CHAPTER 3

SIMPLIFICATION

'BODMAS' RULE

Now a days it has become '**VBODMAS**' where,
'V' stands for "Vinculum"
'B' stands for "Bracket"
'O' stands for "Of"
'D' stands for "Division"
'M' stands for "Multiplication"
'A' stands for "Addition"
'S' stands for "Subtraction"
Same order of operations must be applied during simplification.

BRACKETS

Types of brackets are :

(i) Vinculum or bar : ____
(ii) Parenthesis or small or common brackets : ( )
(iii) Curly or middle brackets : { }
(iv) Square or big brackets : [ ]

- The order for removal of brackets is (), {}, []
- If there is a minus (–) sign before the bracket then while removing bracket, sign of each term will change.

Example 1: Simplify $6 + 5 - 3 \times 2$ of $5 - (15 \div \overline{7-2})$

Solution: $6 + 5 - 3 \times 2$ of $5 - (15 \div \overline{7-2})$

$= 6 + 5 - 3 \times 2$ of $5 - (15 \div 5)$	{Remove vinculum}
$= 6 + 5 - 3 \times 2$ of $5 - 3$	{Remove common bracket}
$= 6 + 5 - 3 \times 10 - 3$	{'Of' is done}
$= 6 + 5 - 30 - 3$	{Multiplication is done}
$= 11 - 33$	{Addition is done}
$= -22$	{Subtraction is done}.

Example 2: Simplify : $7 - 2 + 13 - 5 - 2 + 1$

Solution :

$7 - 2 + 13 - 5 - 2 + 1$
$= 7 + 13 + 1 - 2 - 5 - 2 = 21 - 9 = 12$
$[7 + 13 + 1 = 21$ and $- 2 - 5 - 2 = - 9]$

Example 3: What is the missing figure in the expession given below ?

$$\frac{16}{7}\times\frac{16}{7}-\frac{*}{7}\times\frac{9}{7}+\frac{9}{7}\times\frac{9}{7}=1$$

Solution :

Let the missing figure in the expession be x.

$$\frac{16}{7}\times\frac{16}{7}-\frac{x}{7}\times\frac{9}{7}+\frac{9}{7}\times\frac{9}{7}=1$$

$\Rightarrow 16 \times 16 - 9x + 9 \times 9 = 7 \times 7$

$\Rightarrow 9x = 16 \times 16 + 9 \times 9 - 7 \times 7$
$= 256 + 81 - 49 = 288$

$\Rightarrow x = \frac{288}{9} = 32$

FRACTION

A fraction is a quantity which is expressed in the form p/q where p and q are natural numbers.

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}$$

Example 4: Write a fraction whose numerator is $2^2 + 1$ and denominator is $3^2 - 1$.

Solution :

Numerator $= 2^2 + 1 = 4 + 1 = 5$
Denominator $= 3^2 - 1 = 9 - 1 = 8$

$\therefore \quad \text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}} = \frac{5}{8}$

TYPES OF FRACTIONS

1. Proper Fraction

If numerator is less than its denominator, then it is a proper fraction.

Ex: $\frac{2}{5}, \frac{6}{18}$

2. Improper Fraction

If numerator is greater than or equal to its denominator, then it is an improper fraction.

Ex: $\frac{5}{2}, \frac{18}{7}, \frac{13}{13}$

- If in a fraction, its numerator and denominator are of equal value then fraction is equal to unity i.e., 1.

3. Mixed Fraction

It consists of an integer and a proper fraction.

Ex: $1\frac{1}{2}, 3\frac{2}{3}, 7\frac{5}{9}$

- Mixed fraction can always be changed into improper fraction and vice versa.

Ex: $7\frac{5}{9}=\frac{7\times9+5}{9}=\frac{63+5}{9}=\frac{68}{9}$ and

$\frac{19}{2}=\frac{9\times2+1}{2}=9+\frac{1}{2}=9\frac{1}{2}$

4. Equivalent Fractions / Equal Fractions

Fractions with same value are called equivalent fractions.

Ex: $\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12}$.

- Value of fraction is not changed by multiplying or dividing the numerator or denominator by the same number.

Ex:

(i) $\frac{2}{5}=\frac{2\times5}{5\times5}=\frac{10}{25}$ So, $\frac{2}{5}=\frac{10}{25}$

(ii) $\frac{36}{16}=\frac{36\div4}{16\div4}=\frac{9}{4}$ So, $\frac{36}{16}=\frac{9}{4}$

5. Like Fractions

Fractions with same denominators are called like fractions.

Ex: $\frac{2}{7},\frac{3}{7},\frac{9}{7},\frac{11}{7}$

6. Unlike Fractions

Fractions with different denominators are called unlike fractions.

Ex: $\frac{2}{5},\frac{4}{7},\frac{9}{8},\frac{9}{2}$

- Unlike fractions can be converted into like fractions.

Ex: $\frac{3}{5}$ and $\frac{4}{7}$

$\frac{3}{5}\times\frac{7}{7}=\frac{21}{35}$ and $\frac{4}{7}\times\frac{5}{5}=\frac{20}{35}$

7. Simple Fraction

If in a fraction, both numerator and denominator are integers then it is called a simple fraction.

Ex : $\frac{3}{7}$ and $\frac{2}{5}$.

8. Complex Fraction

If in a fraction, numerator or denominator or both are fractional numbers, then it is called a complex fraction.

Ex : $\frac{2}{\frac{5}{7}}, \frac{2\frac{1}{3}}{5\frac{2}{3}}, \frac{2+\frac{1+\frac{2}{7}}{3}}{2}$

9. Decimal Fraction

If a fraction has denominator in the form of powers of 10, then it is a decimal fraction.

Ex: $\frac{2}{10}=(0.2), \frac{9}{100}=(0.09)$

10. Vulgar Fraction

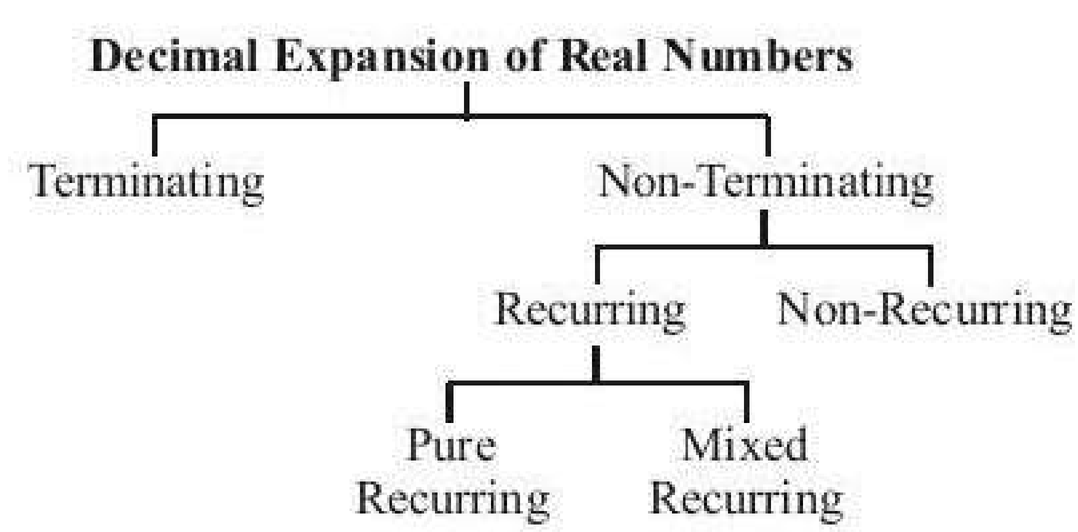
If denominator of any fraction is not in the form of powers of 10, then it is called vulgar fraction.

Ex: $\frac{3}{7},\frac{9}{2},\frac{5}{193}$.

ROUNDING OFF (APPROXIMATION) OF DECIMALS

There are some decimals in which numbers are found upto large number of decimal places.
Ex : 3.4578, 21.358940789.
But many times we require decimal numbers upto a certain number of decimal places. Therefore,

If the digit of the decimal place is five or more than five, then the digit in the preceding decimal place is increased by one and if the digit in the last place is less than five, then the digit in the preceding place remains unchanged.



Terminating (or Finite Decimal Fractions)

Ex : $\frac{7}{8} = 0.875, \frac{21}{5} = 4.2$

Non-Terminating Decimal Fractions

There are two types of Non-terminating decimal fractions.

(i) Non-terminating periodic fractions or non-terminating recurring (repeating) decimal fractions :

Form : $x.a_1a_2a_3 \ldots a_1a_2a_3 \ldots a_1a_2a_3$

Ex: $\frac{10}{3} = 3.333\ldots = 3.\bar{3}$

$$\frac{1}{7} = 0.142857142857... = 0.\overline{142857}$$

(ii) Non-terminating non-periodic fraction or non-terminating non-recurring fractions :

Form : $x.a_1a_2a_3...b_1b_2b_3...c_1c_2c_3...$
Ex : 15.2731259629

- The decimal expansion of a rational number is either terminating or non-terminating recurring. Moreover, a number whose decimal expansion is **terminating or non-terminating recurring** is rational.
- The decimal expansion of an irrational number is **non-terminating non recurring**. Moreover, a number whose decimal expansion is non-terminating non recurring is irrational.

  Ex: $\sqrt{2} = 1.4142135623730950488 0...$

  $\pi = 3.14159265358979323846264 33...$
- We often take $\frac{22}{7}$ as an approximate value of π, but $\pi \neq \frac{22}{7}$.

SQUARE AND SQUARE ROOTS

SQUARE

When a number is multiplied by itself, we get square of that number.

Ex : $4 \times 4 = 16$; we say that the square of 4 is 16.

- x^n is also read as x raised to the power n.
- Square of a even number is always even.
- Square of a odd number is always odd.

Squares of first 30 natural number			
x	x^2	x	x^2
1	1	16	256
2	4	17	289
3	9	18	324
4	16	19	361
5	25	20	400
6	36	21	441
7	49	22	484
8	64	23	529
9	81	24	576
10	100	25	625
11	121	26	676
12	144	27	729
13	169	28	784
14	196	29	841
15	225	30	900

Perfect Square

A natural number is called a perfect square, if it is the square of some natural number.

