Chapter
Units And Measurements

Topic-1: Unit of Physical Quantities

1 MCQs with One Correct Answer

1. The *SI* unit of inductance, the henry can be written as **[1998 - 2 Marks]**

(a) weber/ampere (b) volt-sec/amp
(c) Joule/(ampere)^2 (d) ohm-second

7 Match the Following

2. **Column I** **Column II**

(A) GM_eM_s, G – universal gravitational constant, M_e – mass of the earth, M_s – mass of the Sun (p) (volt) (coulomb)(metre)

(B) $\frac{3RT}{M}$, R – universal gas constant, T – absolute temperature, M – molar mass (q) (kilogram) (metre)^3 (second)$^{-2}$

(C) $\frac{F^2}{q^2B^2}$, F – Force, q – charge, B – magnetic field (r) (metre)^2 (second)$^{-2}$

(D) $\frac{GM_e}{R_e}$, G – universal M_e – mass of the earth, R_e – radius of the earth (s) (farad) (volt)^2 (kg)$^{-1}$ **[2007]**

3. **Column I** **Column II**

(A) Capacitance (i) ohm-second
(B) Inductance (ii) coulomb^2–joule^{-1}
(C) Magnetic Induction (iii) coulomb (volt)$^{-1}$
(iv) newton (amp-metre)$^{-1}$
(v) volt-second (ampere)$^{-1}$

[1990 - 3 Marks]

10 Subjective Problems

4. Give the MKS units for each of the following quantities. **[1980]**
(A) Young's modulus
(B) Magnetic Induction (C) Power of a lens

Topic-2: Dimensions of Physical Quantities

1 MCQs with One Correct Answer

1. A dimensionless quantity is constructed in terms of electronic charge e, permittivity of free space ε_0, Planck's constant h, and speed of light c. If the dimensionless quantity is written as $e^{\alpha}\varepsilon_0{}^{\beta}h^{\gamma}c^{\delta}$ and n is a non-zero integer, then $(\alpha, \beta, \gamma, \delta)$ is given by **[Adv. 2024]**
(a) $(2n, -n, -n, -n)$ (b) $(n, -n, -2n, -n)$
(c) $(n, -n, -n, -2n)$ (d) $(2n, -n, -2n, -2n)$

2. Young's modulus of elasticity Y is expressed in terms of three derived quantities, namely, the gravitational constant G, Planck's constant h and the speed of light c, as $Y = c^{\alpha}h^{\beta}G^{\gamma}$. Which of the following is the correct option? **[Adv. 2023]**
(a) $\alpha = 7, \beta = -1, \gamma = -2$ (b) $\alpha = -7, \beta = -1, \gamma = -2$
(c) $\alpha = 7, \beta = -1, \gamma = 2$ (d) $\alpha = -7, \beta = 1, \gamma = -2$

3. Area of the cross-section of a wire is measured using a screw gauge. The pitch of the main scale is 0.5 mm. The circular scale has 100 divisions and for one full rotation of the circular scale, the main scale shifts by two divisions. The measured readings are listed below. **[Adv. 2022]**

Measurement condition	Main scale reading	Circular scale reading
Two arms of gauge touching each other without wire	0 division	4 divisions
Attempt-1: With wire	4 divisions	20 divisions
Attempt-2: With wire	4 divisions	16 divisions

What are the diameter and cross-sectional area of the wire measured using the screw gauge?

(a) $2.22 \pm 0.02\, mm, \pi(1.23 \pm 0.02) mm^2$
(b) $2.22 \pm 0.01\, mm, \pi(1.23 \pm 0.01) mm^2$
(c) $2.14 \pm 0.02\, mm, \pi(1.14 \pm 0.02) mm^2$
(d) $2.14 \pm 0.01\, mm, \pi(1.14 \pm 0.01) mm^2$

4. Let $[\epsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then: **[2012]**
(a) $\epsilon_0 = [M^{-1} L^{-3} T^2 A]$ (b) $\epsilon_0 = [M^{-1} L^{-3} T^4 A^2]$
(c) $\epsilon_0 = [M^1 L^2 T^1 A^2]$ (d) $\epsilon_0 = [M^1 L^2 T^1 A]$

5. Which of the following set have different dimensions? **[2005S]**
(a) Pressure, Young's modulus, Stress
(b) EMF, Potential difference, Electric potential
(c) Heat, Work done, Energy
(d) Dipole moment, Electric flux, Electric field

6. Pressure depends on distance as, $P = \frac{\alpha}{\beta}\exp\left(-\frac{\alpha z}{k\theta}\right)$, where α, β are constants, z is distance, k is Boltzman's constant and θ is temperature. The dimension of β are **[2004S]**
(a) $M^0L^0T^0$ (b) $M^{-1}L^{-1}T^{-1}$
(c) $M^0L^2T^0$ (d) $M^{-1}L^1T^2$

7. A quantity X is given by $\varepsilon_0 L \frac{\Delta V}{\Delta t}$ where ϵ_0 is the permittivity of the free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of **[2001S]**
(a) resistance (b) charge
(c) voltage (d) current

8. The dimension of $\left(\frac{1}{2}\right)\varepsilon_0 E^2$ (ε_0: permittivity of free space, E electric field) **[2000S]**
(a) MLT^{-1} (b) ML^2T^{-2}
(c) $ML^{-1}T^{-2}$ (d) ML^2T^{-1}

2 Integer Value Answer

9. In a particular system of units, a physical quantity can be expressed in terms of the electric charge e, electron mass m_e, Planck's constant h, and Coulomb's constant $k = \frac{1}{4\pi\epsilon_0}$, where ϵ_0 is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^\alpha [m_e]^\beta [h]^\gamma [k]^\delta$. The value of $\alpha + \beta + \gamma + \delta$ is ____. **[Adv. 2022]**

10. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways-engineer uses dimensions and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is **[Adv. 2014]**

4 Fill in the Blanks

11. The dimension of electrical conductivity is ________. **[1997 - 1 Mark]**

12. The equation of state for real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$. The dimensions of the constant a is ________. **[1997 - 2 Marks]**

13. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. The dimensions of Y in $MKSQ$ sytem are ________, ________. **[1988 - 2 Marks]**

14. Planck's constant has dimension ________. **[1985 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

15. A physical quantity $\vec{S}$ is defined as $\vec{S} = (\vec{E} \times \vec{B})/\mu_0$, where $\vec{E}$ is electric field, $\vec{B}$ is magnetic field and μ_0 is the permeability of free space. The dimensions of $\vec{S}$ are the same as the dimensions of which of the following quantities? **[Adv. 2021]**
(a) $\frac{\text{Energy}}{\text{Charge} \times \text{Current}}$ (b) $\frac{\text{Force}}{\text{Length} \times \text{Time}}$
(c) $\frac{\text{Energy}}{\text{Volume}}$ (d) $\frac{\text{Power}}{\text{Area}}$

16. Sometimes it is convenient to construct a system of units so that all quantities can be expressed in terms of only one physical quantity. In one such system, dimensions of different quantities are given in terms of a quantity X as follows:
[position] = $[X^\alpha]$; [speed] = $[X^\beta]$; [acceleration] = $[X^p]$; [linear momentum] = $[X^q]$; [force] = $[X^r]$. Then **[Adv. 2020]**
(a) $\alpha + p = 2\beta$ (b) $p + q - r = \beta$
(c) $p - q + r = \alpha$ (d) $p + q + r = \beta$

17. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimensions of L, which of the following statement(s) is/are correct? **[Adv. 2019]**
(a) The dimension of force is L^{-2}
(b) The dimension of linear momentum is L^{-1}
(c) The dimension of energy is L^{-2}
(d) The dimension of power is L^{-5}

18. A length-scale (l) depends on the permittivity (ε) of a dielectric material. Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is(are) dimensionally correct? **[Adv. 2016]**
(a) $l = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (b) $l = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$
(c) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (d) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

19. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are) **[Adv. 2015]**

(a) $\mu_0 I^2 = \varepsilon_0 V^2$ (b) $\varepsilon_0 I = \mu_0 V$
(c) $I = \varepsilon_0 cV$ (d) $\mu_0 cI = \varepsilon_0 V$

20. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is(are) **[Adv. 2015]**

(a) $M \propto \sqrt{c}$ (b) $M \propto \sqrt{G}$
(c) $L \propto \sqrt{h}$ (d) $L \propto \sqrt{G}$

21. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of the vacuum, and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current, **[1998 - 2 Marks]**

(a) $[\varepsilon_0] = M^{-1} L^{-3} T^2 I$ (b) $[\varepsilon_0] = M^{-1} L^{-3} T^4 I^2$
(c) $[\mu_0] = M L T^{-2} I^{-2}$ (d) $[\mu_0] = M L^2 T^{-1} I$

22. The pairs of physical quantities that have the same dimensions is (are) : **[1995S]**

(a) Reynolds number and coefficient of friction
(b) Curie and frequency of a light wave
(c) Latent heat and gravitational potential
(d) Planck's constant and torque

23. The dimensions of the quantities in one (or more) of the following pairs are the same. Identify the pair (s) **[1986 - 2 Marks]**

(a) Torque and Work
(b) Angular momentum and Work
(c) Energy and Young's modulus
(d) Light year and Wavelength

7 Match the Following

24. Match List I with List II and select the correct answer using the codes given below the lists: **[Adv. 2013]**

	List I		List II
P.	Boltzmann constant	1.	$[ML^2T^{-1}]$
Q.	Coefficient of viscosity	2.	$[ML^{-1}T^{-1}]$
R.	Planck constant	3.	$[MLT^{-3}K^{-1}]$
S.	Thermal conductivity	4.	$[ML^2T^{-2}K^{-1}]$

Codes:

	P	Q	R	S
(a)	3	1	2	4
(b)	3	2	1	4
(c)	4	2	1	3
(d)	4	1	2	3

25. Match the physical quantities given in column I with dimensions expressed in terms of mass (M), length (L), time (T), and charge (Q) given in column II and write the correct answer against the matched quantity in a tabular form in your answer book. **[1983 - 6 Marks]**

Column I	Column II
Angular momentum	ML^2T^{-2}
Latent heat	ML^2Q^{-2}
Torque	ML^2T^{-1}
Capacitance	$ML^3T^{-1}Q^{-2}$
Inductance	$M^{-1}L^{-2}T^2Q^2$
Resistivity	L^2T^{-2}

8 Comprehension/Passage Based Questions

Passage

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, $[E]$ and $[B]$ stand for dimensions of electric and magnetic fields respectively, while $[\varepsilon_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. $[L]$ and $[T]$ are dimensions of length and time respectively. All the quantities are given in SI units. **[Adv. 2018]**

26. The relation between $[E]$ and $[B]$ is

(a) $[E] = [B][L][T]$ (b) $[E] = [B][L]^{-1}[T]$
(c) $[E] = [B][L][T]^{-1}$ (d) $[E] = [B][L]^{-1}[T]^{-1}$

27. The relation between $[\varepsilon_0]$ and $[\mu_0]$ is

(a) $[\mu_0] = [\varepsilon_0][L]^2[T]^{-2}$ (b) $[\mu_0] = [\varepsilon_0][L]^{-2}[T]^2$
(c) $[\mu_0] = [\varepsilon_0]^{-1}[L]^2[T]^{-2}$ (d) $[\mu_0] = [\varepsilon_0]^{-1}[L]^{-2}[T]^2$

10 Subjective Problems

28. Write the dimensions of the following in terms of mass, time, length and charge **[1982 - 2 Marks]**

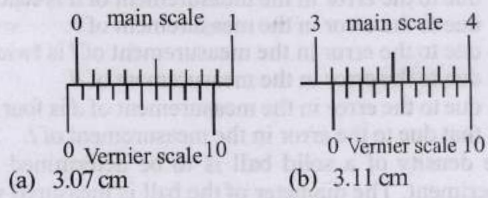
(i) magnetic flux
(ii) rigidity modulus

29. A gas bubble, from an explosion under water, oscillates with a period T proportional to $p^a d^b E^c$. Where 'P' is the static pressure, 'd' is the density of water and 'E' is the total energy of the explosion. Find the values of a, b and c. **[1981 - 3 Marks]**

Topic-3: Errors in Measurements & Experimental Physics

1 MCQs with One Correct Answer

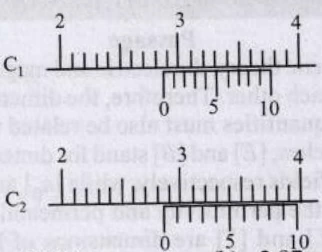
1. The smallest division on the main scale of a Vernier calipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this calipers with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is **[Adv. 2021]**



(a) 3.07 cm (b) 3.11 cm
(c) 3.15 cm (d) 3.17 cm

2. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be $L = 20$ meters. Take the acceleration due to gravity $g = 10\ ms^{-2}$ and the velocity of sound is $300\ ms^{-1}$. Then the fractional error in the measurement, $\delta L/L$, is closest to **[Adv. 2017]**
(a) 0.2% (b) 1% (c) 3% (d) 5%

3. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2, respectively, are **[Adv. 2016]**



(a) 2.85 and 2.82 (b) 2.87 and 2.83
(c) 2.87 and 2.86 (d) 2.87 and 2.87

4. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24^{th} division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is **[Adv. 2013]**
(a) 5.112 cm (b) 5.124 cm
(c) 5.136 cm (d) 5.148 cm

5. In the determination of Young's modulus $\left(Y = \dfrac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $l = 0.25$ mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement **[2012]**
(a) due to the errors in the measurements of d and l are the same.
(b) due to the error in the measurement of d is twice that due to the error in the measurement of l.
(c) due to the error in the measurement of l is twice that due to the error in the measurement of d.
(d) due to the error in the measurement of d is four times that due to the error in the measurement of l.

6. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2 %, the relative percentage error in the density is **[2011]**
(a) 0.9 % (b) 2.4 % (c) 3.1 % (d) 4.2 %

7. A vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is **[2010]**
(a) 0.02 mm (b) 0.05 mm (c) 0.1 mm (d) 0.2 mm

8. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of $\pm$ 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of $\pm$ 0.01 mm. Take $g = 9.8\ m/s^2$ (exact). The Young's modulus obtained from the reading is **[2007]**
(a) $(2.0 \pm 0.3) \times 10^{11}\ N/m^2$ (b) $(2.0 \pm 0.2) \times 10^{11}\ N/m^2$
(c) $(2.0 \pm 0.1) \times 10^{11}\ N/m^2$ (d) $(2.0 \pm 0.05) \times 10^{11}\ N/m^2$

9. A student performs an experiment for determination of $g\left(= \dfrac{4\pi^2 \ell}{T^2}\right)$. The error in length ℓ is $\Delta\ell$ and in time T is ΔT and n is number of times the reading is taken. The measurement of g is most accurate for **[2006 - 3M, –1]**

	$\Delta\ell$	ΔT	n
(a)	5 mm	0.2 sec	10
(b)	5 mm	0.2 sec	20
(c)	5 mm	0.1 sec.	10
(d)	1 mm	0.1 sec	50

10. In a screw gauge, the zero of mainscale coincides with fifth division of circular scale in figure (i). The circular division of screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball in figure (ii) is **[2006 - 3M, –1]**

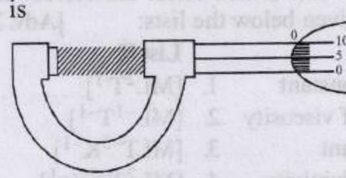


Figure (i)

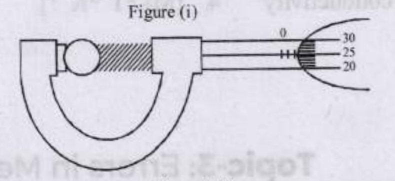


Figure (ii)

(a) 2.25 mm (b) 2.20 mm
(c) 1.20 mm (d) 1.25 mm

11. A wire of length $\ell = 6 \pm 0.06$ cm and radius $r = 0.5 \pm 0.005$ cm and mass $m = 0.3 \pm 0.003$ gm. Maximum percentage error in density is **[2004S]**
(a) 4 (b) 2 (c) 1 (d) 6.8

12. A cube has a side of length 1.2×10^{-2} m. Calculate its volume. **[2003S]**
(a) $1.7 \times 10^{-6}\ m^3$ (b) $1.73 \times 10^{-6}\ m^3$
(c) $1.70 \times 10^{-6}\ m^3$ (d) $1.732 \times 10^{-6}\ m^3$

