{13}$

c) $\frac{-5}{13}$

d) $\frac{7}{11}$

19. **Assertion (A):** Two right-angled triangles are always similar. **[1]**

Reason (R): By Pythagoras Theorem, $H^2 = P^2 + B^2$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The HCF of two numbers is 18 and their product is 3072. Then their LCM = 169. **[1]**

Reason (R): If a, b are two positive integers, then HCF $\times$ LCM = a $\times$ b.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. The sum of a two-digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number. **[2]**

22. If a number x is chosen at random from the numbers - 2 ,- 1 ,0 ,1 ,2 . What is the probability that $x^2 < 2$? **[2]**

23. If α and β are the zeroes of the quadratic polynomial f(s) = $3s^2$ - 6s + 4, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$. **[2]**

24. Find the distance between the points: **[2]**
A(7, -4) and B(-5, 1).

OR

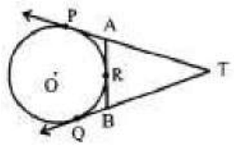
Show that A(-3, 2), B(-5, -5), C(2, - 3) and D(4, 4) are the vertices of a rhombus.

25. Prove that the tangents drawn at the end of a chord of a circle make equal angle with the chord. **[2]**

OR

In the given figure, TP and TQ are tangents from T to the circle with centre O and R is any point on the circle. If AB is a tangent to the circle at R, prove that TA + AR = TB +

BR.



Section C

26. Solve the system of equations graphically: **[3]**
3x - 4y =7
5x + 2y =3
Shade the region between the lines and the y-axis

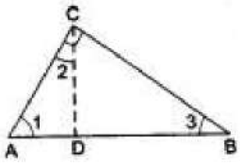
27. If cosec A = 2, find the value of $\frac{1}{\tan A}+\frac{\sin A}{1+\cos A}$ **[3]**

28. On morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps? **[3]**

OR

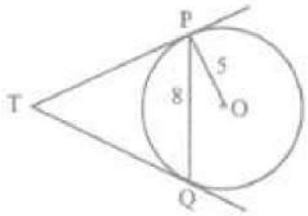
Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

29. In the given figure, $\angle ACB=90^{\circ}$ and $CD\perp AB$. Prove that $CD^2=BD\cdot AD$. **[3]**



30. From a balloon vertically above a straight road, the angles of depression of two cars at an instant are found to be 45° and 60°. If the cars are 100 m apart, find the height of the balloon. **[3]**

31. In the given figure, PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at TP and TQ intersect at point T. Find the length of TP. **[3]**



OR

ABCD is a quadrilateral such that ∠D = 90°. A circle C (O, r) touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, Find r.

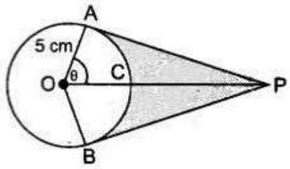
Section D

32. Solve for x: **[5]**
$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}$; $x\neq 1,2,3$

OR

The sum of ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their present ages.

33. An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area. **[5]**



OR

Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending angle of 90° at the centre.

34. In trapezium ABCD, AB ∥ DC and DC = 2AB. EF ∥ AB, where E and F lie on BC and AD respectively, such that $\frac{BE}{EC} = \frac{4}{3}$. Diagonal DB intersects EF at G. Prove that 7EF = 11 AB. **[5]**

35. Find the mean of the following frequency distribution: **[5]**

Class interval	0-8	8-16	16-24	24-32	32-40
Frequency	6	7	10	8	9

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.

(i) If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it

(ii) Find the total money he saved.

OR

How many coins are there in piggy bank on 15th day?

(iii) How much money Akshar saves in 10 days?

37. **Read the text carefully and answer the questions:** **[4]**

Rohan makes a project on coronavirus in science for an exhibition in his school. In this Project, he picks a sphere which has volume 38808 cm^3 and 11 cylindrical shapes each of Volume 1540 cm^3 with 10 cm length.

(i) Find the area covered by cylindrical shapes on the surface of a sphere.

(ii) Find the diameter of the sphere.

(iii) Find the total volume of the shape.

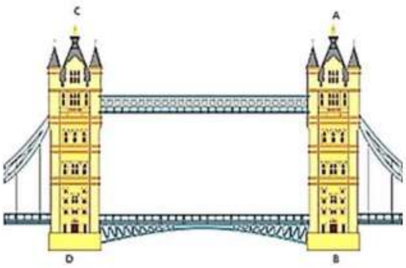
OR

Find the curved surface area of the cylindrical shape.

38. **Read the text carefully and answer the questions:** **[4]**

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60^o and 30^o respectively.



(i) Find the distances of the point from the base of the towers where Neeta was standing while measuring the height.

(ii) Neeta used some applications of trigonometry she learned in her class to find the height of the towers without actually measuring them. What would be the height of the towers she would have calculated?

OR

Find the distance between Neeta and top tower CD?