Ex : Numbers 1, 4, 9, 16, 25, 36, etc. are all perfect squares.

To find out whether the given number is perfect square :

- Express the number as a product of prime factors.
- If it is expressible as the product of pairs of equal factors, then it is a perfect square.

Example 5: **Is 144 a perfect square ?**

Solution :

2 | 144
2 | 72
2 | 36
2 | 18
3 | 9
3 | 3
  | 1

$\therefore\ 144 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

Since both the factors in each group are equal hence it is a perfect square.

Remember

- ✧ A number ending in an odd number of zeros is never a perfect square. For example, 15000.
- ✧ A number ending in 2, 3, 7 or 8 is never a perfect square.
- ✧ The square of a natural number (other than 1) is a multiple of 3 or exceeds a multiple of 3 by 1.

  Ex: $2^2 = 4 = (3 \times 1) + 1$

  $3^2 = 9 = (3 \times 3)$

  $4^2 = 16 = (3 \times 5) + 1$
- ✧ The square of a natural number (other than 1) is a multiple of 4 or exceeds a multiple of 4 by 1.

Ex: $2^2 = 4 = 4 \times 1$

$3^2 = 9 = (4 \times 2) + 1$

$4^2 = 16 = (4 \times 4)$

$5^2 = 25 = (4 \times 6) + 1.$

✧ For a natural number m (other than 1) 2m, $(m^2 - 1)$ and $(m^2 + 1)$ are pythagorean triplets.

Ex: take m = 4 then,

$2m = 2 \times 4 = 8$

$m^2 - 1 = 4^2 - 1 = 15$

$m^2 + 1 = 4^2 + 1 = 17$

$8^2 + 15^2 = 64 + 225 = 289 = 17^2$

So, 8, 15 and 17 are pythagorean triplets.

Alternative Method to Find the Square of a Number

STEP I: Express the given number as a sum or difference of two numbers.

STEP II: Apply any one of the following formulae

$$(a+b)^2 = a^2 + 2ab + b^2; \quad (a-b)^2 = a^2 - 2ab + b^2$$

Example 6: Find the square of 151.

Solution :

$(151)^2 = (150 + 1)^2$

Here a = 150, b = 1

$\therefore\ (151)^2 = (150)^2 + 2 \times 150 \times 1 + (1)^2$

$= 22500 + 300 + 1$

$= 22801$

Example 7: Find the square of 679.

Solution :

$(679)^2 = (700 - 21)^2$

$= 490000 - 2 \times 700 \times 21 + 441$

$= 461041$

SQUARE ROOTS

The square root of a number 'x' is that number which when multiplied by itself gives 'x' as the product.

Square root of x is denoted by the symbol $\sqrt{x}$

Ex:

(i) Square root of 4 is 2 or $\sqrt{4} = 2$

(ii) $\sqrt{2.56} = \sqrt{\dfrac{256}{100}} = \sqrt{\dfrac{2\times2\times2\times2\times2\times2\times2\times2}{10\times10}}$

$= \dfrac{2\times2\times2\times2}{10} = \dfrac{16}{10} = 1.6$

- Whether the number is negative or positive, its square is always positive and so, the square root of a negative number is not possible.

Square roots of first 20 natural numbers

x	$\sqrt{x}$	x	$\sqrt{x}$
1	1.000	11	3.317
2	1.414	12	3.464
3	1.732	13	3.606
4	2.000	14	3.742
5	2.236	15	3.873
6	2.449	16	4.000
7	2.646	17	4.123
8	2.828	18	4.243
9	3.000	19	4.359
10	3.162	20	4.472

Finding the Square Root of a Perfect Square Number by Prime Factorisation Method

Step I : Resolve the given number into prime factors.

Step II : Make pairs of similar factors.

Step III : Take the product of prime factors choosing one out of every pair.

Example 8: Find the square root of 1521.

Solution :

$$\begin{array}{r|l} 3 & 1521 \\ 3 & 507 \\ 13 & 169 \\ 13 & 13 \\ & 1 \end{array}$$

$1521 = \underline{3\times3}\times\underline{13\times13}$

$\sqrt{1521} = 3\times13 = 39$

Example 9: Find the smallest number by which 396 must be multiplied so that the product becomes a perfect square.

Solution :

By Prime factorisation, we get

$$\begin{array}{r|l} 2 & 396 \\ 2 & 198 \\ 3 & 99 \\ 3 & 33 \\ 11 & 11 \\ & 1 \end{array}$$

$396 = \underline{2\times2}\times\underline{3\times3}\times11$

In order to become a perfect square, one more 11 is required.

Example 10: Find the smallest number by which 6300 be divided, so that the quotient is a perfect square.

Solution :

$$\begin{array}{r|l} 2 & 6300 \\ 2 & 3150 \\ 3 & 1575 \\ 3 & 525 \\ 5 & 175 \\ 5 & 35 \\ 7 & 7 \\ & 1 \end{array}$$

$6300 = \underline{2\times2}\times\underline{3\times3}\times\underline{5\times5}\times7$

Since, the prime factor 7 is not paired.

$\therefore$ The given number should be divided by 7.

Finding the Square Root by Long Division Method

STEP I : Mark off the digits in pairs starting from right to left.

STEP II : Find the largest or think of the largest whole number which when multiplied by itself is equal or nearest to the left most pair. The number is taken as the divisor as well as quotient.

STEP III : Subtract the product from left most pair. There will be a remainder, bring down the next pair of digits.

STEP IV : Now, for next divisor add the quotient to previous divisor.

Go on repeating the above steps till all the pairs have been taken up. The quotient so obtained is the required squareroot of the given number.

Example 11: Find the square root of 106276.

Solution : By long division method :

	326
3	$\overline{10}\,\overline{62}\,\overline{76}$ 9
62	162 124
646	3876 3876
	×

$\therefore \sqrt{106276} = 326$

Example 12: What least number must be subtracted from 46687 to get a perfect square ? Also, find the square root of this perfect square.

Solution :

Let's find the square root of 46687.

	216
2	$4\,\overline{66}\,\overline{87}$ 4
41	66 41
426	2587 2556
	31

This shows that $(216)^2$ is less than 46687 by 31. So, in order to get a perfect square, 31 must be subtracted from the given number.

$\therefore$ Required perfect square number = 46687 – 31 = 46656

Also, $\sqrt{46656} = 216$

Example 13: Find the least number of six digits which is a perfect square. Also, find the square root of the number.

Solution : The least number of six digits = 100000

Square root of 100000 :

	316
3	$\overline{10}\,\overline{00}\,\overline{00}$ 9
61	100 61
626	3900 3756
	144

Hence, $(316)^2 < 100000 < (317)^2$

$\therefore$ The least number to be added $= (317)^2 - 100000 = 489$

Hence, the required number = 100000 + 489 = 100489

$\therefore \sqrt{100489} = 317$

Square Roots of Numbers in Decimal form

In the mixed decimal numbers, starting from the decimal point, pairing the integral part from right to left and decimal part from left to right.

Example 14: Find the square root of 8.3521.

Solution :

	2.89
2	$\overline{8}.\overline{35}\,\overline{21}$ 4
48	435 384
569	5121 5121
	×

$\therefore \sqrt{8.3521} = 2.89$

Example 15: Find the square root of 0.976426 correct upto two places of decimal.

Solution :

	0.988
9	$\overline{0}.\overline{97}\,\overline{64}\,\overline{26}$ 81
188	1664 1504
1968	16026 15744
	282

$\therefore \sqrt{0.976426} = 0.988 = 0.99$

Remember

✧ For any positive numbers *a* and *b*,

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b} \text{ and } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Example 16: Find the square root of $3\frac{1}{16}$.

Solution :

$$\sqrt{3\frac{1}{16}} = \sqrt{\frac{49}{16}} = \frac{7}{4} = 1\frac{3}{4}$$

Example 17: Find the square root of $5\frac{551}{1369}$.

Solution :

$$\sqrt{5\frac{551}{1369}} = \sqrt{\frac{7396}{1369}} = \frac{\sqrt{7396}}{\sqrt{1369}}$$

	86
8	$\overline{73}\,\overline{96}$
	64
166	996
	996
	×

	37
3	$\overline{13}\,\overline{69}$
	9
67	469
	469
	×

$\therefore \quad \sqrt{7396} = 86$ and $\sqrt{1369} = 37$

Hence $\sqrt{5\frac{551}{1369}} = \frac{86}{37} = 2\frac{12}{37}$.

Alternative Method to Find the Square Root

Method for small numbers : Take the number n whose square root is required. Subtract from n the odd numbers 1, 3, 5, 7, 9, 11, 13 .... successively. Then, we will get zero at some stage (only if n is a perfect square). Count the number of times we have performed subtraction. This is the required square root of n.

Ex : let us take $n = 36$.

Then, $36 - 1 = 35$
$35 - 3 = 32$
$32 - 5 = 27$
$27 - 7 = 20$
$20 - 9 = 11$
$11 - 11 = 0$

Here, the total number of subtraction is 6

$\therefore \sqrt{36} = 6.$

Remember

✧ If the square ends in 1, then its square root end in either 1 or 9.

Unit digit of square	1	4	5	6	9	00
Units or extreme right digit of square root	1 or 9	2 or 8	5	4 or 6	3 or 7	0

✧ Square root of a number greater than or equal to 1 but less than 100 consists of only one digit. i.e. if $1 \le x < 100$ then $\sqrt{x}$ consists of only one digit.

✧ If $100 \le x < 10000$ then $\sqrt{x}$ consists of two digits.

Finding Square Roots of Exact Squares having upto Four Digits

STEP I : Make pair of digits, starting from the extreme right. The leftmost digit may or maynot be paired-up. The number of such pairs equal the number of digit in the square root.

STEP II : Find the largest number whose square is less than or equal to the number under the left-most bar. This is the left most digit (L) of square root.

STEP III : Guess the unit digit (R) from the above table.

STEP IV : Coose the correct digit by squaring one of them.

