Mathematics
NTSE- Foundation
Surds and Exponents

Surds & Radical

Let a be a rational number and n be a positive integer such that $\sqrt[n]{a}$ is irrational, then $\sqrt[n]{a}$ is called as surd of the order n and a is called the radical.

$\sqrt[n]{a}$ is a surd if a is a rational number and $\sqrt[n]{a}$ is an irrational number.

Laws of radicals:

(i) $\left(\sqrt[n]{a}\right)^n = \left(a^{\frac{1}{n}}\right)^n = a^{\frac{1}{n}\times n} = a.$

(ii) $\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$

For example : $\sqrt[3]{2}.\sqrt[3]{4} = \sqrt[3]{2\times 4} = \sqrt[3]{2^3} = (2^3)^{\frac{1}{3}} = 2$

(iii) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

(iv) $(\sqrt[n]{a})^m = \sqrt[n]{a^m} = a^{m/n}$

(v) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

Rationalizing Factor:
If the product of two surds is a rational number, then each surds is called a rationalising factor (RF) of the other.

Rationalisation of surds:
The process of converting a surd into rational number by multiplying it with a suitable RF, is called the rationalisation of the surd.
Monomial surds and their RF:
The general form o a monomial surd is K $\sqrt[n]{a}$ and its RF is $a^{1-\frac{1}{n}}$.
Binomial surds and their RF:
The surds of the types : $a+\sqrt{b}, a-\sqrt{b}, \sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$ are called binomial surds.
Conjugate Surds :

The binomial surds which differ only in sign between the terms separating them are known as conjugate surds. In binomial surds, the conjugate surds are RF of each other.

For example :

(i) RF of $\sqrt{a}+\sqrt{b}$ is $\sqrt{a}-\sqrt{b}$

(ii) RF of $\sqrt{a}-\sqrt{b}$ is $\sqrt{a}+\sqrt{b}$

Trinomial surds and their RF:
A surd which consists of three terms, at least two of which are monomial surds, is called a trinomial surd.

Exponents

The repeated multiplication of the same factor can be written in a more compact form called exponential form.

Laws of exponents:
If a is any non - zero rational number and m, n are whole numbers, then
(i) On the same base in multiplication, powers are added.

$A^m \times a^n = a^{m+n}$

For example: $3^2 \times 3^4 = 3^{2+4} = 3^6$.
(ii) On the same base in division, powers are subtracted

$$\frac{a^m}{a^n} = a^{m-n}$$

For Example: $\frac{3^5}{3^2} = 3^{5-2} = 3^3$

(iii) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m$

For Example : $\frac{2^3}{2^4} = \frac{1}{2^{4-3}} = \frac{1}{2}$

(iv) $(a^m)^n = a^{mn}$

For Example : $(2^2)^3 = 2^{2\times 3} = 2^6$.
(v) $a^n \times a^{-n} = a^0 = 1$
(vi) $a^m \times b^m = (ab)^m$
For example : $2^2 \times 3^2 = (2\times 3)^2 = 6^2 = 36$.

EXERCISE

1. The value of $\frac{2^{m+3}\times 3^{2m-n}\times 5^{m+n+3}\times 6^{n+1}}{6^{m+1}\times 10^{n+3}\times 15^m}$ is equal to:
(A) 0 (B) 1
(C) 2^m (D) None of these

2. The value of $\left[\left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^3\right]\times 2^6$ is :
(A) 1 (B) 2
(C) 3 (D) 4

3. If $3^x = 500$, then the value of 3^{x-2} is :
(A) $\frac{100}{9}$ (B) $\frac{1000}{9}$
(C) $\frac{500}{9}$ (D) $\frac{500}{3}$

4. If $\frac{p}{q} = \left(\frac{2}{3}\right)^3 \div \left(\frac{3}{2}\right)^{-3}$ then the value of $\left(\frac{p}{q}\right)^{-10}$ is :
(A) 1 (B) 0
(C) cannot be determined (D) None of these

5. $\sqrt{1+\sqrt{1+\sqrt{1+.....}}}$ is :
(A) equal 1 (B) lies between0 and 1
(C) lies between 1 and 2 (D) is greater than 2

6. The value of $\sqrt[3]{343}\times\sqrt[3]{-64}$ is :

(A) 28 (B) -28

(C) 18 (D) -18

7. The value of $(3^0 - 4^0) \times 5^2$ is :

(A) 25 (B) 0

(C) -25 (D) None of these

8. The solution of $3^{3x-5} = -\frac{1}{9^x}$ is :

(A) $\frac{5}{2}$ (B) 5

(C) 1 (D) $\frac{7}{2}$

9. The value of $\left[(-2)^{(-2)}\right]^{(-3)}$ is :

(A) 64 (B) 32

(C) cannot be determined (D) None of these

10. Simplify: $(32)^{\frac{-2}{5}} \div (125)^{\frac{-2}{3}}$

(A) $\frac{4}{25}$ (B) $\frac{25}{4}$

(C) $\frac{2}{5}$ (D) $\frac{5}{2}$

11. The value of $\frac{4\sqrt{3}-4}{\sqrt{3}+1} - \frac{2-\sqrt{3}}{2+\sqrt{3}}$ is :

(A) 0 (B) 1

(C) $8\sqrt{3}$ (D) 15

12. $(64)^{\frac{-2}{3}} \times \left(\frac{1}{4}\right)^{-3}$ equals to :