2 Integer Value Answer

13. The dimensions of a cone are measured using a scale with a least count of 2 mm. The diameter of the base and the height are both measured to be 20.0 cm. The maximum percentage error in the determination of the volume is **[Adv. 2024]**

14. In an experiment for determination of the focal length of a thin convex lens, the distance of the object from the lens is 10 ± 0.1 cm and the distance of its real image from the lens is 20 ± 0.2 cm. The error in the determination of focal length of the lens is n%. The value of n is ______ **[Adv. 2023]**

15. The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2\ s^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of $E(t)$ at $t = 5$ s is **[Adv. 2015]**

16. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is $8 \times 10^{-7}\ m^2$. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is **[Adv. 2014]**

3 Numeric / New Stem Based Questions

17. An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is **[Adv. 2019]**

18. A steel wire of diameter 0.5 *mm* and Young's modulus $2 \times 10^{11}\ N m^{-2}$ carries a load of mass M. The length of the wire with the load is 1.0 *m*. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 *mm*, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 *kg*, the vernier scale division which coincides with a main scale division is __________. Take $g = 10\ m s^{-2}$ and $\pi = 3.2$.

[Adv. 2018]

19. The side of a cube is measured by vernier callipers (10 divisions of a vernier scale coincide with 9 divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 g. Find the density of the cube in appropriate significant figures. **[2005 - 2 Marks]**

20. In Searle's experiment, which is used to find Young's Modulus of elasticity, the diameter of experimental wire is $D = 0.05$ cm (measured by a scale of least count 0.001 cm) and length is $L = 110$ cm (measured by a scale of least count 0.1 cm). A weight of 50 N causes an extension of $X = 0.125$ cm (measured by a micrometer of least count 0.001cm). Find maximum possible error in the values of Young's modulus. Screw gauge and meter scale are free from error. **[2004 - 2 Marks]**

21. A screw gauge having 100 equal divisions and a pitch of length 1 mm is used to measure the diameter of a wire of length 5.6 cm. The main scale reading is 1 mm and 47th circular division coincides with the main scale. Find the curved surface area of wire in cm^2 to appropriate significant figure. (use $\pi = \frac{22}{7}$). **[2004 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

22. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52s, 0.56s, 0.57s, 0.54s and 0.59s. The least count of the watch used for the measurement of time period is 0.01s. Which of the following statement(s) is (are) true? **[Adv. 2016]**

(a) The error in the measurement of r is 10%
(b) The error in the measurement of T is 3.75%
(c) The error in the measurement of T is 2%
(d) The error in the determined value of g is 11%

23. Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then : **[Adv. 2015]**

(a) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm
(b) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm
(c) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm
(d) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm

24. Using the expression $2d \sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0° **[Adv. 2013]**

(a) The absolute error in d remains constant
(b) The absolute error in d increases
(c) The fractional error in d remains constant
(d) The fractional error in d decreases

25. A student uses a simple pendulum of exactly 1m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true? **[2010]**
(a) Error ΔT in measuring T, the time period, is 0.05 seconds
(b) Error ΔT in measuring T, the time period, is 1 second
(c) Percentage error in the determination of g is 5%
(d) Percentage error in the determination of g is 2.5%

8 Comprehension/Passage Based Questions

Passage

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = x/y$. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp (\Delta y/y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\Delta x/x \ll 1$, $\Delta y/y \ll 1$. Therefore, the higher powers of these quantities are neglected. **[Adv. 2018]**

26. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa ($\Delta a/a \ll 1$, then what is the error Δr in determining r?
(a) $\frac{\Delta a}{(1+a)^2}$ (b) $\frac{2\Delta a}{(1+a)^2}$ (c) $\frac{2\Delta a}{(1-a^2)}$ (d) $\frac{2a\Delta a}{(1-a^2)}$

27. In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For $|x| \ll 1$, $\ln(1+x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ, in s^{-1}, is
(a) 0.04 (b) 0.03 (c) 0.02 (d) 0.01

10 Subjective Problems

28. If n^{th} division of main scale coincides with $(n+1)^{th}$ divisions of vernier scale. Given one main scale division is equal to 'a' units. Find the least count of the vernier. **[2003 - 2 Marks]**

Topic-4: Miscellaneous (Mixed Concepts) Problems

8 Comprehension/Passage Based Questions

Passage

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency 'ω_p' which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals. **[2011]**

1. Taking the electronic charge as 'e' and the permittivity as 'ε_0'. Use dimensional analysis to determine the correct expression for ω_p.
(a) $\sqrt{\frac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\frac{m\varepsilon_0}{Ne}}$ (c) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$

2. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}\ m^{-3}$. Taking $\varepsilon_0 = 10^{-11}$ and mass $m \approx 10^{-30}$, where these quantities are in proper SI units.
(a) 800 nm (b) 600 nm (c) 300 nm (d) 200 nm

Answer Key

Topic-1 : Unit of Physical Quantities

1. (a, b, c, d) **2.** $A \rightarrow p, q; B \rightarrow r, s; C \rightarrow r, s; D \rightarrow r, s$

Topic-2 : Dimensions of Physical Quantities

1. (a) **2.** (a) **3.** (c) **4.** (b) **5.** (d) **6.** (c) **7.** (d) **8.** (c) **9.** (4) **10.** (3)
15. (b, d) **16.** (a, b) **17.** (a, b, c) **18.** (b, d) **19.** (a, c) **20.** (a, c, d) **21.** (b, c) **22.** (a, b, c) **23.** (a, d) **24.** (c)
26. (c) **27.** (d)

Topic-3 : Errors in Measurements & Experimental Physics

1. (c) **2.** (b) **3.** (b) **4.** (b) **5.** (a) **6.** (c) **7.** (d) **8.** (b) **9.** (d) **10.** (c)
11. (a) **12.** (a) **13.** (3) **14.** (1) **15.** (4) **16.** (4) **17.** (1.39) **18.** (3.00) **19.** (2.66 g cm^{-3})
20. (1.09×10^{10} N/m^2) **21.** (2.6 cm^2) **22.** (a, b, d) **23.** (b, c) **24.** (d) **25.** (a, c) **26.** (b) **27.** (c)

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. (c) **2.** (b)

Hints & Solutions

Topic-1: Unit of Physical Quantities

1. **(a, b, c, d)**

(a) Inductance, $L = \frac{\text{Flux}(\phi)}{\text{Current}(I)} = \frac{\text{weber}}{\text{ampere}} \Rightarrow \text{henry}$

(b) $\varepsilon = L\left(\frac{di}{dt}\right) \Rightarrow [L] = \frac{\varepsilon}{di/dt} = \frac{\text{volt} \times \text{sec}}{\text{amp}} \Rightarrow H = \frac{Vs}{A}$

(c) $U = \frac{1}{2}LI^2 \Rightarrow L = \frac{2U}{I^2} = \frac{J}{A^2} \Rightarrow H = \frac{J}{A^2}$

(d) $L = \frac{\varepsilon}{di/dt} = \left(\frac{\varepsilon}{di}\right)dt = \text{ohm} \times \text{sec}$

$\therefore$ henry = ohm − sec

2. **A → p, q**

Using $F = \frac{GM_eM_s}{r^2} \Rightarrow GM_eM_s = Fr^2 = \text{Nm}^2 = \text{kg}\frac{m}{s^2} \times m^2$
$= \text{kg m}^3\text{s}^{-2}$

Also (volt) (coulomb) (metre) = (joule) (metre)
($\because$ volt = Joule/coulomb)

$= (N-m)(m) = \text{Nm}^2 = \text{kg m}^3\text{s}^{-2}$

B → r, s

Using $v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{rms}^2 = \frac{3RT}{M}$

Unit of $\frac{3RT}{M}$ is $m^2\, s^{-2}$

Also (farad) (volt)^2 (kg)$^{-1}$ = (joule) kg^{-1} ($\because u = \frac{1}{2}cv^2$)

$= \text{N-m kg}^{-1} = \text{kg ms}^{-2}\text{ m kg}^{-1} = \text{m}^2\text{s}^{-2}$

C → r, s

Using $F = qvB \Rightarrow v^2 = \frac{F^2}{q^2B^2}$

$\therefore$ Unit of v^2 is m^2s^{-2} which is further equal to $FV^2\text{ kg}^{-1}$.