Example 18: Find the square root of 4489.

Solution :

We have $\overline{44}\,\overline{89}$

$\because$ 44 lies between 6^2 and 7^2

$\therefore$ L = 6

$\because$ 89 ends with 9,

So R = 3 or 7

$\therefore$ Square root is either 63 or 67

Now $63^2 = 3969 \ne 4489$

$\therefore \sqrt{4489} = 67$

CUBE AND CUBE ROOTS

CUBE

Cube of a number is that number raised to the power 3.

Ex:

$4^3 = 4 \times 4 \times 4 = 64$; we say the cube of 4 is 64.

Cubes of First 20 Natural Numbers

x	x^3	x	x^3
1	1	11	1331
2	8	12	1728
3	27	13	2197
4	64	14	2744
5	125	15	3375
6	216	16	4096
7	343	17	4913
8	512	18	5832
9	729	19	6859
10	1000	20	8000

Perfect Cube

A natural number is said to be a perfect cube if it is the cube of some natural number.

If m is a natural number, then m^3 is a perfect cube.

Ex : $10^3 = 1000$ is a perfect cube.

Example 19: Is 343 a perfect cube?

Solution :

7	343
7	49
7	7
	1

$\therefore 343 = 7 \times 7 \times 7$

Hence, it is a perfect cube.

- Cubes of all even numbers are even.
- Cubes of all odd numbers are odd.
- Cubes of negative integers are negative.

Finding the Cube of a Number

To find the cube of given number, multiply the number with itself three times.

Ex : $8^3 = 8 \times 8 \times 8 = 512$

Alternative Method to Find the Cube of a Number

STEP I : Express the given number as a sum or difference of two numbers.

STEP II : Apply the one of the following formula :

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Example *20:* **Find the cube of 33.**

Solution :

$(33)^3 = (30+3)^3$

Here a = 30, b = 3

$\therefore\ (33)^3 = 27000 + 3 \times (30)^2 \times 3 + 3(30)(3)^2 + 27$

$= 27000 + 8100 + 810 + 27$

$= 35937$

CUBE ROOT

The cube root of a number x is that number whose cube gives x.

The cube root of x is denoted by the symbol $\sqrt[3]{x}$.

Cube Roots of Some Natural Numbers			
x	$\sqrt[3]{x}$	x	$\sqrt[3]{x}$
1	1.000	20	2.714
2	1.260	30	3.107
3	1.442	40	3.420
4	1.587	50	3.684
5	1.710	60	3.915
6	1.817	70	4.121
7	1.913	80	4.309
8	2.000	90	4.481
9	2.080	100	4.642
10	2.154		

- If cube ends in 1, then its cube root ends in 1.

Remember

✧

Unit digit of cube	1	2	3	4	5	6	7	8	9	0
Unit (or extreme right) digit of cube root	1	8	7	4	5	6	3	2	9	0

Hence, 1, 4, 5, 6, 9 and 0 repeat themselves and 2, 3, 7 and 8 complement of 10 i.e. 8, 7, 3 and 2 respectively.

Finding the Cube Root of a Perfect Cube by Prime Factorisation Method

STEP I : Resolve the given number into prime factors.

STEP II : Make triplet of similar factors.

STEP III: Take the product of prime factors choosing one out of every triplet number.

STEP IV : The product is the required cube root of the given number.

Example *21:* **Find the cube root of 2744.**

Solution :

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$2744 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$

$\therefore\ \sqrt[3]{2744} = 2 \times 7 = 14$

Alternative Method to Find the Cube Roots of Exact Cubes having upto 6 Digits

STEP I : Form the groups of three consecutive digits, starting from the extreme right. The last group may consists of less than three digits. The number of such 3-digit groups equals the number of digits in the cube root.
If the number consists of upto 3 digits, its cube root will be less than 10 and can be easily found from the table.
For number consisting of 4, 5 or 6 digits, the cube root will consist of 2 digits and its right most digit can be found by the table.

STEP II : Find the largest number whose cube is less than or equal to the number under the left most bar. This is the left most digit (L) of cube root.

Example *22:* **Find the cube root of 9261.**

Solution :

We have : $\overline{9}\,\overline{261}$

$\because$ 9 lies between 2^3 and 3^3

So L = 2

Now, 261 ends in 1. So R = 1

$\therefore$ number is 21.

CUBE ROOT OF A NEGATIVE PERFECT CUBE :

If a is a positive integer then – a is a negative integer.

$(-a)^3 = -a^3$

So, $\sqrt[3]{(-a^3)} = -a$

In general, we have $\sqrt[3]{-x} = -\sqrt[3]{x}$.

Example 23: Find the cube root of – 74088.

Solution :

$74088 = \overline{2\times2\times2}\times\overline{3\times3\times3}\times\overline{7\times7\times7}$

So, $\sqrt[3]{74088} = 2\times3\times7 = 42$

Hence $\sqrt[3]{-74088} = -\sqrt[3]{74088} = -42$

Remember

✧ For any two integers *a* and *b*, we have

$\sqrt[3]{ab} = \sqrt[3]{a}\times\sqrt[3]{b}$ and $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Example 24: Find the cube root of 2.744

Solution :

$\sqrt[3]{2.744} = \sqrt[3]{\frac{2744}{1000}} = \frac{\sqrt[3]{2744}}{\sqrt[3]{1000}}$

$2744 = 2\times2\times2\times7\times7\times7$

$\therefore \quad \sqrt[3]{2744} = 2\times7 = 14$

Also $\sqrt[3]{1000} = \sqrt[3]{10\times10\times10} = 10$

So, $\frac{\sqrt[3]{2744}}{\sqrt[3]{1000}} = \frac{14}{10} = 1.4$

Example 25: Find the cube root of 658503.

Solution :

$\sqrt[3]{658503} = 219501\times3$
$= 73167\times3\times3$
$= 24389\times3\times3\times3$
$= 841\times29\times3\times3\times3$
$= 29\times29\times29\times3\times3\times3$
$= 29\times3 = 87$

SURDS AND INDICES

INDICES

When a number '*a*' is multiplied by itself '*m*' times, then we say that '*a* is of m-index'. a^m is read as 'a raised to the power *m*'.

LAWS OF INDICES

1. $a^m\times a^n = a^{m+n}$

Example 26: Simplify : $(1000)^7\times(10)^5$

Solution :

$(10^3)^7\times(10)^5 = 10^{21+5} = 10^{26}$

2. $\frac{a^m}{a^n} = a^{m-n}$

Example 27: Simplify : $(5)^{25}\div(125)^8$

Solution :

$5^{25}\div(5^3)^8 = 5^{25}\div5^{24} = 5^{25-24} = 5$

3. $(a^m)^n = a^{mn}$

Example 28: Simplify : $\left[\left(\sqrt{2}\right)^3\right]^4$

Solution :

$\left[\left(\sqrt{2}\right)^3\right]^4 = (2)^{\frac{3}{2}\times4} = 2^6$

4. $(ab)^n = a^nb^n$

Ex : $(12)^2 = (4\times3)^2 = 4^2\times3^2 = 16\times9 = 144$

5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Ex: $\left(\frac{3}{4}\right)^4 = \frac{3^4}{4^4} = \frac{81}{256}$

6. $a^0 = 1$

Ex : $4^0 = 1$

Remember

✧ $a^m + a^n \neq a^{m+n}$

$a^m\times b^n \neq (ab)^{m+n}$

SURDS

If '*a*' is a rational number and '*n*' is a positive integer such that the n^{th} root of a i.e, $a^{1/n}$ or $\sqrt[n]{a}$ is an irrational number, then $a^{1/n}$ is called a surd or radical of order *n* and *a* is called the radicand.

Ex : $\sqrt{2}$, $\sqrt[3]{3}$, etc.

Consider the real number $\sqrt{2+\sqrt{3}}$. Since $2+\sqrt{3}$ is not a rational number, therefore, $\sqrt{2+\sqrt{3}}$ is not a surd.

Mixed Surds

A rational factor and a surd multiplied together are called mixed surds.

Ex : $3\sqrt{2}$, $5\sqrt[3]{6}$ etc.

Pure Surd

A surd which has unity only as rational factor, the other factor being irrational, is called a pure surd.

Ex: $\sqrt{3}$, $\sqrt[5]{2}$, $\sqrt[4]{7}$.

Quadratic Surd

A surd of order 2 is called a quadratic surd.

Ex : $\sqrt{7} = 7^{1/2}$ and $\sqrt{13} = 13^{1/2}$

Cubic Surd

A surd of order 3 is called a cubic surd.

Ex : $\sqrt[3]{4} = (4)^{1/3}$.

Biquadratic Surd

A surd of order 4 is called a biquadratic surd. A biquadratic surd is also called **quartic surd**.

Ex : $\sqrt[4]{5}$ is a biquadratic surd but $\sqrt[4]{81}$ (= 3) is not a biquadratic surd as it is not a surd.