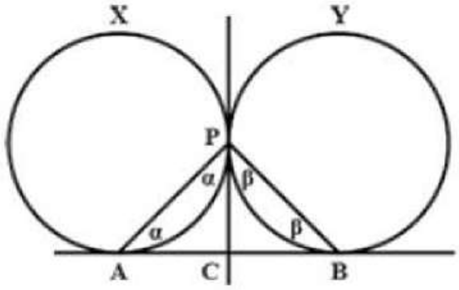
(iii) Find the distance between Neeta and top of tower AB?

Solution

Section A

1. **(c)** 90°

Explanation:



Given X and Y are two circles touch each other externally at P. AB is the common tangent to the circles X and Y at point A and B respectively.

Let $\angle CAP = \alpha$ and $\angle CBP = \beta$

CA = CP [lengths of the tangents from an external point C].

In a triangle PAC, $\angle CAP = \angle APC = \alpha$

Similarly, CB = CP and $\angle CPB = \angle PBC = \beta$

Now in the triangle APB,

$\angle PAB + \angle PBA + \angle APB = 180^{\circ}$ [Sum of the interior angles in a triangle]

$\alpha + \beta + (\alpha + \beta) = 180^{\circ}$

$2\alpha + 2\beta = 180^{\circ}$

$\alpha + \beta = 90^{\circ}$

Therefore, $\angle APB = \alpha + \beta = 90^{\circ}$

2. **(a)** $2 + \sqrt{2}$

Explanation: Let the vertices of $\triangle$ABC be A(0, 0), B(1, 0) and C(0, 1)

Now length of AB $= \sqrt{(1-0)^2 + (0-0)^2}$

$= \sqrt{(1)^2 + 0^2} = \sqrt{1^2} = 1$

Length of AC $= \sqrt{(0-0)^2 + (1-0)^2} = \sqrt{0^2 + (1)^2}$

$= \sqrt{1^2} = 1$

and length of BC $= \sqrt{(0-1)^2 + (1-0)^2}$

$= \sqrt{(1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$

Perimeter of $\triangle$ABC = Sum of sides

$= 1 + 1 + \sqrt{2} = 2 + \sqrt{2}$

3. **(d)** (x, y)

Explanation: AB $= \sqrt{(2x-0)^2 + (0-2y)^2}$

$= \sqrt{4x^2 + 4y^2} = 2\sqrt{x^2 + y^2}$ units

$BO = \sqrt{(0-2x)^2 + (0-0)^2}$

$= \sqrt{4x^2} = 2x$ units

$AO = \sqrt{(0-0)^2 + (0-2y)^2}$

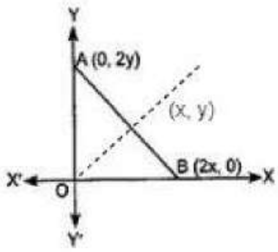
$= \sqrt{4y^2} = 2y$ units

Now, $AB^2 = AO^2 + BO^2 \Rightarrow \left(2\sqrt{x^2+y^2}\right)^2 = (2x)^2 + (2y)^2$

$\Rightarrow 4\left(x^2+y^2\right) = 4\left(x^2+y^2\right)$

Therefore, triangle AOB is an isosceles right-angled triangle.
Since the coordinate of the point which is equidistant from the three vertices of a right-angled triangle is the coordinates of mid-point of its hypotenuse.

$\therefore$ Mid-point of AB = $\left(\frac{0+2x}{2}, \frac{2y+0}{2}\right)$ = (x,y)



4. **(d)** $\frac{3}{4}$

Explanation: Number of Total outcomes ={HH, HT, TH, TT} = 4
Number of possible outcomes = (HH , HT, TH) = 3

$\therefore$ Required Probability = $\frac{3}{4}$

5. **(a)** 7

Explanation: Given that R is the mid- point of the line segment AB.

Th y-coordinate of R = $\frac{5+y}{2}$

$\Rightarrow$ y = 7

6. **(c)** intersecting exactly at one point

Explanation: We have,
2x + 3y – 2 = 0
And, x – 2y – 8 = 0
Here, $a_1 = 2$, $b_1 = 3$ and $c_1 = -2$
And, $a_2 = 1$, $b_2 = -2$ and $c_2 = -8$

$\therefore \frac{a1}{a2} = \frac{2}{1}, \frac{b1}{b2} = \frac{3}{-2}$ and $\frac{c_1}{c_2} = \frac{-2}{-8} = \frac{1}{4}$

Clearly, $\frac{a1}{a2} \neq \frac{b1}{b2}$

Hence, the given system has a unique solution and the lines intersect exactly at one point.

7. **(d)** 6400

Explanation: Volume of the wall = $(800 \times 600 \times 22.5)$ cm^3,

Number of bricks = $\frac{volume\ of\ the\ wall}{volume\ of\ 1\ brick}$

$= \left(\frac{800 \times 600 \times 22.5}{25 \times 11.25 \times 6}\right) = 6400$

8. **(a)** $\frac{1}{2}$

Explanation: Number of possible outcomes = {6, 7, 8, 9, 10} = 5
Number of total outcomes = 10

$\therefore$ Required Probability = $\frac{5}{10} = \frac{1}{2}$

9. **(a)** $\frac{13}{15}$

Explanation: Total number of balls in the bag = 8 + 2 + 5 = 15.
Number of non-black balls = 8 + 5 = 13.