D → r, s

Reason : Escape velocity $v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e^2 = \frac{2GM}{R}$

$\therefore$ Unit of $\frac{GM}{R}$ is m^2 s^{-2}.

3. Capacitance $C = \frac{q}{v} = \frac{q}{\frac{w}{q}}$ coulomb-volt^{-1}, coulomb^2-joule^{-1}

Inductance $\frac{L}{R} = t$ and $R = \frac{v}{I}$ ohm-sec, volt-second (ampere)$^{-1}$

Magnetic Induction $F = IlB \Rightarrow B = \frac{F}{Il}$ newton (ampere-metre)$^{-1}$

4. (i) The M.K.S. unit of Young's modulus $\left(Y = \frac{F}{A}\bigg/\frac{\Delta l}{l_0}\right)$ is Nm^{-2}.

(ii) The M.K.S. unit of magnetic induction $\left(B = \frac{\phi}{A}\right)$ is tesla or wb/m^2.

(iii) The M.K.S. unit of power of lens $P = \frac{1}{f\,(\text{in metre})}$ is dioptre or m^{-1}.

Topic-2: Dimensions of Physical Quantities

1. **(a)** For dimensionless quantity

$e^\alpha \varepsilon_0^\beta h^\gamma c^\delta = M^0L^0T^0A^0$

Substituting dimensions of the physical quantities in the above relation and equating the dimension of LHS and RHS

$\Rightarrow (AT)^\alpha (M^{-1}L^{-3}T^4A^2)^\beta (ML^2T^{-1})^\gamma (LT^{-1})^\delta = A^0M^0L^0T^0$

$\therefore \alpha + 2\beta = 0, \alpha + 4\beta - \gamma - \delta = 0, -\beta + \gamma = 0$ and $-3\beta + 2\gamma + \delta = 0$

$\therefore \alpha = -2\beta, \beta = \gamma$ and $\gamma = \delta$

$\therefore \alpha = 2n, \beta = -n, \gamma = -n, \delta = -n$

2. **(a)** Given, $Y = c^\alpha h^\beta G^\gamma$

$ML^{-1}T^{-2} = (LT^{-1})^\alpha (ML^2T^{-1})^\beta (M^{-1}L^3T^{-2})^\gamma$

Equating Power

$1 = \beta - \gamma$ ...(i)

$-1 = \alpha + 2\beta + 3\gamma$ ...(ii)

$-2 = -\alpha - \beta - 2\gamma$ ...(iii)

$-3 = \beta + \gamma$ ...(iv)

(By adding eq. ii & iii)

Now by adding eq. (i) & (iv) we get,

$-2 = 2\beta$

$\therefore \beta = -1, \gamma = -2$

$-1 = \alpha - 2 - 6 \quad \therefore \alpha = 7$

3. **(c)** Least count $= \frac{\text{Pitch}}{\text{no. of division}} = \frac{2 \times 0.5}{100} = 0.01$ mm

Zero error = 4 × L.C = 0.04 mm

Attempt 1: d_1 = MSR + CSR − zero error

$d_1 = 4 \times 0.5 + 20 \times 0.01 - 0.04 = 2.16$ mm

Attempt 2: $d_2 = 4 \times 0.5 + 16 \times 0.01 - 0.04 = 2.12$ mm

So, $d_{mean} = \frac{2.16 + 2.12}{2} = 2.14$ mm

Now, $\Delta d_1 = 0.02$ mm and $\Delta d_2 = -0.02$ mm

$\overline{\Delta d} = \frac{|0.02| + |-0.02|}{2} = 0.01$ mm

So, d = 2.14 ± 0.02 mm

Area (A) $= \pi R^2 = \pi\left(\frac{d}{2}\right)^2 = \pi(1.07)^2 = 1.14\pi\ mm^2$

$\frac{\Delta A}{A} = \frac{2\Delta R}{R} = \frac{2\Delta d}{d} = 2 \times \frac{0.02}{2.14}$

$\Rightarrow \Delta A = \frac{2 \times 0.02}{2.14} \times 1.14\pi = 0.02\pi$

So, area $= \pi(1.14 \pm 0.02)\ mm^2$

4. (b) As we know, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{R^2} \Rightarrow \varepsilon_0 = \frac{q_1q_2}{4\pi FR^2}$

Hence, $\varepsilon_0 = \frac{C^2}{N.m^2} = \frac{[AT]^2}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4A^2]$

5. (d) Electric flux $\phi_E = \vec{E}.\vec{S}$

$\therefore$ Dimensionally $\phi_E \neq E$ (electric field)

Dipole moment, p = charge (q) × distance (d)

6. (c) The power of an exponent is a number or constant. Therefore, dimensionally $\frac{\alpha z}{k\theta} = [M^\circ L^\circ T^\circ]$

$\therefore \quad \alpha = \frac{k\theta}{z} \ \therefore \alpha = \frac{[ML^2T^{-2}\theta^{-1}][\theta]}{[L]} = \left[MLT^{-2}\right]$

As $k = \frac{\text{joule}}{\text{kelvin}} = [ML^2T^{-2}\theta^{-1}]$ and $z = [L]$

And, dimensionally $P = \frac{\alpha}{\beta} \Rightarrow \beta = \frac{\alpha}{P}$

$\therefore \quad [\beta] = \frac{MLT^{-2}}{ML^{-1}T^{-2}} = [M^0L^2T^0]$

$\left[\because P = \frac{F}{A} = [ML^{-1}T^{-2}]\right]$

7. (d) Capacitance, $C = \frac{\Delta q}{\Delta V} = \frac{\varepsilon_0 A}{L} \Rightarrow \varepsilon_0 = \frac{(\Delta q)L}{A(\Delta V)}$

$\therefore X = \varepsilon_0 L\frac{\Delta V}{\Delta t} = \frac{(\Delta q)L}{A(\Delta V)}L\frac{\Delta V}{\Delta t}$

$= \frac{(\Delta q)}{\Delta V}\frac{L^2}{L^2}\frac{\Delta V}{\Delta t} = \frac{\Delta q}{\Delta t} = I$ (current)

8. (c) Here $\left(\frac{1}{2}\right)\varepsilon_0 E^2$ represents, energy density i.e., energy per unit volume.

$\therefore \left[\frac{1}{2}\varepsilon_0 E^2\right] = \frac{[\text{Energy}]}{[\text{Volume}]} = \frac{ML^2T^{-2}}{L^3} = [ML^{-1}T^{-2}]$

9. (4) $[B] = [e]^\alpha[m_e]^\beta[h]^\gamma[k]^\delta$

$\Rightarrow MT^{-2}A^{-1} = A^\alpha T^\alpha M^\beta M^\gamma L^{2\gamma}T^{-\gamma}M^\delta L^{3\delta}A^{-2\delta}T^{-4\delta}$

$\Rightarrow \beta+\gamma+\delta = 1, -2 = \alpha-\gamma-4\delta, -1 = \alpha-2\delta$

$2\gamma+3\delta = 0$

$\Rightarrow \alpha = 3, \beta = 2, \gamma = -3, \delta = 2 \Rightarrow \alpha+\beta+\gamma+\delta = 4$

10. (3) Let $d \propto \rho^x S^y f^z = K\rho^x S^y f^z$ where K is a dimensionless constant.