Laws of Surds

1. $(\sqrt[n]{a})^n = a$

Example 29: Simplify : (i) $(\sqrt[3]{7})^3$ (ii) $\sqrt[3]{27}$

Solution :

(i) $(\sqrt[3]{7})^3 = [(7)^{1/3}]^3 = 7^{1/3 \times 3} = 7$

(ii) $\sqrt[3]{27} = \sqrt[3]{3^3} = (3^3)^{\frac{1}{3}} = 3$

Example 30: Solve : $\sqrt[4]{3x+1} = 2$

Solution :

$\sqrt[4]{3x+1} = 2$

$\Rightarrow (\sqrt[4]{3x+1})^4 = (2)^4$

$\Rightarrow 3x + 1 = 16$

$\Rightarrow 3x = 15$

$\therefore x = 5$

2. $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$

Example 31: Simplify : $\sqrt[3]{3} . \sqrt[3]{4}$

Solution :

$\sqrt[3]{3}.\sqrt[3]{4} = \sqrt[3]{3.4} = \sqrt[3]{12}$

3. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

Example 32: Find $\sqrt[3]{\frac{125}{64}}$

Solution :

$$\sqrt[3]{\frac{125}{64}} = \frac{\sqrt[3]{125}}{\sqrt[3]{64}} = \frac{\sqrt[3]{5^3}}{\sqrt[3]{4^3}} = \frac{5}{4}$$

4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

Example 33: Simplify : $\sqrt[3]{\sqrt[4]{5}}$

Solution :

$\sqrt[3]{\sqrt[4]{5}} = \sqrt[12]{5}$

CONVERSION OF MIXED SURDS INTO PURE SURDS

Example 34: Express each of the following as a pure surd :

(i) $2\sqrt[3]{4}$ (ii) $\frac{2}{3}\sqrt[3]{108}$

Solution :

(i) $2\sqrt[3]{4} = 2 \times 4^{1/3} = (2^3)^{1/3} \times 4^{1/3}$

$= (8)^{1/3} \times 4^{1/3} = (8 \times 4)^{1/3} = (32)^{1/3} = \sqrt[3]{32}$

(ii) $\frac{2}{3}\sqrt[3]{108} = \frac{2}{3} \times (108)^{1/3}$

$$= \left[\left(\frac{2}{3}\right)^3\right]^{1/3} \times (108)^{1/3}$$

$$= \left(\frac{8}{27}\right)^{1/3} \times (108)^{1/3} = \left(\frac{8}{27} \times 108\right)^{1/3}$$

$= (8 \times 4)^{1/3} = (32)^{1/3} = \sqrt[3]{32}$

CONVERSION OF SURDS INTO SURDS OF THE SAME ORDER

Let the surds be $\sqrt[n_1]{a_1}, \sqrt[n_2]{a_2}, \sqrt[n_3]{a_3}, ...$

STEP I : Compute L.C.M. of $n_1, n_2, n_3, ...$
Let L.C.M. $= n$

STEP II : Compute $\frac{n}{n_1}, \frac{n}{n_2}, \frac{n}{n_3}, ...$

$= m_1, m_2, m_3, ...$

STEP III : Required surds are

$\sqrt[n]{a_1^{m_1}}, \sqrt[n]{a_2^{m_2}}, \sqrt[n]{a_3^{m_3}}, ...$

Example 35: Express $\sqrt[4]{2}, \sqrt[3]{3}, \sqrt[5]{4^2}$ as surds of the same order.

Solution :

Here, $n_1 = 4, n_2 = 3, n_3 = 5$

$\therefore n =$ L.C.M. of $(4, 3, 5) = 60$

$$m_1 = \frac{n}{n_1} = \frac{60}{4} = 15,\ m_2 = \frac{60}{3} = 20,\ m_3 = \frac{60}{5} = 12$$

$\therefore \sqrt[4]{2} = \sqrt[60]{2^{15}}$; $\sqrt[3]{3} = \sqrt[60]{3^{20}}$

$\sqrt[5]{4^2} = \sqrt[60]{(4^2)^{12}} = \sqrt[60]{4^{24}}$.

COMPARISON OF SURDS

STEP I: Convert each surd into a surd of same order.

STEP II: Compare the radicand of the surds. The surd with larger radicand is the largest of the given surds.

Example 36: **Which is greater $\sqrt[3]{6}$ or $\sqrt[4]{8}$?**

Solution :

L.C.M. of 3 and 4 is 12.

$\sqrt[3]{6} = \sqrt[12]{6^4} = \sqrt[12]{1296}$

$\sqrt[4]{8} = \sqrt[12]{8^3} = \sqrt[12]{512}$

$1296 > 512$

$\therefore \quad \sqrt[3]{6} > \sqrt[4]{8}$

Example 37: **Arrange in increasing order relation among the surds $a = \sqrt{5}$, $b = \sqrt[3]{11}$ and $c = 2\sqrt[6]{3}$.**

Solution :

The order of each surds are different, so we convert each of them into the surd of order 6

$a = \sqrt{5} = \sqrt[6]{5^3} = \sqrt[6]{125}$

$b = \sqrt[3]{11} = \sqrt[6]{11^2} = \sqrt[6]{121}$

$c = 2\sqrt[6]{3} = \sqrt[6]{3 \times 2^6} = \sqrt[6]{192}$

Sign, $\quad 121 < 125 < 192$

$\therefore \sqrt[6]{121} < \sqrt[6]{125} < \sqrt[6]{192} \Rightarrow b < a < c$.

ADDITION AND SUBTRACTION OF SURDS

Example 38: **Simplify : $4\sqrt{3} - 3\sqrt{12} + 2\sqrt{75}$**

Solution :

$3\sqrt{12} = 3\sqrt{2\times2\times3} = 3\times2\sqrt{3} = 6\sqrt{3}$

$2\sqrt{75} = 2\sqrt{5\times5\times3} = 2\times5\sqrt{3} = 10\sqrt{3}$

$\therefore \quad 4\sqrt{3} - 3\sqrt{12} + 2\sqrt{75}$

$= 4\sqrt{3} - 6\sqrt{3} + 10\sqrt{3}$

$= (4 - 6 + 10)\sqrt{3} = 8\sqrt{3}$

MULTIPLICATION AND DIVISION OF SURDS

Surds of same order can be multiplied and divided according to the following laws :

(i) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ (ii) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

Example 39: **Simplify :**

(i) $\sqrt[3]{2} . \sqrt[4]{2} . \sqrt[12]{32}$ (ii) $\sqrt[6]{12} \div (\sqrt{3}\sqrt[3]{2})$

Solution :

(i) L.C.M. of 3, 4 and 12 is 12

$\sqrt[3]{2} = \sqrt[12]{2^4} = \sqrt[12]{16}$

$\sqrt[4]{2} = \sqrt[12]{2^3} = \sqrt[12]{8}$

$\sqrt[12]{32} = \sqrt[12]{32}$

$\therefore \sqrt[3]{2} . \sqrt[4]{2} . \sqrt[12]{32} = \sqrt[12]{16} \times \sqrt[12]{8} \times \sqrt[12]{32}$

$= \sqrt[12]{16\times8\times32} = \sqrt[12]{2^4\times2^3\times2^5} = \sqrt[12]{2^{12}} = 2.$

(ii) L.C.M. of 2 and 3 is 6.

$\sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}$

$\sqrt[3]{2} = \sqrt[6]{2^2} = \sqrt[6]{4}$

$\therefore \sqrt[6]{12} \div (\sqrt{3}\sqrt[3]{2}) = \sqrt[6]{12} \div (\sqrt[6]{27}\ \sqrt[6]{4})$

$= \sqrt[6]{12} \div \sqrt[6]{27\times4} = \sqrt[6]{12} \div \sqrt[6]{108}$

$= \sqrt[6]{\dfrac{12}{108}} = \sqrt[6]{\dfrac{1}{9}} = \sqrt[6]{\left(\dfrac{1}{3}\right)^2} = \sqrt[3]{\dfrac{1}{3}}.$

RATIONALISING FACTOR

If the product of two surds is a rational number, then each one of them is called the rationalising factor (R.F.) of the other.

In a binomial surd of the form $\sqrt{a} \pm \sqrt{b}$, the rationalising factors are $\sqrt{a} \mp \sqrt{b}$.

Example 40: **Find the simplest rationalising factor of :**

(i) $\sqrt[3]{32}$ (ii) $\sqrt{5} - \sqrt{3}$

Solution :

(i) $\sqrt[3]{32} = \sqrt[3]{2\times2\times2\times2\times2} = 2\sqrt[3]{2\times2}$

Now, since $(2\times\sqrt[3]{2\times2})\times\sqrt[3]{2} = 2\times\sqrt[3]{2\times2\times2}$

$= 2\times2 = 4$

$\therefore$ Simplest R.F. of $\sqrt[3]{32}$ is $\sqrt[3]{2}$.

(ii) $\sqrt{5} - \sqrt{3} = (\sqrt{5} - \sqrt{3}) \times (\sqrt{5} + \sqrt{3})$

$= (\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$

$\therefore$ Simplest R.F. of $\sqrt{5} - \sqrt{3}$ is $\sqrt{5} + \sqrt{3}$.

Example 41: **Simplify by rationalising the denominator :**

$$\frac{7 + 3\sqrt{5}}{7 - 3\sqrt{5}}$$

Solution :

$$\frac{7+3\sqrt{5}}{7-3\sqrt{5}} \times \frac{7+3\sqrt{5}}{7+3\sqrt{5}} = \frac{(7+3\sqrt{5})^2}{(7)^2 - (3\sqrt{5})^2}$$

$$= \frac{(7)^2 + (3\sqrt{5})^2 + 2\times7\times3\sqrt{5}}{49 - 45}$$

$$= \frac{49 + 45 + 42\sqrt{5}}{49 - 45}$$

$$= \frac{94 + 42\sqrt{5}}{4} = \frac{47 + 21\sqrt{5}}{2}$$

IMPORTANT SERIES TYPE FORMULAE

(i) Value of $\sqrt{P + \sqrt{P + \sqrt{p + ...\infty}}} = \frac{\sqrt{4P+1}+1}{2}$

(ii) Value of $\sqrt{P - \sqrt{P - \sqrt{p - .....\infty}}} = \frac{\sqrt{4P+1}-1}{2}$

(iii) Value of $\sqrt{P.\sqrt{P.\sqrt{p.....\infty}}} = P$

(iv) Value of $\sqrt{P\sqrt{P\sqrt{P\sqrt{P\sqrt{P}}}}} = P^{(2^n - 1) \div 2^n}$

where n $\rightarrow$ no. of times P repeated.

ALGEBRAIC IDENTITIES

Consider the equality $(x + 2)(x + 3) = x^2 + 5x + 6$

Let us evaluate both sides of this equality for some value of variable x say $x = 4$

LHS $= (x + 2)(x + 3) = (4 + 2)(4 + 3) = 6 \times 7 = 42$

RHS $= (4)^2 + 5 \times 4 + 6 = 16 + 20 + 6 = 42$

So for $x = 4$, LHS = RHS

Let us calculate LHS and RHS for $x = -3$

LHS $= (-3 + 2)(-3 + 3) = 0$

RHS $= (-3)2 + -(-3) + 6 = 9 - 15 + 6 = 0$

$\therefore$ for $x = -3$, LHS = RHS

If we take any value of variable x, we can find that LHS = RHS

Such an equality which is true for every value of the variable present in it is called an identity. Thus $(x + 2)(x + 3) = x^2 + 5x + 6$, is an identity.

