Gravitation

Exercise Solutions

Solution 1:

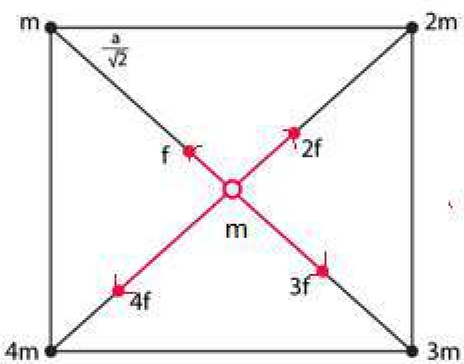
The mass of each ball = m = 10 kg

Distance of separation = r = 10 cm or 0.010 cm

Force = $GMm/r^2 = [6.67\times10^{-11}\times10^2]/(0.010)^2$

$= 6.67 \times 10^{-7}$

Solution 2:



The gravitational force at the center = vector sum of all the forces acting on it.

The distance between the center particle with others, say $r = a/\sqrt{2}$

Force acting between particles of mass m and center particle = $F_m = GMm/r^2 = 2Gm^2/a^2$

Force acting between particles of mass m and center particle = $F_2m = GM(2m)/r^2 = (4Gm^2)/r^2 = 2F_m$

Similarly, we can calculate of mass 3m and 4m along with the center particle:

$F_{3m} = 3\,F_m$ and $F_{4m} = 4\,F_m$

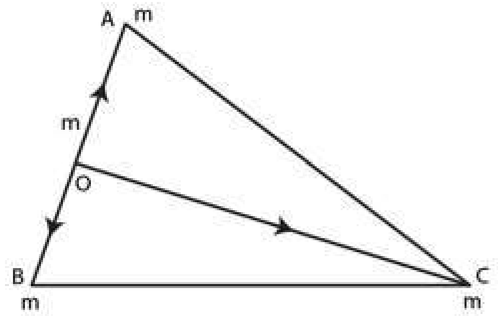
The net force:

$F_{net} = 2\,F_m \cos\theta = 4\,F_m\,(1/\sqrt{2}) = 2\sqrt{2}\,F_m$

$\Rightarrow F_{net} = 2\sqrt{2}\,F_m = 2\sqrt{2} \times 2Gm^2/a^2 = [4\sqrt{2}\,Gm^2]/a^2$

Solution 3:

(a)



If "m" is the mid point of a side, then

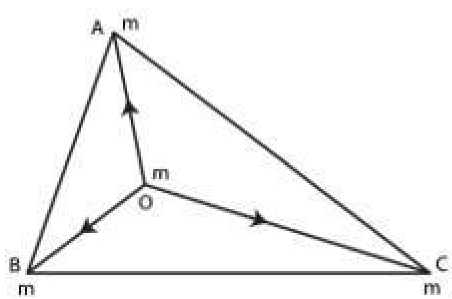
$F_{OA} = 4Gm^2/a^2$ in OA direction
$F_{OB} = 4Gm^2/a^2$ in OB direction

=> $F_{OC} = 4Gm^2/3a^2$ in OC direction

[As, Equal and opposite cancel each other]

So, net gravitational force on m is $4Gm^2/a^2$

(b)



If point "O" is the centroid, then

$F_{OA} = 3Gm^2/a^2$ and $F_{OB} = 3Gm^2/a^2$

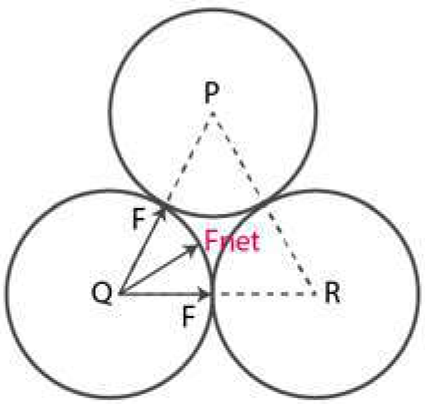
So, resultant force is

$$= \sqrt{2(\frac{3Gm^2}{a^2})^2 - 2(\frac{3Gm^2}{a^2})^2 \times \frac{1}{2}} = \frac{3Gm^2}{a^2}$$

Since $F_{OC} = 3Gm^2/a^2$
[Equal and opposite to F cancel each other]

=> Net gravitational force become zero.

