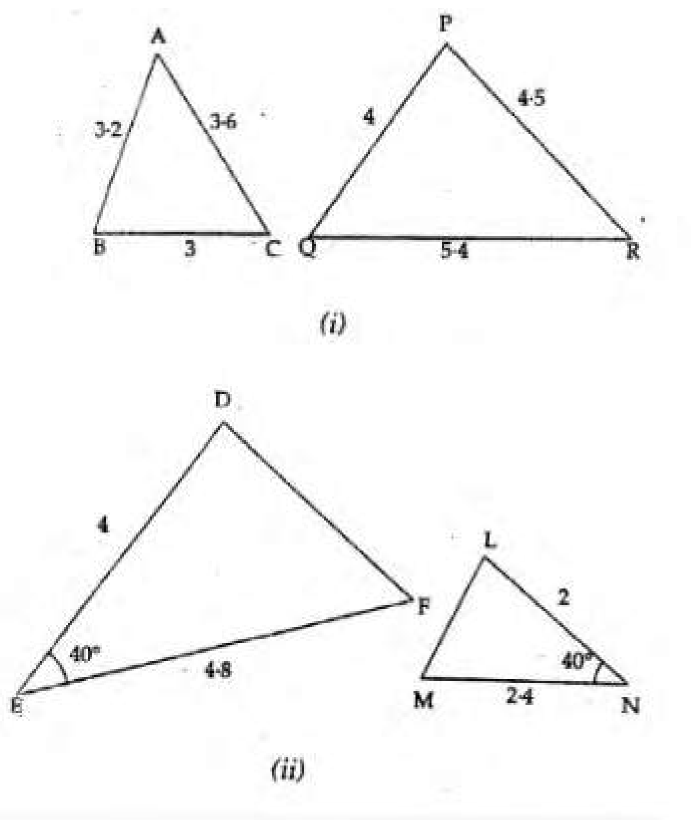
Chapter 13
Similarity

Exercise 13.1

1. State which pairs of triangles in the figure given below are similar. Write the similarly rule used and also write the pairs of similar triangles in symbolic form (all lengths of sides are in cm):



Solution : -

(i) From the ΔABC and ΔPQR

$\frac{AB}{PQ} = \frac{3.2}{4}$

$= \frac{32}{40}$

Divide both numerator and denominator by 8 we get,

$= \frac{4}{5}$

$\frac{AC}{PR} = \frac{3.6}{4.5}$

$= \frac{36}{45}$

Divide both mumerator and denominator by 9 we get,

$= \frac{4}{5}$

$\frac{BC}{QR} = \frac{3}{5.4}$

$= \frac{30}{54}$

Divide both numerator and denominator by 6 we get,

$= \frac{5}{9}$

By comparing all the results, the side are not equal,

Therefore, the triangles are not equal.

(ii) By comparing all the results, the side are not equal.

Therefore, the triangles are not equal.

(iii) From the ΔDEF and Δ LMN

$\angle E = \angle N = 40^{\circ}$

Then, $\frac{DE}{LN} = \frac{4}{2}$

Divide both numerator and denominator by 2 we get, = 2

$\frac{EF}{MN} = \frac{4.8}{2.4}$

$= \frac{48}{24}$

Divide both numerator and denominator by 24 we get, = 2

Therefore,

$\Delta DEF \sim \Delta LMN$

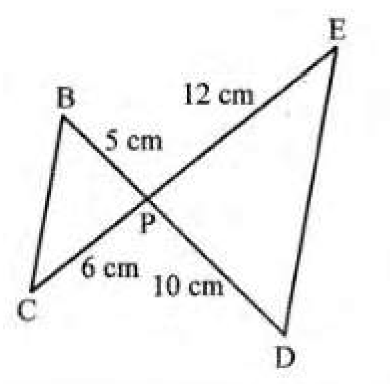
2. It it given that $\Delta DEF \sim \Delta RPQ$. Is is true to say that $\angle D = \angle R$ and $\angle F = \angle P$? Why ?

Solution:

From the question is given that, $\Delta DEF \sim \Delta RPQ$

$\angle D = \angle R$ and $\angle F = \angle Q$ not $\angle P$
No, $\angle F \neq \angle P$

3. If in two right triangles, one of the acute angle of one triangle is equal to an acute angle of the other triangle, can you say that the two triangles are similar ? Why ?



Solution:

From the figure, two line segments are intersecting each other at P.

In ΔBCP and ΔDPE

$\frac{5}{10} = \frac{6}{12}$

Dividing LHS and RHS by 2 we get,

$\frac{1}{2} = \frac{1}{2}$

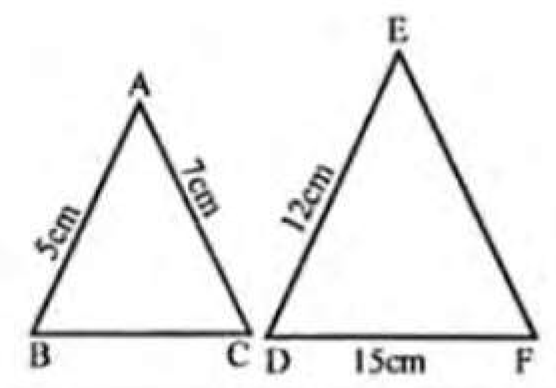
Therefore, $\Delta BCD \sim \Delta DEP$

5. It is given that $\Delta ABC \sim \Delta EDF$ such that AB = 5cm, AC= 7cm, DF = 15cm and DE = 12cm.

Find the lengths of the remaining sides of the triangles.

Solution:

As per the dimensions give in the question,



From the question it is given that,

$\Delta DEF \sim \Delta LMN$

So, $\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$

Consider $\frac{AB}{ED} = \frac{AC}{EF}$

$\frac{5}{12} = \frac{7}{EF}$

By cross multiplication,

EF = $\frac{(7\times12)}{5}$

EF = 16.8 cm

Now, consider $\frac{AB}{ED} = \frac{BC}{DF}$

$\frac{5}{12} = \frac{BC}{15}$

$BC = \frac{(5\times15)}{12}$

$BC = \frac{75}{12}$

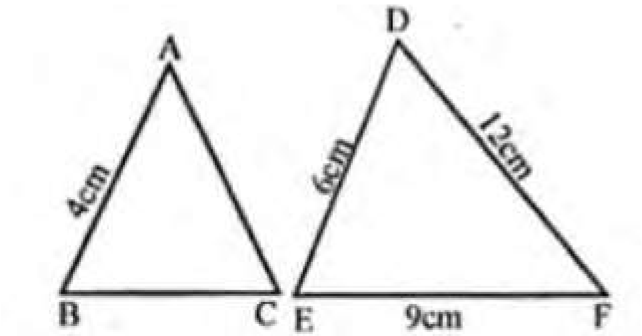
BC = 6.25

BC = 6.25

6. If $\Delta ABC \sim \Delta DEF$, AB = 4cm, DE = 6cm, EF = 9cm and FD = 12cm, then find the perimeter of ΔABC.

Solution:

As per the dimensions given in the questions,



Now, we have to find out the perimeter of ΔABC

Let $\Delta ABC \sim \Delta DEF$

So, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Consider, $\frac{AB}{DE} = \frac{AC}{DE}$

$\frac{4}{6} = \frac{AC}{12}$

By cross multiplication we get,

AC = $\frac{(4\times12)}{6}$

AC = $\frac{48}{6}$

AC = 8 cm

Then, consider $\frac{AB}{DE} = \frac{BC}{EF}$

$\frac{4}{6} = \frac{BC}{9}$

BC = $\frac{(4\times 9)}{6}$

BC = $\frac{36}{6}$

BC = 6cm

Therefore, the perimeter $\Delta ABC = AB + BC + AC$

= 4 + 6 + 8

= 18 cm

(b) If $\Delta ABC \sim \Delta PQR,$ perimeter of $\Delta ABC = 32cm,$ perimeter of $\Delta PQR = 48$ cm and PR = 6 cm, then find the length of AC.

Solution:

From the question it is given that,

$\Delta ABC \sim \Delta PQR$

Perimeter of ΔABC = 32cm

Perimeter of ΔPQR = 48cm

So, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

Then, perimeter of $\frac{\Delta ABC}{perimeter\ of\ \ \Delta PQR} = \frac{AC}{PR}$

$\frac{32}{48} = \frac{AC}{6}$

AC = $\frac{(32\times 6)}{48}$

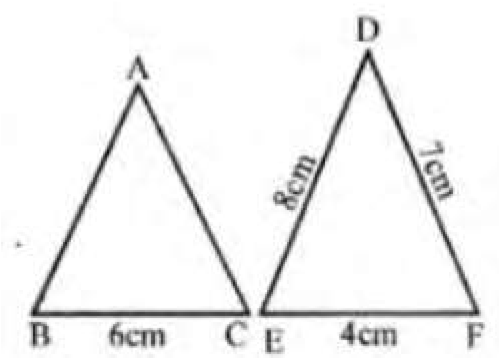
AC = 4

Therefore, the length of AC =4 cm.

7. Calculate the Other sides of a triangle whose shortest side is 6cm and which is similar to a triangle whose sides aree 4cm, 7cm and 8cm.

Solution:

Let us assume that, $\Delta ABC \sim \Delta DEF$



Δ ABC is BC = 6cm

Δ ABC $\sim \Delta DEF$

So, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Consider $\frac{AB}{DE} = \frac{BC}{EF}$

$\frac{AB}{8} = \frac{6}{4}$

AB = $\frac{(6\times8)}{4}$

AB = $\frac{48}{4}$

AB = 12

Now, consider $\frac{BC}{EF} = \frac{AC}{DF}$

$\frac{6}{4} = \frac{AC}{7}$

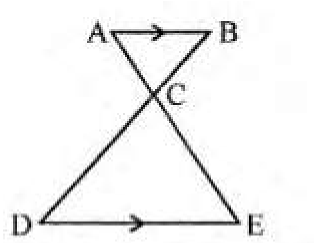
AC = $\frac{(6\times7)}{4}$

AC = $\frac{42}{4}$

AC = $\frac{21}{2}$

AC = 10.5 cm

8. (a) In the figure given below, AB || DE, AC = 3 cm, CE = 7.5 cm and BD = 14cm. Calculate CB and DC.



Solution:

From the question it is given that,

AB || DE

AC = 3 cm

CE = 7.5 cm

BD = 14 cm

From the figure,

$\angle ACB = \angle DCE$ [because vertically opposite angles]

$\angle BAC = \angle CED$ [alternate angles]

Then, $\Delta ABC \sim \Delta CDE$

So, $\frac{AC}{CE} = \frac{BC}{CD}$

$\frac{3}{7.5} = \frac{BC}{CD}$

By cross multiplication we get,

7.5BC = 3CD

Let us assume BC = x and CD = 14 – x

$7.5 \times x = 3 \times (14 - x)$

$7.5x = 42 - 3x$

$7.5x + 3x = 42$

$10.5x = 42$

$x = \frac{42}{10.5}$

$x = 4$

Therefore, BC = x = 4cm

CD = $14 - x$

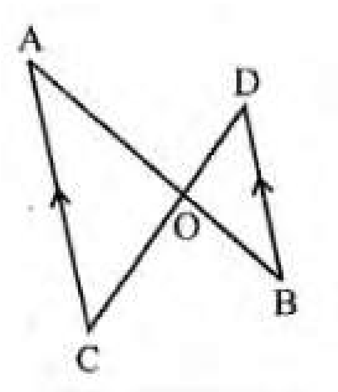
= 14 – 4

= 10 cm

(b) In the figure (2) given below, CA || BD, the lines AB and CD meet at G.

(i) Prove that $\Delta ACO \sim \Delta BDO$.

(ii) If BD = 2.4 cm, OD = 4cm, OB = 3.2cm and AC = 3.6 cm, calculate OA and OC.



Solution:

(i) We have to prove that, $\Delta ACO \sim \Delta BDO$.

So, from the figure

Consider ΔACO and ΔBDO

Then,

$\angle$AOC = $\angle BOD$ [from vertically opposite angles]

$\angle$A = $\angle B$

Therefore, $\Delta ACO = \Delta BDO$

Given, BD = 2.4cm, OD = 4cm, OB = 3.2 cm, AC = 3.6 cm,

$\Delta ACO \sim \Delta BOD$

So, $\frac{AO}{OB} = \frac{CO}{OD} = \frac{AC}{BD}$

Consider $\frac{AC}{BD} = \frac{AO}{OB}$

$\frac{3.6}{2.4} = \frac{AO}{3.2}$

AO = $\frac{(3.6 \times 3.2)}{2.4}$

AO = 4.8 cm

Now, consider $\frac{AC}{BD} = \frac{CO}{OD}$

$\frac{3.6}{2.4} = \frac{CO}{4}$

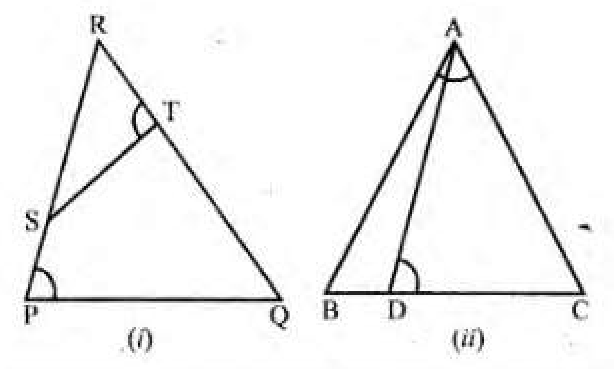
CO = $\frac{(3.6 \times 4)}{2.4}$

CO = 6 cm

9. (a) In the figure

(i) given below, $\angle P = \angle RTS$.

Prove that ΔRPQ~ΔRTS.



Solution:

From the given figure, $\angle P = \angle RTS$

So we have to prove that $\Delta RPQ \sim \Delta RTS$

So we have to prove that ΔRPQ ~ ΔRTS

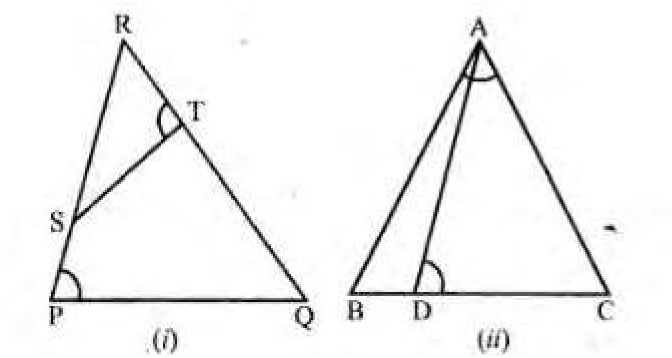
In ΔRPQ and ΔRTS

∠R = ∠R (common angle for both triangle)

∠P = ∠RTS (from the question)

ΔRPQ ~ ΔRTS

(b) In the figure (ii) given below, $\angle ADC = \angle BAC. Prove\ that$ $CA^2 = \text{DC} \times BC$



Solution:

From the figure, $\angle ADC = \angle BAC$

So, we have to prove that $CA^2 = DC \times BC$

In ΔABC and ΔADC

$\angle C = \angle C$ (common angle for both triangle)

$\angle BAC = \angle ADC$ (from the question)

$\Delta ABC \sim \Delta ADC$

Therefore, $\frac{CA}{DC} = \frac{BC}{CA}$

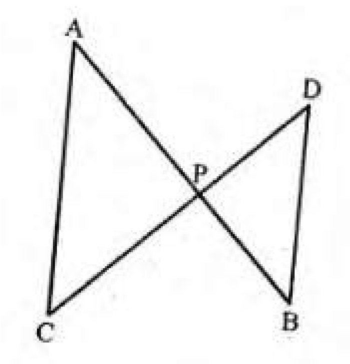
We know that, corresponding sides are proportional,

Therefore, $CA^2 = DC \times BC$

10. (a) In the figure(1) given below, AP = 2PB and CP = 2PD.

(i) Prove that ΔACP is similar to ΔBDP and AC || BD.

(ii) If AC = 4.5 cm, Calculate the length of BD.



Solution:

From the question it is given that,

AP = 2PB, CP = 2PD

(i) We have to prove that, ΔACP is similar to ΔBDP and AC || BD

AP = 2PB, CP = 2PD

(i) We have to prove that, ΔACP is similar to ΔBDP and AC||BD AP = 2PB

$\frac{AP}{PB} = \frac{2}{1}$

Then, CP = 2PD

$$\frac{CP}{PD} = \frac{2}{1}$$

$\angle APC = \angle BPD$ [from vertically opposite angles]

So, $\Delta ACP \sim \Delta BDP$

Therefore, $\angle CAP = \angle PBD$ [alternate angles]

Hence, AC ||BD

(ii) $\frac{AP}{PB} = \frac{AC}{BD} = \frac{2}{1}$

AC = 2BD

2BD = 4.5 cm

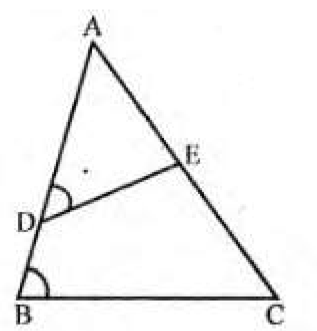
BD = $\frac{4.5}{2}$

BD = 2.25 cm

(b) In the figure (2) given below,

$\angle ADE = \angle ACB$.

(i) Prove that ΔS ABC and AED are similar.
(ii) If AE = 3cm, BD = 1 cm and AB = 6cm, calculate AC.