Identities differ from equations in the following manners.

An equation is a statement of equality of two algebraic expression involving one or more variables and it is true for certain values of the variable.

Ex:

$4x + 3 = x - 3$ ... (1)

$\Rightarrow 3x = -6 \Rightarrow x = -2$

Thus equality (1) is true only for $x = -2$, no other value of x satisfy equation (1).

Standard Identities

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(v) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Some More Identities

We have dealt with identities involving squares. Now we will see how to handle identities involving cubes.

(i) $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$\Rightarrow (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(ii) $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$\Rightarrow (a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(iii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(iv) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(v) $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

EXERCISE

1. The pure form of the surd $\frac{3}{2}\sqrt[4]{\frac{32}{243}}$ is

(a) $\sqrt[3]{\frac{2}{3}}$ (b) $\sqrt[4]{\frac{2}{3}}$ (c) $\sqrt{\frac{2}{3}}$ (d) 1

2. If $9^x = \frac{9}{3^x}$, then x is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 3 (d) $\frac{4}{3}$

3. $1.236\times10^{15} - 5.23\times10^{14}$ equals :

(a) 7.13×10^{14} (b) 7.13×10^{15}
(c) 71.3×10^{14} (d) -3.994

4. If $3^{4X-2} = 729$, then the value of X is
(a) 4 (b) 3
(c) 2 (d) 5

5. If $2^{x+4} - 2^{x+2} = 3$ then x is equal to
(a) –2 (b) 0
(c) 2 (d) 4

6. The value of $\left(\frac{-1}{216}\right)^{-\frac{2}{3}}$ is :

(a) $\frac{1}{36}$ (b) $-\frac{1}{36}$
(c) –36 (d) 36

7. The value of $\sqrt{\frac{(0.03)^2+(0.21)^2+(0.065)^2}{(0.003)^2+(0.021)^2+(0.0065)^2}}$ is :

(a) 0.1 (b) 10
(c) 10^2 (d) 10^3

8. The value of $\sqrt[3]{\sqrt{0.000064}}$ is :
(a) 0.02 (b) 0.2
(c) 2.0 (d) None of these

9. The value of 9821 – [48 + {12 × (153 + 24)}] is
(a) 9312 (b) 8647
(c) 8749 (d) 7649

10. The value of $\frac{\frac{1}{2}\div\frac{1}{2}\text{ of }\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}\text{of }\frac{1}{2}}$ is:

(a) 8/3 (b) 2
(c) 4/3 (d) 3

11. $\frac{3}{5}$ of $\frac{4}{7}$ of $\frac{5}{9}$ of $\frac{21}{24}$ of 504 = ?
(a) 63 (b) 69
(c) 96 (d) None of these

12. The value of $\frac{(243)^{0.13}\times(243)^{0.07}}{(7)^{0.25}\times(49)^{0.075}\times(343)^{0.2}}$ is :

(a) $\frac{3}{7}$ (b) $\frac{7}{8}$ (c) $1\frac{3}{7}$ (d) $2\frac{2}{7}$

13. A man plants 15376 apple trees in his garden and arranges them so that there are as many rows as there are apples trees in each row. The number of rows is :
(a) 124 (b) 126 (c) 134 (d) 144

14. If $\sqrt{2} = 1.4142$, the value of $\frac{7}{(3+\sqrt{2})}$ is:
(a) 1.5858 (b) 3.4852
(c) 3.5858 (d) 4.4142

15. The fourth root of $28 + 16\sqrt{3}$ is
(a) $4+2\sqrt{3}$ (b) $2\sqrt{2}+\sqrt{3}$
(c) $\sqrt{3}+1$ (d) $\sqrt{3}-\sqrt{2}$

16. The sum of three fractions is $2\frac{11}{24}$. When the largest fraction is divided by the smallest, the fraction thus obtained is $\frac{7}{6}$ which is $\frac{1}{3}$ more than the middle one. The fractions are:

(a) $\frac{3}{5},\frac{4}{7},\frac{2}{3}$ (b) $\frac{7}{8},\frac{5}{6},\frac{3}{4}$
(c) $\frac{7}{9},\frac{2}{3},\frac{3}{5}$ (d) None of these

17. A gardener has 1000 plants. He wants to plant them in such a way that the number of rows and the number of columns remains the same. What is the minimum number of plants that he needs more for this purpose?
(a) 14 (b) 24
(c) 32 (d) 34

18. Find the value of $x^a + y^b$, if $a^y = 19683$, where y is a multiple of a and $b^x = 1024$, where b is a factor of x.a, b, x and y being positive integers-
(a) 1081 (b) 829
(c) 181 (d) 1729

19. If $5^a = 3125$, then the value of $5^{(a-3)}$ is –
(a) 25 (b) 125
(c) 625 (d) 1625

20. The value of

$3\div\left[(8-5)\div\left\{(4-2)+\left(2+\frac{8}{13}\right)\right\}\right]$ is

(a) $\frac{15}{17}$ (b) $\frac{13}{17}$ (c) $\frac{15}{19}$ (d) $\frac{13}{19}$

21. When $\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\right)$ is divided by $\left(\frac{2}{5}-\frac{5}{9}+\frac{3}{5}-\frac{7}{18}\right)$, the result is :

(a) $2\frac{1}{18}$ (b) $3\frac{1}{6}$

(c) $3\frac{3}{10}$ (d) $5\frac{1}{10}$

22. Find the value of * in the following.

$1\frac{2}{3}\div\frac{2}{7}\times\frac{*}{7}=1\frac{1}{4}\times\frac{2}{3}\div\frac{1}{6}$

(a) 0.006 (b) $\frac{1}{6}$

(c) 0.6 (d) 6

23. $2.002+7.9\{(2.8-6.3(3.6-1.5)+15.6)\}=?$

(a) 2.002 (b) 4.2845

(c) 40.843 (d) 42.845

24. $9-1\frac{2}{9}$ of $3\frac{3}{11}\div5\frac{1}{7}$ of $\frac{7}{9}=?$

(a) $\frac{5}{4}$ (b) 8 (c) $8\frac{32}{81}$ (d) 9

25. Evaluate $\frac{\sqrt{24}+\sqrt{6}}{\sqrt{24}-\sqrt{6}}$

(a) 2 (b) 3

(c) 4 (d) 5

26. Arranging the following in descending order $2^{57}, 4^{38}, 15^{19}$ we get

(a) $2^{57}>4^{38}>15^{19}$ (b) $4^{38}>15^{19}>2^{57}$

(c) $15^{19}>2^{57}>4^{38}$ (d) $2^{57}>15^{19}>4^{38}$

27. Arranging the following in ascending order $2^{10000}, 10^{3000}, 3^{6000}, 7^{4000}$ we get

(a) $3^{6000}<10^{3000}<2^{10000}<7^{4000}$

(b) $2^{10000}<7^{4000}<10^{3000}<3^{6000}$

(c) $10^{3000}<3^{6000}<7^{4000}<2^{10000}$

(d) $7^{4000}<3^{6000}<2^{10000}<10^{3000}$

28. If all the fractions $\frac{3}{5},\frac{1}{8},\frac{8}{11},\frac{4}{9},\frac{2}{7}$ and $\frac{5}{12}$ are arranged in the descending order of their values, which one will be the third?

(a) $\frac{1}{8}$ (b) $\frac{4}{9}$ (c) $\frac{5}{12}$ (d) $\frac{8}{11}$

29. Which one of the following is the least?

$\sqrt{3},\sqrt[3]{2},\sqrt{2}$ and $\sqrt[3]{4}$

(a) $\sqrt{2}$ (b) $\sqrt[3]{4}$

(c) $\sqrt{3}$ (d) $\sqrt[3]{2}$

30. The smallest of $\sqrt{8}+\sqrt{5}, \sqrt{7}+\sqrt{6}, \sqrt{10}+\sqrt{3}$ and $\sqrt{11}+\sqrt{2}$ is

(a) $\sqrt{8}+\sqrt{5}$ (b) $\sqrt{7}+\sqrt{6}$

(c) $\sqrt{10}+\sqrt{3}$ (d) $\sqrt{11}+\sqrt{2}$

31. $\left[\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}+\frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}\right]$ in simplified form equals to:

(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 0

32. The value of

$\frac{\sqrt{2}\left(\sqrt{3}+1\right)\left(2-\sqrt{3}\right)}{\left(\sqrt{2}-1\right)\left(3\sqrt{3}-5\right)\left(2+\sqrt{2}\right)}$ is

(a) 1 (b) $2-\sqrt{3}$

(c) $2+\sqrt{3}$ (d) $\sqrt{3}-2$

33. $\frac{1}{10}$ of a pole is coloured red, $\frac{1}{20}$ white, $\frac{1}{30}$ blue, $\frac{1}{40}$ black, $\frac{1}{50}$ violet, $\frac{1}{60}$ yellow and the rest is green. If the length of the green portion of the pole is 12.08 metres, then the length of the pole is

(a) 16 m (b) 18 m

(c) 20 m (d) 30 m

34. Arranging the following in ascending order $3^{34}, 2^{51}, 7^{17}$, we get

(a) $3^{34}>2^{51}>7^{17}$ (b) $7^{17}>2^{51}>3^{34}$

(c) $3^{34}>7^{17}>2^{51}$ (d) $2^{51}>3^{34}>7^{17}$

35. What is $3.\overline{76}-1.4\overline{576}$ equal to ?

(a) $2.3\overline{100191}$ (b) $2.3\overline{101091}$

(c) $2.3\overline{110091}$ (d) $2.3\overline{110901}$

36. What is the value of $0.00\overline{7}+17.\overline{83}+310.020\overline{2}$?

(a) $327.86\overline{638}$ (b) $327.866\overline{38}$

(c) $327.866\overline{83}$ (d) $327.866\overline{8}$

37. What is the value of 0.242424... ?

(a) 23/99 (b) 8/33

(c) 7/33 (d) 47/198

38. Representation of $0.2\overline{341}$ in the form $\frac{p}{q}$, where p and q are integers, $q \neq 0$, is

(a) $\frac{781}{3330}$ (b) $\frac{1171}{4995}$

(c) $\frac{2341}{9990}$ (d) $\frac{2339}{9990}$

39. Let p be a prime number other than 2 or 5. One would like to express the vulgar fraction $1/p$ in the form of a recurring decimal. Then the decimal will be