$\therefore$ P (getting a non-black ball) = $\frac{13}{15}$

10. **(b)** 5

Explanation: Let the roots of the equation $\left(5x^2 + 13x + k = 0\right)$ be α and $\frac{1}{\alpha}$

Product of the roots = $\frac{c}{a}$

$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{k}{5}$

$\Rightarrow 1 = \frac{k}{5}$

$\Rightarrow$ k = 5.

11. **(c)** -2 < b < 2

Explanation: In the equation

$x^2 - bx + 1 = 0$

$D = b^2 - 4ac = (-b)^2 - 4 \times 1 \times 1$

$= b^2 - 4$

$\because$ it is given that the roots are not real, D < 0

$\Rightarrow b^2 - 4 < 0$

$\Rightarrow b^2 < 4 \Rightarrow b^2 < (\pm 2)^2$

$\therefore$ b < 2 and b > -2 or -2 < b

$\therefore$ -2 < b < 2

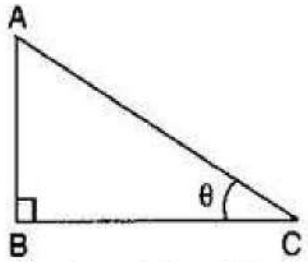
12. **(b)** an irrational number

Explanation: Rational Numbers say $\frac{4}{9}$, $\frac{p}{q}$, $\sqrt{4}$, fraction, whole numbers, terminating decimal, repeating decimal, perfect square, can be expressed as a ratio of two integers

provided the denominator is not equal to zero
Irrational Numbers $\sqrt{2}$,$\sqrt{5}$,$\sqrt{7}$, π not a fraction, decimal does not repeat, decimal does not end, non-perfect square, we cannot express as a ratio but both can be expressed as decimal numbers
The difference between a rational and an irrational number is always an irrational number.
e.g. rational - irrational = irrational say 2 - $\sqrt{2}$ = irrational

13. **(d)** 2
Explanation: 2

14. **(d)** 0
Explanation: $= \frac{1-tan^2 45^0}{1+tan^2 45^0} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

15. **(a)** 20 m
Explanation:


Given: Height of pole = AB = 20 m
And the angle of elevation θ = 45°
Let length of shadow of pole = BC = x meters
$\therefore \tan 45^\circ = \frac{AB}{BC}$
$\Rightarrow 1 = \frac{20}{x}$
$\Rightarrow x = 20$ m
Therefore, the length of the shadow of the pole is 20 m.

16. **(b)** co-prime numbers
Explanation: If two numbers do not have a common factor (other than 1), then they are called co-primer numbers. We know that two numbers are coprime if their common factor (greatest common divisor) is 1. e.g. co-prime of 12 are 11, 13.

17. **(b)** 315
Explanation: Class having maximum frequency is the modal class.
hence, modal class : 150-155
$\therefore$ Lower limit of the modal class = 150
Also, N = 60 $\Rightarrow \frac{N}{2} = 30$
The cumulative frequency just greater than 30 is 37.
Hence, the median class is 160-165.
$\therefore$ Upper limit of the median class = 165
Required sum = 150 + 165 = 315

18. **(b)** $\frac{5}{13}$
Explanation: Let the fraction be $\frac{x}{y}$.

According to question

$x + y = 18$ ... (i)

And $\frac{x}{y+2} = \frac{1}{3}$

$\Rightarrow$ $3x = y + 2$

$\Rightarrow$ $3x - y = 2$ ... (ii)

On solving eq. (i) and eq. (ii), we get

$x = 5, y = 13$

Therefore, the fraction is $\frac{5}{13}$

19. **(d)** A is false but R is true.

Explanation: Two right-angled triangle are similar only if at least one more angle is also equal.

20. **(d)** A is false but R is true.

Explanation: We know that for any two numbers, Product of the two numbers = HCF $\times$ LCM

HCF $\times$ LCM = 18 $\times$ 169 = 3042 $\neq$ 3072

So, A is false but R is true.

Section B

21. Let the ten's and unit's digits of the required number be x and y respectively.

Then, the number = $(10x + y)$.

The number obtained on reversing the digits = $(10y + x)$.

As per given condition

The sum of a two-digit number and the number obtained by reversing the order of its digits is 99.

$\therefore$ $(10y + x) + (10x + y) = 99$

$\Rightarrow$ $11(x + y) = 99$

$\Rightarrow$ $x + y = 9$.

The digits differ by 3

So, $(x - y) = \pm 3$.

Thus, we have

$x + y = 9$ .... (i)

$x - y = 3$ .... (ii)

or $x + y = 9$ .... (iii)

$x - y = -3$ .... (iv)

From (i) and (ii), we get $x = 6, y = 3$.

From (iii) and (iv), we get $x = 3, y = 6$.

Hence, the required number is 63 or 36.

22. Clearly, number x can take any one of the five given values.

So, total number of elementary events = 5

We observe that $x^2 < 2$ when x takes any one of the following three values - 1, 0 and 1.