$M^0L^1T^0 = M^xL^{-3x}M^yT^{-3y}T^{-z}$

Equating dimensions both sides

$M^0L^1T^0 = M^{x+y}L^{-3x}T^{-3y-z}$

$\therefore x+y = 0,\ -3x = 1 \ \therefore\ x = -\frac{1}{3}$ and $y = \frac{1}{3}\ \therefore n = 3$

11. Conductivity, $\sigma = \frac{J}{E} = \frac{i/A}{F/q} = \frac{i^2t}{FA} = \left[M^{-1}L^{-3}T^3A^2\right]$

12. Here, $\left[\frac{a}{V^2}\right] = [P] \Rightarrow [a] = [PV^2] = \frac{MLT^{-2}}{L^2}L^6 = ML^5T^{-2}$

13. $[X] = [C] = [M^{-1}L^{-2}T^2Q^2]$

$[Z] = [B] = [MT^{-1}Q^{-1}]$ $\left(\because Y = \frac{X}{3Z^2} \text{ given}\right)$

$\therefore \quad [Y] = \frac{[M^{-1}L^{-2}T^2Q^2]}{[MT^{-1}Q^{-1}]^2} = [M^{-3}L^{-2}T^4Q^4]$

14. Using $E = h\nu \Rightarrow$ Planck's constant,

$h = \frac{E}{\nu} = \frac{E}{\frac{1}{T}} = \frac{\left[ML^2T^{-2}\right]}{\left[T^{-1}\right]} = \left[ML^2T^{-1}\right]$

15. (b, d) Given physical quantity,

$\vec{S} = [\vec{E} \times \vec{B}]\frac{1}{\mu_0}$

Here S is known as poynting vector and it represents flow of energy per unit area per unit time

$\vec{S} = \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{\text{Power}}{\text{Area}} = \frac{\text{Force}}{\text{Length} \times \text{Time}}$

16. (a, b) Given position, $L = [X^\alpha]$

Speed, $LT^{-1} = [X^\beta]$

Acceleration, $LT^{-2} = [X^p]$

Linear momentum, $MLT^{-1} = [X^q]$

Force, $MLT^{-2} = [X^r]$

$\frac{\text{Position}}{\text{Speed}} = \text{time},\ T = \frac{[X^\alpha]}{[X^\beta]} = X^{\alpha-\beta}$

Acceleration $= \frac{\text{Speed}}{\text{Time}} = \frac{X^\beta}{X^{\alpha-\beta}} = X^p$

$X^{\alpha-\beta+p} = X^\beta \ \therefore\ \alpha+p = 2\beta$

Hence option (a) is correct.

Force $= \frac{\text{linear momentum}}{\text{time}}$

$[X^r] = \frac{[X^q]}{[X^{\alpha-\beta}]} \Rightarrow r = q+\beta-\alpha \Rightarrow r = q+\beta-(2\beta-p)$

$\Rightarrow r = q-\beta+p \Rightarrow p+q-r = \beta$

Hence option (b) is correct.

17. (a, b, c) According to question, dimensions of angular momentum $[mvr] = M^0L^0T^0$ and of mass $[m] = M^0L^0T^0$

$ML^2T^{-1} = ML^0T^0 \Rightarrow T = L^2$

Momentum $\quad p = mv = \frac{mvr}{r} = \frac{M^0L^0T^0}{L} = L^{-1}$

Energy $\quad E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = L^{-2}$

Power $P = \frac{E}{t} = \frac{L^{-2}}{T} = \frac{L^{-2}}{L^2} = L^{-4}$

Force $F = \frac{E}{x} = \frac{L^{-2}}{L} = L^{-3}$

18. **(b, d)**

Dimensions of $[n] = [L^{-3}]$, $[q] = [AT]$

$\left[\frac{q^2}{\varepsilon}\right] = [Fr^2] = [U \times r] = [ML^3T^{-2}]$

$[k_BT] = [U] = [ML^2T^{-2}]$

Dimension of $l = [L]$

(a) R.H.S. $= \sqrt{\left(\frac{q^2}{\varepsilon}\right) \times \frac{1}{(k_BT)} \times n}$

$= \sqrt{[U \times r] \times \frac{1}{[U]} \times n} = \sqrt{[n] \times [r]} = \sqrt{[L^{-3}][L]} = [L^{-1}]$

(b) R.H.S. $= \sqrt{\frac{\varepsilon(k_BT)}{nq^2}} = \sqrt{\frac{(k_BT)}{n(q^2/\varepsilon)}} = \sqrt{\frac{[U]}{[n][U \times r]}}$

$= \sqrt{\frac{1}{[n][r]}} = \sqrt{\frac{1}{[L^{-3}][L]}} = [L]$

(c) R.H.S. $= \sqrt{\left(\frac{q^2}{\varepsilon}\right)\frac{1}{(k_BT)} \times \frac{1}{n^{2/3}}}$

$= \sqrt{[U \times r]\frac{1}{[U]}\frac{1}{[L^{-2}]}} = [L^{3/2}]$

(d) R.H.S. $= \sqrt{\left(\frac{q^2}{\varepsilon}\right) \times \frac{1}{(k_BT)} \times \frac{1}{n^{1/3}}}$

$= \sqrt{[U \times r] \times \frac{1}{[U]} \times \frac{1}{[n^{1/3}]}} = \sqrt{[r] \times \frac{1}{[n^{1/3}]}}$

$= \sqrt{[L]\frac{1}{[L^{-1}]}} = [L]$

19. **(a, c)** Using, $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ and $R = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

(a) $\mu_0 I^2 = \varepsilon_0 V^2 \Rightarrow \frac{\mu_0}{\varepsilon_0} = \frac{V^2}{I^2} = R^2 = \sqrt{\left(\frac{\mu_0}{\varepsilon_0}\right)^2}$

(b) $\varepsilon_0 I = \mu_0 V \Rightarrow \frac{\mu_0}{\varepsilon_0} = \frac{I}{V} = \frac{1}{R}$ $(\because V = RI)$

Dimensionally incorrect.

(c) $I = \varepsilon_0 C V$

$\therefore \frac{1}{\varepsilon_0 C} = \frac{V}{I} = R \therefore \frac{1}{\varepsilon_0 \frac{1}{\sqrt{\mu_0\varepsilon_0}}} = R$

Dimensionally correct.

(d) $\mu_0 C I = \varepsilon_0 V$

$\therefore \frac{\mu_0}{\varepsilon_0} = \frac{V}{IC} = \frac{R}{C} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \times \frac{1}{\frac{1}{\sqrt{\mu_0\varepsilon_0}}} = \mu_0$

Dimensionally incorrect.

20. **(a, c, d)**

As Planck's constant h, speed of light c and gravitational constant G are used as basic units for length L and Mass M so, $L \propto h^x c^y G^z$ .... (i)

and $M \propto h^p c^q G^r$ .... (ii)

Dimensions of $[h] = [ML^2T^{-1}]$, $[c] = [LT^{-1}]$

$[G] = [M^{-1}L^3T^{-2}]$

Using principle of homogeneity of dimensions

For eqn. (i)

$[M^0LT^0] = [M^x L^{2x} T^{-x}][L^y T^{-y}][M^{-z} L^{3z} T^{-2z}]$

$M^0 L T^0 = M^{(x-z)} L^{(2x+y+3z)} T^{(-x-y-2z)}$

On comparing powers from both sides, we get

$x - z = 0, 2x + y + 3z = 1, -x - y - 2z = 0$

On solving these eqns., we get

$x = \frac{1}{2}, y = -\frac{3}{2}, z = \frac{1}{2}$

$\therefore L = K\sqrt{\frac{hG}{c^3}}$; K is a constant.

In the same way solving eqn. (ii) we get,

$M = K'\sqrt{\frac{hc}{G}}$; K' is a constant.

21. **(b, c)** Using $F = \frac{Q_1Q_2}{(4\pi\varepsilon_0)r^2}$ and $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Hence, $[\varepsilon_0] = \frac{[Q]^2}{[F][r^2]} = \frac{I^2T^2}{MLT^{-2}.L^2} = M^{-1}L^{-3}T^4I^2$

$[\mu_0] = \frac{1}{[\varepsilon_0][C]^2} = \frac{1}{[M^{-1}L^{-3}T^4I^2][LT^{-2}]^2}$

$= [MLT^{-2}I^{-2}]$

22. **(a, b, c)**

Reynold's number

= Coefficient of friction = $[M^0L^0T^0]$ i.e., dimensionless.

Curie is the unit of radioactivity (number of atoms decaying per second) and frequency also has the unit per second *i.e.*, $[M^0L^0T^{-1}]$.

Latent heat = $\frac{Q}{m}$ and Gravitation potential = $\frac{W}{m}$.