(A) 4 (B) $\frac{1}{4}$

(C) 1 (D) 16

13. The value of $\frac{(5)^{0.25}\times(125)^{0.25}}{(256)^{0.10}\times(256)^{0.15}}$ is :

(A) $\frac{\sqrt{5}}{2}$ (B) $\frac{5}{4}$

(C) $\frac{25}{2}$ (D) $\frac{25}{16}$

14. If $x+\frac{1}{x}=2$ then $x^8+\frac{1}{x^8}$ is equal to :

(A) 1 (B) 3

(C) 2 (D) $\sqrt{2}$

15. The value of $\frac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1}\left(\frac{3}{2}\right)^3+\left(\frac{-1}{3}\right)^{-1}}$ is :

(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $-\frac{1}{2}$

16. The value of $\frac{\sqrt[6]{0.001}\sqrt[6]{x^{216}}}{\sqrt{10}}$ is :

(A) $(10x)^{36}$ (B) $\frac{x^6}{10^3}$

(C) $\frac{x^{36}}{10}$ (D) None of these

17. If $a=\frac{1}{3-2\sqrt{2}}, b=\frac{1}{3+2\sqrt{2}}$ then the value of $a^2 + b^2$ is:

(A) 34 (B) 35

(C) 36 (D) 37

18. If $a=\frac{1}{3-2\sqrt{2}}, b=\frac{1}{3+2\sqrt{2}}$ then the value of $a^3 + b^3$ is

(A) 193 (B) 194

(C) 198 (D) 196

19. If $x=(7+4\sqrt{4})$ then the value of $\sqrt{x}+\frac{1}{\sqrt{x}}$ is :

(A) 8 (B) 6

(C) 5 (D) 4

20. The simples rationalising factor of $\sqrt[4]{48}$ is :

(A) $\sqrt[4]{9}$ (B) $\sqrt[4]{27}$

(C) $\sqrt[3]{9}$ (D) None of these

21. The exponential form of $\sqrt{\sqrt{2}\times\sqrt{3}}$ is :

(A) $6^{-1/2}$ (B) $6^{1/2}$ (C) $6^{1/4}$ (D) 6

22. The value of $\frac{6^n\times 2^{2n}\times 3^{3n}}{30^n\times 3^{2n}\times 3^{3n}}$ is equal to :

(A) 1 (B) $(0.3)^n$ (C) 3^{-n} (D) $(0.3)^n$

23. The value of $\frac{2^{1/2}\times 3^{1/3}\times 4^{1/4}}{10^{-1/5}\times 5^{3/5}} \div \frac{4^{-2/3}\times 5^{-7/5}}{4^{-3/5}\times 6^{-1/3}}$ is :

(A) 10 (B) 1 (C) 6 (D) 18

24. The value of $\left[\left(\frac{-1}{3}\right)^{-2}\right]^{-2} \times \left[\left(\frac{2}{3}\right)^2\right]^{-2} \div \left[\left(\frac{3}{2}\right)^{-1}\right]^{-2}$ is

[NTSE Stage-I/Raj. /2007]

(A) 81 (B) 36 (C) $\frac{1}{81}$ (D) $\frac{1}{36}$

25. If $(4)^3 \times (6)^4 \times (10)^5 = 2^x \times 3^y \times 5^z$, then the value of x + y + z is : **[NTSE Stage - I/raj./2008]**

(A) 12 (B) 15 (C) 20 (D) 24

26. Which of the following is equal to 1 ?

[NTSE Stage - II/raj./2007]

(A) $\frac{(0.11)}{(1.1)^2\times 0.1}$ (B) $\frac{(1.1)^2}{(11)^2\times 0.1}$

(C) $\frac{(0.011)^2}{(1.1)^2\times(0.01)^2}$ (D) $\frac{(0.11)^2}{(11)^2\times 0.01}$

27. If $\left(\frac{1}{17}\right)^0 - (64)^{-\frac{1}{2}} - (-32)^{-\frac{4}{5}} = \frac{b}{a}$, where a and b are positive integers, then the smallest value of a - b is :

[NTSE Stage - II/2007]

(A) 1 (B) 2 (C) 3 (D) 4

ANSWER – KEY

SURDS & EXPONENTS

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	C	A	C	B	B	C	A	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	A	B	C	B	C	A	C	D	B
Que.	21	22	23	24	25	26	27			
Ans.	C	D	A	D	D	C	C			