MULTIPLE AND SUBMULTIPLE ANGLES

- $\sin 2A = 2\sin A\cos A = \dfrac{2\tan A}{1+\tan^2 A} = \dfrac{2\cot A}{1+\cot^2 A}$
  - $\sin A = 2\sin\dfrac{A}{2}\cos\dfrac{A}{2} = \dfrac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}} = \dfrac{2\cot\frac{A}{2}}{1+\cot^2\frac{A}{2}}$
- $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \dfrac{1-\tan^2 A}{1+\tan^2 A} = \dfrac{\cot^2 A - 1}{\cot^2 A + 1}$
  - $\cos A = \cos^2\dfrac{A}{2} - \sin^2\dfrac{A}{2} = 2\cos^2\dfrac{A}{2} - 1 = 1 - 2\sin^2\dfrac{A}{2}$

    $= \dfrac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}} = \dfrac{\cot^2\frac{A}{2}-1}{\cot^2\frac{A}{2}+1}$
- $\sqrt{1+\sin 2A} = \pm(\cos A + \sin A)$

  $\Rightarrow \cos A + \sin A = \pm\sqrt{1+\sin 2A}$
  - $\sqrt{1-\sin 2A} = \pm(\cos A - \sin A)$

    $\Rightarrow \cos A - \sin A = \pm\sqrt{1-\sin 2A}$
  - $\sqrt{1+\sin A} = \pm\left(\cos\dfrac{A}{2} + \sin\dfrac{A}{2}\right)$

    $\Rightarrow \cos\dfrac{A}{2} + \sin\dfrac{A}{2} = \pm\sqrt{1+\sin A}$
  - $\sqrt{1-\sin A} = \pm\left(\cos\dfrac{A}{2} - \sin\dfrac{A}{2}\right)$

    $\Rightarrow \cos\dfrac{A}{2} - \sin\dfrac{A}{2} = \pm\sqrt{1-\sin A}$
- $\sqrt{\dfrac{1+\cos 2A}{2}} = \pm\cos A$

  $\Rightarrow \cos A = \pm\sqrt{\dfrac{1+\cos 2A}{2}}$
  - $\sqrt{\dfrac{1+\cos A}{2}} = \pm\cos\dfrac{A}{2}$

    $\Rightarrow \cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$
  - $\sqrt{\dfrac{1-\cos 2A}{2}} = \pm\sin A$

    $\Rightarrow \sin A = \pm\sqrt{\dfrac{1-\cos 2A}{2}}$
  - $\sqrt{\dfrac{1-\cos A}{2}} = \pm\sin\dfrac{A}{2}$

    $\Rightarrow \sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}$
- $\tan\left(\dfrac{\pi}{4}+\dfrac{A}{2}\right) = \dfrac{1+\tan\frac{A}{2}}{1-\tan\frac{A}{2}} = \dfrac{\cot\frac{A}{2}+1}{\cot\frac{A}{2}-1}$

  $= \dfrac{\cos\frac{A}{2}+\sin\frac{A}{2}}{\cos\frac{A}{2}-\sin\frac{A}{2}} = \sqrt{\dfrac{1+\sin A}{1-\sin A}}$

  $= \sqrt{\dfrac{\operatorname{cosec} A+1}{\operatorname{cosec} A-1}} = \dfrac{1+\sin A}{\cos A} = \dfrac{\cos A}{1-\sin A}$

  $= \dfrac{\operatorname{cosec} A+1}{\cot A} = \dfrac{\cot A}{\operatorname{cosec} A-1}$

  $= \sec A + \tan A = \cot\left(\dfrac{\pi}{4}-\dfrac{A}{2}\right)$
- $\tan\left(\dfrac{\pi}{4}-\dfrac{A}{2}\right) = \dfrac{1-\tan\frac{A}{2}}{1+\tan\frac{A}{2}} = \dfrac{\cot\frac{A}{2}-1}{\cot\frac{A}{2}+1}$

  $= \dfrac{\cos\frac{A}{2}-\sin\frac{A}{2}}{\cos\frac{A}{2}+\sin\frac{A}{2}} = \sqrt{\dfrac{1-\sin A}{1+\sin A}}$

  $= \sqrt{\dfrac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}} = \dfrac{1-\sin A}{\cos A} = \dfrac{\cos A}{1+\sin A}$

  $= \dfrac{\operatorname{cosec} A-1}{\cot A} = \dfrac{\cot A}{\operatorname{cosec} A+1}$