So, favourable number of elementary events = 3

Hence, $P\left(x^2 < 2\right) = \frac{3}{5}$

23. Since, α and β are the zeroes of the quadratic polynomial $f(s) = 3s^2 - 6s + 4$

Compare f(s) with standard form $f(x) = ax^2 + bx + c$

$a = 3, b = -6$ and $c=4$

Sum of the zeroes = $\alpha + \beta = -\frac{b}{a} = \frac{6}{3}$

Product of the zeroes = $\alpha \times \beta = \frac{c}{a} = \frac{4}{3}$

Now,

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta.$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left[\frac{\alpha + \beta}{\alpha\beta}\right] + 3\alpha\beta$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left[\frac{\alpha + \beta}{\alpha\beta}\right] + 3\alpha\beta$$

By substituting the values of sum and product of the zeroes, we will get

$$\frac{\alpha}{\beta} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta = 8$$

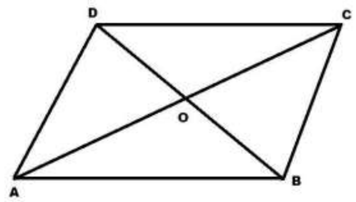
24. The given points are A(7, -4) and B(-5, 1)
Then, $(x_1 = 7, y_1 = -4)$ and $(x_2 = -5, y_2 = 1)$
Therefore, by distance formula, we have,

$$\therefore AB = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$= \sqrt{(-5-7)^2 + (1+4)^2} = \sqrt{(-12)^2 + (5)^2}$$

$$= \sqrt{144 + 25} = \sqrt{169} = 13 \textit{ units}$$

OR



We know that all the sides of a rhombus are equal and diagonals are not equal.
i.e. In rhombus ABCD, AB = BC = CD = DA and $AC \neq BD$

Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here A(-3, 2), B(-5, -5), C(2, - 3) and D(4, 4)

$$AB = \sqrt{\{-5 - (-3)\}^2 + (-5 - 2)^2} = \sqrt{4 + 49} = \sqrt{53}$$

$$BC = \sqrt{\{2 - (-5)\}^2 + \{-3 - (-5)\}^2} = \sqrt{49 + 4} = \sqrt{53}$$

$$CD = \sqrt{(4 - 2)^2 + \{4 - (-3)\}^2} = \sqrt{4 + 49} = \sqrt{53}$$

$$AD = \sqrt{\{4 - (-3)\}^2 + (4 - 2)^2} = \sqrt{49 + 4} = \sqrt{53}$$

Thus, AB = CD = BC = AD i.e. all sides are equal.
Now,

$AC = \sqrt{(2+3)^2 + (-3-2)^2}$

$= \sqrt{5^2 + 5^2} = = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$

$BD = \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{9^2 + 9^2} = \sqrt{81 + 81} = 9\sqrt{2}$

Thus, AC $\neq$ BD i.e diagonals are not equal.

$\therefore$ ABCD is a rhombus.

25. Let AB be a chord of the circle and P and Q are the tangents to the circle at A and B respectively.

Let the tangent P and Q, when produced meet at C.

Now, CA and CB are tangent to the circle at A and B from an external point C.

$\therefore$ CA = CB [lengths of tangents from P are equal]

In $\triangle$ACB,

$\angle$ CAB = $\angle$CBA [$\because$ In a triangle angles opposite to equal sides are equal]

OR

Length of tangents from same external point are equal.

$\therefore$ TP = TQ

AP = AR

and BR = BQ

We have, $TP = TQ$

$\Rightarrow TA + AP = TB + BQ$

$\Rightarrow TA + AR = TB + BR$

Hence proved.

Section C

26. $3x - 4y = 7$ *and* $5x + 2y = 3$

The given system of linear equation is $3x - 4y = 7$ *and* $5x + 2y = 3$

Now, $3x - 4y = 7$

$y = \frac{3x-7}{4}$

When x = 1 then, y = -1

When x = -3 then y = -4

x	1	-3
y	-1	-4

Now, 5x + 2y = 3

$y = \frac{3-5x}{2}$

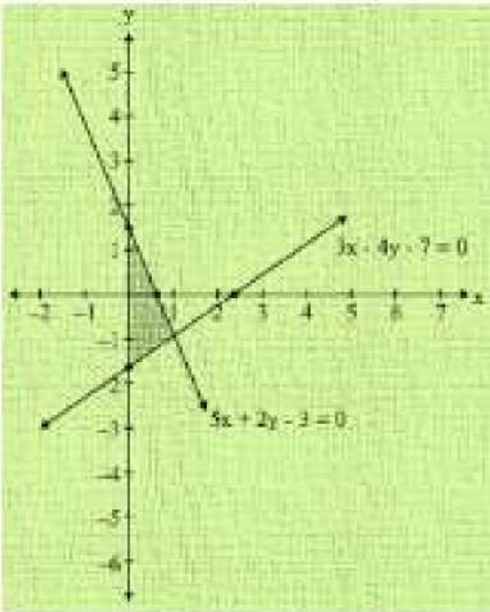
When x = 1 then, y = -1

When x = 3 then y = -6

Thus, we have the following table

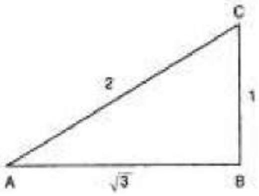
x	1	3
y	-1	-6

Graph of the given system of equations are



Clearly the two lines intersect at A(1, -1)

Hence, x = 1 and y = -1 is the solution of the given system of equations.