Solution:

From the given figure,

(i) $\angle A = \angle A$ (common angle for both triangles)

$\angle ACB = \angle ADE$ [given]

Therefore, $\Delta ABC \sim \Delta AED$

(ii) from (i) proved that, $\Delta ABC \sim \Delta AED$

So, $\frac{BC}{DE} = \frac{AB}{AE} = \frac{AC}{AD}$

AD = AB – BD

= 6 – 1 = 5

Consider, $\frac{AB}{AE} = \frac{AC}{AD}$

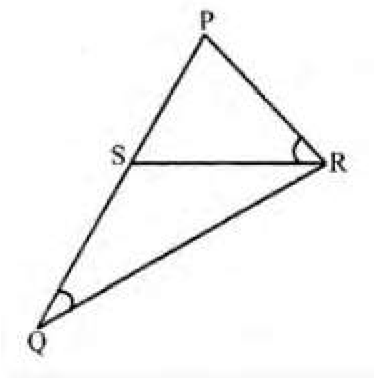
$\frac{6}{3} = \frac{AC}{5}$

AC = $\frac{(6\times5)}{3}$

AC = $\frac{30}{3}$

AC = 10 cm

(c) In the figure (3) given below, $\angle PQR = \angle PRS$. Prove that triangles PQR and PRS are similar. If PR = 8cm, PS = 4cm, calculate PQ.



Solution:-

From the figure,

$\angle P = \angle P$ (common angle for both triangles)

$\angle PQR = \angle PRS$ [from the question]

So, $\Delta PQR \sim \Delta PRS$

Then, $\frac{PQ}{PR} = \frac{PR}{PS} = \frac{QR}{SR}$

Consider $\frac{PQ}{PR} = \frac{PR}{PS}$

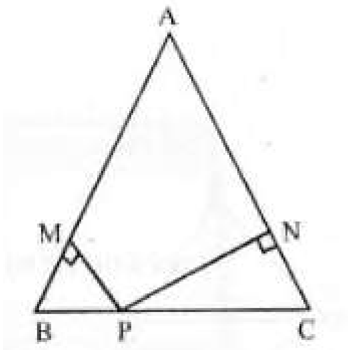
$\frac{PQ}{8} = \frac{8}{4}$

$PQ = \frac{(8\times8)}{4}$

$PQ = \frac{64}{8}$

PQ = 16 cm

11. In the given figure, ABC is a triangle in which AB = AC. P is a point on the side BC such that PM ⊥ AB and PN ⊥ AC. Prove that BM × NP = CN × MP.



Solution:

From the question it is given that, ABC is a triangle in which AB = AC.

P is a point on the side BC such that PM ⊥ AB and PN ⊥ AC.

We have to prove that, BM $\times NP = CN \times MP$

Consider the ΔABC

AB = AC ……….. [from the question]

$\angle B = \angle C$ … [angles opposite to equal sides]

Then, consider ΔBMP and ΔCNP

$\angle M = \angle N$

Therefore, $\Delta BMP \sim \Delta CNP$

So, $\frac{BM}{CN} = \frac{MP}{NP}$

By cross multiplication we get,

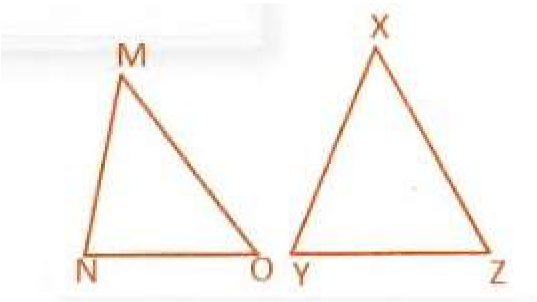
BM $\times NP = CN \times MP$

Hence it is proved.

12. Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.

Solution:

Consider the two triangles, ΔMNO and ΔXYZ



From the question it is given that, two triangles are similar triangles

So, $\Delta MNO \sim \Delta XYZ$

If two triangles are similar, the corresponding angles are equal and their corresponding sides are proportional.

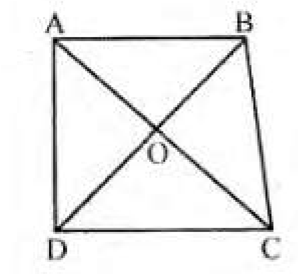
$\frac{MN}{XY} = \frac{NO}{YZ} = \frac{MO}{XZ}$

Perimeter of ΔMNO = MN + NO + MO

Therefore, $\left(\frac{MN}{XY} = \frac{NO}{YZ} = \frac{MO}{XZ}\right) = \left(\frac{MN}{XY} = \frac{NO}{YZ} = \frac{MO}{XZ}\right)$

= Perimeter of $\frac{\Delta MNO}{perimeter\ of\ \Delta XYZ}$

13. In the adjoining figure, ABCD is a trapezium in which AB || DC. The diagonals AC and BD in intersect at O. Prove that $\frac{AO}{OC} = \frac{BO}{OD}$



Using the above result, find the values of x if OA = $3x - 19, OB = x - 4,\ OC = x - 3\ and\ OD = 4$.

Solution:

From the given figure, ABCD is a trapezium in which AB || DC,

The diagonals AC and BD intersect at O.

So we have to prove that, $\frac{AO}{OC} = \frac{BO}{OD}$

Consider the ΔAOB and ΔCOD,

$\angle AOB = \angle COD$ ……. [vertically opposite angles

$\angle OAB = \angle OCD$

Therefore, $\Delta AOB \sim \Delta COD$

So, $\frac{OA}{OC} = \frac{OB}{OD}$

Now by using above result we have to find the value of x if OA =$3x - 19, OB = x - 4,\ \ OC = x - 3$ and OD = 4.

$\frac{OA}{OC} = \frac{OB}{OD}$

$\frac{(3x-19)}{(x-3)} = \frac{(x-4)}{4}$

By cross multiplication we get,

$(x - 3)(x - 4) = 4(3x - 19)$

$X^2 - 4x - 3x + 12 = 12x - 76$

$X^2 - 7x + 12 - 12x + 76 = 0$

$X^2 - 19x + 88 = 0$

$X^2 - 8x - 11x + 88 = 0$

$X(x - 8) - 11(x - 8) = 0$

$(x - 8)(x - 11) = 0$

Take $x - 8 = 0$

$X = 8$

Then, $x - 11 = 0$

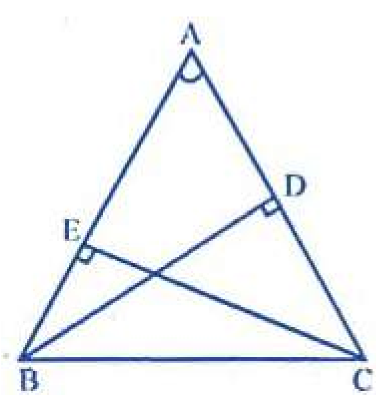
X = 11

Therefore, the value of x is 8 and 11.

14. In $\Delta ABC, \angle A$ is acute. BD and CE are perpendicular on AC and AB respectively. Prove that AB $\times AE = AC \times AD$.

Solution:

Consider the ΔABC



So, we have to prove that, AB ×AE = AC × AD

Now, consider the ΔADB and $\Delta AEC,$

$\angle A = \angle A$ [common angle for both triangles]

$\angle ADB = \angle AEC$ [both angles are equal to 90°]

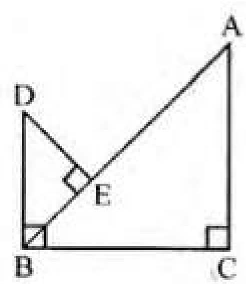
$\Delta ADB \sim \Delta AEC$

So, $\frac{AB}{AC} = \frac{AD}{AE}$

By cross multiplication we get,

AB $\times AE = AC \times AD$

15. In the given figure, DB $\perp BC, DE \perp AB$ and AC $\perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$



Solution :

From the figure, DB $\perp BC,\ DE \perp AB\ and$ AC $\perp BC$

We have to prove that, $\frac{BE}{DE} = \frac{AC}{BC}$

Consider the ΔABC and ΔDEB,

$\angle C = 90^\circ$

$\angle A + \angle ABC = 90^\circ$ [from the figure equation (i)]

Now in ΔDEB

$\angle DBE + \angle ABC = 90^{\circ}$ [from the figure equation (ii)]

From equation (i), we get

$\angle A = \angle DBE$

Then, in ΔABC and ΔDBE

$\angle C = \angle E$ [both angles are equal to 90°]

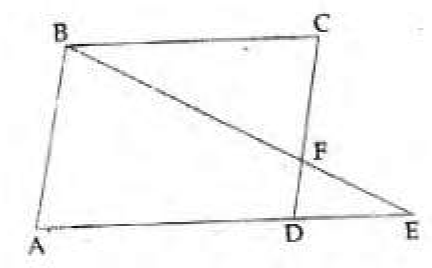
So, $\Delta ABC \sim \Delta DBE$

Therefore, $\frac{AC}{BE} = \frac{BC}{DE}$

By cross multiplication, we get

$\frac{AC}{BC} = \frac{BE}{DE}$

16. (a) In the figurre (1) given below, E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. show that $\Delta ABE \sim \Delta CFB$.



Solution:

From the figure, ABCD is a parallelogram,

Then, E is a point on AD and produced and BE intersects CD at F.

We have to prove that $\Delta ABE \sim \Delta CFB$

Consider ΔABE and ΔCFB

$\angle A = \angle C$ [Opposite angles of a parallelogram]

$\angle ABE = \angle BFC$ [O

$\angle$ ABE = $\angle BFC$ [Alternate angles are equal]

$\Delta ABE \sim \Delta CFB$

(b) In the figure (2) given below, PQRS is a parallelogram; PQ = 16cm, QR = 10cm. L is a point on PR such that RL : LP = 2 : 3. QL produced meetes RS at M and PS produced at N.

(i) Prove that triangle RLQ is similar to triangle PLN. Hence, find PN.

Solution:

From the question it is give that,

$\angle RLQ = \angle NLP$ [vertically opposite angles are equal]

$\angle RQL = \angle LNP$ [alternate angle are equal]

Therefore, $\Delta RLQ \sim \Delta PLN$

So, $\frac{QR}{PN} = \frac{RL}{LP} = \frac{2}{3}$

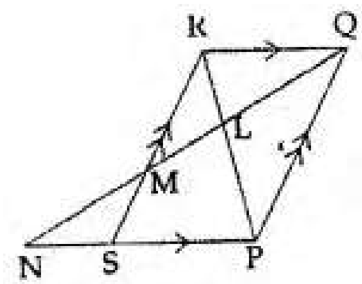
$\frac{QR}{PN} = \frac{2}{3}$

$\frac{10}{PN} = \frac{2}{3}$

PN = $\frac{(10\times 3)}{2}$

$PN = \frac{30}{2}$
PN = 15cm
Therefore, PN = 15cm

(ii) Name a triangle similar to triangle RLM. Evaluate RM.



Solution:

From the figure,

Consider ΔRLM and ΔQLP

Then, $\angle RLM = \angle QLP$ [verticallu opposite angles are equal]

$\angle$ LRM = $\angle LPQ$ [alternate angles are equal]

Therefore, $\Delta RLM \sim \Delta QLP$

Then, $\frac{RM}{PQ} = \frac{RL}{LP} = \frac{2}{3}$

So, $\frac{RM}{16} = \frac{2}{3}$

$RM = \frac{(16\times2)}{3}$

$RM = \frac{32}{3}$

RM = $10\frac{2}{3}$

17. The altitude BN and CM of ΔABC meet at H. Prove that

(i) $CN \times HM = BM \times HN$

(ii) $\frac{HC}{HB} = \sqrt{[(CN \times HN)/(BM \times HM)]}$

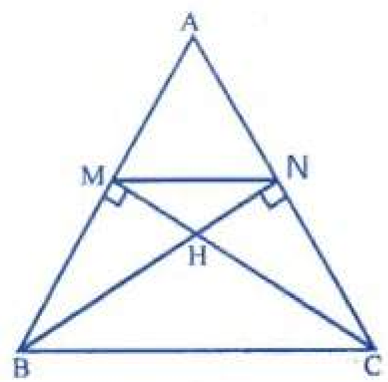
(iii) $\Delta MHN \sim \Delta BHC$

Solution:

Consider the $\Delta ABC,$

Where, the altitude BN and CM of ΔABC meet at H.

and construction : Join MN



(i) We have to prove that, $CN \times HM = BM \times HN$

In $\Delta\, BHM$ and ΔCHN

$\angle BHM = \angle CHN$ [because vertically opposite angles are equal]

$\angle M = \angle N$ [both angles are equal to 90°]

Therefore, $\Delta BHM \sim \Delta CHN$

So, $\frac{HM}{HN} = \frac{BM}{CN} = \frac{HB}{HC}$

Then, by cross multiplication we get

$CN \times HM = BM \times HN$

(ii) Now, $\frac{HC}{HB} = \sqrt{\frac{(HN \times CN)}{(HM \times BM)}}$

$= \sqrt{\frac{(CN \times HN)}{(BM \times HM)}}$

Because, M and N divide AB and AC in the same ratio.

(iii) Now consider ΔMHN and ΔBHC

∠MHN = ∠BHC [because vertically opposite angles are equal]

∠MNH = ∠HBC [because alternate angles are equal]

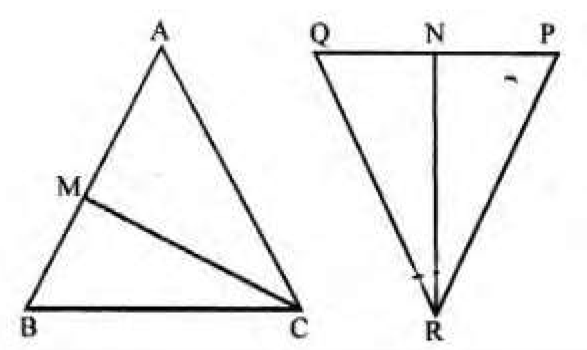
Therefore, ΔMHN ~ ΔBHC

18. In the given figure, CM and RN are respectively the medians of ΔABC and ΔPQR. If $\Delta ABC \sim \Delta PQR$, prove that :

(i) $\Delta AMC \sim \Delta PQR$

(ii) $\frac{CM}{RN} = \frac{AB}{PQ}$

(iii) $\Delta CMB \sim \Delta RNQ$



Solution :-

From the given figure it is given that, CM and RN are respectively the medians of ΔABC and ΔPQR.

(i) We have to prove that, $\Delta AMC \sim \Delta PQR$

Consider the ΔABC and ΔPQR

As ΔABC~ΔPQR

$\angle A = \angle P,\ \angle B = \angle Q$ and $\angle C = \angle R$

And also corresponding sides are proportional

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$

Then, consider the ΔAMC and ΔPNR,

$\angle A = \angle P$

$\frac{AC}{PR} = \frac{AM}{PN}$

Because, $\frac{AB}{PQ} = \frac{\frac{1}{2}AB}{\frac{1}{2}PQ}$

$\frac{AB}{PQ} = \frac{AM}{PN}$

Therefore, ΔAMC~ΔPNR

(ii) From solution

(i) $\frac{CM}{RN} = \frac{AM}{PN}$

$\frac{CM}{RN} = \frac{2AM}{2PN}$

$\frac{CM}{RN} = \frac{AB}{PQ}$

(iii) Now consider the ΔCMB and ΔRNQ

$\angle B = \angle Q$

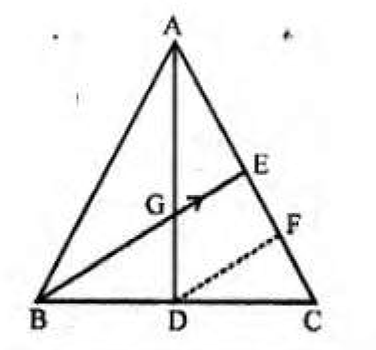
$\frac{\boldsymbol{BC}}{\boldsymbol{QP}} = \frac{\boldsymbol{BM}}{\boldsymbol{QN}}$

Therefore, $\Delta CMB \sim \Delta RNQ$

19. In the adjoining figure, medians AD and BE of ΔABC meet at the point G, and DF is drawn parallel to BE. Prove that

(i) EF = FC

(ii) AG : GD = 2 : 1



Solution :

From the figure it is given that, medians AD and BE of ΔABC meet at the point G, and DF is drawn parallel to BE.

(i) We have to prove that, EF = FC

From the figure, D is the midpoint of BC and also DF parallel to BE.

So, F is the midpoint of EC

Therefore, EF = FC

$= \frac{1}{2} EC$

Therefore, EF = FC

$= \frac{1}{2} EC$

EF = $\frac{1}{2} AE$

(ii) Now consider the ΔAGE and ΔADF

Then, (BG or GE) ||DF

Therefore, $\Delta AGE \sim \Delta ADF$

So, $\frac{AG}{GD} = \frac{AE}{EF}$

$\frac{AG}{GD} = \frac{1}{1/2}$

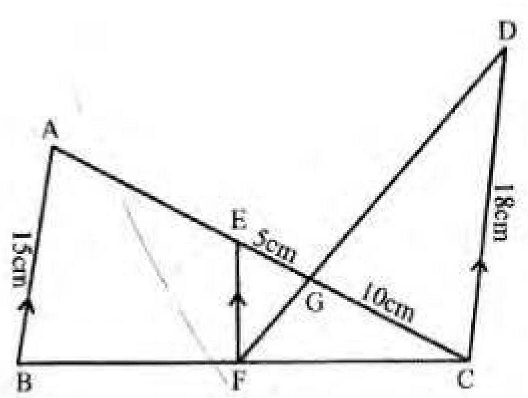
$\frac{AG}{GD} = 1 \times \left(\frac{2}{1}\right)$

Therefore, AG : GD = 2 : 1

20. In the figure given below, AB, EF and CD are parallel lines. Given that AB = 15cm, EG = 5cm, GC = 10cm and DC = 18 cm. Calculate

(i) EF

(ii) AC



Solution:-

From the figure it is given that, AB, EF and CD are parallel lines.