Solution 4:
Distance between the centers of two spheres = r= 2a



Force on one sphere due to another = $F = GM^2/4a^2$

Net force = $F_{net} = 2F \cos\theta = 2F \cos 30^o$

$= 2 \times \sqrt{3}/2 \times GM^2/4a^2$

=> $F_{net} = \sqrt{3}GM^2/4a^2$

Solution 5:
Let A, B, C and D are four particles of mass , moving in a circle of radius R.

Force between A and B = $F_{AB} = GM^2/(\sqrt{2}R)^2 = GM^2/2R^2$

Force between A and D = $F_{AD} = GM^2/(\sqrt{2}R)^2 = GM^2/2R^2 = F_{AB}$

Net force in downward direction = $F_D = 2F_{AB} \cos 45^o = \sqrt{2}\, F_{AE}$

Force between A and C = $F_{AC} = GM^2/(2R)^2 = GM^2/4R^2$

Now,
Net force on particle A = $F_{net} = F_D + F_{AC}$

$$F_{net} = \left(\frac{2\sqrt{2}+1}{4}\right)\frac{GM^2}{R^2}$$

For moving along the circle, F_net = mv^2/R

$$\frac{Mv^2}{R} = \left(\frac{2\sqrt{2}+1}{4}\right)\frac{GM^2}{R^2}$$

$$v = \sqrt{\left(\frac{2\sqrt{2}+1}{4}\right)\frac{GM}{R}}$$

Solution 6:

Mass of the moon= M = 7.4× 10^{22} kg
Radius =R = 1740 km and Distance of the point from surface= R' = 1000 km

Total distance from the center = r = 1740+1000 = 2740 km

Now,
Find Acceleration due to gravity:
$g = GM/r^2$

$$g = \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{(2740 \times 10^3)^2} = 0.65 m/s^2$$

Solution 7:
let m_1 and m_2 masses of bodies, where m_1 = 10 kg and m_2 = 20 kg

Initial separation, say r_1 = 10 m and Final separation, say r_2 = 0.5 m
Let v_1 be the initial velocity and v_2 be the final velocity, where $v_1 = v_2 = 0$ m/s

Let us consider v_1', v_2' are the final velocities.

Now,
$m_1 v_1' + m_2 v_2' = 0$

=> $v_1' = -(20/10)v_2' = -2 v_2'$

[From momentum conservation]

Again, from using the conservation of energy:
$PE_{initial} + KE_{initial} = PE_{final} + KE_{final}$

$$=> \frac{Gm_1m_2}{r_1^2} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{Gm_1m_2}{r_2^2} + \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

Substituting the values, we get

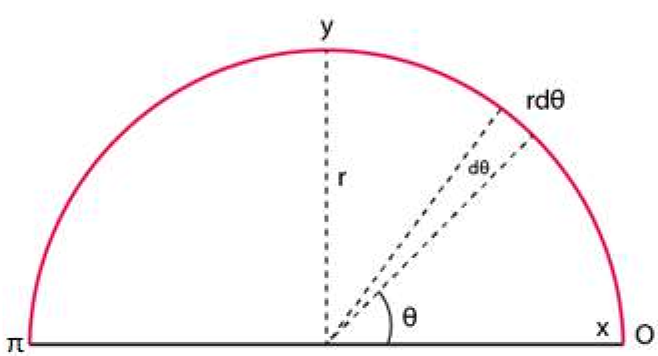
$$=> v_2'^2 = \frac{2 \times 6.67 \times 10^{-11}}{3} = 44.47 \times 10^{-11} = 2.10 \times 10^{-5} m/s$$

and,

$$v_1' = 2v_2' = 4.20 \times 10^{-5} m/s$$

Solution 8:
Let us take a small element on the wire. The arc length of the element is r dθ .
=> Mass of the element = dM = (M/L) r dθ
Also, r = L/π

=> dM = Mdθ/π



The force on particle due to element = dF = $GmdM/r^2$= $(GMm\pi d\theta)/L^2$
Therefore,

$$F = \int dF = \int_0^{\pi} \frac{GMm\pi}{L^2} \cos\theta d\theta$$

$$F = \frac{2\pi GM m}{L^2}$$

Solution 9:
A small section of rod is at "x" distance mass of the element = dm = (M/L).dx

$dE_1 = [G(dm)]/(d^2+x^2) = dE_2$

So resultant $dE = 2\ dE_1 \sin\theta$

$$2 \times \frac{Gdm}{d^2 + x^2} \times \frac{d}{\sqrt{d^2 + x^2}} = \frac{2(GM)dx}{L(d^2 + x^2)(\sqrt{d^2 + x^2})}$$