  $= \sec A - \tan A = \cot\left(\dfrac{\pi}{4}+\dfrac{A}{2}\right)$

- $\tan\frac{A}{2} = \operatorname{cosec} A - \cot A = \frac{1-\cos A}{\sin A}$

$$= \frac{\sin A}{1+\cos A} = \sqrt{\frac{1-\cos A}{1+\cos A}} = \sqrt{\frac{\sec A - 1}{\sec A + 1}}$$

$$= \frac{\sec A - 1}{\tan A} = \frac{\tan A}{\sec A + 1}$$

- $\cot\frac{A}{2} = \operatorname{cosec} A + \cot A = \frac{1+\cos A}{\sin A}$

$$= \frac{\sin A}{1-\cos A} = \sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{\sec A + 1}{\sec A - 1}}$$

$$= \frac{\sec A + 1}{\tan A} = \frac{\tan A}{\sec A - 1}$$

- • $\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\cot A}{\cot^2 A - 1}$

  • $\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}} = \frac{2\cot\frac{A}{2}}{\cot^2\frac{A}{2} - 1}$

$$= \sqrt{\frac{1-\cos 2A}{1+\cos 2A}} = \sqrt{\frac{\sec 2A - 1}{\sec 2A + 1}} = \frac{\sec 2A - 1}{\tan 2A}$$

$$= \frac{\tan 2A}{\sec 2A + 1} = \frac{1-\cos 2A}{\sin 2A}$$

$$= \operatorname{cosec} 2A - \cot 2A = \frac{\sin 2A}{1+\cos 2A}$$

  • $\cot 2A = \frac{\cot^2 A - 1}{2\cot A}$

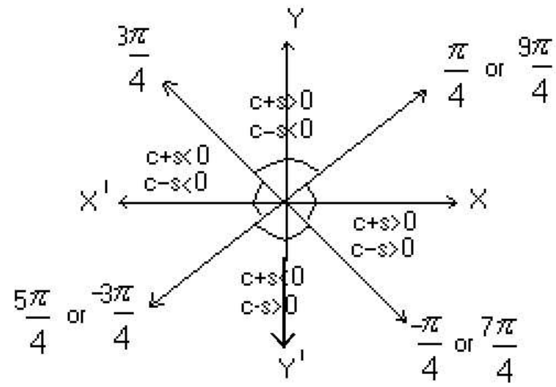
  • $\cot A = \frac{\cot^2\frac{A}{2} - 1}{2\cot\frac{A}{2}} = \sqrt{\frac{1+\cos 2A}{1-\cos 2A}} = \sqrt{\frac{\sec 2A + 1}{\sec 2A - 1}}$

$$= \frac{\sec 2A + 1}{\tan 2A} = \frac{\tan 2A}{\sec 2A - 1} = \frac{1+\cos 2A}{\sin 2A}$$

$$= \frac{\sin 2A}{1-\cos 2A} = \operatorname{cosec} 2A + \cot 2A$$

- $\frac{\tan A + \tan B}{\tan A - \tan B} = \frac{\sin(A+B)}{\sin(A-B)}$

- $C = \cos\frac{A}{2};\ S = \sin\frac{A}{2}$



$$c + s > 0\ in\ \left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$$

$$c + s < 0\ in\ \left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$$

$$c - s > 0\ in\ \left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$$

$$c - s < 0\ in\ \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$$

- • cot A + tan A = 2 cosec 2A
  • cot A - tan A = 2 cot 2A
- tan A + 2 tan 2A + ...+ 2^{n-1} tan 2^{n-1} A + 2^n cot 2^n A = cot A (or)
  cot A - tan A - 2 tan 2A........... -2^{n-1} tan 2^{n-1} A = 2^n cot 2^n A
- • sin 3A = 3 sin A - 4 $\sin^3$ A
  • cos 3A = 4 $\cos^3$A - 3 cos A

  • $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$,

  $A, 3A \neq (2n+1)\frac{\pi}{2},\ n \in N$

  • $\cot 3A = \frac{3\cot A - \cot^3 A}{1 - 3\cot^2 A},\ A, 3A \neq n\pi, n \in N$

- $\sin\theta.\sin(\alpha-\theta).\sin(\alpha+\theta) = \frac{1}{4}\sin 3\theta$ where
  $\alpha =$ 60^0 (or) 120^0 (or) 240^0 (or) 300^0