(i) Consider the ΔEFG and ΔCGD

$\angle EGF = \angle CGD$ [Because vertically opposite angles are equal]

$\angle FEG = \angle GCD$ [alternate angles are equal]

Therefore, $\Delta EFG \sim \Delta CGD$

Then, $\frac{EG}{GC} = \frac{EF}{CD}$

$\frac{5}{10} = \frac{EF}{18}$

EF = $\frac{(5\times18)}{10}$

Therefore, EF = 9 cm

(ii) Now, consider the ΔABC and ΔEFC

EF || AB

So, $\Delta ABC \sim \Delta EFC$

Then, $\frac{AC}{EC} = \frac{AB}{EF}$

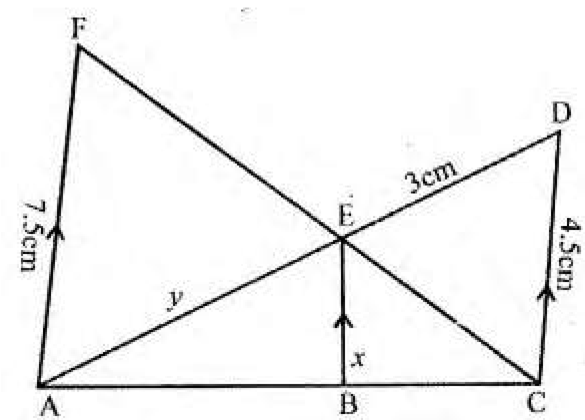
$\frac{AC}{(5+10)} = \frac{15}{9}$

$\frac{AC}{(15)} = \frac{15}{9}$

$AC = \frac{(15\times5)}{9}$

Therefore, AC = 25cm

(b) In the figure given below, AF, BE and CD are parallel line. Given that AF = 7.5 cm, CD = 4.5cm, ED = 3cm, BE = x and AE = y. Find the values of x and y.



Solution:

From the figure, AF, BE and CD are parallel lines.

Consider the ΔAEF and ΔCED

$\angle AEF$ and $\angle CED$ [because vertically opposite angles are equal]

$\angle F = \angle C$ [altenate angles are equal]

Therefore, $\Delta AEF \sim \Delta CED$

So, $\frac{AF}{CD} = \frac{AE}{ED}$

$= \frac{7.5}{4.5} = \frac{y}{3}$

By cross multiplications,

$y = \frac{(7.5\times3)}{4.5}$

$y = 5\ cm$

So, similarly in $\Delta ACD,$ BE || CD

Therefore, $\Delta ABE \sim \Delta ACD$

$\frac{EB}{CD} = \frac{AE}{AD}$

$\frac{x}{CD} = \frac{y}{y} + 3$

$\frac{x}{4.5} = \frac{5}{(5+3)}$

$\frac{x}{4.5} = \frac{5}{8}$

$x = \frac{(4.5 \times 5)}{8}$

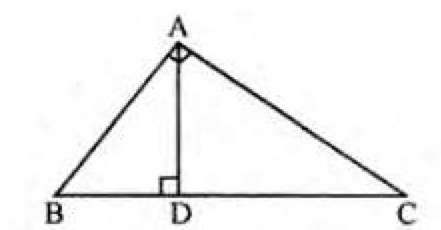
$x = \frac{22.5}{8}$

$x = \frac{225}{80}$

$x = \frac{45}{16}$

$x = 2\frac{13}{16}$

21. In the given figure, $\angle A = 90^\circ$ and AD$\perp$ BC If BD = 2 cm and CD = 8cm, find AD.



Solution:

From the figure, consider $\Delta ABC,$

So, $\angle A = 90^\circ$

And AD $\perp$ BC

$\angle BAC = 90^\circ$

Then, $\angle BAD + \angle DAC = 90^{\circ}$ …. [equation (i)]

Now, consider ΔADC

$\angle ADC = 90^{\circ}$

So, $\angle DCA + \angle DAC = 90^{\circ}$ …..[equation (ii)]

From equation (i) and equation (ii)

we have

$\angle BAD + \angle DAC = \angle DCA + \angle DAC$

$\angle BAD = \angle DCA$ ….[equation (iii)]

So, from ΔBDA and ΔADC

$\angle BDA = \angle ADC$ …[both the angles are equal to 90°]

$\angle BAD = \angle DCA$ ….[from equation (iii)]

Therefore, $\Delta BDA \sim \Delta ADC$

$\frac{BD}{AD} = \frac{AD}{DC} = \frac{AB}{AC}$

Because, corresponding sides of similar triangles are proportional

$\frac{BD}{AD} = \frac{AD}{DC}$

By cross multiplication we get,

$AD^2 = BD \times CD$

$AD^2 = 2 \times 8 = 16$

AD = $\sqrt{16}$

AD = 4

22. A 15 metres high tower casts a shadow of 24 metres long at a certain time and at the same time, a telephone pole casts a shadow 16 metres long. Find the height of the telephone pole.

Solution:

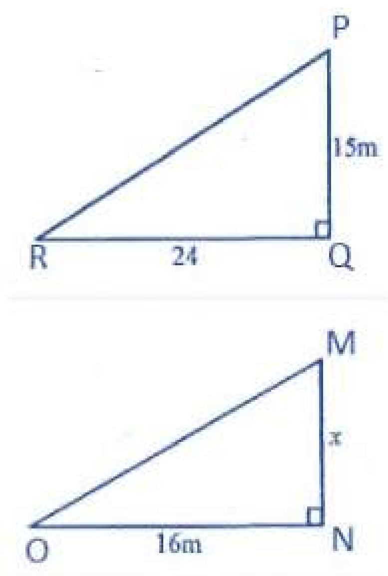
From the question it is gien that,

Height of a tower PQ = 15m

It's shadow QR = 24m

Let us assume the height of a telephone pole MN = x

It's shadow NO = 16 m



Given at the same time,

$\Delta PQR \sim \Delta MNO$

Therefore, $\frac{PQ}{MN} = \frac{ON}{RQ}$

$\frac{15}{x} = \frac{24}{16}$

By cross multiplication we get,

$x = \frac{(15\times16)}{24}$

$x = \frac{240}{24}$

$x = 10$

Therefore, height of pole = 10m.

23. A street light bulb is fixed on a pole 6m above the level of street. If a women of height casts a shadow of 3m, find how far she is away from the base of the pole ?

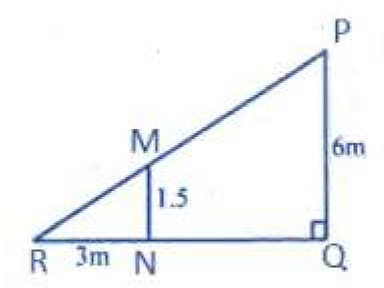
Solution:

From the question it is given that,

Height of pole (PQ) = 6m

Height of a women = 1.5m

So, shadow NR = 3m



Therefore, pole and woman are standing in the same line

PM || MR

$\Delta PRQ \sim \Delta MNR$

So, $\frac{RQ}{RN} = \frac{PQ}{MN}$

$$\frac{(3+x)}{3} = \frac{6}{1.5}$$

$$\frac{(3+x)}{3} = \frac{60}{15}$$

$$\frac{(3+x)}{3} = \frac{4}{1}$$

$$(3 + x) = 12$$

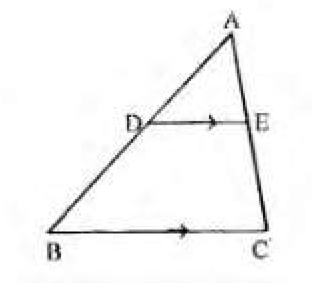
$$X = 12 - 3$$

$$X = 9m$$

Therefore, women is 9m away from the pole.

Exercise 13.2

1. (a) In the figure (i) given below if DE || BG, AD = 3cm, BD = 4cm and BC = 5cm. Find (i) AE : EC (ii) DE.



Solution:

From the figure,

DE || BG, AD = 3cm, BD = 4cm and BD = 5 cm

(i) AE : EC

So, $\frac{AD}{BD} = \frac{AE}{EC}$

$\frac{AE}{EC} = \frac{AD}{BD}$

$\frac{AE}{EC} = \frac{3}{4}$

(ii) Consider ΔADE and ΔABC

$\angle D = \angle B$

$\angle E = \angle C$

Therefore, $\Delta ADE \sim \Delta ABC$

Then, $\frac{DE}{BC} = \frac{AD}{AB}$

$\frac{\boldsymbol{DE}}{\mathbf{5}} = \frac{\mathbf{3}}{(\mathbf{3+4})}$

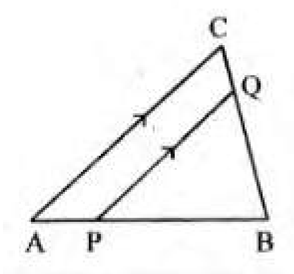
$\frac{\boldsymbol{DE}}{\mathbf{5}} = \frac{\mathbf{3}}{\mathbf{7}}$

DE = $\frac{(3\times5)}{7}$

DE = $\frac{15}{7}$

DE = $2\frac{1}{7}$

(b) In the figure (ii) given below, PQ || AC, AP = 4cm, PB = 6 cm and BC = 8cm, Find CQ and BQ.



Solution :

From the figure,

PQ || AC, AP = 4cm, PB = 6cm and BC = 8cm

$\angle BQP = \angle BCA$ …..[because alternate angles are equal]

Also, $\angle B = \angle B$ ….[common for both the triangles]

Therefore, $\Delta ABC \sim \Delta BPQ$

Then, $\frac{BQ}{BC} = \frac{BP}{AB} = \frac{PQ}{AC}$

$\frac{BQ}{BC} = \frac{6}{(6+4)} = \frac{PQ}{AC}$

$\frac{BQ}{BC} = \frac{6}{(10)} = \frac{PQ}{AC}$

$\frac{BQ}{8} = \frac{6}{(10)} = \frac{PQ}{AC}$ .....[because BC = 8cm given]

Now, $\frac{BQ}{8} = \frac{6}{(10)}$

BQ = $\frac{(6\times10)}{8}$

BQ = $\frac{48}{10}$

BQ = 4.8 cm

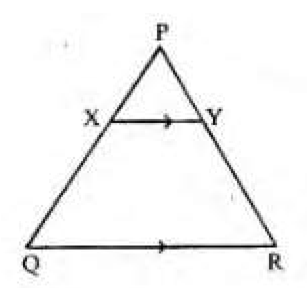
Also, CQ = BC - BQ

CQ = $(8 - 4.8)$cm

CQ =3.2 cm

Therefore, CQ = 3.2 cm and BQ = 4.8 cm

(c) In the figure (iii) given below, if XY || QR, PX = 1 cm, QX = 3cm, YR = 4.5 cm and QR = 9cm, find PY and XY.



Solution :

From the figure,

XY || QR, PX = 1cm, QX = 3cm, YR = 4.5 cm and QR = 9cm, So, $\frac{PX}{QX} = \frac{PY}{YR}$

$= \frac{1}{3} = \frac{PY}{4.5}$

By cross multiplication we get,

$\frac{(4.5\times1)}{3} = PY$

$PY = \frac{45}{30}$

PY = 1.5

Then, $\angle X = \angle Q$

$\angle Y = \angle R$

So, $\Delta PXY \sim \Delta PQR$

Therefore, $\frac{XY}{QR} = \frac{PX}{PQ}$

$\frac{XY}{9} = \frac{1}{(1+3)}$

$\frac{XY}{9} = \frac{1}{(4)}$

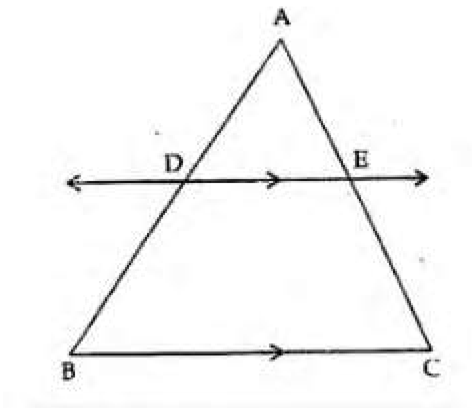
$XY = \frac{9}{4}$

$XY = 2.25$

2. In the given figure, DE || BC.

(i) If AD = x, DB = $x - 2$, AE = $x + 2$ and $EC = x - 1$, find the value of x.

(ii) If DB = $x - 3, AB = 2x, EC = x - 2$ and AC = $2x + 3$, find the value of x.



Solution:

(i) From the figure, it is given that,

Consider the ΔABC,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{x}{(x-2)} = \frac{(x+2)}{(x-1)}$

By cross multiplication we get,

$X(x-1) = (x-2)(x+2)$

$x^2 - x = x^2 - 4$

-x = -4

$x = 4$

(ii) From the question it is given that,

DB = $x - 3$, AB = $2x$, EC = $x - 2$ and AC = $2x + 3$

Consider the ΔABC,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{2x}{(x-2)} = \frac{(2x+3)}{(x-3)}$$

By cross multiplication we get,

$2x(x-2) = (2x+3)(x-3)$

$2x^2 - 4x = 2x^2 - 6x + 3x - 9$

$2x^2 - 4x - 2x^2 + 6x - 3x = -9$

$-7x + 6x = -9$

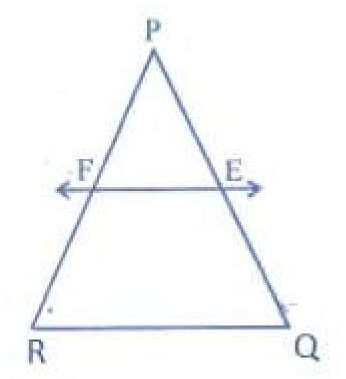
$-x = -9$

$x = 9$

3. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR :
(i) PE = 3.9 cm, EQ = 3cm, PF = 8cm and RF = 9cm.

Solution:
From the given dimensions,

Consider the ΔPQR



So, $\frac{PE}{EQ} = \frac{3.9}{3}$

$= \frac{39}{30}$

$= \frac{13}{10}$

Then, $\frac{PF}{FR} = \frac{8}{9}$

By comparing both the results,

$\frac{13}{10} \neq \frac{8}{9}$

Therefore, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

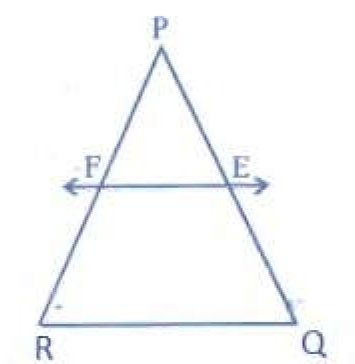
So, EF is not parallel to QR

(ii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18cm and PF = 0.36 cm.

Solution:

From the dimensions given in the question,

Consider the ΔPQR



So, $\frac{PQ}{PE} = \frac{1.28}{0.18}$

$= \frac{128}{18}$

$= \frac{64}{9}$

Them $\frac{PR}{RF} = \frac{2.56}{0.36}$

$= \frac{256}{36}$

$= \frac{64}{9}$

By comparing both the results,

$\frac{64}{9} = \frac{64}{9}$

Therefore, $\frac{PQ}{PE} = \frac{PR}{PF}$

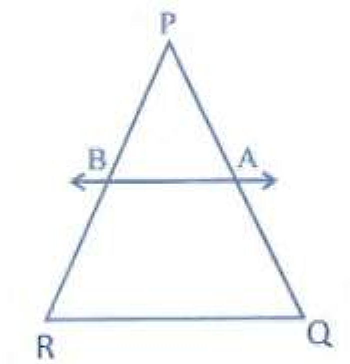
So, EF is parallel to QR.

4. A and B are respectively the points on the sides PQ and PR of a triangle PQR such that PQ = 12.5cm, PA= 5cm, BR = 6cm and PB = 4cm. Is AB || QR ? Give reasons for your answer.

Solution:

From the dimensions given in the question,

Consider the ΔPQR



So, $\frac{PQ}{PA} = \frac{12.5}{5}$

$= \frac{2.5}{1}$

$\frac{PR}{PB} = \frac{(PB+BR)}{PB}$

$= \frac{(4+6)}{4}$

$= \frac{10}{4}$

= 2.5

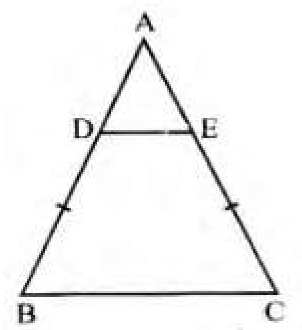
by comparing both the results,

2.5 = 2.5

Therefore, $\frac{PQ}{PA} = \frac{PR}{PB}$

So, AB is parallel to QR.

5. (a) In figure (i) given below, DE || BC and BD = CE. Prove that ABC in an isosceles triangle.