Now, the total gravitational force:

$$E = \int_0^{\frac{L}{2}} \frac{2Gmddx}{L(d^2 + x^2)^{3/2}}$$

Solving above equation, we get

$E = 2GM/[d\ \sqrt{(L^2+4d^2)}]$

Solution 10:

The gravitational force on m due to shell of M_2 is zero. Ma is at distance $(R_1 + R_2)/2$

The gravitational force:

$$F = \frac{GM_1 m}{r^2} = \frac{GM_1 m}{[(R_1 + R_2)/2]^2}$$

$$F = \frac{4GM_1 m}{(R_1 + R_2)^2}$$

Solution 11:

Let us assume that tunnel doesn't change the gravitational field distribution of earth.
Mass of the sphere:

$$\frac{M'}{\frac{4}{3\pi x^3}} = \frac{M_e}{\frac{4}{3\pi R^3}}$$

or

$$M' = \frac{x^3}{R^3} M_e$$

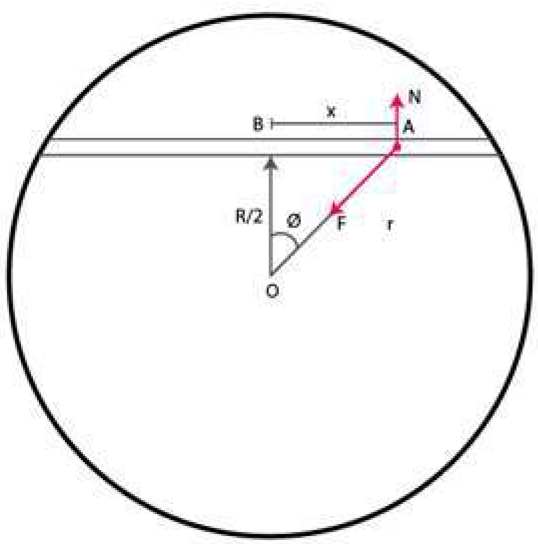
Where M_e is the mass of the earth .

The gravitational force on the particle at distance x,

$F = GMM'/x^2 = GM_e/R^3$

Solution 12:

Let M_E be the mass of the earth.



From figure,

$N = F \cos \phi$

here, $\cos \phi = R/2r$ and $F = GM_E\, mr/R^3$

thus, $N = GM_E\, mr/R^3 \times R/2r$

or $N = GMm/2R^2$

Solution 13:

(a) distance of the particle from the center of solid sphere:

$l = x - r$

Gravitational force on the object:
$F = Gmm'/r^3$

Here, the mass of the sphere "m" and m' is the place at distance x from O.

=> $F = Gmm'(x-r)/r^3$

(b) $2r < x < 2R$, then F is due to only sphere

$F = Gmm'/(x-r)^2$

(c) If $x > 2R$, the gravitational force is due to both shell and sphere,

Force due to shell: $F = GMm'/(x-R)^2$

Force due to sphere: $F = GMm'/(x-r)^2$

So, resultant force = $GMm'/(x-R)^2 + GMm'/(x-r)^2$

Solution 14:

At point P_1
Gravitational force due to sphere = $M = GM/(3a+a)^2 = GM/16a^2$

At point P_2,Gravitational force due to sphere and shell

$= GM/(a+4a+a)^2 + GM/(4a+a)^2$

$= (61/900)\ GM/a^2$

Solution 15:

we know, the field inside the shell is zero. Let the gravitational field at A due to the first part be E and the gravitational field at B due to the second part be E'.

Therefore, E + E' = 0

or E = -E'

Hence, the fields are equal is magnitude and opposite in direction

Solution 16:

Let the mass of 0.10 kg be at a distance x from 2 kg mass and at the distance of (2-x) from the 4 kg mass.

Force between 0.1 kg mass and 4 kg mass = Force between 0.1 kg mass and 2 kg mass

$(2x0.1)/x^2 = -(4x0.1)/(2-x)^2$

x = 2/2.414

or x = 0.83 m from the 2 kg mass.