- $\cos\theta.\cos(\alpha-\theta).\cos(\alpha+\theta)=\frac{1}{4}\cos 3\theta$ where $\alpha = 60^0$(or) 120^0 (or) 240^0 (or) 300^0
- $\left.\begin{array}{l}\sin^2\theta+\sin^2(\alpha-\theta)+\sin^2(\alpha+\theta)\\ \cos^2\theta+\cos^2(\alpha-\theta)+\cos^2(\alpha+\theta)\end{array}\right\} = \frac{3}{2}$

  where $\alpha = 60^0$(or)120^0(or)240^0(or) 300^0
- $\left.\begin{array}{l}\sin^3\theta+\sin^3(60^0-\theta)-\sin^3(60^0+\theta)\\ \sin^3\theta-\sin^3(120^0-\theta)+\sin^3(120^0+\theta)\\ \sin^3\theta-\sin^3(240^0-\theta)+\sin^3(240^0+\theta)\\ \sin^3\theta+\sin^3(300^0-\theta)-\sin^3(300^0+\theta)\end{array}\right\} = -\frac{3}{4}\sin 3\theta$
- $\left.\begin{array}{l}\cos^3\theta-\cos^3(60^0-\theta)-\cos^3(60^0+\theta)\\ \cos^3\theta+\cos^3(120^0-\theta)+\cos^3(120^0+\theta)\\ \cos^3\theta+\cos^3(240^0-\theta)+\cos^3(240^0+\theta)\\ \cos^3\theta-\cos^3(300^0-\theta)-\cos^3(300^0+\theta)\end{array}\right\} = \frac{3}{4}\cos 3\theta$
- $\left.\begin{array}{l}\sin^3\theta-\sin^3(60^0+\theta)+\sin^3(120^0+\theta)\\ \sin^3\theta+\sin^3(120^0+\theta)+\sin^3(240^0+\theta)\\ \sin^3\theta+\sin^3(240^0+\theta)-\sin^3(300^0+\theta)\\ \sin^3\theta-\sin^3(60^0+\theta)-\sin^3(300^0+\theta)\end{array}\right\}$

  $= -\frac{3}{4}\sin 3\theta$
- $\left.\begin{array}{l}\cos^3\theta-\cos^3(60^0+\theta)+\cos^3(120^0+\theta)\\ \cos^3\theta+\cos^3(120^0+\theta)+\cos^3(240^0+\theta)\\ \cos^3\theta+\cos^3(240^0-\theta)-\cos^3(300^0+\theta)\\ \cos^3\theta-\cos^3(60^0+\theta)-\cos^3(300^0+\theta)\end{array}\right\} = \frac{3}{4}\cos 3\theta$
- $\tan\theta.\tan(\alpha-\theta).\tan(\alpha+\theta)=\tan 3\theta$ where $\alpha = 60^0$ (or) 120^0 (or) 240^0 (or) 300^0
- $\cot\theta.\cot(\alpha-\theta).\cot(\alpha+\theta)=\cot 3\theta$ where $\alpha = 60^0$ (or) 120^0 (or) 240^0 (or) 300^0
- • $\tan\theta+\tan(\theta-\alpha)+\tan(\theta+\alpha)=3\tan\theta$ where $\alpha = 60^0$ (or) 120^0 (or) 240^0 (or) 300^0

  ii) $\left.\begin{array}{l}\tan\theta+\tan(60^0+\theta)+\tan(120^0+\theta)\\ \tan\theta+\tan(120^0+\theta)+\tan(240^0+\theta)\\ \tan\theta+\tan(240^0+\theta)+\tan(300^0+\theta)\\ \tan\theta+\tan(60^0+\theta)+\tan(300^0+\theta)\end{array}\right\}$

  $= 3\tan 3\theta$
- $\cot\theta+\cot(\theta-\alpha)+\cot(\theta+\alpha)=3\cot\theta$ where $\alpha = 60^0$ (or) 120^0 (or) 240^0 (or) 300^0
- $\left.\begin{array}{l}\cot\theta+\cot(60^0+\theta)+\cot(120^0+\theta)\\ \cot\theta+\cot(120^0+\theta)+\cot(240^0+\theta)\\ \cot\theta+\cot(240^0+\theta)+\cot(300^0+\theta)\\ \cot\theta+\cot(60^0+\theta)+\cot(300^0+\theta)\end{array}\right\} = 3\cot 3\theta$
- α, β are the solutions of $a\cos\theta+b\sin\theta=c \Rightarrow$
  - $\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{b}{a}$
  - $\sin(\alpha+\beta)=\frac{2ab}{a^2+b^2}$
  - $\cos(\alpha+\beta)=\frac{a^2-b^2}{a^2+b^2}$
  - $\tan(\alpha+\beta)=\frac{2ab}{a^2-b^2}$
  - $\sin\alpha+\sin\beta=\frac{2bc}{a^2+b^2}$
  - $\sin\alpha.\sin\beta=\frac{c^2-a^2}{a^2+b^2}$
  - $\cos\alpha+\cos\beta=\frac{2ca}{a^2+b^2}$
  - $\cos\alpha\cos\beta=\frac{c^2-b^2}{a^2+b^2}$
  - $\tan\alpha+\tan\beta=\frac{2ab}{c^2-b^2}$
  - $\tan\alpha\tan\beta=\frac{c^2-a^2}{c^2-b^2}$