Solution :

From the question it is given that,

DE || BC and BD = CE

So, we have to prove that ABC is an isosceles triangle.

Consider the triangle ABC,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Given, $DB = EC$ ….. [equation (i)]

Then AD = AE …. [equation (ii)]

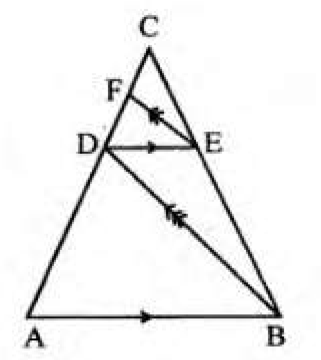
By adding equation (i) and equation (ii) we get,

AD + DB = AE + EC

So, AB = AC

Therefore, ΔABC is an isosceles triangle.

(b) In figure (ii) given below, AB || DE and BD || EF. Prove that $DC^2 = CF \times AC$.



Solution:

From the figure it is given that, AB || DE and BD || EF.

We have to prove that, $DC^2 = CF \times AC$

Consider the $\Delta ABC,$

$\frac{DC}{CA} = \frac{CE}{CB}$ ….[equation (i)]

Now, consider ΔCDE

$\frac{CF}{CD} = \frac{CE}{CB}$ ….[equation (ii)]

From equation (i) and equation (ii),

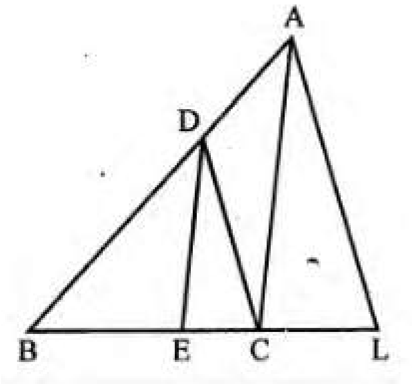
$\frac{DC}{CA} = \frac{CF}{CD}$

$\frac{DC}{AC} = \frac{CF}{DC}$

By cross multiplication we get,

$DC^2 = CF \times AC$

6. (a) In the figure (i) given below, CD || LA and DE || AC. Find the length of CL if BE = 4cm and EC = 2cm.



Solution:

From the given figure, CD || LA and DE || AC,

Consider the ΔBCA,

$\frac{BE}{BC} = \frac{BD}{BA}$

By using the corollary of baisc proportionality theorem,

$\frac{BE}{(BE+EC)} = \frac{BD}{AB}$

$\frac{4}{(4+2)} = \frac{BD}{AB}$ …. [equation (i)]

Then, consider the ΔBLA

$\frac{BC}{BL} = \frac{BD}{AB}$

By using the corollary of basic proportionality theorem,

$\frac{6}{(6+CL)} = \frac{BD}{AB}$ ….[equation (ii)]

Now, combining the equation (i) and equation (ii), we get

$\frac{6}{(6+CL)} = \frac{4}{6}$

By cross multiplication we get,

$6 \times 6 = 4 \times (6 + CL)$

$24 + 4CL = 36$

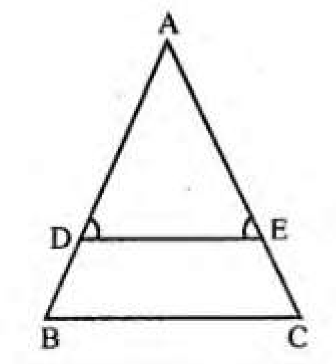
$4CL = 36 - 24$

$CL = \frac{12}{4}$

$CL = 3\ cm$

Therefore, the length of CL is 3 cm.

(b) In the give figure, $\angle D = \angle E$ and $\frac{AD}{BD} = \frac{AE}{EC}$. Prove that BAC is an isosceles triangle.



Solution:

From the given figure, $\angle D = \angle E$ and $\frac{AD}{BD} = \frac{AE}{EC}$.

We have to prove that, BAC is an isosceles triangle

So, consider the ΔADE

$\angle D = \angle E$ …[from the question]

AD = DE…. [sides opposite to equal angles]

Consider the $\Delta ABC,$

Then, $\frac{AD}{DB} = \frac{AE}{EC}$ ….[equation (i)]

Therefore, DE parallel to BC

Because AD = DE

DB = EC ……[equation (ii)]

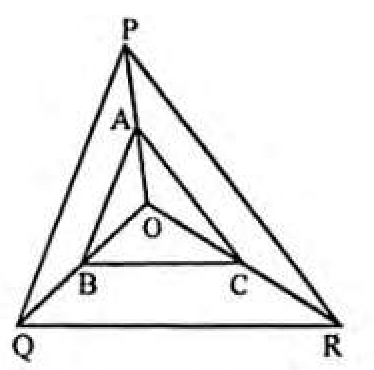
By adding equation (i) and equation (ii) we get,

$AD + DB = AE + EC$

$AB = AC$

Therefore, ΔABC is an isosceles triangle.

7. In the adjoining given below, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. show that BC || QR.



Solution:

Consider the ΔPOQ

AB || PQ ………… [given]

So, $\frac{OA}{AP} = \frac{OB}{BQ}$ ….. [equation (i)]

Then, consider the ΔOPR

AC || PR

$\frac{OA}{AP} = \frac{OC}{CR}$ ….[equation (ii)]

Now by comparing both equation (i) and equation (ii),

$\frac{OB}{BQ} = \frac{OC}{CR}$

Then, in ΔOQR

$\frac{OB}{BQ} = \frac{OC}{CR}$

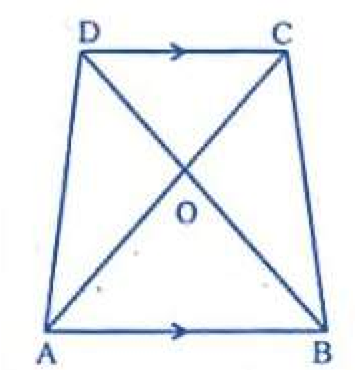
Therefore, BC || QR

8. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at O. Using Basic Proportionality theorem, Prove that $\frac{AO}{BO} = \frac{CO}{DO}$

Solution:

From the question It is given that,

ABCD is a trapezium in which AB || DC and its diagonal intersect each other at O



Now consider the ΔOAB and $\Delta OCD,$

$\angle AOB = \angle COD$ [because vertically opposite angles are equal]

$\angle OBA = \angle ODC$ [because alternate angles are equal]

Therefore, $\Delta OAB \sim \Delta OCD$

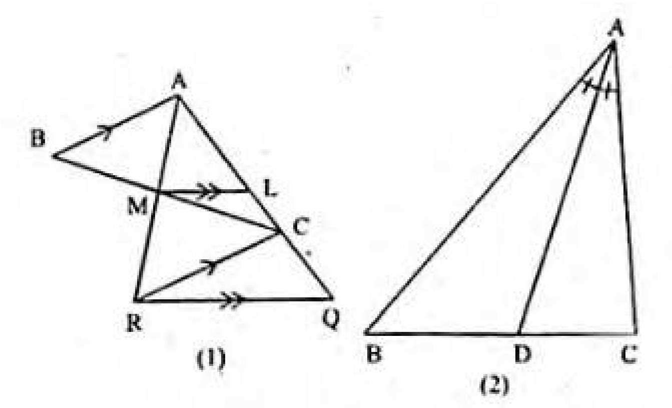
Then, $\frac{OA}{OC} = \frac{OB}{OD}$

$\frac{AO}{OB} = \frac{CO}{DO}$ .. [by alternate angles]

9. (a) In the figure (i) given below, AB || CR and LM || QR.

(i) Prove that $\frac{BM}{MC} = \frac{AL}{LQ}$

(ii) Calculate LM : QR, given that BM : MC = 1 : 2.



Solution:

From the question it is given that, AB || CR and LM || QR

(i) We have to prove that, $\frac{BM}{MC} = \frac{Al}{LQ}$

Consider the ΔARQ

LM || QR….[from the question]

So, $\frac{AM}{MR} = \frac{AL}{LQ}$ …..[equation (i)]

Now, consider the ΔAMB and ΔMCR

$\angle AMB = \angle CMR$ ….[because vertically opposite angles are equal]

$\angle MBA = \angle MCR$ ….. [because alternate angles are equal]

Therefore, $\frac{AM}{MR} = \frac{BM}{MC}$ ……[equation (ii)]

From equation (i) and equation (ii) we get,

$\frac{BM}{MR} = \frac{AL}{LQ}$

(ii) Given, BM : MC = 1 : 2

$\frac{AM}{MR} = \frac{BM}{MC}$

$\frac{AM}{MR} = \frac{1}{2}$ ...[equation (iii)]

LM || QR …. [given from equation]

$\frac{AM}{MR} = \frac{LM}{QR}$ …[equation (iv)]

$\frac{AR}{AM} = \frac{QR}{LM}$

$\frac{(AM+MR)}{AM} = \frac{QR}{LM}$

$1 + \frac{MR}{AM} = \frac{QR}{LM}$

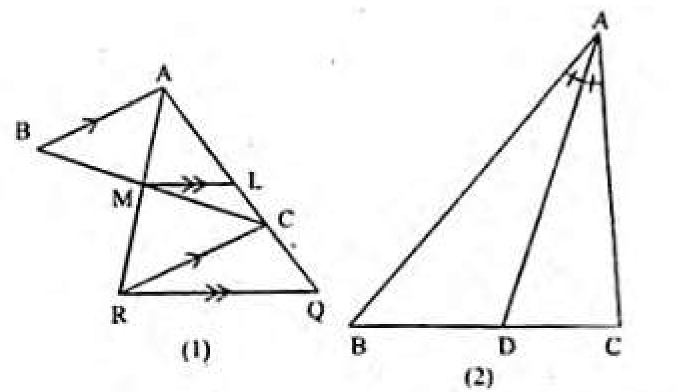
$1 + \frac{2}{1} = \frac{QR}{LM}$

$\frac{3}{1} = \frac{QR}{LM}$

$\frac{LM}{QR} = \frac{1}{3}$

Therefore, the ratio of LM : QR is 1 : 3.

(b) In the figure (2) given below AD is bisector of $\angle BAC$. If AB = 6cm, AC = 4cm and BD = 3cm, find BC



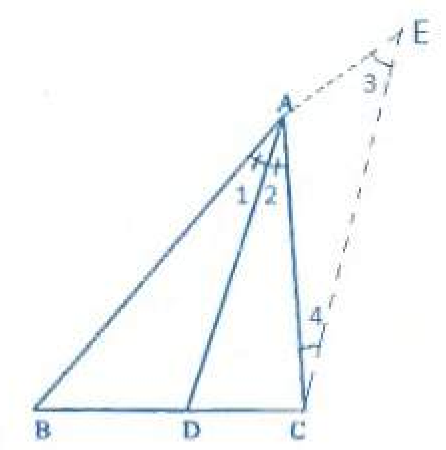
Solution:

From the question it is given that,

AD is bisector of $\angle BAC$

AB = 6cm, AC = 4cm and BD = 3cm

Construction, from C draw a straight line CE parallel to DA and join AE



$\angle 1 = \angle 2$ …[equation (i)]

By construction CE || DE

So, $\angle 2 = \angle 4$ … [because alternate angles are equal] [equation (ii)]

Again by construction CE || DE

$\angle 1 = \angle 3$ …[because corresponding angles are equal] [equation (iii)]

By comparing eqution(i), equation (ii) and equation (iii) we get,

$\angle 3 = \angle 4$

So, AC = AE ...[equation (iv)]

Now, consider the $\Delta BCE,$

CE || DE

$\frac{BD}{DC} = \frac{AB}{AE}$

$\frac{BD}{DC} = \frac{AB}{AC}$

$\frac{3}{DC} = \frac{6}{4}$

by cross multiplication we get,

$3 \times 4 = 6 \times DC$

$$DC = \frac{(3 \times 4)}{6}$$

$$DC = \frac{(12)}{6}$$

DC = 2

Therefore, BC = BD + DC

= 3 + 2

= 5cm

Exericse 13. 3

1. Given that Δs ABC and PQR are similar :

Find :

(i) The ratio of the area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 1 : 3.

(ii) The ratio of their corresponding sides if area of ΔABC : area of $\Delta PQR = 25 : 36.$

Solution:-

From the question it is given that,

(i) The ratio of the area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 1 : 3.

Then, $\Delta ABC \sim \Delta PQR$

area of $\frac{\Delta ABC}{area\ of \Delta PQR} = \frac{BC^2}{QR^2}$

So, BC : QR = 1 : 3

Therefore, $\frac{\Delta ABC}{area\ of\ \Delta PQR} = \frac{12}{32}$

$= \frac{1}{9}$

Hence the ratio of the area of ΔABC to the area of ΔPQR is 1 : 9.

(ii) The area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 25 : 36.
Then, $\Delta ABC \sim \Delta PQR$

Area of $\frac{\Delta ABC}{area\ of\ \Delta PQR} = \frac{BC^2}{QR^2}$

area of $\frac{\Delta ABC}{area\ of\ \Delta PQR} = \frac{25}{36}$

$= \left(\frac{BC}{QR}\right)^2 = \left(\frac{5}{6}\right)^2$

$= \frac{BC}{QR} = \frac{5}{6}$

Hence the ratio of their corresponding sides is 5 : 6.

2. $\Delta ABC \sim \Delta DEF$. If area of ΔABC = 9 sq. cm., area of $\Delta DEF = 16sq$.cm and BC = 2.1 cm., find the length of EF.

Solution:

From the question it is given that,
$\Delta ABC \sim \Delta DEF$

Area of ΔABC = 9 sq. cm
Area of ΔDEF = 16 sq. cm
We know that,

$\frac{area\ of\ \Delta ABC}{area\ of\ \Delta DEF} = \frac{BC^2}{EF^2}$

$\frac{9}{16} = \frac{BC^2}{EF^2}$

$\frac{9}{16} = \frac{(2.1)^2}{x^2}$

$\frac{2.1}{x} = \frac{\sqrt{9}}{\sqrt{16}}$

$\frac{2.1}{x} = \frac{3}{4}$

By cross multiplication we get,

$2.1 \times 4 = 3 \times x$

$8.4 = 3x$

$x = \frac{8.4}{3}$

$x = 2.8$

Therefore, EF = 2.8 cm

3. ΔABC ~ΔDEF. If BC = 3cm, EF = 4cm and area of $\Delta ABC = 54$ sq. cm. Determine the area of ΔDEF.

Solution:

From the question it is given that,

ΔABC ~ΔDEF

BC = 3cm, EF = 4cm

Area of $\Delta ABC = 54$ sq. cm.

We know that,

$$\frac{Area\ of \Delta ABC}{area\ of\ \Delta DEF} = \frac{BC^2}{EF^2}$$

$$\frac{54}{area\ of\ \Delta DEF} = \frac{3^2}{4^2}$$

$$\frac{54}{area\ of\ \Delta DEF} = \frac{9}{16}$$

By cross multiplication we get,

Area of $\Delta DEF = \frac{(54 \times 16)}{9}$

$= 6 \times 16$

= 96 cm

4. The area of two similar triangles are $36cm^2$ and $25cm^2$. If an altitude of the firsst triangle is 2.4cm, find the corresponding altitude of the other triangle.

Solution :

From the question it is given that,

The area of two similar triangles are $36cm^2$ and $25cm^2$

Let us assume $\Delta PQR \sim \Delta XYZ,$ PM and XN are their altitudes.

So, area of $\Delta PQR = 36cm^2$

Area of ΔXYZ $= 25cm^2$

PM = 2.4 cm

Assum XN = a

we Know that,

$$\frac{\text{area of } \Delta PQR}{\text{area of } \Delta XYZ} = \frac{PM^2}{XN^2}$$

$$\frac{36}{25} = \frac{(2.4)^2}{a^2}$$

By cross multiplication we get,

$36a^2 = 25(2.4)^2$

$a^2 = 5.76 \times \frac{25}{36}$

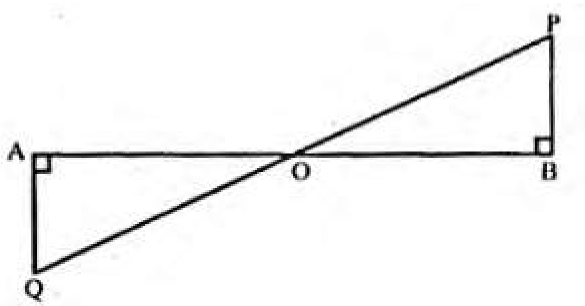
$a^2 = \frac{144}{36}$

$a^2 = \sqrt{4}$

$a = 2$ cm

Therefore, altitude of the other triangle XN = 2 cm.

5. (a) In the figure, (i) given below, PB and QA perpendiculars to the line segment AB. If PO = 6cm, QO = 9cm and the area of $\Delta POB = 120cm^2$, find the area of ΔQOA.