Now,

The gravitational potential energy is given by

$$V = \sum_{i \neq j} \frac{Gm_i m_j}{r_{ij}} = \frac{G0.1 \times 2}{0.83} + \frac{G0.1 \times 4}{1.17} + \frac{G2 \times 4}{2}$$

$$= 0.24GJ$$

$$= -3.06 \times 10^{-10}$$

Solution 17:

work done = W = $U_f - U_i$

Where U_f = Final potential energy and U_i = Initial potential energy

Here, $U_f = -3Gm^2/2a$ and $U_i = -3Gm^2/a$

Now, $W = 3Gm^2/2a$

Solution 18:

U_f = Final potential energy = 0

[As the particle is to be taken away, we assume the final point to be approximately at infinite distance]

and U_i = Initial potential energy = $(-GM_s m)/r$

Here, m = Mass of the particle = 100 g or 0.1 kg

M_s = Mass of sphere = 10 kg

And r = radius of sphere = 10 cm or 0.1 m

On putting values, we get

$U_i = -6.67 \times 10^{-10}$

Now, work done = $W = -(U_f - U_i)$

=> $W = 6.67 \times 10^{-10}$ J

Solution 19:

(a) force on the particle

F = mE = 2[5i+12j] = 10i + 24j N

[given mass of the particle = m = 2 kg]

Magnitude of F = 26 N

(b) Potential at (12, 0):

$V = -E.r = -12i\ [5i+12j] = -60$ J/kg

Potential at (0, 5):

$V = -E.r = -5i\ [5i+12j] = -60$ J/kg

(c) potential energy at (12,5) m:

$V = [5i+12j]\ [2i + 5j] = -120$ J/kg

And potential energy at the origin is zero.

Therefore, the change in potential energy is -240 J.

(d) Change in potential energy = 0
[from part (b), potential energy of the particle would be same at both the points.]

Solution 20:

(a) $V = 20N\ Kg^{-1}\ (x+y)$

Dimension of $V = [MLT^{-2}]/M \times L = L^2T^{-2}$

Dimension of $j/kg = [ML^2T^{-2}]/M = L^2T^{-2}$

Hence dimensions are correct.

(b)

$$E = -\frac{\partial V}{\partial x} - \frac{\partial V}{\partial y}$$

Or $E = -20i - 20j$ N/kg

E is independent of the coordinate.

(c) Force = mE
$= 0.5 \times [-20(i+j)]$
$= -10i - 10j$

Magnitude of Force = $10\sqrt{2}$ N

Solution 21:

Electric field = E = 2i + 3j

Angle made by E with the x-axis:

$$\cos\theta = \frac{E.i}{|E||i|} = \frac{(2i + 3i).i}{\sqrt{13}} = \frac{2}{\sqrt{13}}$$

$$\sec\theta = \frac{\sqrt{13}}{2}$$

we know,

$$\tan^2\theta = \sec^2\theta - 1$$

$$= \left(\frac{\sqrt{13}}{2}\right)^2 - 1$$

or

$$\tan\theta = \frac{3}{2}$$

Equation of line is y = -(2/3)x + 5/3 made angle with x-axis is

tanϕ = -2/3

Now,

tan(ϕ - θ) = [tanϕ - tanθ]/[1+tanϕ tanθ]

= infinity

Now, the angle between the electric filed and the line = ϕ - θ = 90^o
Since product of both the slope is -1, the direction of field and the displacement are perpendicular, is done by the particle on the line.

Solution 22:
Let h be the height

Therefore, (1/2) $Gm/R^2 = GM/(R+h)^2$

or $2R^2 = (R+h)^2$

or h = (√2 - 1)R

Solution 23:
Height of Mount Everest = h = 8848 m or 8.848 km

Acceleration due to gravity at a height h, say g'

$g' = g(1 - 2h/R)$

$= 9.8(1 - 640/(6400 \times 10^3)$

$= 9.799 \text{ m/s}^2$

Solution 24:
Let g' be the acceleration due to gravity

$g' = g (1 - 2h/R)$

$= 98(1-0.64/6400)$

$= 9.799 \text{ m/s}^2$

Solution 25:
Let g' be the acceleration due to gravity at equation and that of pole g

Angular velocity of earth = $\omega = 2\pi/T$

$= 2\pi/(24 \times 3600)$ rad/s

Now, acceleration due to gravity at equator:

$g' = g - \omega^2 R$

$= 9.8 - [2\pi/(24 \times 3600)]^2 \times 64000$

$= 9.767 \text{ m/s}^2$

And the weight at equator = mg' = 1 x 9.767 N = 0.997 kg

Solution 26:

Acceleration due to gravity at equator = $g' = g - \omega^2 R$

Acceleration due to gravity at a height above south pole = $g'' = g(1-2h/R)$

Now $g' = g''$

=> $g - \omega^2 R = g(1-2h/R)$

=> $h = \omega^2 R^2/2g$

Or $h = [4 \pi \times 6400000^2]/[(4 \times 3600)^2 \times 2 \times 9.8]$

= 10 km (approx)

Solution 27:

for apparent g at equator be zero.