27. 

According to the question,

$\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{2}{1}$

So, we draw a right triangle, right angled at B such that

Perpendicular $= BC = 1$, Hypotenuse $= AC = 2$

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad 2^2 = 1^2 + AB^2$

$\Rightarrow \quad 4 - 1 = AB^2$

$\Rightarrow \quad AB = \sqrt{3}$

$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}, \sin A = \frac{BC}{AC} = \frac{1}{2}$ and , $\cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

$$\therefore \frac{1}{\tan A} + \frac{\sin A}{1 + \cos A} = \frac{1}{\frac{1}{\sqrt{3}}} + \frac{1/2}{1 + \frac{\sqrt{3}}{2}}$$

$$\Rightarrow \frac{1}{\tan A} + \frac{\sin A}{1 + \cos A} = \frac{\sqrt{3}}{1} + \frac{1/2}{\frac{2+\sqrt{3}}{2}} = \frac{\sqrt{3}}{1} + \frac{1}{2+\sqrt{3}}$$

$$\Rightarrow \quad \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \sqrt{3} + \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\Rightarrow \quad \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \sqrt{3} + \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} = \sqrt{3} + \frac{2-\sqrt{3}}{4-3} = \sqrt{3} + (2-\sqrt{3}) = 2$$

28. Since, the three persons start walking together.

∴ The minimum distance covered by each of them in complete steps = LCM of the measures of their steps

$40 = 8 \times 5 = 2^3 \times 5$

$42 = 6 \times 7 = 2 \times 3 \times 7$

$45 = 9 \times 5 = 3^2 \times 5$

Hence LCM (40, 42, 45)

$= 2^3 \times 3^3 \times 5 \times 7 = 8 \times 9 \times 5 \times 7 = 2520$

∴ The minimum distance each should walk so that each can cover the same distance = 2520 cm = 25.20 meters.

OR

Let the $\frac{2+\sqrt{3}}{5}$ be a rational number.

Therefore it can be written in form of p/q (q not = 0 and p, q are co. Prime)

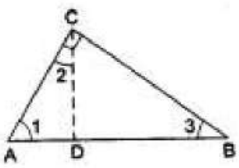
$$\frac{2+\sqrt{3}}{5} = \frac{p}{q}$$

$$\sqrt{3} = \frac{5p}{q} - 2$$

$$\sqrt{3} = \frac{5p-2q}{q}$$

Here LHS i.e, $\sqrt{3}$ is an irrational and RHS is a rational number. This is not possible. Hence our supposition is wrong. So the given number $\frac{2+\sqrt{3}}{5}$ Is irrational.

29.



It is given that in $\triangle ABC$, $\angle ACB = 90^\circ$ and $CD \perp AB$.

To Prove: $CD^2 = BD \cdot AD$

Proof : In right $\triangle ADC$, we have

$\angle 1 + \angle 2 = 90^\circ$......................(i)

In right $\triangle ACB$, we have

$\angle 1 + \angle 3 = 90^\circ$.........................(ii)

From (i) and (ii) we have

$\angle 1 + \angle 2 = \angle 1 + \angle 3$

$\Rightarrow \angle 2 = \angle 3$

In $\triangle ADC$ and $\triangle CDB$, we have

$\angle 2 = \angle 3$ (proved)

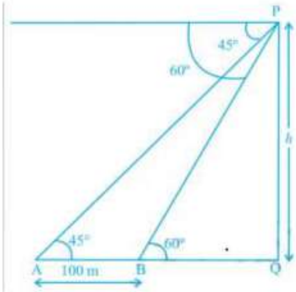
and $\angle ADC = \angle CDB = 90°$

$\therefore \quad \triangle ADC \sim \triangle CDB$ [by AA-similarity]

$\therefore \quad \frac{AD}{CD} = \frac{CD}{BD}$

Hence, $CD^2 = BD \cdot AD$.

30. Let the height of the balloon at P be h meters (see Fig).
Let A and B be the two cars.
Thus AB = 100 m. From ΔPAQ, AQ = PQ = h



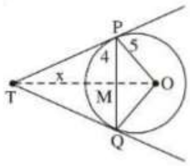
Now from ΔPBQ, $\frac{PQ}{BQ} = \tan 60°$

or $\frac{h}{h-100} = \sqrt{3}$

or $h = \sqrt{3}(h - 100)$

Therefore, $h = \frac{100\sqrt{3}}{\sqrt{3}-1} = 50(3 + \sqrt{3})$

i.e, Height of the balloon is $50(3 + \sqrt{3})m$

31. 