Solution:

From the question it is given that, PO = 6cm, QO = 9 cm and the area of $\Delta POB = 120cm^2$

From the figure,

Consider the ΔAOQ and $\Delta BOP,$

$\angle OAQ = \angle OBP$ …[both angles are equal to 90°]

$\angle AOQ = \angle BOP$ ..[because vertically opposite angles are equal]

Therefore, $\Delta AOQ \sim \Delta BOP$

Then, $\frac{area\ of\ \Delta AOQ}{area\ of\ \Delta BOP} = \frac{OQ^2}{PO^2}$

$$\frac{Area\ of\ \Delta AOQ}{120} = \frac{9^2}{6^2}$$

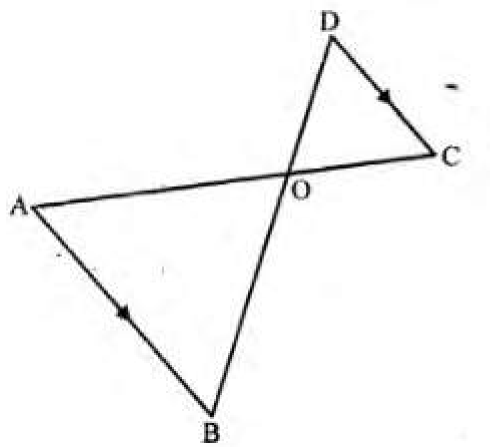
$$\frac{Area\ of\ \Delta AOQ}{120} = \frac{81}{36}$$

$$Area\ of\ \Delta AOQ = \frac{(81 \times 120)}{36}$$

$$Area\ of\ \Delta AOQ = 270\ cm^2$$

b) In the figure (ii) given below, AB || DC. AO = 10cm, OC = 5cm, AB = 6.5 cm and OD = 2.8 cm.

(i) Prove that $\Delta OAB \sim \Delta OCD$.
(ii) Find CD and OB.
(iii) find the ratio of areas of ΔOAB and ΔOCD



Solution:
from the qustion it is given that,
AB || DC. AO = 10cm, OC = 5 cm, AB = 6.5cm and OD = 2.8 cm
(i) We have to prove that, $\Delta OAB \sim \Delta OCD$
So, consider the ΔOAB and ΔOCD
$\angle AOB = \angle COD$ ... [$because\ vertically\ opposite\ angles\ are\ equal$]
$\angle OBA = \angle OCD$ ... [because alternate angles are equal]
Therefore, $\Delta OAB \sim \Delta OCD$ ….[from AAA axiom]

(ii) Consider the ΔOAB and ΔOCD
$$\frac{OA}{OC} = \frac{OB}{OD} = \frac{AB}{CD}$$

Now consider $\frac{OA}{OC} = \frac{OB}{OD}$

$\frac{10}{5} = \frac{OB}{2.8}$

OB = $\frac{(10 \times 2.8)}{5}$

OB = 2 × 2.8

OB = 5.6 cm

Then, consider $\frac{OA}{OC} = \frac{AB}{CD}$

$\frac{10}{5} = \frac{6.5}{CD}$

CD = $\frac{(6.5 \times 5)}{10}$

$CD = \frac{32.5}{10}$

CD = 3.25 cm

(iii) We have to find the ratio of areas of ΔOAB and ΔOCD.

From (i) we proved that, $\Delta OAB \sim \Delta OCD$

Then, $\frac{area\ of\ (\Delta OAB)}{areaof\ \Delta\ OCD}$

$\frac{AB^2}{CD^2} = \frac{(6.5)^2}{(3.25)^2}$

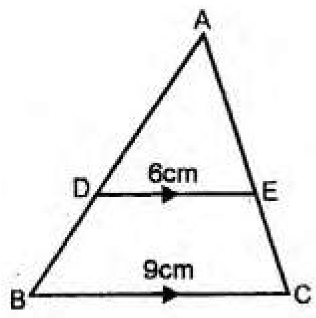
$= \frac{(6.5 \times 6.5)}{(3.25 \times 3.25)}$

$= 2\ \times \frac{2}{1}$

$= \frac{4}{1}$

Therefore, the ratio of areas of ΔOAB and $\Delta OCD = 4:1$

6. (a) In the figure (i) given below, DE || BC. If DE = 6cm, BC = 9cm and area of $\Delta ADE = 28$ sq. cm, find the area of ΔABC.



Solution:

From the question it is given that,

DE || BC, DE = 6cm, BC = 9 cm and area of $\Delta ADE = 28\ sq.cm$

From the fig. $\angle D = \angle B$ and $\angle E = \angle C$ … [corresponding angles are equal]

Now, consider the ΔADE and ΔABC,

$\angle A = \angle A$ …[common angles for both triangles]

Therefore, $\Delta ADE \sim \Delta ABC$

Then, $\frac{\text{area of } \Delta ADE}{area\ of\ \Delta ABC} = \frac{(DE)^2}{(BC)^2}$

$\frac{28}{area\ of\ \Delta ABC} = \frac{(6)^2}{(9)^2}$

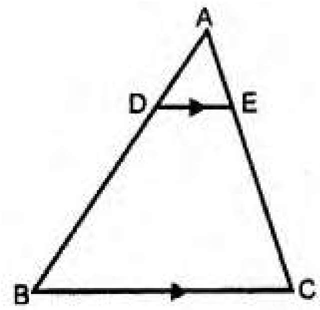
$\frac{28}{area\ of\ \Delta ABC} = \frac{36}{81}$

$area\ of\ \Delta ABC = \frac{(28\times81)}{36}$

$area\ of\ \Delta ABC = \frac{2268}{36}$

$area\ of\ \Delta ABC = 63cm^2$

(b) In the figure (ii) given below, DE || BC and AD : DB = 1 : 2, find the ratio of the areas of ΔADE and trapezium DBCE.



Solution :

From the question it is given that, DE || BC and AD : DB = 1 : 2,

$\angle D = \angle B, \angle E = \angle C$ ….[corresponding angles are equal]

Consider the ΔADE and ΔABC,

$\angle A = \angle A$ ….[common angles for both triangles]

Therefore, ΔADE ~ΔABC

But, $\frac{AD}{DB} = \frac{1}{2}$

Then, $\frac{DB}{AD} = \frac{2}{1}$

Now, adding 1 for both side LHS and RHS,

$\left(\frac{DB}{AD} + 1\right) = \left(\frac{2}{1} + 1\right)$

$\left(\frac{DB+AD}{AD}\right) = (2 + 1)$

Therefore, ΔADE~ ΔABC

Then, $\frac{area\ of\ \Delta\ ADE}{area\ of\ \Delta ABC} = \frac{AD^2}{AB^2}$

$\frac{area\ of\ \Delta\ ADE}{area\ of\ \Delta ABC} = \left(\frac{1}{3}\right)^2$

$\frac{area\ of\ \Delta\ ADE}{area\ of\ \Delta ABC} = \frac{1}{9}$

$area\ of\ \Delta ABC = 9\ area\ of\ \ \Delta\ ADE$

Area of trapezium DBCE

Area of ΔABC - Area of ΔADE

9 area of $\Delta ADE - area\ of\ \Delta ADE$

8 area of ΔADE

Therefore, $\frac{area\ of\ \Delta ADE}{area\ of\ trapezium} = \frac{1}{8}$

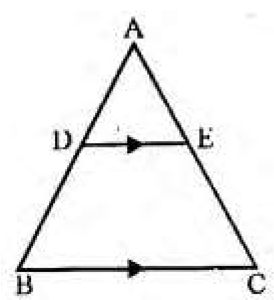
Then area of ΔADE : area of trapezium DBCE = 1 : 8

7. In the given figure, DE || BC.

(i) Prove that ΔADE and ΔABC are similar.

(ii) Given that AD = $\frac{1}{2}$ BD, calculate DE if BC = 4.5 cm.

(iii) If area of $\Delta ABC = 18cm^2$, find the area of trapezium DBCE



Solution:

(i) From the question it is given that, DE || BC

We have to prove that, ΔADE and ΔABC are similar

$\angle A = \angle A$ ... ... [Common angle for both triangles]

$\angle ADE = \angle ABC$ …[because corresponding angles are equal]

Therefore, $\Delta ADE \sim \Delta ABC$… [AA axiom]

(ii) From (i) we proved that, $\Delta ADE \sim \Delta ABC$

Then, $\frac{AD}{AB} = \frac{AB}{AC} = \frac{DE}{BC}$

So, $\frac{AD}{(AD+BD)} = \frac{DE}{BC}$

$\frac{\left(\frac{1}{2}BD\right)}{\left(\left(\frac{1}{2}BD\right)+BD\right)} = \frac{DE}{4.5}$

$\frac{\left(\frac{1}{2}BD\right)}{\left(\frac{3}{2}BD\right)} = \frac{DE}{4.5}$

$\frac{1}{2} \times \left(\frac{2}{3}\right) = \frac{DE}{4.5}$

$\frac{1}{3} = \frac{DE}{4.5}$

Therefore, DE = $\frac{4.5}{3}$

DE = 1.5 cm

(iii) From the question it is given that, area of $\Delta ABC = 18cm^2$

Then, $\frac{area\ of\ \Delta ADE}{area\ of\ \Delta ABC} = \frac{DE^2}{BC^2}$

$\frac{area\ of\ \Delta ADE}{18} = \left(\frac{DE}{BC}\right)^2$

$\frac{area\ of\ \Delta ADE}{18} = \left(\frac{AD}{AB}\right)^2$

$\frac{area\ of\ \Delta ADE}{18} = \left(\frac{1}{3}\right)^2$

$\frac{area\ of\ \Delta ADE}{18} = \frac{1}{9}$

$area\ of\ \Delta ADE = 18 \times \frac{1}{9}$

$area\ of\ \Delta ADE = 2$

So, area of trapezium DBCE = area of $\Delta ABC - area\ of\ \Delta ADE$

= 18 – 2

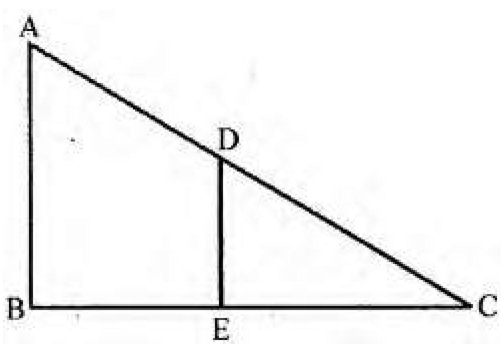
= 16 cm^2

8. In the given figure, AB and DE are perpendicular to BC.

(i) Prove that $\Delta ABC \sim \Delta DEC$

(ii) If AB = 6cm: DE = 4cm and AC = 15 cm, calculate CD.

(iii) Find the ratio of the area of ΔABC : area of ΔDEC.



Solution:

(i) Consider the ΔABC and ΔDEC.

$\angle ABC = \angle DEC$… [both angles are equal to 90°]

$\angle C = \angle C$.. [common angle for both triangles]

Therefore, $\Delta ABC \sim \Delta DEC$ …. [by AA axiom]

(ii) $\frac{AC}{CD} = \frac{AB}{DE}$

Corresponding sides of similar triangles are proportional

$\frac{15}{CD} = \frac{6}{4}$

CD = $\frac{(15\times4)}{6}$

CD = $\frac{60}{6}$

CD = 10 cm

(iii) we know that, $\frac{\text{area of } \Delta\text{ABC}}{area\ of\ \Delta DEC} = \frac{AB^2}{DE^2}$

$\frac{\text{area of } \Delta\text{ABC}}{area\ of\ \Delta DEC} = \frac{6^2}{4^2}$

$\frac{\text{area of } \Delta\text{ABC}}{area\ of\ \Delta DEC} = \frac{36}{16}$

$\frac{\text{area of } \Delta\text{ABC}}{area\ of\ \Delta DEC} = \frac{9}{4}$

Therefore, the ratio of the area of ΔABC : area of ΔDEC is 9 : 4.

Area of ΔABC = 9 area of ΔADE

Area of trapezium DBCE

Area of ΔABC - Area of ΔADE

9 area of ΔADE − area of ΔADE

8 area of ΔADE

Therefore, $\frac{\text{area of } \Delta\text{ADE}}{\text{Area of trapezium}} = \frac{1}{8}$

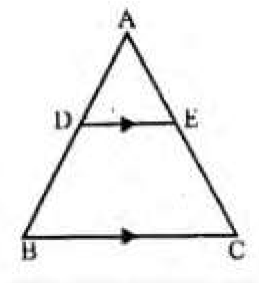
Then area of ΔADE : Area of trapezium DBCE = 1 : 8

7. In the given figure, DE || BC.

(i) Prove that ΔADE and ΔABC are similar.

(ii) Given that AD = $\frac{1}{2}BD,$ calculate DE if BC = 4.5 cm.

(iii) If area of $\Delta ABC = 18cm^2$, find the area of trapezium DBCE



Solution:

(i) From the question it is given that, DE || BC

We have to prove that, ΔADE and ΔABC are similar

$\angle A = \angle A$ …[common angle for both triangles]

$\angle ADE = \angle ABC$ ..[because corresponding angles are equal]

Therefore, ΔADE ~ΔABC…..[AA axiom]

(ii) From (i) we proved that, $\angle ADE \sim \angle ABC$

Then, $\frac{AD}{AB} = \frac{AB}{AC} = \frac{DE}{BC}$

So, $\frac{AD}{(AD+BD)} = \frac{DE}{BC}$

$$\frac{\left(\frac{1}{2}BD\right)}{\left(\left(\frac{1}{2}BD\right)+BD\right)} = \frac{DE}{4.5}$$

$$\frac{\left(\frac{1}{2}BD\right)}{\left(\left(\frac{3}{2}BD\right)\right)} = \frac{DE}{4.5}$$

$\frac{1}{2} \times \left(\frac{2}{3}\right) = \frac{DE}{4.5}$

$\frac{1}{3} = \frac{DE}{4.5}$

Therefore, DE = $\frac{4.5}{3}$

DE = 1.5 cm

(iii) From the question it is given that, area of $\Delta ABC = 18cm^2$

Then, $\frac{\text{area of } \Delta\text{ADE}}{area\ of\ \Delta ABC} = \frac{DE^2}{BC^2}$

$\frac{\text{area of } \Delta\text{ADE}}{18} = \left(\frac{DE}{BC}\right)^2$

$\frac{\text{area of } \Delta\text{ADE}}{18} = \left(\frac{AD}{AB}\right)^2$

$\frac{\text{area of } \Delta\text{ADE}}{18} = \left(\frac{1}{3}\right)^2$

$\frac{\text{area of } \Delta\text{ADE}}{18} = \frac{1}{9}$

$\text{area of } \Delta\text{ADE} = 18 \times \frac{1}{9}$

$\text{area of } \Delta\text{ADE} = 2$

So, area of trapezium DBCE = area of $\Delta ABC - area\ of\ \Delta\text{ADE}$

= 18 – 2

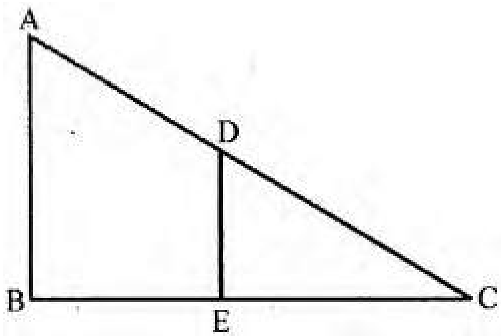
= $16cm^2$

8. In the given figure, AB and DE are perpendicular to BC.

(i) Prove that $\Delta ABC \sim \Delta DEC$

(ii) If AB = 6cm: DE = 4cm and AC = 15 cm, calculate CD.

(iii) Find the ratio of the area of $\Delta ABC : area\ of\ \Delta DEC$.



Solution:

(i) Consider the ΔABC and ΔDEC,

$\angle ABC = \angle DEC$ .... [both angles are equal to 90°]

$\angle C = \angle C$ ...[common angle for both triangles]

Therefore, $\Delta ABC \sim \Delta DEC$... [by AA axiom]

(ii) $\frac{AC}{CD} = \frac{AB}{DE}$

Corresponding sides of similar triangles are proportional

$\frac{15}{CD} = \frac{6}{4}$

CD = $\frac{(15\times 4)}{6}$

CD = $\frac{60}{6}$

CD = 10 cm

(iii) we know that, $\frac{area\ of\ \Delta ABC}{area\ of\ \Delta DEC} = \frac{AB^2}{DE^2}$

$\frac{area\ of\ \Delta ABC}{area\ of\ \Delta DEC} = \frac{6^2}{4^2}$

$\frac{area\ of\ \Delta ABC}{area\ of\ \Delta DEC} = \frac{36}{16}$

$\frac{area\ of\ \Delta ABC}{area\ of\ \Delta DEC} = \frac{9}{4}$

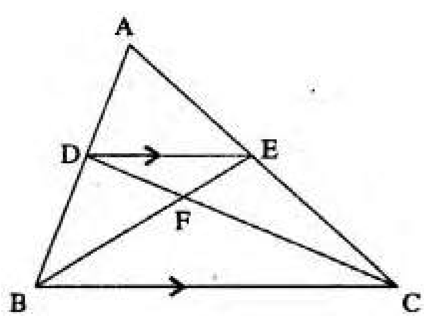
Therefore, the ratio of the area of ΔABC : area of ΔDEC is 9 : 4.

9. In the adjoining figure, ABC is a triangle. DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$.

(i) Determin the ratios $\frac{AD}{DB}$, $\frac{DE}{BC}$

(ii) Prove that ΔDEF is similar to ΔCBF. Hence, find $\frac{EF}{FB}$.

(iii) What is the ratio of the areas of ΔDEF and ΔCBF ?