(a) a pure recurring decimal and its period will be necessarily $(p-1)$

(b) a mixed recurring decimal and its period will be necessarily $(p-1)$

(c) a pure recurring decimal and its period will be some factor of $(p-1)$

(d) a mixed recurring decimal and its period will be some factor of $(p-1)$

40. What is one of the square roots of $9-2\sqrt{14}$?

(a) $\sqrt{7}-\sqrt{3}$ (b) $\sqrt{6}-\sqrt{3}$
(c) $\sqrt{7}-\sqrt{5}$ (d) $\sqrt{7}-\sqrt{2}$

41. What is the square root of $\frac{0.324\times0.64\times129.6}{0.729\times1.024\times36}$?

(a) 4 (b) 3
(c) 2 (d) 1

42. What is the smallest number that must be added to 1780 to make it a perfect square?

(a) 39 (b) 49
(c) 59 (d) 69

43. What is $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ equal to ?

(a) 16 (b) 8
(c) 4 (d) $\sqrt{15}$

44. If $a^x=b, b^y=c$ and xyz = 1, then what is the value of c^z?

(a) a (b) b (c) ab (d) $\frac{a}{b}$

45. If $196\,x^4=x^6$, then x^3 is equal to which one of the following ?

(a) $x^6/14$ (b) $14x^4$
(c) $x^2/14$ (d) $14x^2$

46. If $a=2+\sqrt{3}$, then what is the value of $\left(a^2+a^{-2}\right)$?

(a) 12 (b) 14
(c) 16 (d) 18

47. If $3^{x+y}=81$ and $81^{x-y}=3$, then what is the value of x ?

(a) $\frac{17}{16}$ (b) $\frac{17}{8}$ (c) $\frac{17}{4}$ (d) $\frac{15}{4}$

48. If $\sqrt{10+\sqrt[3]{x}}=4$, then what is the value of x ?

(a) 150 (b) 216
(c) 316 (d) 450

49. The least number of four digits which is a perfect square is

(a) 1204 (b) 1024
(c) 1402 (d) 1420

50. What is the value of $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+...}}}}$?

(a) 2 (b) 3
(c) 3.5 (d) 4

51. If $16\times8^{n+2}=2^m$, then m is equal to

(a) $n+8$ (b) $2n+10$
(c) $3n+2$ (d) $3n+10$

52. The expression $\left[\left(\sqrt{2}\right)^{\sqrt{2}}\right]^{\sqrt{2}}$ gives

(a) a natural number
(b) an integer and not a natural number
(c) a rational number but not an integer
(d) a real number but not a rational number

53. Which is the smallest number among the following? **(CDS)**

(a) $\left[\left(5^{-2}\right)^{-2}\right]^{-2}$ (b) $\left[\left(5^{-2}\right)^{2}\right]^{-2}$

(c) $\left[\left(2^{-5}\right)^{-2}\right]^{-2}$ (d) $\left[\left(2^{-5}\right)^{2}\right]^{-2}$

54. Consider the following in respect of the numbers $\sqrt{2},\sqrt[3]{3}$ and $\sqrt[6]{6}$ **(CDS)**

I. $\sqrt[6]{6}$ is the greatest number.

II. $\sqrt{2}$ is the smallest number.

Which of the above statements is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

55. The square root of $\frac{(0.75)^3}{1-0.75}+[0.75+(0.75)^2+1]$ is

(a) 1 (b) 2 **(CDS)**
(c) 3 (d) 4

56. What is the value of $\frac{\sqrt{0.0032}}{\sqrt{0.32}}$? **(CDS)**

(a) 0.0001 (b) 0.001
(c) 0.01 (d) 0.1

57. The value of $(0.\overline{63}+0.\overline{37})$ is **(CDS)**

(a) 1 (b) $\frac{100}{91}$

(c) $\frac{100}{99}$ (d) $\frac{1000}{999}$

58. If $x=\frac{91}{216}$, then the value of $3-\frac{1}{(1-x)1/3}$ is **(CDS)**

(a) $\frac{9}{5}$ (b) $\frac{5}{9}$ (c) $\frac{4}{9}$ (d) $\frac{4}{5}$

59. The sum of first 47 terms of the series **(CDS)**

$\frac{1}{4}+\frac{1}{5}-\frac{1}{6}-\frac{1}{4}-\frac{1}{5}+\frac{1}{6}+\frac{1}{4}+\frac{1}{5}-\frac{1}{6}.....$ is

(a) 0 (b) $-\frac{1}{6}$

(c) $\frac{1}{6}$ (d) $\frac{9}{20}$

60. If $x=\sqrt{3}+\sqrt{2}$, then the value of $x^3+x+\frac{1}{x}+\frac{1}{x^3}$ is

(a) $10\sqrt{3}$ (b) $20\sqrt{3}$ **(CDS)**
(c) $10\sqrt{2}$ (d) $20\sqrt{2}$

HINTS & SOLUTIONS

1. (b) $\frac{3}{2}\sqrt[4]{\frac{32}{243}} = \frac{3}{2}\times\left(\frac{32}{243}\right)^{\frac{1}{4}} = \left(\frac{2}{3}\right)^{-1}\left\{\left(\frac{2}{3}\right)^{5}\right\}^{\frac{1}{4}}$

$= \left(\frac{2}{3}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{2}{3}}$

2. (b) We have, $9^x \times 3^x = 9$

or $(3)^{2x}\times(3)^x = 9$

or $(3)^{3x} = (3^2)$

or $3x = 2$ or $x = \frac{2}{3}$

3. (a) $1.236\times10^{15} - 5.23\times10^{14}$

$= 10^{14}(12.36 - 5.23) = 7.13\times10^{14}$

4. (c) $729 = 9^3 = 3^6$, Now $4X - 2 = 6 \Rightarrow X = 2$.

5. (a) $2^{x+4} - 2^{x+2} = 3$

$\Rightarrow 2^{x+2}(2^2 - 1) = 3$

$\Rightarrow 2^{x+2} = 2^0$

$\therefore x + 2 = 0 \Rightarrow x = -2$

6. (d) $\left(\frac{-1}{216}\right)^{-\frac{2}{3}} = \left(\frac{-1}{6^3}\right)^{-\frac{2}{3}} = \left(-\frac{1}{6}\right)^{-2} = (-6)^2 = 36$

7. (b) Given exp. $= \sqrt{\frac{(0.03)^2 + (0.21)^2 + (0.065)^2}{\left(\frac{0.03}{10}\right)^2 + \left(\frac{0.21}{10}\right)^2 + \left(\frac{0.065}{10}\right)^2}}$

$= \sqrt{\frac{100\left[(0.03)^2 + (0.21)^2 + (0.065)^2\right]}{(0.03)^2 + (0.21)^2 + (0.065)^2}}$

$= \sqrt{100} = 10$

8. (b) $\sqrt[3]{\sqrt{0.000064}} = \sqrt[3]{0.008} = \left((0.2^3)\right)^{1/3} = 0.2$

9. (d) $9821 - [48 + \{12\times177\}]$
$= 9821 - [48 + 2124] = 9821 - 2172 = 7649$

10. (a) $\frac{\frac{1}{2}\div\frac{1}{4}}{\frac{1}{2}+\frac{1}{4}} = \frac{\frac{1}{2}\times4}{\frac{3}{4}} = \frac{\frac{2}{3}}{4} = \frac{8}{3}$

11. (d) Given exp. $= \left(\frac{3}{5}\times\frac{4}{7}\times\frac{5}{9}\times\frac{21}{24}\times504\right) = 84.$

12. (a) $\frac{(243)^{0.13}\times(243)^{0.07}}{7^{0.25}\times(49)^{0.075}\times(343)^{0.2}}$

$= \frac{(243)^{(0.13+0.07)}}{7^{0.25}\times(7^2)^{0.075}\times(7^3)^{0.2}}$

$= \frac{(243)^{(0.2)}}{7^{0.25}\times(7)^{(2\times0.075)}\times(7)^{(3\times0.2)}}$

$= \frac{(3^5)^{0.2}}{7^{0.25}\times7^{0.15}\times7^{0.6}}$

$= \frac{(3)^{(5\times0.2)}}{7^{(0.25+0.15+0.6)}} = \frac{3^1}{7^1} = \frac{3}{7}$

13. (a)

```
    1 | 15376 (124
      | 1
      |-----
   22 | 53
      | 44
      |-----
  244 |  976
      |  976
      |-----
      |   ×
```

$\therefore$ Number of rows = 124.

14. (a) $\frac{7}{(3+\sqrt{2})} = \frac{7}{(3+\sqrt{2})}\times\frac{(3-\sqrt{2})}{(3-\sqrt{2})} = \frac{7(3-\sqrt{2})}{(9-2)}$

$= (3-\sqrt{2}) = (3 - 1.4142) = 1.5858$

15. (c) $\sqrt[4]{(28+16\sqrt{3})} = \sqrt{\sqrt{(28+16\sqrt{3})}}$

$= \sqrt{\sqrt{(28+2\sqrt{192})}} = \sqrt{\sqrt{\left(\sqrt{16}+\sqrt{12}\right)^2}}$

$= \sqrt{\sqrt{16}+\sqrt{12}} = \sqrt{(4+2\sqrt{3})}$

$= \sqrt{\left(\sqrt{3}+1\right)^2} = \sqrt{3}+1$

16. (b) Let the largest fraction be x and the smallest be y.

Then, $\frac{x}{y} = \frac{7}{6}$ or $y = \frac{6}{7}x$.