$g' = g - \omega^2 R = 0$

or $g = \omega^2 R$

=> $\omega = \sqrt{(g/R)} = \sqrt{(9.8/6400000)}$

= 1.237×10^{-3} rad/s

Now, $T = 2 \pi / \omega$

= $[2 \times 3.14]/[1.237 \times 10^{-3} \times 3600]$

= 1.4 h (approx.)

Solution 28:

a) the speed of the ship is equal to earth's rotation when the ship is stationary point.

speed $= \omega R$

(b) tension in the string at the equator

$T_0 = mg' = mg - m\omega^2 R$

$mg - T_0 = m\omega^2 R$

Difference between T_0 and the earth's attraction on the bob.

(c) angular speed of the ship is v/R about its center.

Total angular speed $= \omega' = \omega - v/R$

And $T = mg - m\omega'^2 R$ [tension given]

$\Rightarrow T = mg - m(\omega - V/R)^2 R$

$\Rightarrow T = mg - [m\omega^2 + mv^2/R^2 - 2m\omega v/R)R]$

$\Rightarrow \ T = mg - m\omega^2 R - mv^2/R + 2m\omega v$

From part (b),

$T_0 = mg' = mg - m\omega^2 R$

$\Rightarrow T = T_0 - mv^2/R + 2m\omega v$

$\Rightarrow T = T_0 + 2m\omega v$

Neglect, mv^2/R, As small quantity.

Solution 29:

From Kepler's third law, the time period of an orbit is proportional to the cube of the radius of the orbit.

$T^2 \ \alpha \ R^3$

$T_m^2/T^2 = R_{MS}^3/R_{SE}^3$

$R_{MS}/R_{SE} = (3.534)^{1/3} = 1.52$

Solution 30:
For an orbit, the time period:

$T^2 = 4\pi^2a^3/GM$

[here a = 3.84×10^5 km and T = 27.3 x 24x3600 sec]

Or M = 6.02×10^{24} kg

Solution 31:

For an orbit, the time period:

$T^2 = 4\pi^2a^3/GM$

Or $M = 4\pi^2a^3/G T^2$ …….(1)
Where M = mass of mars.

Here, Radius of mars=a = 9.4×10^3 km or 9.4×10^6 m and Time = T = 27540 s

Now, (1)=> $M = [4\pi^2 (9.4 \times 10^6)^3]/[6.67 \times 10^{-11} \times 27540^2]$

Or M = 6.5×10^{23} kg

Solution 32:

(a) Radius of the orbit = a = 2000 + 6400 = 8400 km or 8.4×10^6 m

Therefore, the speed = $v = \sqrt{(GM)}/a$

$v = \sqrt{[(6.67 \times 10^{-11} \times 6 \times 10^{24})/(8.4 \times 10^6)]}$ = 6.9 km/s (approx.)

(b) KE = $(1/2)mv^2$
Here m = 1000 kg (mass of satellite)

KE = $(1/2) \times 1000 \times 6900^2 = 2.38 \times 10^{10}$

(c) Potential energy at infinity is zero. Hence, the potential energy at a radius, a

$PE = -GMm/a$

$= [-6.67 \times 10^{-11} \times 6 \times 10^{24} \times 1000]/[8.4 \times 10^6]$

$= -4.76 \times 10^{10}$

(d) Time period

$T^2 = 4\pi^2 a^3/GM$

$= [4\pi^2(8.4 \times 10^6)^3]/[6.67 \times 10^{-11} \times 6 \times 10^{24}]$

= 2.12 hours

Solution 33:

(a) Time period of revolution of satellite: T = 24x3600 = 86400 sec

Let "a" be the radius of the orbit.

$T^2 = 4\pi^2 a^3/GM$

Or $a^3 = GM\, T^2/4\pi^2$

$= [6.67 \times 10^{-11} \times 6 \times 10^{24} \times 86400^2]/[4\pi^2]$

$= 7.56 \times 10^{22}$

Or a = 42300 km (approx.)