Solution:

(i) We have to find the ratios $\frac{AD}{AB}$, $\frac{DE}{BC}$,

From the question it is given that, $\frac{AD}{DB} = \frac{3}{2}$,

Then, $\frac{DB}{AD} = \frac{2}{3}$

Now add 1 for both LHS and RHS we get,

$\left(\frac{DB}{AD} + 1\right) = \left(\frac{2}{3} + 1\right)$

$\frac{(DB+AD)}{AD} = \frac{(2+3)}{3}$

From the figure (DB +AD) = AB

So, $\frac{AB}{AD} = \frac{5}{3}$

Now, consider the ΔADE and ΔABC,

$\angle ADE = \angle B$ …[corresponding angles are equal]

$\angle AED = \angle C$ …[corresponding angles are equal]

Therefore, $\Delta ADE \sim \Delta ABC$ ..[by AA similarity]

Then, $\frac{AD}{DB} = \frac{DE}{BC} = \frac{3}{5}$

(ii) Now consider the ΔDEF and ΔCBF

$\angle EDF = \angle BCF$ …[because alternate angles are equal]

$\angle DEF = \angle FBC$ ….[because alternate angles are equal]

$\angle DFE = \angle ABFC$ ….. [because vertically opposite angles are equal]

Therefore, $\Delta DEF \sim \Delta CBF$

So, $\frac{EF}{FB} = \frac{DE}{BC} = \frac{3}{5}$

(iii) we have to find the ratio of the areas of ΔDEF and $\Delta CBF,$

We know that, $\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta BFC} = \frac{DE^2}{BC^2}$

$\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta BFC} = \left(\frac{DE}{BC}\right)^2$

$\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta BFC} = \left(\frac{3}{5}\right)^2$

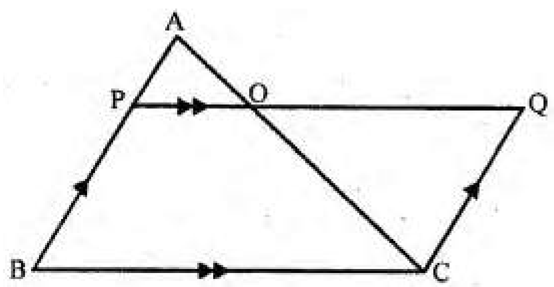
$\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta BFC} = \frac{9}{25}$

Therefore, the ratio of the areas of ΔDEF and ΔCBF is 9 : 25.

10. In ΔABC, AP : PB = 2 : 3. PO is parallel to BC and is extended to Q so that CQ is parallel to BA. find:

(i) Area ΔAPO : $area$ ΔABC.

(ii) Area ΔAPO : Area ΔCQO.



Solution:

From the question it is given that,

PB = 2 : 3

PO is parallel to BC and is extended to Q so that CQ is parallel to BA.

(i) we have to find the area $\Delta APO : area\ \Delta ABC,$

Then,

$\angle A = \angle A$ ..... [common angles for both triangles]

∠APO = ∠ABC …[because corresponding angles are equal]

Then, $\Delta APO \sim \Delta ABC$ ….[AA axiom]

We know that, $\frac{area\ of\ \Delta APO}{area\ of\ \Delta ABC} = \frac{AP^2}{AB^2}$

$= \frac{AP^2}{(\boldsymbol{AP+PB})^2}$

$= \frac{2^2}{(\boldsymbol{2+3})^2}$

$= \frac{4}{5^2}$

$= \frac{4}{25}$

Therefore, area $\Delta APO : Area\ \Delta ABC\ is\ 4 : 25$

(ii) we have to find the area $\Delta APO : area\ \Delta\ CQO$

Then, $\angle AOP = \angle COQ$ ... [because vertically opposite angles are equal]

$\angle APQ = \angle OQC$ …[becausee alternate angles are equal]

Therefore, $\frac{area\ of\ \Delta APO}{area\ of\ \Delta CQO} = \frac{AP^2}{CQ^2}$

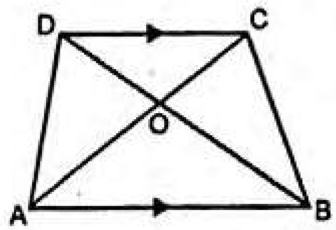
$\frac{area\ of\ \Delta APO}{area\ of\ \Delta CQO} = \frac{AP^2}{PB^2}$

$\frac{area\ of\ \Delta APO}{area\ of\ \Delta CQO} = \frac{2^2}{3^2}$

$\frac{area\ of\ \Delta APO}{area\ of\ \Delta CQO} = \frac{4}{9}$

Therefore, area ΔAPO : area $\Delta CQO\ \ is\ 4 : 9.$

11. (a) In the figure (i) given below, ABCD is a trapezium in which AB || SC and AB = 2CD. Determine the ratio of the areas of ΔAOB and ΔCOD.



Solution:

From the question it is given that,

ABCD is a trapezium in which AB || DC and AB = 2CD,

Then, $\angle OAB = \angle OCD$ ... [$because\ alternate\ angles\ are\ equal$]

$\angle OBA = \angle ODC$

Then, $\Delta AOB \sim \Delta COD$

So, $\frac{area\ of\ \Delta AOB}{area\ of\ \Delta COD} = \frac{AB^2}{CD^2}$

$= \frac{(2CD)^2}{CD^2}$.... [because AB = 2CD]

$= \frac{4CD^2}{CD^2}$

$= \frac{4}{1}$

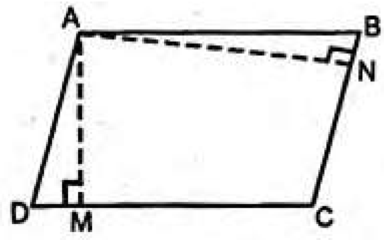
Therefore, the ratio of the areas of ΔAOB and ΔCOD is 4 :1.

(b) In the figure (ii) given below, ABCD is a parallelogram. AM⊥ DC and AN ⊥ CB. If AM = 6cm, AN = 10 cm and the area of parallelogram ABCD is $45cm^2$, find

(i) AB

(ii) BC

(iii) area of $\Delta ADM : area\ of\ \Delta ANB$.



Solution:

From the question it is given that,

ABCD is a parallelogram, AM ⊥ DC and AN⊥ CB

AM = 6 cm

AN = 10 cm

The area of parallelogram ABCD is $45cm^2$

Then, area of paralelogram ABCD = AC × $AM = BC \times AN$

45 = DC × $6 = BC \times 10$

(i) DC = $\frac{45}{6}$

Divide both numerator and denominator by 3 we get,

= $\frac{15}{2}$

= 7.5 cm

Therefore, AB = DC = 7.5 cm

(ii) $BC \times 10 = 45$

BC = $\frac{45}{10}$

BC = 4.5 cm

(iii) Now, consider ΔADM and ΔABN

$\angle D = \angle B$ …[because opposite angles of a parallelogram]

$\angle M = \angle N$ ... [both angles are equal to 90°]

Therefore, $\Delta ADM \sim \Delta ABN$

Therefore, $\frac{area\ of\ \Delta ADM}{area\ of\ \Delta ABN} = \frac{AD^2}{AB^2}$

$= \frac{BC^2}{AB^2}$

$= \frac{4.5^2}{7.5^2}$

$= \frac{20.25}{56.25}$

$= \frac{2025}{5625}$

$= \frac{81}{225}$

$= \frac{9}{25}$

Therefore, area of $\Delta ADM : area\ of\ \Delta ANB\ is\ 9: 25$

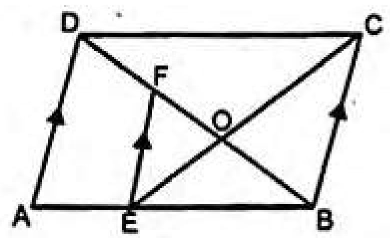
(c) In the figure (iii) given below, ABCD is a parallelogram. E is a point on AB, CE intersects the diagonal BD at O and EF || BC. If AE : EB = 2 : 3, find

(i) EF : AD

(ii) Area of $\Delta\, BEF$: $area\ of\ \Delta\, ABD$

(iii) area of $\Delta\, ABD$: area of trapezium AFED

(iv) area of ΔFEO : area of ΔOBC.



Solution:

From the question it is given that, ABCD is a parallelogram.

E is a point on AB, CE intersects the diagonal BD at O.

AE : EB = 2 : 3

(i) We have to find EF : AD

So, $\frac{AB}{BE} = \frac{AD}{EF}$

$\frac{EF}{AD} = \frac{BE}{AB}$

$\frac{AE}{EB} = \frac{2}{3}$ …[given]

Now add 1 to both LHS and RHS we get,

$\frac{(AE+EB)}{1} = \left(\frac{2}{3}\right) + 1$

$\frac{(AE+EB)}{EB} = \left(\frac{2+3}{3}\right)$

$\frac{AB}{EB} = \frac{5}{3}$

$\frac{EB}{AB} = \frac{3}{5}$

Therefore, EF : AD is 3 : 5

(ii) we have to find area of $\Delta BEF : area\ of\ \Delta ABD,$

Then, $\frac{area\ of \Delta\ BEF}{area\ of\ \Delta ABD} = \frac{(EF)^2}{(AD)^2}$

$\frac{area\ of \Delta\ BEF}{area\ of\ \Delta ABD} = \frac{(3)^2}{(5)^2}$

$= \frac{9}{25}$

Therefore, area of $\Delta BEF : area\ of\ \Delta ABD\ is\ 9 : 25$

(iii) from(ii) $\frac{area\ of\ \Delta ABD}{area\ of\ \Delta BEF} = \frac{25}{9}$

25 area of $\Delta BEF = 9\ area\ of\ \Delta ABD$

25 (area of $\Delta ABD -$ area of trapezium AEFD) = 9 area of ΔABD

25 area of ΔABD - 25 area of trapezium AEFD = 9 area of ΔABD

25 area of trapezium AEFD == 25 area of ΔABD $-$9 area of ΔABD

25 area of trapezium AEFD = 16 area of ΔABD

$\frac{area\ of\ \Delta ABD}{\text{area of trapezium AEFD}} = \frac{25}{16}$

Therefore, area of ΔABD : area of trapezium AFED = 25 : 16

(iv) Now we have to find area of $\Delta FEO : area\ of\ \Delta OBC$

So, consider ΔFEO and ΔOBC,

$\angle EOF = \angle BOC$.. [because vertically opposite angles are equal]

$\angle F = \angle OBC$.. [because alternate angles are equal]

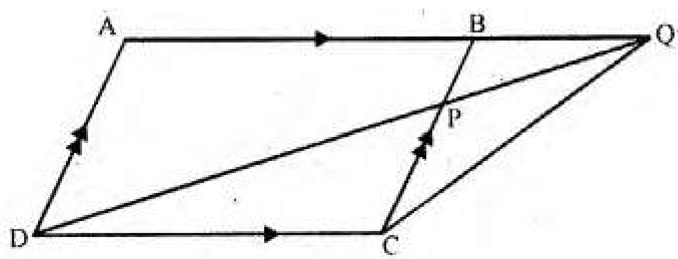
$\Delta FEO \sim \Delta OBC$

Then, $\frac{area\ of\ FEO}{area\ of\ \Delta OBC} = \frac{EF^2}{BC^2}$

$\frac{EF^2}{AD^2} = \frac{9}{25}$

Therefore, area of $\Delta FEO : area\ of\ \Delta OBC = 9 : 25$

12. In the adjoining figure, ABCD is a parallelogram. P is a point on BC such that BP : PC = 1 : 2 and DP produced meets AB produced at Q. If area of $\Delta CPQ = 20\ cm^2$, find



(i) area of ΔBPQ.

(ii) area ΔCDP.

(iii) area of parallelogram ABCD.

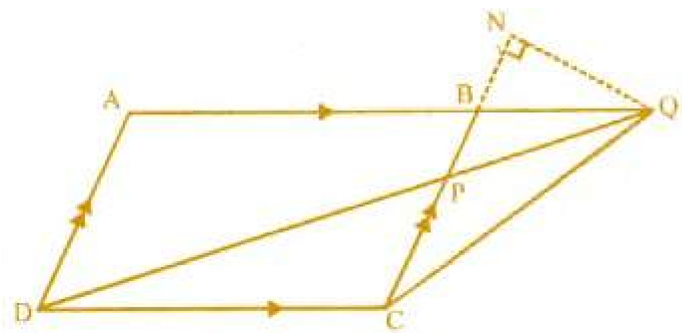
Solution:

From the question it is given that, ABCD is a parallelogram.

BP : PC = 1 : 2

area of $\Delta CPQ = 20cm^2$

Construction : deaw QN perpendicular CB and Join BN.



Then, $\frac{area\ of\ \Delta BPQ}{area\ of\ \Delta CPQ} = \frac{\left(\left(\frac{1}{2}BP\right)\times QN\right)}{\left(\left(\frac{1}{2}PC\right)\times QN\right)}$

$= \frac{BP}{PC} = \frac{1}{2}$

(i) So, area $\Delta BPQ = \frac{1}{2}$ area of ΔCPQ

$= \frac{1}{2} \times 20$

Therefore, area of $\Delta BPQ = 10cm^2$

(ii) Now we have to find area of $\Delta CDP,$

Consider the ΔCDP and $\Delta BQP,$

Then, $\angle CPD = \angle QPD$.. [because vertically opposite angles are eqaul]

$\angle PDC = \angle PQB$ ... $[because\ alternate\ angles\ are\ equal]$

Therefore, $\Delta CDP \sim \Delta BQP$ …[AA axiom]

$\frac{area\ of\ \Delta CDP}{area\ of\ \Delta BQP} = \frac{PC^2}{BP^2}$

$$\frac{area\ of\ \Delta CDP}{area\ of\ \Delta BQP} = \frac{2^2}{1^2}$$

$$\frac{area\ of\ \Delta CDP}{area\ of\ \Delta BQP} = \frac{4}{1}$$

$area\ of\ \Delta CDP = 4 \times area\ \Delta$BQP

Therefore, area of $\Delta CDP = 4\ \times 10$

= 40cm^2

(iii) We have to find the area of parallelogram ABCD,

Area of paralllelogram ABCD = 2 area of ΔDCQ

= 2 area $(\Delta DCP\ + \Delta CPQ)$

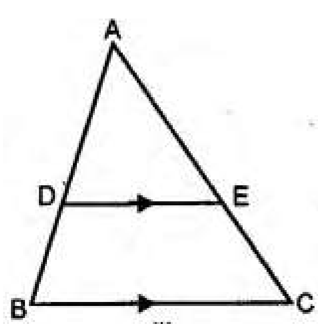
= 2 (40 + 20) cm^2

= 2 $\times 60\ cm^2$

= 120cm^2

Therefore, the area of parallelogram ABCD is 120 cm^2.

13. (a) In the figure (i) given below, DE || BC and the ratio of the areas of ΔADE and trapezium DBCE is 4 : 5. Find the ratio of DE: BC.



Solution:

From the question it is given that,

DE || BC

The ratio of the areas of ΔADE and trapezium DBCE is 4 : 5

Now, consider the $\Delta ABC\ and\ \Delta\ ADE$

$\angle A = \angle A$ …[common angle for both triangles]

$\angle D = \angle B$ and

$\angle E = \angle C$ …. [because corresponding angles are equal]

Therefore, $\Delta\ ADE \sim\ \Delta ABC$

So, $\frac{area\ of\ \Delta\ ADE}{area\ of\ \Delta ABC} = \frac{(DE)^2}{(BC)^2}$ …..[equation (i)]

Then, $\frac{area\ of\ \Delta ADE}{area\ of\ trapezium\ DBCE} = \frac{4}{5}$

$$\frac{area\ of\ trapezium\ DBCE}{area\ of\ \Delta ADE} = \frac{5}{4}$$

Add 1 for both LHS and RHS we get,

$$\left(\frac{area\ \ of\ trapezium\ DBCE}{area\ of\ \Delta ADE}\right) + 1 = \left(\frac{5}{4}\right) + 1$$

$$\frac{(area\ of\ trapezium\ DBCE + area\ of\ \Delta ADE)}{area\ of\ \Delta ADE} = \frac{(5+4)}{4}$$

$$\frac{(area\ of \Delta ABC\ )}{area\ of\ \Delta ADE} = \frac{9}{4}$$

$$\frac{area\ of\ \Delta ADE}{area\ of \Delta ABC} = \frac{4}{9}$$

From equation (i),

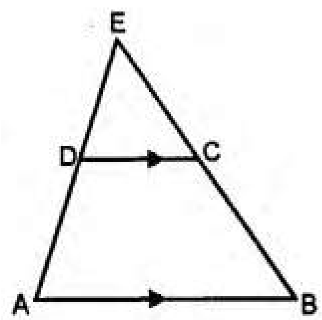
$$\frac{area\ of\ \Delta ADE}{area\ of \Delta ABC} = \frac{(DE)^2}{(BC)^2}$$

$$\frac{area\ of\ \Delta ADE}{area\ of \Delta ABC} = \frac{(4)^2}{(9)^2}$$

$$\frac{area\ of\ \Delta ADE}{area\ of \Delta ABC} = \frac{2}{3}$$

Therefore, DE : BC = 2 : 3

(b) In the figure (ii) given below, AB || DC and AB = 2 DC. If AD = 3cm, BC = 4cm and AD, BC produced meet at E, find (i) ED (ii) BE (iii) area of ΔEDC : area of trapezium ABCD.