Let the middle one be z. Then, $x + \frac{6}{7}x + z = \frac{59}{24}$ or

$z = \left(\frac{59}{24} - \frac{13x}{7}\right)$.

$\therefore \frac{59}{24} - \frac{13x}{7} + \frac{1}{3} = \frac{7}{6} \Rightarrow \frac{13x}{7} = \frac{59}{24} + \frac{1}{3} - \frac{7}{6} = \frac{39}{24}$

$\Rightarrow \quad x = \left(\frac{39}{24} \times \frac{7}{13}\right) = \frac{7}{8}.$

So, $x = \frac{7}{8}, y = \frac{6}{7} \times \frac{7}{8} = \frac{3}{4}$ and $z = \frac{59}{24} - \frac{13}{7} \times \frac{7}{8} = \frac{20}{24}$

$= \frac{5}{6}$

Hence, the fractions are $\frac{7}{8}, \frac{5}{6}$ and $\frac{3}{4}$.

17. (b) If the number of rows and columns are to be equal, then the total number of trees would represent a perfect square. Since, 1000 is not a perfect square, we need to check for a perfect square above and nearest to 1000. It's 1024, which is square of 32. So he needs 24 more trees to get 1024.

18. (a) $a^y = 19683$
$\Rightarrow 3^9 = 27^3 = (19683)^1 = 19683$
As y is a multiple of a, a = 3, y = 9
$b^x = 1024$
$\Rightarrow 2^{10} = 4^5 = 32^2 = (1024)^1 = 1024$
As b is a factor of x, b = 2, x = 10
Now, $x^a + y^b$
$\Rightarrow 10^3 + 9^2 = 1000 + 81 = 1081$

19. (a) $5^a = 3125 \Rightarrow 5^a = 5^5 \Rightarrow a = 5$
$\Rightarrow 5^{(a-3)} = 5^{(5-3)} = 5^2 = 25$

20. (b) $3 \div \left[(8-5) \div \left\{(4-2) \div \left(2 + \frac{8}{13}\right)\right\}\right]$

$\Rightarrow 3 \div \left[(3) \div \left(2 \div \frac{34}{13}\right)\right]$

$\Rightarrow 3 \div \left[(3) \div \left(2 \times \frac{13}{34}\right)\right]$

$\Rightarrow 3 \div \left[\frac{3 \times 34}{13 \times 2}\right]$

$\Rightarrow \frac{3 \times 13 \times 2}{3 \times 34} = \frac{13}{17}$

21. (d) $\frac{\left(\frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right)}{\left(\frac{2}{5} - \frac{5}{9} + \frac{3}{5} - \frac{7}{18}\right)} = \frac{\left(\frac{30-15+12-10}{60}\right)}{\left(\frac{2}{5} + \frac{3}{5}\right) - \left(\frac{5}{9} + \frac{7}{18}\right)} = \frac{\left(\frac{17}{60}\right)}{1 - \frac{17}{18}}$

$= \left(\frac{17}{60} \times 18\right) = \frac{51}{10} = 5\frac{1}{10}$

22. (d) Let $\frac{5}{3} \div \frac{2}{7} \times \frac{x}{7} = \frac{5}{4} \times \frac{2}{3} \div \frac{1}{6}$. Then,

$\frac{5}{3} \times \frac{7}{2} \times \frac{x}{7} = \frac{5}{4} \times \frac{2}{3} \times 6 \Leftrightarrow \frac{5}{6}x = 5' \Leftrightarrow x = \left(\frac{5 \times 6}{5}\right) = 6.$

23. (d) Given exp.$= 2.002 + 7.9\,(2.8 - 6.3 \times 2.1 + 15.6)$
$= 2.002 + 7.9\,(2.8 - 13.23 + 15.6) = 2.002 + 7.9 \times 5.17$
$= 2.002 + 40.843 = 42.845$

24. (b) Given exp. $= 9 - \frac{11}{9}$ of $\frac{36}{11} \div \frac{36}{7}$ of $\frac{7}{9}$ $= 9 - 4 \div 4$
$= 9 - 1 = 8$

25. (b) $\frac{\sqrt{24} + \sqrt{6}}{\sqrt{24} - \sqrt{6}} = \frac{2\sqrt{6} + \sqrt{6}}{2\sqrt{6} - \sqrt{6}} = \frac{3\sqrt{6}}{\sqrt{6}} = 3$

26. (b) $2^{57} = (2^3)^{19} = 8^{19}$
$4^{38} = (4^2)^{19} = 16^{19}$
$4^{38} > 15^{19} > 2^{57}$

27. (a) $2^{10000} = (2^{10})^{1000} = (1024)^{1000}$
$(10)^{3000} = (10^3)^{1000} = (1000)^{1000}$
$3^{6000} = (3^6)^{1000} = (729)^{1000}$
$7^{4000} = (7^4)^{1000} = (2401)^{1000}$
$3^{6000} < 10^{3000} < 2^{10000} < 7^{4000}$

28. (b) $\frac{3}{5} = 0.6, \frac{4}{9} = 0.44$

$\frac{1}{8} = 0.0125, \frac{2}{7} = 0.28$

$\frac{8}{11} = 0.727, \frac{5}{12} = 0.41$

therefore, the descending order is

$\frac{8}{11} > \frac{3}{5} > \frac{4}{9} > \frac{5}{12} > \frac{2}{7} > \frac{1}{8}$

So, the third fraction $= \frac{4}{9}$

29. (d) The smallest number is $\sqrt[3]{2}$

30. (d) Here, $\left(\sqrt{8} + \sqrt{5}\right)^2$

$= \left(\sqrt{8}\right)^2 + \left(\sqrt{5}\right)^2 + 2 \times \sqrt{8} \times \sqrt{5}$

$= 8 + 5 + 2 \times \sqrt{8 \times 5} = 13 + 2\sqrt{40}$

Similary,

$\left(\sqrt{7} + \sqrt{6}\right)^2 = 7 + 6 + 2 \times \sqrt{7 \times 6} = 13 + 2\sqrt{42}$

$\left(\sqrt{10} + \sqrt{3}\right)^2$

$= 10 + 3 + 2 \times \sqrt{10 \times 3} = 13 + 2\sqrt{30}, \left(\sqrt{11} + \sqrt{2}\right)^2$

$= 11 + 2 + 2\ 2\sqrt{11 \times 2} = 13 + 2\sqrt{22}$

Clearly, $13 + 2\sqrt{22}$ is the smallest among these.

$\therefore \sqrt{11} + \sqrt{2}$ is the smallest.

31. (c) $\frac{1}{\sqrt{2} + \sqrt{3} - \sqrt{5}}$

$= \frac{\sqrt{2} + \sqrt{3} + \sqrt{5}}{\left[\sqrt{2} + \sqrt{3} + \sqrt{5}\right]} = \frac{\sqrt{2} + \sqrt{3} + \sqrt{5}}{2 + 3 + 2\sqrt{6} - 5}$

$$= \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{2\sqrt{6}}$$

Similary, $\frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}$

$$= \frac{\sqrt{2}-\sqrt{3}+\sqrt{5}}{\left[\left(\sqrt{2}-\sqrt{3}\right)-\sqrt{5}\right]\left[\left(\sqrt{2}-\sqrt{3}\right)+\sqrt{5}\right]}$$

$$= \frac{\sqrt{2}-\sqrt{3}+\sqrt{5}}{-2\sqrt{6}}$$

∴ Expression

$$= \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{2\sqrt{6}} - \frac{\sqrt{2}-\sqrt{3}+\sqrt{5}}{2\sqrt{6}}$$

$$= \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}-\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}} = \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}}$$

32. (c) Expression

$$= \frac{\sqrt{2}\left(\sqrt{3}+1\right)\left(2-\sqrt{3}\right)}{\left(\sqrt{2}-1\right)\sqrt{2}\left(\sqrt{2}+1\right)\left(3\sqrt{3}-5\right)}$$

$$= \frac{\sqrt{2}\left(2\sqrt{3}-3+2-\sqrt{3}\right)}{\sqrt{2}\left(2-1\right)\left(3\sqrt{3}-5\right)}$$

$$= \frac{\sqrt{3}-1}{3\sqrt{3}-5} = \frac{\sqrt{3}-1}{\left(3\sqrt{3}-5\right)} \times \frac{3\sqrt{3}+5}{3\sqrt{3}+5}$$

$$= \frac{9-3\sqrt{3}+5\sqrt{3}-5}{27-25}$$

$$= \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

33. (a) Green portion $= \left[1-\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{30}+\frac{1}{40}+\frac{1}{50}+\frac{1}{60}\right)\right]$

$$= \left[1-\frac{1}{10}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)\right] = 1-\frac{1}{10}\times\frac{147}{60}$$

$$= 1-\frac{147}{600} = \frac{453}{600}$$

Let the length of the pole be x metres.

Then, $\frac{453}{600}x = 12.08 \Leftrightarrow x = \left(\frac{12.08\times 600}{453}\right) = 16$

34. (b) $3^{34} = (3^2)^{17} = 9^{17}$
$2^{51} = (2^3)^{17} = 8^{17}$
Clearly, $7^{17} > 8^{17} > 9^{17}$
or $7^{17} > 2^{51} > 3^{34}$

35. (a) $3.\overline{76} - 1.4\overline{576} = 3 + 0.\overline{76} - 1 - 0.4\overline{576}$

$$= 3+\left(\frac{76-0}{99}\right)-1-\left(\frac{4576-4}{9990}\right)$$

$$= 3+\frac{76}{99}-1-\frac{4572}{9990} = 2+\left(\frac{76}{99}-\frac{4572}{9990}\right)$$

$$= 2+\frac{1}{9}\left(\frac{76}{11}-\frac{4572}{1110}\right) = 2+\frac{1}{9}\times\frac{(84360-50292)}{12210}$$

$$= 2+\frac{1}{9}\times\frac{34068}{12210} = 2+\frac{11356}{36630}$$

$$= 2+0.3\overline{100191} = 2.3\overline{100191}$$

36. (b) $0.00\overline{7}+17.\overline{83}+310.020\overline{2}$

$$= \frac{7}{900}+\frac{1783-17}{99}+\frac{3100202-310020}{9000}$$

$$= \frac{7}{900}+\frac{1766}{99}+\frac{2790182}{9000}$$

$$= \frac{770+1766000+30692002}{99000}$$

$$= \frac{32458772}{99000} = 327.866\overline{38}$$

37. (b) Given that,

$$0.242424...\ ? = 0.\overline{24} = \frac{24}{99} = \frac{8}{33}$$

Write down as many 9's in the denominator as the number of digits in the period of decimal number.