Join OT and OQ.
TP = TQ (As length of tangents from a point outside the circle is equal)
$\therefore$ TM $\perp$ PQ and bisects PQ
Hence PM = 4 cm
Therefore OM = $\sqrt{25 - 16} = \sqrt{9} = 3$ cm

Let TM = x

From $\triangle$PMT, $PT^2 = x^2 + 16$

From $\triangle$POT, $PT^2 = (x + 3)^2 - 25$

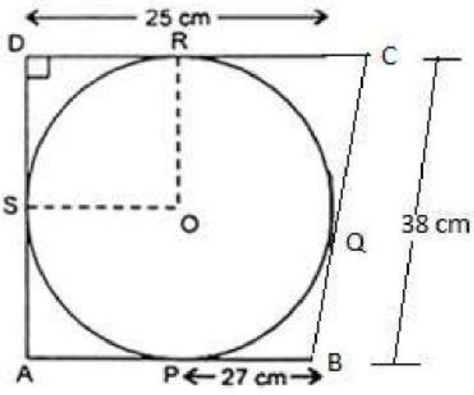
Hence $x^2 + 16 = x^2 + 9 + 6x - 25$

$\Rightarrow \ 6x = 32 \ \Rightarrow \ x = \frac{16}{3}$

Hence, $PT^2 = \frac{256}{9} + 16 = \frac{400}{9}$

$\therefore\ PT = \frac{20}{3}$ cm = 6.667 cm

OR



Given that ABCD is a quadrilateral such that ∠D = 90°.
BC = 38 cm,CD = 25cm and BP = 27 cm
∴ From the figure,
BP = BQ = 27 cm [Tangents from an external point are equal]
Now, BC = 38
⇒ BQ + QC = 38
⇒ 27 + QC = 38
⇒ QC = 38 - 27
⇒ QC = 11 cm
∴ QC = 11 cm = CR [Tangents from an external point are equal]
CD = 25 cm
CR + RD=25
⇒ 11+ RD = 25
⇒ RD = 25-11
⇒ RD = 14 cm
Also,
RD = DS = 14 cm [Tangents from an external point are equal]
OR and OS are radii of the circle.
From tangents R and S, ∠ORD = ∠OSD = 90°
Thus, ORDS is a square.
OR = DS = 14 cm
Hence, the radius of the circle, r = OR = 14 cm

Section D

32. Given, $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$

$$\frac{(x-3)+(x-1)}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$

$$\frac{x-3+x-1}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$

$$\frac{2x-4}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$

$$\frac{2(x-2)}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$

$$\frac{2}{(x-1)(x-3)}=\frac{2}{3}$$

$(x - 1)(x - 3) = 3$

$x^2 - 4x + 3 = 3$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$x = 0$, $x - 4 = 0$

$x = 0$, $x = 4$

OR

Let the present age of father be x years.

Son's present age = (45 - x) years.

Five years ago:

Father's age = (x - 5) years

Son's age = (45 - x - 5) years = (40 - x) years.

According to question,

$\therefore$ $(x - 5)(40 - x) = 124$

$\Rightarrow$ $40x - x^2 - 200 + 5x = 124$

$\Rightarrow$ $x^2 - 45x + 324 = 0$

Spilting the middle term,

$\Rightarrow$ $x^2 - 36x - 9x + 324 = 0$

$\Rightarrow$ $x(x - 36) - 9(x - 36) = 0$

$\Rightarrow$ $(x - 9)(x - 36) = 0$

$\Rightarrow$ $x = 9$, or 36

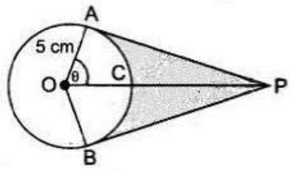
We can't take father age as 9 years

So, x = 36, we have

Father's present age = 36 years

Son's present age = 9 years

Hence, Father's present age = 36 years and Son's present age = 9 years.

33. 

$\cos\theta = \frac{1}{2}$ or, $\theta = 60°$

Reflex $\angle AOB = 120°$

$\therefore$ ADB $= \dfrac{2 \times 3.14 \times 5 \times 240}{360} = 20.93$ cm

Hence length of elastic in contact = 20.93 cm

Now, AP = $5\sqrt{3}$cm

$$a(\triangle OAP) = \frac{1}{2} \times \text{ base } \times \text{ height} = \frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25\sqrt{3}}{2}$$

$$\text{Area } (\triangle OAP + \triangle OBP) = 2 \times \frac{25\sqrt{3}}{2} = 25\sqrt{3} = 43.25 \text{ cm}^2$$

$$\text{Area of sector OACB} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{25 \times 3.14 \times 120}{360} = 26.16 \text{ cm}^2$$

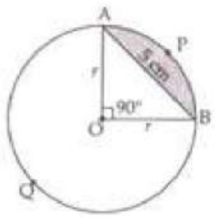
Shaded Area = 43.25 - 26.16 = 17.09 cm^2

OR

Chord AB = 5 cm divides the circle into two segments minor segment APB and major segment AQB. We have to find out the difference in area of major and minor segment.