Dimensionally same $[M^0L^2T^{-2}]$

23. **(a, d)** Torque, $\tau = F \times r \times \sin\theta$

and work done, $W = F \times d \times \cos\theta$

Dimensionally, $\tau = W = [ML^2T^{-2}]$

Dimensionally, light year = wavelength = $[L]$

24. **(c)** Boltzmannn constant $[K_B] = \frac{u}{\theta}$ $\left(\because u = \frac{1}{2}K_BT\right)$

$\therefore [K_B] = ML^2T^{-2}K^{-1}$

Coefficient of viscosity $[\eta] = \frac{F}{6\pi rv} = \frac{MLT^{-2}}{L \times LT^{-1}} = ML^{-1}T^{-1}$

Planck constant, $h = \frac{E}{\nu} = \frac{ML^2T^{-2}}{T^{-1}} = ML^{-2}T^{-1}$

Thermal conductivity $K_{conductivity} = \frac{H\ell}{tA\Delta T}$

$= \frac{ML^2T^{-2} \times L}{T \times L^2 \times K} = MLT^{-3}K^{-1}$

25. Angular Momentum = $mvr = [ML^2T^{-1}]$
Latent heat $[L] = Q/m$ $[L] = [L^2T^{-2}]$
Torque $\tau = F \times r$ $[\tau] = [ML^2T^{-2}]$

Capacitance $C = \frac{1}{2}\frac{q^2}{U}$; $[C] = [M^{-1}L^{-2}T^2Q^2]$

Inductance $L = \frac{2U}{i^2}$; $[L] = [ML^2Q^{-2}]$

Resistivity $\rho = \frac{RA}{l}$ and $R = \frac{L}{t}$ $\therefore$ $[\rho] = [ML^3T^{-1}Q^{-2}]$

26. **(c)** Using, $C = \frac{E}{B}$ where C = speed of light
$\therefore$ $E = CB = LT^{-1}B$

27. **(d)** We know that
$C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ $\therefore C^2 = \frac{1}{\mu_0\varepsilon_0}$
$\therefore \mu_0 = \varepsilon_0^{-1}L^{-2}T^2$ $[\because C = LT^{-1}]$

28. (i) Magnetic Flux $\phi = BA = \frac{F}{qv}A = [M^1L^2T^{-1}Q^{-1}]$

(ii) Modulus of Rigidity $= \frac{F}{A} = [ML^{-1}T^{-2}]$

29. As per question, $T \propto P^a d^b E^c$ or $[T] = [P]^a[d]^b[E]^c$
or $[M^0L^0T^1] = [ML^{-1}T^{-2}]^a\,[ML^{-3}]^b\,[ML^2T^{-2}]^c$
or $M^0L^0T^1 = M^{a+b+c}\,L^{-a-3b+2c}\,T^{-2a-2c}$
$\therefore$ $a + b + c = 0$ ..... (i)
$-a - 3b + 2c = 0$ ..... (ii)
$-2a - 2c = 1$ ..... (iii)
Solving, eqns. (i), (ii) and (iii) we get
$a = -5/6, b = 1/2, c = 1/3.$

Topic-3: Errors in Measurements & Experimental Physics

1. **(c)** Since 10 VSD = 9 MSD $\therefore$ 1 VSD $= \frac{9}{10}$ MSD

Least count (L.C.) = 1 MSD – 1VSD $= \left(1 - \frac{9}{10}\right)$ MSD

= 0.1 MSD = 0.1 × 0.1 cm = 0.01 cm
Since 0 of Vernier scale lie before 0 of main scale.
Zero error = – [10 – 6] L.C. = – 4 × 0.01 cm
= – 0.04 cm $\therefore$ Correct diameter of the sphere
= Reading – Zero error = 3.1 + 1 × L.C. – (– 0.04)
= 3.1 + 1 × 0.01 + 0.04 = 3.11 + 0.04 cm = 3.15 cm

2. **(b)** Depth of the well = L = 20 m
$\therefore$ Time taken by stone to reach the bottom of well,

$t_1 = \sqrt{\frac{2L}{g}}$ $\left(\text{using, } L = \frac{1}{2}gt^2\right)$

Time taken by impact sound to reach the person,

$t_2 = \frac{L}{v}$

Total time taken in the process is given by,

$T = t_1 + t_2 = \sqrt{\frac{2L}{g}} + \frac{L}{v}$ $\therefore \delta T = \sqrt{\frac{2}{g}}\frac{1}{2}\frac{\delta L}{\sqrt{L}} + \frac{\delta L}{v}$

On substituting the given values we get,

$\delta T = \frac{16}{300}\delta L$

$\therefore$ Fractional error, $\frac{\delta L}{L} \times 100 = \frac{300}{16}\frac{\delta T}{L} \times 100$

$= \frac{300}{16} \times \frac{0.01}{20} \times 100 \simeq 1\%$

3. **(b)** For C_1 vernier calliper,
L.C. = 1MSD – 1VSD = 1mm – 0.9 mm = 0.1 mm = 0.01 cm
[$\because$ 10 VS D = 9 MSD = 9 mm]
Reading = MSR + L.C. × VSR = 2.8 + (0.01) × 7 = 2.87 cm
For C_2 vernier calliper,
L.C. = 1 mm – 1.1 mm [$\because$ 10 VSD = 11 MSD = 11 mm]
L.C = – 0.1 mm = – 0.01 cm
Reading = 2.8 + (10 – 7) × 0.01 = 2.83 cm

4. **(b)** In the measurement, the diameter of cylinder,
D = M.S.R + (V.S.R) × (L.C.), L.C. = (1MSD – 1VSD)
$= (5.15 - 5.10) - \left(\frac{2.45}{50}\right) = .001$ cm
$\therefore$ D = 5.10 + 24 × 0.001 = 5.124 cm

5. **(a)** The maximum possible error in Y due to l and d

$\frac{\Delta Y}{Y} = \frac{\Delta l}{l} + \frac{2\Delta d}{d}$

Least count $= \frac{\text{Pitch}}{\text{No. of division on circular scale}}$

$= \frac{0.5}{100}$ mm = 0.005 mm
Here, $\Delta d = \Delta l$ = 0.005 mm

Error contribution of l $= \frac{\Delta l}{l} = \frac{0.005 \text{ mm}}{0.25 \text{ mm}} = \frac{1}{50}$

Error contribution of d $= \frac{2\Delta d}{d} = \frac{2 \times 0.005 \text{ mm}}{0.5 \text{ mm}} = \frac{1}{50}$

Hence contribution to the maximum possible error in the measurement of y due to l and d is the same.

6. **(c)** Least count of screw gauge

$= \frac{\text{Pitch}}{\text{divisions on circular scale}} = \frac{0.5}{50} = 0.01 \text{ mm} = \Delta r$
Diameter, r = M.S.R. + (C.S.R) × (L.C.)

Diameter, $r = 2.5 \text{ mm} + 20 \times \frac{0.5}{50} = 2.70 \text{ mm}$

$\frac{\Delta r}{r} = \frac{0.01}{2.70}$ or $\frac{\Delta r}{r} \times 100 = \frac{1}{2.7}$

Now, density, $d = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi\left(\frac{r}{2}\right)^3}$

$\therefore$ Percentage error in density, $\frac{\Delta d}{d} \times 100$

$= \left\{\frac{\Delta m}{m} + 3\left(\frac{\Delta r}{r}\right)\right\} \times 100 = \frac{\Delta m}{m} \times 100 + 3 \times \left(\frac{\Delta r}{r}\right) \times 100$

$= 2\% + 3 \times \frac{1}{2.7} = 3.11\%$

7. **(d)** 20 divisions on the vernier scale
= 16 divisions of main scale
$\therefore$ 1 division on the vernier scale

$= \frac{16}{20}$ divisions of main scale $= \frac{16}{20} \times 1\text{mm} = 0.8$ mm

We know that least count, L.C. = 1MSD – 1VSD
= 1 mm – 0.8 mm = 0.2 mm

8. **(b)** $Y=\frac{F/A}{\ell/L}=\frac{4mgL}{\pi D^2\ell}=\frac{4\times1\times9.8\times2}{\pi\left(0.4\times10^{-3}\right)^2\times\left(0.8\times10^{-3}\right)}$
$= 2.0\times10^{11}\ \text{N/m}^2$.

