

Number System and its Operations

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BOOK

MATHEMATICS

NOTES

FUNDAMENTAL

- A number which can be expressed in the form of $\frac{p}{q}$, Where P and q are integers and $q \neq 0$ is called a rational number.

Example:- $\frac{1}{2}, \frac{1}{3}, \frac{2}{5}$ etc.

Representation of Rational Number as Decimals.

- **Case I:-** When remainder becomes zero $\frac{1}{2} = .5, \frac{1}{4} = .25, \frac{1}{8} = .125$ it is a terminating Decimal expansion.
- **Case II:-** When Remainder never becomes zero..

Example:- $\frac{1}{3} = .3333, \frac{2}{3} = .6666$ it is a non - terminating Decimal expansion.

- There are infinitely rational numbers between any two given rational numbers.
- **Irrational Number:** The number which cannot be part in form of $\frac{p}{q}$ and neither there are terminating nor recurring are known as irrational Number.

Example:- $\sqrt{2}, \sqrt{3}$ etc.

- **Rationalization:** "Changing of an irrational number into rational number is called rationalization and the factor by which we multiply and divide the number is called rationalizing factor.

Example:- Rationalizing factor of $\frac{1}{2-\sqrt{3}}$ is $2+\sqrt{3}$.

Rationalizing factor of $\sqrt{3} + \sqrt{2}$ is $\sqrt{3} - \sqrt{2}$

LAW OF EXPONENTS FOR REAL NUMBERS

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$

Some useful results on irrational number

- Negative of an irrational number is an irrational number.
- The sum of a rational and an irrational number is an irrational number.
- The product of a non - zero rational number and an irrational number is an irrational number.

Some results on square roots

- $(\sqrt{x})^2 = x, x \geq 0$
- $\sqrt{x} \times \sqrt{y} = \sqrt{xy}, x \geq 0 \text{ and } y \geq 0$
- $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = x - y, (x \geq 0 \text{ and } y \geq 0)$
- $(\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy}, (x \geq 0 \text{ and } y \geq 0)$
- $(\sqrt{x} - \sqrt{y})^2 = x + y - 2\sqrt{xy}, (x \geq 0 \text{ and } y \geq 0)$
- $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}, (x \geq 0 \text{ and } y \geq 0)$
- $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b, (b \geq 0)$
- $(\sqrt{a} + \sqrt{b})(\sqrt{a} + \sqrt{b}) = \sqrt{ac} + \sqrt{bc} + \sqrt{ad} + \sqrt{bd}, (a \geq 0, b \geq 0, c \geq 0 \text{ and } d \geq 0)$