- $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}=\frac{2b}{c+a}$
- $\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}=\frac{c-a}{c+a}$
- $\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}=\frac{2b}{c-a}$

- If n is odd, $n\neq 1$, then the sides

  n, $\frac{n^2-1}{2},\ \frac{n^2+1}{2}$

  If n is even, $n\neq 2$, then the sides

  $n,\ \frac{n^2}{4}-1,\ \frac{n^2}{4}+1$ form right angled triangles

- $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a+b}{a-b}=\frac{c+d}{c-d}$
- $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}\Rightarrow\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{a+c+e}{b+d+f}$

  $=\frac{a+c}{b+d}=\frac{c+e}{d+f}=\frac{a+e}{b+f}=\frac{a-c}{b-d}=\frac{c-e}{d-f}$

  $=\frac{a-e}{b-f}$

- $\sin 22\frac{1}{2}^0=\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}=\frac{1}{2}\sqrt{2-\sqrt{2}}=\cos 67\frac{1}{2}^0$
- $\cos 22\frac{1}{2}^0=\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}=\frac{1}{2}\sqrt{2+\sqrt{2}}=\sin 67\frac{1}{2}^0$
- $\tan 22\frac{1}{2}^0=\sqrt{2}-1=\cot 67\frac{1}{2}^0$
- $\cot 22\frac{1}{2}^0=\sqrt{2}+1=\tan 67\frac{1}{2}^0$

- $\sin 18^0=\frac{\sqrt{5}-1}{4}=\cos 72^0$
- $\cos 36^0=\frac{\sqrt{5}+1}{4}=\sin 54^0$
- $\cos 18^0=\frac{\sqrt{10+2\sqrt{5}}}{4}=\sin 72^0$
- $\sin 36^0=\frac{\sqrt{10-2\sqrt{5}}}{4}=\cos 54^0$

- $\sin 7\frac{1}{2}^0=\sqrt{\frac{2\sqrt{2}-\sqrt{3}-1}{4\sqrt{2}}}=\frac{\sqrt{4-\sqrt{6}-\sqrt{2}}}{2\sqrt{2}}$

  $=\cos 82\frac{1}{2}^0$

- $\cos 7\frac{1}{2}^0=\sqrt{\frac{2\sqrt{2}+\sqrt{3}+1}{4\sqrt{2}}}=\frac{\sqrt{4+\sqrt{6}+\sqrt{2}}}{2\sqrt{2}}$

  $=\sin 82\frac{1}{2}^0$

- $\tan 7\frac{1}{2}^0=\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{2}-1\right)=\cot 82\frac{1}{2}^0$
- $\tan 37\frac{1}{2}^0=\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{2}+1\right)=\cot 57\frac{1}{2}^0$
- $\tan 52\frac{1}{2}^0=\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{2}-1\right)=\cot 37\frac{1}{2}^0$
- $\tan 82\frac{1}{2}^0=\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{2}+1\right)=\cot 7\frac{1}{2}^0$

- $\tan 18^0=\sqrt{4\sqrt{5}-8}$ ii) $\tan 36^0=\sqrt{5-2\sqrt{5}}$

LEVEL-I

1. $\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=$

   1. $\tan\frac{\theta}{2}$ 2. $\cot\frac{\theta}{2}$ 3. $\sec\frac{\theta}{2}$ 4. $\operatorname{cosec}\frac{\theta}{2}$

2. $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}=$

   1. $\tan\frac{\theta}{2}$ 2. $\cot\frac{\theta}{2}$ 3. $\tan\theta$ 4. $\cot\theta$