Now $\frac{\Delta Y}{Y}=\frac{2\Delta D}{D}+\frac{\Delta\ell}{\ell}$

[$\because$ the value of m, g and L are exact]

$= 2\times\frac{0.01}{0.4}+\frac{0.05}{0.8}=2\times0.025+0.0625$

$=0.05+0.0625=0.1125$

$\Rightarrow \Delta Y= Y\times0.1125=2\times10^{11}\times0.1125=0.225\times10^{11}$
$=0.2\times10^{11}\ \text{N/m}^2$

$\therefore$ Reading of Young's modulus $=(Y+\Delta Y)$
$=(2\pm0.2)\times10^{11}\ \text{N/m}^2$

9. **(d)** Relative error in g, $\frac{\Delta g}{g}=\frac{\Delta\ell}{\ell}+2\frac{\Delta T}{T}$

$\Delta\ell$ and ΔT are least and number of readings taken are maximum in option (d), therefore the measurement of g is most accurate.

10. **(c)** Least count $=\frac{\text{Pitch}}{\text{no. of divisions on circular scale}}$
$=\frac{0.5}{50}=0.01\text{mm}$

Zero error $= 5\times L.C. = 5\times0.01$ mm $= 0.05$ mm

Diameter of ball = [Reading on main scale] + [Reading on circular scale $\times$ $L.C.$] – Zero error

$=0.5\times2+25\times0.01-0.05=1.20$ mm

11. **(a)** Density, $\rho=\frac{m}{v}=\frac{m}{A\ell}=\frac{m}{\ell\pi r^2}$

Taking log and differentiate for errors

$$\frac{\Delta\rho}{\rho}=\frac{\Delta m}{m}+\frac{2\Delta r}{r}+\frac{\Delta\ell}{\ell} \quad \text{....(i)}$$

From question, putting the values of

$\Delta\ell=0.06$ cm, $\ell=6$ cm; $\Delta r=0.005$ cm; $r=0.5$ cm,

$m=0.3$ g; $\Delta m=0.003$ g in eqn. (i) we get $=\frac{\Delta\rho}{\rho}=\frac{4}{100}$

$\therefore$ Percentage error in density,

$\frac{\Delta\rho}{\rho}\times100=\frac{4}{100}\times100=4\%$.

12. **(a)** Volume of cube, $V=\ell^3=(1.2\times10^{-2}\ \text{m})^3$
$=1.728\times10^{-6}\text{m}^3\Rightarrow V=1.7\times10^{-6}\text{m}^3$.

As length has two significant figures so volume has also two significant figures.

13. **(3)** Volume of a cone $=\text{V}=\frac{1}{3}\pi(\text{R})^2\,\text{H}$

$\Rightarrow \frac{\text{dV}}{\text{V}}=2\frac{\text{dR}}{\text{R}}+\frac{\text{dH}}{\text{H}}$

$\therefore$ Percentage error $=\left[2\times\frac{0.2}{20}+\frac{0.2}{20}\right]\times100$

14. **(1)** Given: Differentiating lens formula, $u=10\pm0.1$ cm, $v=20\pm0.2$ cm

$\frac{1}{\text{v}}-\frac{1}{\text{u}}=\frac{1}{\text{f}}\Rightarrow\frac{1}{\text{v}^2}\text{dv}+\frac{1}{\text{u}^2}\text{du}=-\frac{1}{\text{f}^2}\text{df}$

$\frac{1}{20}+\frac{1}{10}=\frac{1}{\text{f}}\Rightarrow\frac{1}{\text{f}}=\frac{3}{20}\Rightarrow\text{f}=\frac{20}{3}\text{cm}$

$\Rightarrow\frac{1}{(20)^2}(0.2)+\frac{1}{(10)^2}(0.1)=\frac{9}{400}\text{df}$

$\text{df}=\frac{1}{9}\left(\frac{400}{400}\times0.2+\frac{400}{100}\times0.1\right)$

$\Rightarrow\text{df}=\frac{1}{9}(0.2+0.4)\Rightarrow\text{df}=\frac{0.6}{9}$

$\Rightarrow\frac{\text{df}}{\text{f}}=\frac{0.6}{9}\times\frac{3}{20}=\frac{1}{100}$

$\therefore$ % error $=\frac{\text{df}}{\text{f}}\times100=\frac{1}{100}\times100=1\,\%$

15. **(4)** $E_{(t)}=A^2e^{-\alpha t}=A^2e^{-0.2t}$

$\therefore \log_e E=2\log_e A-0.2t$

On differentiating we get $=\frac{dE}{E}=2\frac{dA}{A}-0.2\frac{dt}{t}\times t$

As errors always add up

$\therefore \frac{dE}{E}\times100=2\left(\frac{dA}{A}\times100\right)+0.2t\left(\frac{dt}{t}\times100\right)$

$\Rightarrow \frac{dE}{E}\times100=2\times1.25\%+0.2\times5\times1.5\%=4\%$

16. **(4)** Young's modulus $Y=\frac{FL}{A\times l}$

Here F, A and L are accurately known.

$\therefore$ Percentage error in Young's modulus

$l=(45-20)\times\text{L.C}=25\times10^{-5}$ m

$\frac{\Delta Y}{Y}\times100=\frac{\Delta l}{l}\times100=\frac{1.0\times10^{-5}}{25\times10^{-5}}\times100=4\%$

17. **(1.39)** $u\pm\Delta u=(75-45)\pm\left(\frac{1}{4}+\frac{1}{4}\right)=(30\pm0.5)$ cm

$v\pm\Delta v=(135-75)\pm\left(\frac{1}{4}+\frac{1}{4}\right)=(60\pm0.5)$ cm

We know that

$$\frac{1}{\text{v}}-\frac{1}{\text{u}}=\frac{1}{\text{f}}\ \therefore+\frac{\Delta\text{v}}{\text{v}^2}+\frac{\Delta\text{u}}{\text{u}^2}=\frac{\Delta\text{f}}{\text{f}^2} \quad \text{....(i)}$$

Now $\frac{1}{\text{v}}-\frac{1}{\text{u}}=\frac{1}{\text{f}}\Rightarrow\frac{1}{60}-\frac{1}{-30}=\frac{1}{\text{f}}$

$\Rightarrow\frac{1}{\text{f}}=\frac{1}{20}\ \therefore\ \text{f}=20\text{cm}$

Substituting the values in eqn. (i)

$\frac{0.5}{(60)^2}+\frac{0.5}{(30)^2}=\frac{\Delta\text{f}}{(20)^2}\ \therefore\Delta\text{f}=0.277$

Hence percentage error in the measurement of focal length

$\frac{\Delta\text{f}}{\text{f}}\times100=\frac{0.277}{20}\times100=1.388\%=1.39\%$

18. **(3.00)** We know that $\Delta l=\frac{Fl}{AY}$

$$= \frac{1.2\times10\times1}{\pi\left(\frac{5\times10^{-4}}{2}\right)^2\times2\times10^{11}} \simeq 0.3\text{ mm}$$

L.C. of vernier calliper = 1 MSD – 1 VSD

$$= \left(1-\frac{9}{10}\right) = 0.1\text{ mm}$$

As 9 MSD = 10 VSD

The third marking of vernier scale will coincide with the main scale because least count is 0.1 mm.

19. **(2.66 g cm^{-3})**

L.C. = 1 *MSD* – 1 *VSD*

$$= 1MSD - \frac{9}{10}MSD \quad (\because\ 9\,MSD = 10\,VSD)$$

$$= \left(1-\frac{9}{10}\right)MSD = \frac{1}{10}MSD = \left(\frac{1}{10}\times1\right)\text{mm} = 0.1\text{ mm}$$

$(\because\ 1\text{ MSD} = 1\text{ mm})$

Reading of side, l = MSR + VSR (LC) = 10 mm + 1 × 0.1 mm = 10.1 mm = 1.01 cm

$$\text{Now, density} = \frac{\text{mass}}{\text{volume}} = \frac{m}{l^3} = \frac{2.736\,g}{(1.01)^3} = 2.66\text{ g/cm}^3$$

(Rounding off to 3 significant figures)

20. **(1.09 × 10^{10} N/m^2)**

$$\text{Young's modulus, } Y = \frac{F}{A}\Big/\frac{X}{L} = \frac{F}{\frac{\pi D^2}{4}}\times\frac{L}{X}$$

Maximum error in Y

$$\left(\frac{\Delta Y}{Y}\right)_{max} = 2\left(\frac{\Delta D}{D}\right) + \frac{\Delta X}{X} + \frac{\Delta L}{L}$$

$$= 2\left(\frac{0.001}{0.05}\right) + \left(\frac{0.001}{0.125}\right) + \left(\frac{0.1}{110}\right) = 0.0489$$