Solution:

From the question it is given that,

AB || DC

AB = 2DC, AD = 3cm, BC = 4cm

Now consider $\Delta EAB,$

$$\frac{EA}{DA} = \frac{EB}{CB} = \frac{AB}{DC} = \frac{2DC}{DC} = \frac{2}{1}$$

(i) EA = 2, DA = $2 \times 3 = 6cm$

Then, ED = EA – DA

= 6 – 3

= 3 cm

(ii) $\frac{EB}{CB} = \frac{2}{1}$

EB = 2 CB

EB = 2 × 4

EB = 8 cm

(iii) Now, consider the $\Delta EAB, DC \parallel AB$

So, $\Delta EDC \sim \Delta EAB$

Therefore, $\frac{area\ of\ \Delta EDC}{area\ of\ \Delta ABE} = \frac{DC^2}{AB^2}$

$\frac{area\ of\ \Delta EDC}{area\ of\ \Delta ABE} = \frac{DC^2}{(2DC)^2}$

$\frac{area\ of\ \Delta EDC}{area\ of\ \Delta ABE} = \frac{DC^2}{4DC^2}$

$\frac{area\ of\ \Delta EDC}{area\ of\ \Delta ABE} = \frac{1}{4}$

Therefore, area of ABE = 4 area of ΔEDC

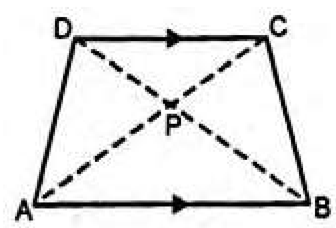
Then, area of ΔEDC + area of trapezium ABCD = 4 area of ΔEDC

Area of trapezium ABCD = 3 area of ΔEDC

So, $\frac{\text{area of } \Delta EDC}{\text{area of } trapezium\ ABCD} = \frac{1}{3}$

Therefore, area of ΔEDC: area of trapezium ABCD $= 1 : 3$

14. (a) In the figure given below, ABCD is a trapezium in which DC is parallel to AB. If AB = 9 cm, DC =6cm and BB = 12 cm. find (i) BP (ii) the ratio of areas of ΔAPB and ΔDPC.



Solution:

From the question it is given that,

DC is parallel to AB

AB = 9 cm, DC = 6cm and BB = 12cm

(i) Consider the $\Delta APB \ and \ \Delta \ CPD$

$\angle APB = \angle CPD$ ..[because vertically opposite angles are equal]

$\angle PAB = \angle PCD$ ... [because alternate angles are equal]

So, $\Delta APB \sim \Delta CPD$

Then, $\frac{BP}{PD} = \frac{AB}{CD}$

$\frac{BP}{(12-BP)} = \frac{9}{6}$

6BP = 108 – 9BP

6BP + 9BP = 108

15BP =108

BP = $\frac{108}{15}$

Therefore, BP = 7.2 cm

(ii) We know that, $\frac{area\ of\ \Delta APB}{area\ of\ \Delta CPD} = \frac{AB^2}{CD^2}$

$\frac{area\ of\ \Delta APB}{area\ of\ \Delta CPD} = \frac{9^2}{6^2}$

$\frac{area\ of\ \Delta APB}{area\ of\ \Delta CPD} = \frac{81}{36}$

By dividing both numerator and denominator by 9, we get,

$\frac{area\ of\ \Delta APB}{area\ of\ \Delta CPD} = \frac{9}{4}$

Therefore, the ratio of areas of ΔAPB and ΔDPC is 9 :4.

(b) In the figure given below, $\angle ABC = \angle DAC$ and AB = 8 cm, AC = 4 cm, AD = 5 cm.

(i) Prove that ΔACD is similar to ΔBCA

(ii) Find BC and CD

(iii) Find the area of ΔACD : $area\ of\ \Delta ABC$.

Solution:

From the question it is given that,

$\angle ABC = \angle DAC$

AB = 8cm, AC = 4cm, AD = 5cm

(i) Now, consider ΔACD and ΔBCA

$\angle C = \angle C$ .... [common angle for both triangles]

$\angle ABC = \angle CAD$ ..[common angle for both triangles]

So, $\Delta ACD \sim \Delta BCA$..[by AA axiom]

(ii) $\frac{AC}{BC} = \frac{CD}{CA} = \frac{AD}{AB}$

Consider $\frac{AC}{BC} = \frac{AD}{AB}$

$\frac{4}{BC} = \frac{5}{8}$

BC = $\frac{(4\times8)}{5}$

BC = $\frac{32}{5}$

BC = 6.4 cm

Then, consider $\frac{CD}{CA} = \frac{AD}{AB}$

$\frac{CD}{4} = \frac{5}{8}$

CD = $\frac{(4\times5)}{8}$

CD = $\frac{20}{8}$

CD = 2.5 cm

(iii) from (i) we proved that, $\Delta ACD \sim \Delta BCA$

$\frac{area\ of\ \Delta\ ACB}{area\ of\ \Delta BCA} = \frac{AC^2}{AB^2}$

$= \frac{4^2}{8^2}$

$= \frac{16}{64}$

By dividing both numerator and denominator by 16, we get, $= \frac{1}{4}$

Therefore, the area of $\Delta ACD : area\ of\ \Delta ABC\ is\ 1 : 4$.

15. ABC is a right angles triangle with $\angle ABC = 90^{\circ}$. D is any point on AB and DE is perpendicular to AC. Prove that :

(i) $\Delta ADE \sim \Delta ACB$.

(ii) If AC = 13 cm, BC = 5cm and AE = 4cm. Find DE and AD.

(iii) Find, area of $\Delta ADE\ : area$ of quadrilateral BCED.

Solution:

From the question it is given that,

$\angle ABC = 90^{\circ}$

AB and DE is perpendicular to AC

(i) Consider the ΔADE and ΔACB.

$\angle A = \angle A$.....[common angle for both triangle]

$\angle B = \angle E$ .... [both angles are equal to 90°]

Therefore, $\Delta ADE \sim \Delta ACB$

(ii) from (i) we proved that, $\Delta ADE \sim \Delta ACB$

So, $\frac{AE}{AB} = \frac{AD}{AC} = \frac{DE}{BC}$ …[eqution (i)]

Consider the $\Delta ABC,$ is a right angle triangle

From Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$13^2 = AB^2 + 5^2$

$169 = AB^2 + 25$

$169 - 25 = AB^2$

$144 = AB^2$

AB = $\sqrt{144}$

AB = 12 cm

Consider the equation (i),

$\frac{AE}{AB} = \frac{AD}{AC} = \frac{DE}{BC}$

Take, $\frac{AE}{AB} = \frac{AD}{AC}$

$$\frac{4}{12} = \frac{AD}{13}$$

$$\frac{1}{3} = \frac{AD}{13}$$

$$\frac{(1 \times 13)}{3} = AD$$

AD = 4.33 cm

Now, take $\frac{AE}{AB} = \frac{DE}{BC}$

$\frac{4}{12} = \frac{DE}{5}$

$\frac{1}{3} = \frac{DE}{5}$

DE = $\frac{(5 \times 1)}{3}$

DE = $\frac{5}{3}$

DE = 1.67 cm

(iii) Now, we have to find area of ΔADE: area of quadrilateral BCED,

We know that, Area of $\Delta ADE = \frac{1}{2} \times AE \times DE$

$= \frac{1}{2} \times 4 \times \left(\frac{5}{3}\right)$

$= \frac{10}{3} cm^2$

Then, area of quadrilateral BCED = area of $\Delta ABC - area\ of\ \Delta ADE.$

$= \frac{1}{2} \times BC \times AB - \frac{10}{3}$

$= \frac{1}{2} \times BC \times AB - \frac{10}{3}$

$= 1 \times 5 - 6 = \frac{10}{3}$

= 30 - $\frac{10}{3}$

$= \frac{(90-10)}{3}$

$= \frac{80}{3} cm^2$

So, the ratio of area of $\Delta ADE : area\ of\ quadrilateral\ BCED$

$= \frac{\left(\frac{10}{3}\right)}{\left(\frac{80}{3}\right)}$

$= \left(\frac{10}{3}\right) \times \left(\frac{3}{80}\right)$

$= \frac{(10\times3)}{(3\times80)}$

$= \frac{(1\times1)}{(1\times8)}$

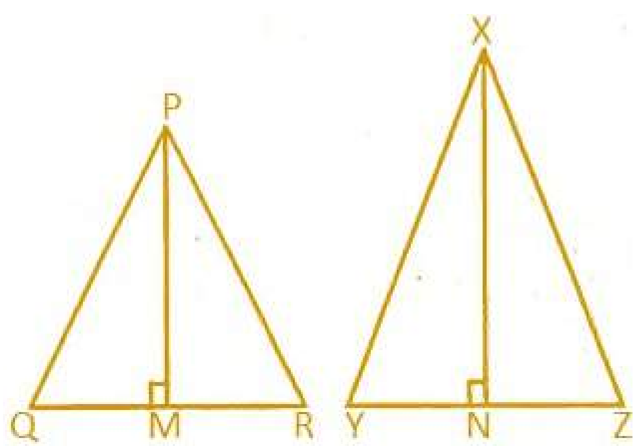
$= \frac{1}{8}$

Therefore, area of $\Delta ADE : area\ of\ quadrilateral\ BCED$ is 1 :8.

16. Two isosceles triangles have equal vertical angles and their areas are in the ratio 7 : 16. Find the ratio of their corresponding height.

Solution:

Consider the two isosceles triangle PQR and XYZ.



$\angle P = \angle X \ldots\ldots [from\ the\ question]$

So, $\angle Q + \angle R = \angle Y + \angle Z$

$\angle Q = \angle R$ and $\angle Y = \angle Z$ [because opposite angles of equal sides]

Therefore, $\angle Q = \angle Y$ and $\angle R = \angle Z$

$\Delta PQR \sim \Delta XYZ$

Then, $\frac{area\ of\ \Delta PQR}{area\ of\ \Delta XYZ} = \frac{PM^2}{XN^2}$ ….[from corollary of theorem]

$\frac{PM^2}{XN^2} = \frac{7}{16}$

$\frac{PM}{XN} = \frac{\sqrt{7}}{\sqrt{16}}$

$\frac{PM}{XN} = \frac{\sqrt{7}}{4}$

Therefore, ratio of PM : DM = $\sqrt{7} : 4$

17. On a map drawn to a scale of 1 : 250000, a triangular plot of land has the following measurements : AB = 3cm, BC = 4 cm and $\angle ABC = 90^\circ$. Calculate

(i) the actual length of AB in km.

(ii) the area of the plot in sq. km :

Solution:

From the question it is given that,

Map drawn to a scale of 1 : 250000

AB = 3cm, BC = 4cm and $\angle ABC = 90^\circ$

(i) We have to find the actual length of AB in km.

Let us assum scale factor K = 1 : 250000

K = $\frac{1}{250000}$

Then, length of AB of actual plot = $\frac{1}{k} \times length\ of\ AB\ on\ the\ map$

= $\left(\frac{1}{(250000)} \times 3\right)$

= 250000 × 3

To covert cm into km divide by 100000

= $\frac{(25000\times3)}{(100\times1000)}$

= $\frac{15}{2}$

length of AB of actual plot = 7.5 km

(ii) We have to find the area of the plot in sq. km

Area of plot on the map = $\frac{1}{2} \times AB \times BC$

= $\frac{1}{2} \times 3 \times 4$

= $\frac{1}{2} \times 12$

= 1×6

= 6 cm^2

Then, area of actual plot = $\frac{1}{k^2} \times 6cm^2$

= $250000^2 \times 6$

To convert cm into km divide by $(100000)^2$

= $\frac{(25000\times25000\times6)}{100000\times100000}$

$= \left(\frac{25}{4}\right) \times 6$

$= \frac{75}{2}$

$= 3.75\ km^2$

18. On a map drawn to a scale of 1 : 25000, a rectangular plot of land, ABCD has the following measurements AB = 12cm and BG = 16 cm. Calculate :

(i) The distance of a diagonal of the plot in km.

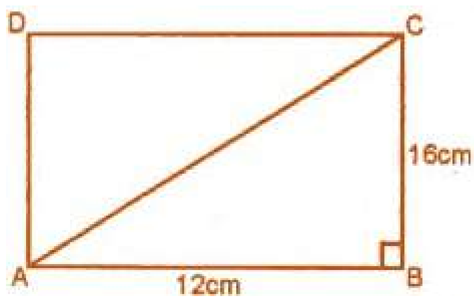
(ii) The area of the plot in sq. km.

Solution :

From the question it is given that,

Map drawn to a scale of 1 : 25000

AB = 12cm, BG = 16cm



Consider the $\Delta ABC,$

From the Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

AC = $\sqrt{(AB^2 + BC^2)}$

AC = $\sqrt{((12)^2 + (16)^2)}$

$= \sqrt{144 + 256}$

$= \sqrt{400}$

= 20 cm

Then, area of rectangular plot ABCD = AB $\times BC$

$= 12 \times 16$

= 192 cm^2

(i) We have to find the distance of a diagonal of the plot in km.

Let us assum scale factor K = 1 : 25000

K = $\frac{1}{250000}$

Then, length of AB of actual plot = $\frac{1}{k} \times$ $length\ of\ diagonal\ of\ rectangular\ plot$

$= \left(\frac{1}{\left(\frac{1}{25000}\right)}\right) \times 3$

= 25000 $\times$ 20

To convert cm into km divide by 100000

$= \frac{(25000 \times 20)}{(100 \times 1000)}$

= 5 km

(ii) We have to find the area of the plot in sq. km.

Then, area of actual plot = $\frac{1}{k^2} \times 192 cm^2$

$= 25000^2 \times 192$

To covert cm into divide by $(100000)^2$

$= \frac{(25000 \times 25000 \times 192)}{(100000 \times 100000)}$

$= 12\ km^2$

19. The model of a building is constructed with she scale factor 1 : 30.

(i) If the height of the model is 80 cm, find the actual height of the building in metres.

(ii) If the actual volume of a tank at the top of the building is $27m^3$, find the volume of the tank on the top of the model.

Solution:

From the question it is given that,

The model of a building is constructed with the scale factor 1 : 30

So,

$$\frac{\textbf{Height of the model}}{\textbf{Height of actual building}} = \frac{1}{30}$$

(i) Given, the height of the model is 80 cm

Then, $\frac{80}{H} = \frac{1}{30}$

H = (80×30)

H = 2400 cm

H = $\frac{2400}{100}$

H = 24 m

(ii) Given, the actual volume of a tank at the top of the building is $27m^3$

$\frac{Volume\ of\ model}{Volume\ of\ tank} = \left(\frac{1}{30}\right)^3$

$\frac{V}{27} = \frac{1}{27000}$

$$V = \frac{27}{27000}$$

V = $\frac{1}{1000}m^3$

Therefore, Volume of model = 1000 cm^3

20. A model of a ship is made to a scale of 1 : 200.

(i) If the length of the model is 4m, find the length of the ship.

(ii) If the area of the deck of the ship is 16000 m^2, find the area of the deck of the model.

(iii) If the volume of the model is 200 litres, find the volume of the ship in m^3. (100 litres = $1m^3$)

Solution:

From the question it is given that, a model of a ship is made to a scale of 1 : 200

(i) Given, the length of the model is 4 m

Then, length of the ship = $\frac{(4\times200)}{1}$

= 800 m

(ii) Given, the area of the deck of the ship is 160000 m^2

Then, area of deck of the model = 160000 × $\left(\frac{1}{200}\right)^2$

= 160000 × $\left(\frac{1}{40000}\right)$

= 4 m^2

(iii) Given, the volume of the model is 200 liters

Then, Volume of ship = 200 × $\left(\frac{200}{1}\right)^3$

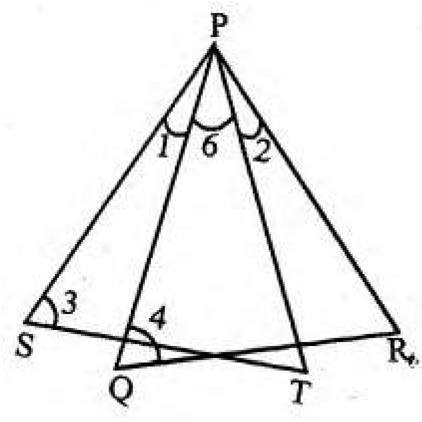
= 200 × 8000000

= $\frac{(200 \times 8000000)}{100}$

= 1600000 m^3

Chapter Test

1. In the adjoining figure, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Show that PT $\times$ $QR = PR \times ST$.