38. (d) Let $x = 0.2\overline{341}$

Here multiply by 10 both sides,

$10x = 2.\overline{341}$ ....(i)

Now, multiply by 1000 both sides,

$10000x = 2341.\overline{341}$ ......(ii)

Now, substract equation (i) from equation (ii),

$9990x = 2341 - 2 = 2339$

$\therefore \quad x = \frac{2339}{9990}.$

- **Shortcut:**

$$0.2\overline{341} = \frac{2341-2}{9990} = \frac{2339}{9990}$$

39. (a) **Pure recurring decimal:-**

A decimal fraction in which all the figures occur repeatedly is called a pure recurring decimal as 7.4444 ..., 2.666 ...., etc.

Let P be prime number

So P = 7, 11, 13,...

$\frac{1}{7} = .142857142857....$

$\frac{1}{11} = .09090909....$

$\frac{1}{13} = 0.0769230769230...$

All above example are pure recurring decimal and its period will (p –1)

40. (d) $\sqrt{9-2\sqrt{14}} = \sqrt{7+2-2\times\sqrt{7}\times\sqrt{2}}$

$=\sqrt{\left(\sqrt{7}-\sqrt{2}\right)^2}=\sqrt{7}-\sqrt{2}$

41. (d) $\sqrt{\dfrac{0.324\times0.64\times129.6}{0.729\times1.024\times36}}$

$=\sqrt{\dfrac{324\times64\times1296}{729\times1024\times36}}$

$=\dfrac{18\times8\times36}{27\times32\times6}=1$

42. (d) We know that, $(42)^2 = 1764$ and $(43)^2 = 1849$
But 1780 lies between 1764 and 1849.
Now, smallest number = 1849 – 1780 = 69

43. (b) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$=\dfrac{\left(\sqrt{5}+\sqrt{3}\right)^2+\left(\sqrt{5}-\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$

$=\dfrac{2\left\{\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2\right\}}{5-3}=\dfrac{2(5+3)}{2}=8$

44. (a) equ $a^x = b$
Multiplying both sides by y in power
$(a^x)^y = b^y$

$\Rightarrow (a)^{x^y} = c \quad (\because b^y = c)$

Again multiplying both sides by z in power

$\left(a^{xy}\right)^z = c^z$

$\Rightarrow a^{xyz} = c^z$

But $xyz = 1$ Given
So $a = c^z$

45. (d) Given, $196x^4 = x^6$

$\Rightarrow \left(14x^2\right)^2 = \left(x^3\right)^2 \Rightarrow 14x^2 = x^3$

46. (b) Given that, $a = 2+\sqrt{3}$, $\dfrac{1}{a} = 2-\sqrt{3}$

Now, $a^2+a^{-2} = \left(a+\dfrac{1}{a}\right)^2-2$

$=\left(2+\sqrt{3}+2-\sqrt{3}\right)^2-2$

$=(4)^2-2=16-2=14$

47. (b) Given that $3^{x+y} = 81$ or $3^{x+y} = 3^4$

$\Rightarrow \quad x+y=4 \quad$ (i)

and $81^{x-y} = 3$ or $(3^4)^{x-y} = 3^1$

$\Rightarrow \quad x-y=\dfrac{1}{4} \quad$ (ii)

On solving eqs. (i) and (ii), we get

$x+y=4 \Rightarrow x-y=\dfrac{1}{4}$

$2x=\dfrac{17}{4} \Rightarrow x=\dfrac{17}{8}$

48. (b) Given, $\sqrt{10+\sqrt[3]{x}}=4$
On squaring both sides
$10+\sqrt[3]{x}=16$
$\Rightarrow \sqrt[3]{x}=6$
On cubic both sides
$x=(6)^3=216$

49. (b) Factor of 1024 = 32 × 32 = 32
So, 1024 is a perfect square number.

50. (b) Given that $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+...}}}}$

It is of the form $\sqrt{P+\sqrt{P+\sqrt{P+...}}}$

$=\dfrac{\sqrt{4P+1}+1}{2}$

Here P = 6, $\therefore \dfrac{\sqrt{4\times6+1}+1}{2}=\dfrac{5+1}{2}=3$

51. (d) Given that, $16\times8^{n+2}=2^m$
$\Rightarrow (2)^4\times2^{3(n+2)}=2^m$
$\Rightarrow (2)^{(4+3n+6)}=2^m$
$\Rightarrow 2^{(3n+10)}=2^m$
Here base is same, so
$3n+10=m$
$\Rightarrow m=3n+10$

52. (d) Let $\dfrac{a}{b}=\left[\left(\sqrt{2}\right)^{\sqrt{2}}\right]^{\sqrt{2}}$

Squaring both sides

$\dfrac{a^2}{b^2}=\left[\left(\sqrt{2}\right)^{\sqrt{2}}\right]^2$

$\Rightarrow \dfrac{a^2}{b^2}=\left(\sqrt{2}\right)^2$

$\Rightarrow \dfrac{a^2}{b^2}=2 \Rightarrow \dfrac{a}{b}=\sqrt{2}$

So $\sqrt{2}$ is a real number but not a rational number.

53. (c) From option (a) $\left[\left(5^{-2}\right)^{-2}\right]^{-2} = 5^{-8} = \frac{1}{5^8}$

From option (b) $\left[\left(5^{-2}\right)^{2}\right]^{-2} = 5^{8} = 5^{8}$

From option (c) $\left[\left(2^{-5}\right)^{-2}\right]^{-2} = 5^{-20} = \frac{1}{5^{20}}$

From option (d) $\left[\left(2^{-5}\right)^{2}\right]^{-2} = 5^{20} = 5^{20}$

Now, smallest number $\left[\left(2^{-5}\right)^{-2}\right]^{-2}$

54. (d) LCM of 2, 3 and 6 = 12

Now, $\sqrt{2} = 2^{\frac{1}{2}\times\frac{12}{12}} = 12\sqrt{2^6} = \sqrt[12]{64}$

$\sqrt[3]{3} = 3^{\frac{1}{3}\times\frac{12}{12}} = 12\sqrt{3^4} = \sqrt[12]{81}$

$\sqrt[6]{6} = 6^{\frac{1}{6}\times\frac{12}{12}} = \sqrt[12]{6^2} = \sqrt[12]{36}$

So, $\sqrt{2}$ is not smallest and $\sqrt[6]{6}$ is not greatest. So neither I nor II correct.

55. (d) $\frac{(.75)^3}{1-0.75} + [.75 + (.75)^2 + 1]$

$= \frac{(.75)^3 + (1-.75)(.75+(.75)^2+1)}{(1-.75)}$

$= \frac{(.75)^3 + (1)^3 - (.75)^3}{.25}$

$= \frac{1}{.25}\times 1 = 4$

56. (d) $\frac{\sqrt{0.0032}}{\sqrt{0.32}} = \frac{\sqrt{0.32}}{\sqrt{100}}\times\frac{1}{\sqrt{0.32}} = \frac{1}{10} = 0.1$

57. (c) $0.\overline{63} = 0.636363$ ------

Let $x = 0.636363$ ------

$\Rightarrow 100x = 63.6363$ ------

$\Rightarrow 99x = 63$

$\Rightarrow x = \frac{63}{99}$

$0.\overline{37} = 0.373737$ ------

Let $y = 0.37\ 37\ 37$ ------

$\Rightarrow 100y = 37.37\ 37$ ------

$\Rightarrow 99y = 37$

$\Rightarrow y = \frac{37}{99}$

$x + y = \frac{63}{99} + \frac{37}{99} = \frac{100}{99}$

So, option (c) is correct.

58. (a) $x = \frac{91}{216}$

$\Rightarrow 3 - \frac{1}{(1-x)^{\frac{1}{3}}}$

$\Rightarrow 3 - \frac{1}{\left(1-\frac{91}{216}\right)^{\frac{1}{3}}}$

$\Rightarrow 3 - \frac{1}{\left(\frac{125}{216}\right)^{\frac{1}{3}}}$

$\Rightarrow 3 - \frac{1}{\left(\frac{5}{6}\right)} = 3 - \frac{6}{5} = \frac{15-6}{5} = \frac{9}{5}$

So, option (a) is correct.

59. (b) Given series is-

$\frac{1}{4}+\frac{1}{5}-\frac{1}{6}-\frac{1}{4}-\frac{1}{5}+\frac{1}{6}+\frac{1}{4}+\frac{1}{5}-\frac{1}{6}-\frac{1}{4}-\frac{1}{5}+\frac{1}{6}+----$

These 6 terms are repeating which are resulting to zero.

so 1st 42 terms of this series will result is zero and after that series will be upto 47 terms –

$\Rightarrow \frac{1}{4}+\frac{1}{5}-\frac{1}{6}-\frac{1}{4}-\frac{1}{5}$

$\Rightarrow -\frac{1}{6}$

So, option (b) is correct.

60. (b) $x = \sqrt{3}+\sqrt{2}$

$\frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}$

$= \sqrt{3}-\sqrt{2}$

$\Rightarrow x^3 + x + \frac{1}{x} + \frac{1}{x^3}$

$= x + \frac{1}{x} + x^3 + \frac{1}{x^3}$

$= \left(x+\frac{1}{x}\right) + \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$= \left(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}\right) + \left(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}\right)^3$
$-3\left(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}\right)$

$= 2\sqrt{3} + \left(2\sqrt{3}\right)^3 - 3\left(2\sqrt{3}\right) = 2\sqrt{3} + 24\sqrt{3} - 6\sqrt{3}$

$= 20\sqrt{3}$

So, option (b) is correct