Since, $W = 50$ N; $D = 0.05$ cm $= 0.05\times10^{-2}$m;
$X = 0.125$ cm $= 0.125\times10^{-2}$m;
$L = 110$ cm $= 110\times10^{-2}$m

$$\therefore\ Y = \frac{50\times4\times110\times10^{-2}}{3.14(0.05\times10^{-2})\times(0.125\times10^{-2})} = 2.24\times10^{11}\text{N/m}^2$$

$\therefore$ Maximum possible error in the measurement of Y

$\Delta Y = (0.0489)Y = 0.0489\times2.24\times10^{11} = 1.09\times10^{10}\text{ N/m}^2$

21. **(2.6 cm^2)** Least count, L.C. $= \frac{1\text{ mm}}{100} = 0.01$ mm

Diameter = *MSR* + *CSR* × (L.C.)
= 1 mm + 47 × (0.01) mm = 1.47 mm

$$\text{Curved surface area} = 2\pi rl = 2\pi\frac{D}{2}l = \pi Dl$$

$$= \frac{22}{7}\times1.47\times56\text{ mm}^2 = 2.58724\text{ cm}^2$$

= 2.6 cm^2 (Rounding off to two significant figures)

22. **(a, b, d)**

$$T_{mean} = \frac{0.52+0.56+0.57+0.54+0.59}{6} = 0.556 \approx 0.56\text{ S}$$

Error in reading $T_{mean} - T_1$; $T_{mean} - T_2$; $T_{mean} - T_3$; $T_{mean} - T_4$; $T_{mean} - T_5$

$$\text{Mean error} = \frac{0.04+0+0.01+0.02+0.03}{6} = 0.016 \approx 0.02\text{S}$$

$\therefore$ % error in the measurement of 'T'

$$\frac{\Delta T}{T}\times100 = \frac{0.02}{0.56}\times100 = 3.57\%$$

% error in the measurement of g

$$\frac{\Delta g}{g}\times100 = 2\frac{\Delta T}{T}\times100 + \left(\frac{\Delta R+\Delta r}{R-r}\right)\times100$$

$$= 2(3.57) + \left(\frac{1+1}{60-10}\right)\times100 \approx 11\%$$

% error in the measurement of r

$$\frac{\Delta r}{r}\times100 = \frac{1}{10}\times100 = 10\%$$

23. **(b, c) Vernier callipers**

$$1\text{ MSD} = \frac{1\text{cm}}{8} = 0.125\text{cm}$$

$$5\text{ VSD} = 4\text{MSD} \therefore 1\text{ VSD} = \frac{4}{5}\times0.125 = 0.1\text{cm}$$

L.C. = 1MSD – 1VSD
= 0.125cm – 0.1cm = 0.025cm

Screw gauge

If the pitch of screw gauge is twice the L.C of vernier callipers then pitch = 2 × L.C. of vernier calliper = 2 × 0.025 = 0.05cm.

L.C. of screw gauge

$$= \frac{\text{Pitch}}{\text{Total no. of divisions of circular scale}}$$

$$= \frac{0.05}{100}\text{cm} = 0.0005\text{cm} = 0.005\text{ mm.}$$

Now if the least count of the linear scale of the screw gauge is twice the least count of venier callipers then L.C. of linear scale of screw gauge = 2 × 0.025 = 0.05cm.

Then pitch = 2 × 0.05 = 0.1cm.

$$\therefore\ \text{L.C. of screw gauge} = \frac{0.1}{100}\text{cm} = 0.001\text{cm} = 0.01\text{mm.}$$

24. **(d)** $\because\ 2d\sin\theta = \lambda \quad \therefore\ d = \frac{\lambda}{2}\operatorname{cosec}\theta$ ...(i)

$$\therefore\ \frac{d(d)}{d\theta} = \frac{\lambda}{2}[-\operatorname{cosec}\theta\cot\theta]$$

$$\therefore\ d(d) = -\frac{\lambda}{2}\operatorname{cosec}\theta\cot\theta\,d\theta \quad \text{...(ii)}$$

Dividing (ii) by (i) we get

$$\left|\frac{d(d)}{d}\right| = \cot\theta\,d\theta$$

As θ increases from 0° to 90°, cot θ decreases and therefore $\left|\frac{d(d)}{d}\right|$ decreases

i.e., the fractional error in d decreases.

25. **(a, c)** The length of the string of simple pendulum is exactly 1 m (given), therefore the error in length $\Delta l = 0$.

Further the possibility of error in measuring time is 1s in 40s as the least count of stop watch is 1s.

$$\therefore\ \frac{\Delta t}{t} = \frac{\Delta T}{T} = \frac{1}{40}$$

Time period $T = \frac{\text{total time}}{\text{no. of oscillations}} = \frac{40}{20} = 2$ seconds

$\therefore \frac{\Delta T}{T} = \frac{1}{40} \Rightarrow \frac{\Delta T}{2} = \frac{1}{40} \Rightarrow \Delta T = 0.05\,\text{sec}$

Now for measuring error in '*g*'

using, $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2\frac{l}{g} \qquad \therefore g = 4\pi^2\frac{l}{T^2}$

$\therefore$ Percentage error in '*g*'

$\frac{\Delta g}{g}\times 100 = \frac{\Delta l}{l}\times 100 + 2\frac{\Delta T}{T}\times 100$

$\therefore \frac{\Delta g}{g}\times 100 = 0 + 2\left(\frac{1}{40}\right)\times 100 = 5\%$

26. **(b)** $r = \left(\frac{1-a}{1+a}\right) \Rightarrow \frac{\Delta r}{r} = \frac{\Delta(1-a)}{(1-a)} + \frac{\Delta(1+a)}{(1+a)}$

$\Rightarrow \frac{\Delta r}{r} = \frac{\Delta a}{1-a} + \frac{\Delta a}{1+a} = \frac{\Delta a(1+a+1-a)}{(1-a)(1+a)}$

$\therefore \Delta r = \frac{2\Delta a}{(1-a)(1+a)}\frac{(1-a)}{(1+a)} = \frac{2\Delta a}{(1+a)^2}$

27. **(c)** Using, $N = N_0 e^{-\lambda t} \Rightarrow \ln N = \ln N_0 - \lambda t$

Differentiation with respect to λ

$\Rightarrow \frac{1}{N}\frac{dN}{d\lambda} = 0 - t \Rightarrow \Delta\lambda = \frac{\Delta N}{Nt} = \frac{40}{2000\times 1} = 0.02$

28. $(n+1)$ V.S.D $= n$ M.S.D $\therefore$ 1 V.S.D $= \frac{n}{n+1}$ M.S.D

Least count, (L.C.) = 1 M.S.D – 1V.S.D

$= \left(1 - \frac{n}{n+1}\right)$ M.S.D

$= \frac{1}{n+1}$ M.S.D. $= \frac{a}{n+1}$ units [$\because$ 1 MSD = a units]

Topic-4: Miscellaneous (Mixed Concepts) Problems

1. **(c)** Charge, e = It = [AT], angular frequency,

$\omega = \frac{1}{T} = [T^{-1}]$

No. of electrons per unit volume, N = $[L^{-3}]$, Permittivity of free space, $\epsilon_o = [M^{-1}\,L^{-3}\,A^2\,T^4]$

Now substituting the dimensions of above quantities we get,

$\sqrt{\frac{Ne^2}{m\epsilon_o}} = \sqrt{\frac{L^{-3}A^2T^2}{M\,M^{-1}L^{-3}A^2T^4}} = \sqrt{T^{-2}} = T^{-1} = \omega_p$

2. **(b)** $\omega_p = \sqrt{\frac{Ne^2}{m\epsilon_o}} = 2\pi\nu = 2\pi\frac{c}{\lambda} \Rightarrow \lambda = 2\pi c\sqrt{\frac{m\epsilon_o}{Ne^2}}$

$= 2\times\frac{22}{7}\times 3\times 10^8\sqrt{\frac{10^{-30}\times 10^{-11}}{4\times 10^{27}\times(1.6\times 10^{-19})^2}} = 600\,\text{nm}$