Here, we are given that $\theta = 90°$

$$\text{Area of } \triangle OAB = \frac{1}{2}\text{Base } \times \text{ Altitude} = \frac{1}{2}\text{r } \times \text{ r} = \frac{1}{2}r^2$$



Area of minor segment APB

$$= \frac{\pi r^2 \theta}{360°} - \text{Area of } \triangle AOB$$

$$= \frac{\pi r^2 90°}{360°} - \frac{1}{2}r^2$$

$$\Rightarrow \text{ Area of minor segment} = \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right) \ldots \text{(i)}$$

Area of major segment AQB = Area of circle – Area of minor segment

$$= \pi r^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2}\right]$$

$$\Rightarrow \text{ Area of major segment AQB } = \left[\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right] \ldots \text{(ii)}$$

Difference between areas of major and minor segment

$$= \left(\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$$

$$= \frac{3}{4}\pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$$

$\Rightarrow$ Required area $= \frac{2}{4}\pi r^2 + r^2 = \frac{1}{2}\pi r^2 + r^2$

In right $\triangle$OAB,

$r^2 + r^2 = AB^2$

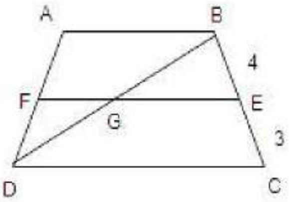
$\Rightarrow 2r^2 = 5^2$

$\Rightarrow r^2 = \frac{25}{2}$

Therefore, required area $= \left[\frac{1}{2}\pi \times \frac{25}{2} + \frac{25}{2}\right] = \left[\frac{25}{4}\pi + \frac{25}{2}\right] cm^2$

34. $\frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$

$\Rightarrow \frac{AD}{FD} = \frac{AF+FD}{3} = \frac{4+3}{3} = \frac{7}{3}$



In $\triangle DFG$ and $\triangle DAB$

$\angle FDG \cong \angle ADB$ ($\because$ common angle)

$\angle DFG \cong \angle DAB$ ($\because$ FG $\parallel$ AB)

So, $\triangle DFG \sim \triangle DAB$ (AA similarity)

Also, $\triangle BGE \sim \triangle BDC$

$\Rightarrow \frac{DF}{DA} = \frac{FG}{AB}$ and $\frac{GE}{DC} = \frac{BE}{BC}$

$\Rightarrow \frac{3}{7} = \frac{FG}{AB}$ and $\frac{GE}{2AB} = \frac{4}{7}$

Now, $EF = FG + GE = \frac{3}{7}AB + \frac{8}{7}AB$

$\Rightarrow EF = (\frac{3}{7} + \frac{8}{7})AB$

$\Rightarrow EF = (\frac{3+8}{7})AB$

$\Rightarrow EF = (\frac{11}{7})AB$

$\Rightarrow 7EF = 11AB$

35.

Class interval	Mid- value	$d_i = x_i - 20$	$u_i = \left(\frac{x_i - 20}{8}\right)$	f_i	$f_i u_i$
0-8	4	-16	-2	6	-12
8-16	12	-8	-1	7	-7
16-24	20	0	0	10	0
24-32	28	8	1	8	8
32-40	36	16	2	9	18
				N = 40	$\sum f_i u_i = 7$

Let the assumed mean (A) is = 20

h = 8

$$\text{Mean} = A + h\left(\frac{\sum f_i u_i}{N}\right)$$

$$= 20 + 8\left(\frac{7}{40}\right)$$

= 20 + 1.4

= 21.4

Section E

36. **Read the text carefully and answer the questions:**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.

(i) Child's Day wise are,

$\underbrace{5}_{1\text{ coin}}, \underbrace{10}_{2\text{ coins}}, \underbrace{15}_{3\text{ coins}}, \underbrace{20}_{4\text{ coins}}, \underbrace{25}_{5\text{ coins}}$ .... to $\underbrace{n\text{ days}}_{n\text{ coins}}$

We can have at most 190 coins

i.e., 1 + 2 + 3 + 4 + 5 + ... to n term = 190

$\Rightarrow \frac{n}{2}[2 \times 1 + (n - 1)1] = 190$

$\Rightarrow$ n(n + 1) = 380 $\Rightarrow$ $n^2 + n - 380 = 0$

$\Rightarrow$ (n + 20)(n − 19) = 0 $\Rightarrow$ (n + 20)(n - 19) = 0

$\Rightarrow$ n = −20 or n = 19 $\Rightarrow$ n = -20 or n = 19

But number of coins cannot be negative

$\therefore$ n = 19 (rejecting n = -20)

So, number of days = 19

(ii) Total money she saved = 5 + 10 + 15 + 20 + ... = 5 + 10 + 15 + 20 + ... upto 19 terms

$= \frac{19}{2}[2 \times 5 + (19 - 1)5]$

$= \frac{19}{2}[100] = \frac{1900}{2} = 950$

and total money she shaved = ₹950

OR

Number of coins in piggy bank on 15th day

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{15} = \frac{15}{2}[2 \times 5 + (15 - 1) \times 5]$

$\Rightarrow S_{15} = \frac{15}{2}[2 + 14]$

$\Rightarrow S_{15} = 120$

So, there are 120 coins on 15th day.

(iii)Money saved in 10 days

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 5 + (10 - 1) \times 5]$

$\Rightarrow S_{10} = 5[10 + 45\}]$

$\Rightarrow S_{10} = 275$

Money saved in 10 days = ₹275

37. **Read the text carefully and answer the questions:**

Rohan makes a project on coronavirus in science for an exhibition in his school. In this Project, he picks a sphere which has volume 38808 cm^3 and 11 cylindrical shapes each of Volume 1540 cm^3 with 10 cm length.