(b) A complete revolution takes 24 hours, therefore a quarter of revolution is 24/4 = 6 hours

Solution 34:

Weight at north pole, $W_p \propto 1/R^2$

Let h is distance of the satellite from earth.

Weight of satellite at equator, $W_e \propto 1/(R+h)^2$

Now, ratio is $W_p / W_e = (R+h)^2/R^2$

$W_e = [W_p R^2]/[(R+h)^2]$

We are given, $W_p = 10$ N and h = 36000 km [Height of the geostationary satellite,]

Therefore, $W_e = [10 \times 6400^2]/[(6400+36000)^2] = 0.23$ N(approx.)

Solution 35:

The time period of revolution: $T^2 = 4\pi^2 a^3/GM$

Or $GM = 4\pi^2 (R_2)^3/ T^2$

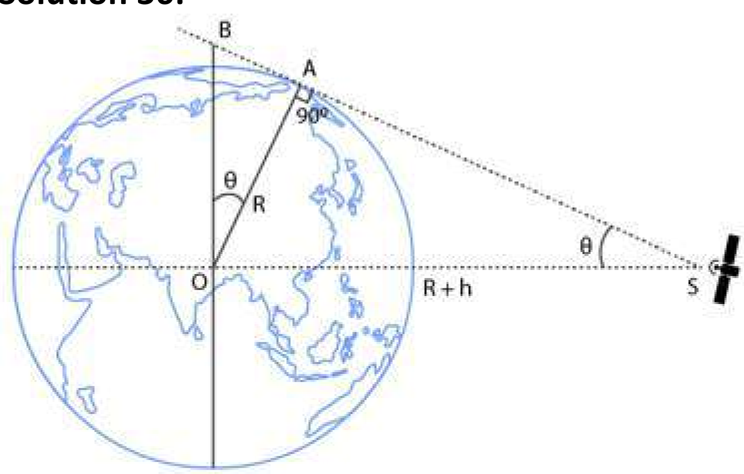
Or $GM/R_1^2 = 4\pi^2 (R_2)^3/ T^2 R_1^2$

Now, Acceleration due to gravity :

$g = GM/ R_1^2$

or $g = 4\pi^2 (R_2)^3/ T^2 R_1^2$

Solution 36:



form figure, angle BOA = angle OSA

In triangle AOS

$\sin\theta = AO/OS$

$= R/(R+h)$

$= 6400/(6400+36000)$

$= 0.15$ (approx)

or $\theta = \sin^{-1}(0.15)$

Solution 37:
Let KE_i be initial KE and PE_i initial potential energy of the system.

$KE_i = (1/2)mv^2$ and $PE_i = -GMm/R$

$KE_f = 0$ [at the maximum height]

And PE at height h is h = 6400 km
And $PE_f = -GMm/(R+h)$

Now, From conservation of energy, $KE_i + PE_i = KE_f + PE_f$

$\Rightarrow (1/2)mv^2 – GMm/R = -GMm/(R+h)$

$\Rightarrow (1/2)mv^2 = GMm[(-R+R+h)/R(R+h)] = GMmh/R(R+h)$

Or $v^2 = 2GMh/R(R+h)$

$= [2x6.67x10^{-11}x6x10^{24}x6400x10^3]/[2x(6400x10^3)^2]$

$= 7.9$ km/s

Solution 38:
Let KE_i be initial KE and PE_i initial potential energy of the system.

$KE_i = (1/2)mv_i^2$ and $PE_i = -GMm/R$

Final KE = $KE_f = (1/2)mv_f^2$

Final potential energy = $PE_f = 0$

Using energy conservation, we have
$KE_i + PE_i = KE_f + PE_f$

$$\Rightarrow \frac{1}{2}mv_i^2 - \frac{GMm}{R} = \frac{1}{2}mv_f^2$$

$$\Rightarrow \frac{15^2}{2} - \frac{6.67 \times 10^{-7} \times 6 \times 10^{24}}{6400} = \frac{1}{2}v_f^2$$

$$\Rightarrow v_f = \times 10^4 m/s = 10km/s$$

Solution 39:
We have, $(1/2)\ mv^2 = GMm/R$

or $R = 2GM/v^2$

$R = [2x6.67x10^{-11}x6x10^{24}]/[(3x10^8)^2]$

R = 9 mm (approx.)