Solution :

From the question it is given that,

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

We have to prove that, PT $\times$ $QR = PR \times ST$

Given, $\angle 1 = \angle 2$

Adding $\angle 6$ to both LHS and RHS we get,

$\angle 1 + \angle 6 = \angle 2 + \angle 6$

$\angle SPT = \angle QPR$

Consider the ΔPQR and ΔPST,

From above $\angle SPT = \angle QPR$

$\angle 3 = \angle 4$

Therefore, $\Delta PQR \sim \Delta PST$

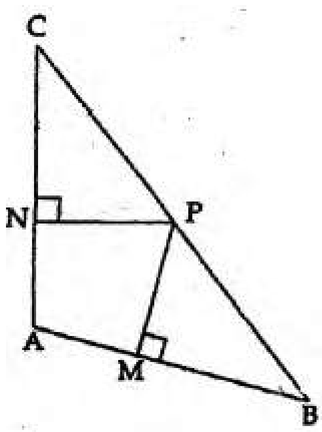
So, $\frac{PT}{PR} = \frac{ST}{QR}$

By cross multiplication we get,

$PT \times QR = PR \times ST$

Hence, it is proved that PT $\times QR = PR \times ST$

2. In the adjoining figure, AB = AC. If PM $\perp AB$ and PN $\perp AC$, show that PM $\times PC = PN \times PB$.



Solution:

From the given figure,

AB = AC, If PM $\perp AB$ and PN $\perp AC$

We have to show that, PM $\times PC = PN \times PB$

Consider the ΔABC,

AB = AC…[given]

$\angle B = \angle C$

Then, consider ΔCPN and ΔBPM

$\angle N = \angle M$ …. [both angles are equal to 90°]

$\angle C = \angle B$ …..[from above]

Therefore, $\Delta CPN \sim \Delta BPM$ ….[from AA axiom]

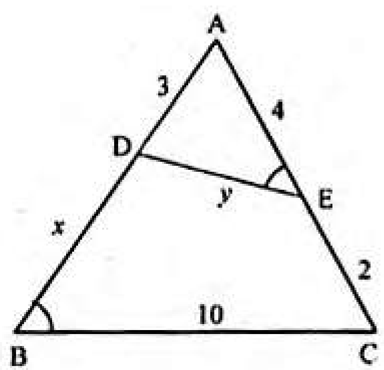
So, $\frac{PC}{PB} = \frac{PN}{PM}$

By cross multiplication we get,

$PC \times PM = PN \times PB$

Therefore, it is proved that, $PM \times PC = PN \times PB$

3. (a) In the figure given below. $\angle AED = \angle ABC$. Find the values of x and y.



Solution:

From the figure it is given that,

$\angle AED = \angle ABC$

Consider the ΔABC and ΔADE

$\angle AED = \angle ABC$ ….[from the figure]

$\angle A = \angle A$ ..[common angle for both triangles]

Therefore, $\Delta ABC \sim \Delta ADE$ ... [by AA axiom]

Then, $\frac{AD}{AC} = \frac{DE}{BC}$

$\frac{3}{(4+2)} = \frac{y}{10}$

$\frac{3}{(6)} = \frac{y}{10}$

By cross multiplication we get,

$y = \frac{(3 \times 10)}{6}$

$y = \frac{30}{6}$

$y = 5$

Now, consider $\frac{AB}{AE} = \frac{BC}{DE}$

$\frac{(3+x)}{4} = \frac{10}{y}$

Substitute the value of y,

$\frac{(3+x)}{4} = \frac{10}{5}$

By cross multiplication,

$5(3 + x) = 10 \times 4$

$15 + 5x = 40$

$5x = 40 - 15$

$5x = 25$

$x = \frac{25}{5}$

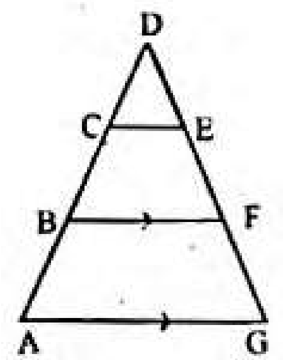
$x = 5$

Therefore, the value of x = 5 cm and y = 5cm

(b) In the figure given below, CD = $\frac{1}{2}$AC, B is mid-point of AC and E is mid-point of DF. If BF || AG, Prove that :

(i) CE || AG

(ii) 3ED = GD



Solution:

From the question it is given that,

CD = $\frac{1}{2}AC$

BF || AG

(i) We have to prove that CE || AG

Consider, CD = $\frac{1}{2}AC$

AC = 2BC….[because from the figure B is mid-point of AC]

So, CD = $\frac{1}{2}(2BC)$

CD = BC

Hence, CE || BF …..[equation (i)]

Given, BF || AG ..[equation (ii)]

By comparing the results of equation (i) and equation (ii) we get,

CE || AG

(ii) We have to prove that, 3ED = GD

Consider the $\Delta AGD,$

CE || AG ….above it is proved]

So, $\frac{ED}{GD} = \frac{DC}{AD}$

AD = AB + BC + DC

= DC + DC + DC

= 3DC

So, $\frac{ED}{GD} = \frac{DC}{(3DC)}$

$\frac{ED}{GD} = \frac{1}{(3(1))}$

$\frac{ED}{GD} = \frac{1}{3}$

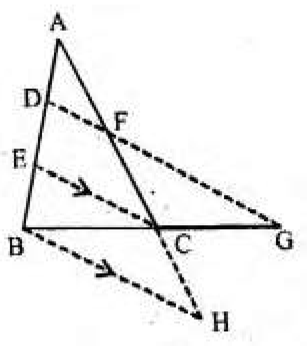
3ED = GD

Hence it is proved that, 3ED = GD

4. In the ajoining figure, 2AD = BD, E is mid-point of BD and F is mid-point of AC and EC || BH. Prove that :

(i) DF || BH

(ii) AH = 3 AF.



Solution:

From the question it is given that, 2AD = BD, ED || BH

(i) given, E is mid-point of BD

2DE = BD ….[equation (i)]

2AD = BD….[equation (ii)]

from equation (i) and equation (ii) we get,

2DE = AD

DE = AD

Also given that, F is mid-point of AC

DF || EC ….[equation (iii)]

Given, EC || BH ….[equation (iv)]

By comparing equation (iii) and equation (iv) we get,

DF || BH

(ii) We have to prove that, AH = 3 AF.

Given, E is mid- point of BD and EC || BH.

And c is midpoint of AH,

Thenm FC = CH ….[equation (v)]

Also given F is mid-point of AC

AF = FC …[equation (vi)]

By comparing both equation (v) and equation (vi) we get,

FC = AF = CH

AF = $\left(\frac{1}{3}\right)$AH

By cross multiplication we get,

3AF = AH

Therefore, it is proved that 3AF = AH

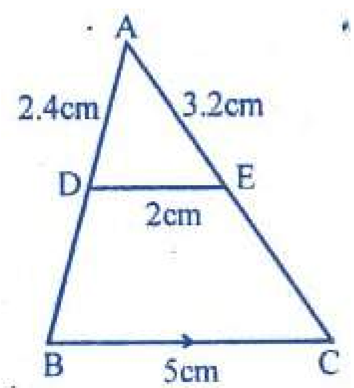
5. In a $\Delta ABC,$ D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2cm and BC = 5cm, find BD and CE.

Solution:

From the question it is given that, In a $\Delta ABC,$ D and E are points on the sides AB and AC respectively.

DE || BC

AD = 2.4 cm, AE = 3.2cm, DE = 2cm and BC = 5cm



Consider the $\Delta ABC,$

Given, DE || BC

So, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

Now, consider $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{2.4}{AB} = \frac{2}{5}$

AB = $\frac{(2.4\times5)}{2}$

AB = $\frac{12}{2}$

AB = 6 cm

Then, consider $\frac{AE}{AC} = \frac{DE}{BC}$

$\frac{3.2}{AC} = \frac{2}{5}$

AC = $\frac{(3.2\times5)}{2}$

AC = $\frac{16}{2}$

AC = 8 cm

Hence, BD = AB – AD

= 6 – 2.4

= 3.6 cm

CE = AC – AE

= 8 – 3.2

= 4.8 cm

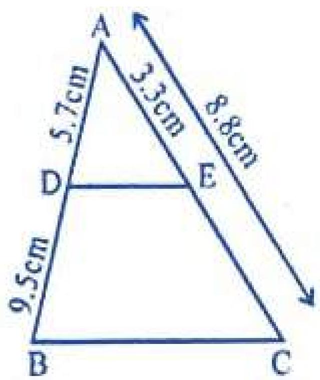
6. In a $\Delta ABC, D\ and\ E\ are\ points$ on the sides AB and AC respectively such that AD = 5.7 cm, BD = 9.5cm, AE = 3.3 cm and AC = 8.8 cm. Is DE || BC ? Justify your answer.

Solution:

From the question it is given that,

In a $\Delta ABC, D\ and\ E\ are\ points$ on the sides AB and AC respectively.

AD = 5.7 cm, BD = 9.5 cm, AE = 3.3cm and AC = 8.8cm



Consider the $\Delta ABC,$

EC = AC - AE

= 8.8 – 3.3

=5.5cm

Then, $\frac{AD}{DB} = \frac{5.7}{9.5}$

$= \frac{57}{95}$

By dividing both numerator and denominator by 19 we get,

$= \frac{3}{5}$

$\frac{AE}{EC} = \frac{3.3}{5.5}$

$= \frac{33}{55}$

By dividing both numerator and denominator by 11 we get,

$= \frac{3}{5}$

So, $\frac{AD}{DB} = \frac{AE}{EC}$

Therefore, $\frac{DE}{BC}$

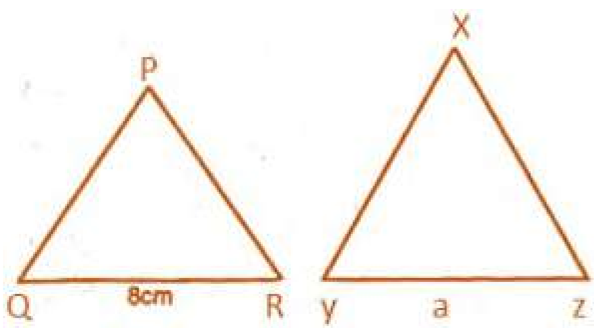
7. If the areas of two similar triangles are 360 cm^2 and 250cm^2 and if one side of the first triangle is 8cm, find the length of the corresponding side of the second triangle.

Solution:

From the question it is given that, the areas of two similar triangles are 360 cm^2 and 250cm^2

One side of the first triangle is 8 cm

So, PQR and XYZ are two similar triangles,



So, let us assume area of $\Delta PQR = 360\ cm^2, QR = 8cm$

And area of $\Delta XYZ = 250cm^2$

Assume YZ = a

We know that, $\frac{\text{area of } \Delta\text{PQR}}{area\ of\ \Delta XYZ} = \frac{QR^2}{yz^2}$

$\frac{\mathbf{360}}{\mathbf{250}} = \frac{(8)^2}{a^2}$

$\frac{\mathbf{360}}{\mathbf{250}} = \frac{64}{a^2}$

By cross multiplication we get,

$a^2 = \frac{(250 \times 64)}{360}$

$a^2 = \frac{400}{9}$

$a = \sqrt{\frac{400}{9}}$

$a = \frac{20}{3}$

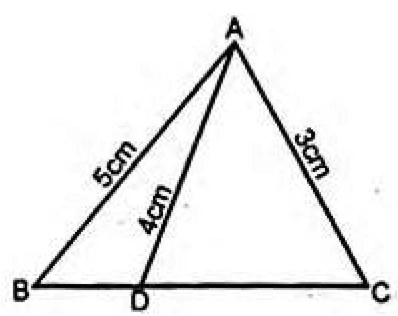
$a = 6\frac{2}{3}$

8. In the adjoining figure, D is a point on BC such that $\angle ABD = \angle CAD$. If AB = 5cm, AC = 3cm and AD = 4cm, find

(i) BC

(ii) DC

(iii) area of ΔACD : area of ΔBCA.



Solution:

From the question it is given that,

$\angle ABD = \angle CAD$

AB = 5cm, AC = 3cm and AD = 4 cm

Now, consider the ΔABC and ΔACD

$\angle C = \angle C$ … [common angle for both triangles]

$\angle ABC = \angle CAD$ .. [from the question]

So, $\angle ABC \sim \Delta ACD$

Then, $\frac{AB}{AD} = \frac{BC}{AC} = \frac{AC}{DC}$

(i) Consider $\frac{AB}{AD} = \frac{BC}{AC}$

$\frac{5}{4} = \frac{BC}{3}$

BC = $\frac{(5\times3)}{4}$

BC = $\frac{15}{4}$

BC = 3.75 cm

(ii) Consider $\frac{AB}{AD} = \frac{AC}{DC}$

$\frac{5}{4} = \frac{3}{DC}$

DC = $\frac{(3\times4)}{5}$

DC = $\frac{12}{5}$

DC = 2.4 cm

(iii) Consider the ΔABC and ΔACD

$\angle CAD = \angle ABC$ .... [from the question]

$\angle ACD = \angle ACB$ ...[common angle for both triangle]

Therefore, $\Delta ACD \sim \Delta ABC$

Then, $\frac{area\ of\ \Delta ACD}{area\ of \Delta ABC} = \frac{AD^2}{AB^2}$

$= \frac{4^2}{5^2}$

$= \frac{16}{25}$

Therefore, area of ΔACD : area of ΔBCA is $16 : 25$.

9. In the adjoining figure, the diagonals of a parallelogram intersect at O. OE is drawn parallel to CB to meet AB at E, find area of ΔAOE : area of parallelogram ABCD.

Solution:

From the given figure,

The diagonals of a parallelogram intersect at O.

OE is drawn parallel to CB to meet AB at E.

In the figure four triangles have equal area.

So, area of $\Delta OAB = \frac{1}{4}$ area of parallelogram ABCD

Then, O is midpoint of AC of ΔABC and DE || CB

E is also midpoint of AB

Therefore, OE is the median of ΔOAB

Area of $\Delta AOE = \frac{1}{2}$ area of ΔAOB

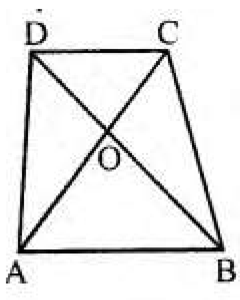
$= \frac{1}{2} \times \frac{1}{4}$ area of parallelogram ABCD

$= \frac{1}{8}$ area of parallelogram ABCD

So, $\frac{\text{area of } \Delta AOE}{\text{area of parallelogram ABCD}} = \frac{1}{8}$

Therefore, area of $\Delta AOE : area\ of\ parallelogram\ ABCD\ is\ 1 : 8$.

10. In the given figure, ABCD is a trapezium in which AB || DC. If 2AB = 3DC, find the ratio of the areas of ΔAOB and ΔCOD.



Solution:

From the question it is given that, ABCD is a trapezium in which AB || DC. If 2AB = 3DC.

So, 2AB = 3DC

$\frac{AB}{DC} = \frac{3}{2}$

Now, consider ΔAOB and ΔCOD

$\angle AOB = \angle COD$ .. [because vertically opposite angles are equal]

$\angle OAB = \angle OCD$ .... [because alternate angles are equal]

Therefore, $\Delta AOB \sim \Delta COD$ …[from AA axiom]

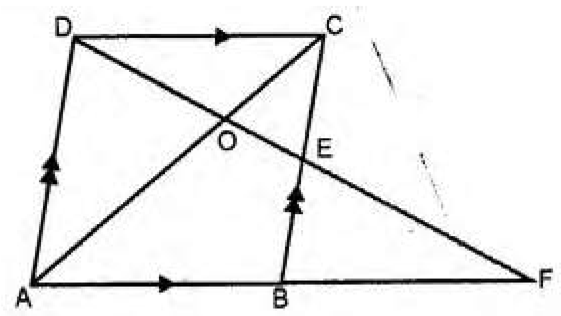
Then, $\frac{area\ of \Delta AOB}{area\ of\ \Delta COD} = \frac{AB^2}{DC^2}$

$= \frac{3^2}{2^2}$

$= \frac{9}{4}$

Therefore, the ratio of the areas of ΔAOB and ΔCOD is 9:4

11. In the adjoining figure, ABCD is a parallelogram. E is mid-point of BC. DE meets the diagonal AC at O and meet AB (produced) at F. Prove that



Solution:

From the question it is given that,

ABCD is a parallelogram. E is mid-point of BC.

De meets the diagonal AC at O.

(i) Now consider the ΔAOD and ΔEDC

$\angle AOD = \angle EOC$ ….[$because\ Vertically\ opposite\ angles\ are\ equal$]

$\angle OAD = \angle OCB$ ….. [because alternate angles are equal]

Therefore, $\angle AOD \sim \angle EOC$

Then, $\frac{OA}{OC} = \frac{DO}{OE} = \frac{AD}{EC} = \frac{2EC}{EC}$

$\frac{OA}{OC} = \frac{DO}{OE} = \frac{2}{1}$

Therefore, OA : OC = 2 : 1

(ii) From (i) we proved that $\Delta AOD \sim \Delta EOC$

So, $\frac{area\ of\ \Delta OEC}{area\ of\ \Delta AOD} = \frac{OE^2}{DO^2}$

$\frac{area\ of\ \Delta OEC}{area\ of\ \Delta AOD} = \frac{1^2}{2^2}$

$\frac{area\ of\ \Delta OEC}{area\ of\ \Delta AOD} = \frac{1}{4}$

Therefore, area of ΔOEC : $area\ of\ \Delta AOD$ is 1: 4.

13. A model of a ship is made to a scale of 1 :250 calculate :

(i) The length of the ship, if the length of model is 1.6m.

(ii) The area of the deck of the ship, if the area of the deck of model is $2.4m^2$.

(iii) The volume of the model, if the volume of the ship is $1km^3$.

Solution;

From the question it is given that, a model of a ship is made to a scale of 1 : 250

(i) Given, the length of the model is 1.6m

Then, length of the ship = $\frac{(1.6\times250)}{1}$

= 400 m

(ii) Given, the area of the deck of the ship is $2.4m^2$

Then, area of deck of the model = $2.4 \times \left(\frac{1}{250}\right)^2$

= 1,50,000 $m^2 = 4m^2$

(iii) Given, the volume of the model is $1km^3$

Then, Volume of ship = $\left(\frac{1}{250^3}\right) \times 1\ km^3$

= $\left(\frac{1}{250^3}\right) \times 1000^3$

= 4^3

= 64 m^3

Therefore, volume of ship is 64 m^3.