(i) Given Volume of cylinder = $1540 cm^3$.

Surface covered by cylindrical shapes on sphere is area of circular base of cylinder

Volume of cylinder = $\pi r^2 h = 1540$

$\Rightarrow 1540 = \frac{22}{7} \times r^2 \times 10$

$\Rightarrow r^2 = \frac{1540 \times 7}{22 \times 10} = 49$

$\Rightarrow r = 7$ cm

Surface area covered by cylindrical shapes = $11\pi r^2$

$\Rightarrow S = 11 \times \frac{22}{7} \times 7 \times 7$

$\Rightarrow$ S = 1694 cm^2

Surface covered by cylindrical shapes on sphere = 1694cm^2

(ii) Volume of sphere = 38808cm^3

Volume of sphere $= \frac{4}{3} \times \pi \times r^3$

$\Rightarrow 38808 = \frac{4}{3} \times \frac{22}{7} \times r^3$

$\Rightarrow r^3 = \frac{38808 \times 3 \times 7}{22 \times 4} = 21^3$

$\Rightarrow$ r = 21 cm

$\Rightarrow$ Diameter = 42 cm

(iii) Given Volume of Sphere = 38808 cm^3 and Volume of each cylinder = 1540cm^3

Total volume of shape = volume of sphere + 11 × volume of cylinder

= 38808 + 11 × 1540

= 38808 +16940

= 55748 cm^3

OR

For cylinder height = h = 10cm and radius = r = 7cm

Curved surface area of cylinder = $2\pi rh$

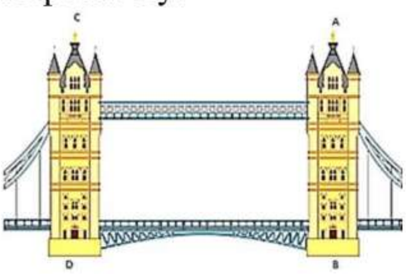
$\Rightarrow CSA = 2 \times \frac{22}{7} \times 7 \times 10$

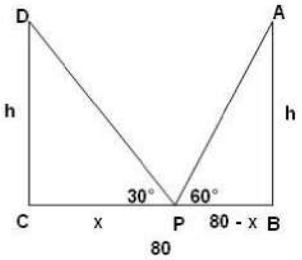
$\Rightarrow$ CSA = 440 cm^2

38. **Read the text carefully and answer the questions:**

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60^o and 30^o respectively.



(i) 

Suppose AB and CD are the two towers of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m, therefore BP = (80 – x).

Also, ∠APB = 60^{o} and ∠DPC = 30^{o}

In right angled triangle DCP,

$$\tan 30^{o} = \frac{CD}{CP}$$

$$\Rightarrow \frac{h}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{x}{\sqrt{3}} \text{ ....(i)}$$

In right angled triangle ABP,

$$\tan 60^{o} = \frac{AB}{AP}$$

$$\Rightarrow \frac{h}{80-x} = \sqrt{3}$$

$$\Rightarrow h = \sqrt{3}(80-x)$$

$$\Rightarrow \frac{x}{\sqrt{3}} = \sqrt{3}(80-x)$$

$\Rightarrow$ x = 3(80 – x)

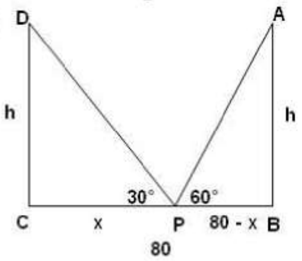
$\Rightarrow$ x = 240 – 3x

$\Rightarrow$ x + 3x = 240

$\Rightarrow$ 4x = 240

$\Rightarrow$ x = 60

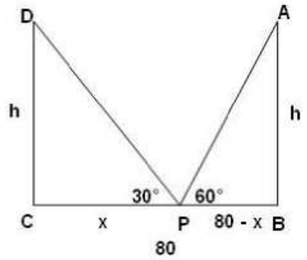
Thus, the position of the point P is 60 m from C.

(ii) 

Height of the tower, h = $\frac{x}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$

The height of each tower is $20\sqrt{3}$ m.

OR



The distance between Neeta and top of tower CD.

In △CDP

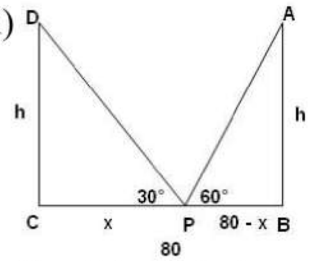
$$\sin 30^o = \frac{CD}{PD}$$

$$\Rightarrow PD = \frac{CD}{\sin 30^0}$$

$$\Rightarrow PD = \frac{20\sqrt{3}}{\frac{1}{2}} = 40\sqrt{3}$$

$$\Rightarrow PD = 40\sqrt{3}$$

(iii)



The distance between Neeta and top of tower AB.

In △ABP

$$\sin 60^o = \frac{AB}{AP}$$

$$\Rightarrow AP = \frac{AB}{\sin 60^0}$$

$$\Rightarrow AP = \frac{20\sqrt{3}}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow AP = 40 \text{ m}$$