3. $\frac{\cos A-\cos 3A}{\cos A}+\frac{\sin A+\sin 3A}{\sin A}=$

   1. 1 2. 2 3. 3 4. 4

4. $\cos^3 A\sin 3A+\sin^3 A\cos 3A=k\sin 4A\Rightarrow k=$

   1. $\frac{1}{4}$ 2. $\frac{3}{4}$ 3. $\frac{1}{2}$ 4. $\frac{5}{2}$

5. $\cos^6 A+\sin^6 A=1-k\sin^2(2A)\Rightarrow k=$

   1. $\frac{1}{4}$ 2. $\frac{1}{2}$ 3. $\frac{3}{4}$ 4. 1

6. $\frac{1}{\tan 3A-\tan A}-\frac{1}{\cot 3A-\cot A}=k\cot 2A \Rightarrow k=$

1. 4 2. 3 3. 2 4. 1

7. $\tan A=\frac{1-\cos B}{\sin B} \Rightarrow \tan 2A-\tan B=$

1. 0 2. 1 3. 1/2 4. 1/4

8. $\cos^3 10^0+\cos^3 110^0+\cos^3 130^0=$

1. $\frac{3}{4}$ 2. $\frac{3}{8}$ 3. $\frac{3\sqrt{3}}{8}$ 4. $\frac{3\sqrt{3}}{4}$

9. $\cos^2 25^0+\cos^2 95^0+\cos^2 145^0=$

1. $\frac{1}{2}$ 2. $\frac{3}{2}$ 3. $\frac{3}{4}$ 4. $\frac{1}{\sqrt{2}}$

10. $\cos^2 10^0+\cos^2 50^0+\cos^2 70^0=$

1. $\frac{1}{2}$ 2. 1 3. $\frac{3}{2}$ 4. 2

11. $\sin^2 160^0+\sin^2 140^0+\sin^2 100^0=$

1. $\frac{1}{2}$ 2. $\frac{3}{2}$ 3. $\frac{5}{2}$ 4. $\frac{7}{2}$

12. $x^2+y^2=1 \Rightarrow (3x-4x^3)^2+(3y-4y^3)^2=$

1. 1 2. 2 3. 3 4. 4

13. $(\cos\alpha-\cos\beta)^2+(\sin\alpha-\sin\beta)^2=$

$k\sin^2\left(\frac{\alpha-\beta}{2}\right) \Rightarrow k=$

1. 4 2. 3 3. $\frac{3}{2}$ 4. $\frac{1}{4}$

14. $(\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2=$

$k\cos^2\left(\frac{\alpha-\beta}{2}\right) \Rightarrow k=$

1. 4 2. 3 3. 2 4. 1

15. $\frac{\sin A+\sin 3A+\sin 5A+\sin 7A}{\cos A+\cos 3A+\cos 5A+\cos 7A}=$

$\tan x \Rightarrow x=$

1. 4A 2. 3A 3. 2A 4. A

16. $x=\cos A+\cos 2A+\cos 3A$

$y=\sin A+\sin 2A+\sin 3A \Rightarrow \frac{x}{y}=$

1. cot A 2. cot 2A 3. cot 3A 4. cot 4A

17. $\frac{\cos 6A+6\cos 4A+15\cos 2A+10}{\cos 5A+5\cos 3A+10\cos A}=$

$k\cos A \Rightarrow k=$

1. 4 2. 3 3. 2 4. 1

18. $x=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \Rightarrow \frac{2x}{1-x^2}=$

1. $\sin\theta$ 2. $\cos\theta$ 3. $\tan\theta$ 4. $\cot\theta$

19. $\frac{\cos^3 21^0+\cos^3 39^0}{\cos 21^0+\cos 39^0}=$

1. $\frac{3}{2}$ 2. $\frac{2}{3}$ 3. $\frac{3}{4}$ 4. $\frac{4}{3}$

20. $\tan(45^0+A)+\tan(45^0-A)=$

1. 2 cosec 2A 2. 2 sec 2A
3. 2 tan 2A 4. 2 cot 2A

21. $\sec(45^0+A)\sec(45^0-A)=$

1. sec 2A 2. cos 2A
3. 2 cos 2A 4. 2 sec 2A

22. $\tan B+\cot B=2\sec(2A) \Rightarrow A+B=$

1. $\frac{\pi}{2}$ 2. $\frac{\pi}{3}$ 3. $\frac{\pi}{6}$ 4. $\frac{\pi}{4}$

23. $2A+B=\frac{\pi}{2} \Rightarrow \sqrt{\frac{1+\sin B}{1-\sin B}}$

1. tan A 2. cot A 3. tan B 4. cot B

24. $\frac{\cos^2\left(\frac{\pi}{4}-A\right)-\sin^2\left(\frac{\pi}{4}-A\right)}{\cos^2\left(\frac{\pi}{4}+A\right)+\sin^2\left(\frac{\pi}{4}+A\right)}$

1. cos 2A 2. tan 2A 3. sin 2A 4. cot 2A

25. $\frac{\cos A+\sin A}{\cos A-\sin A}-\frac{\cos A-\sin A}{\cos A+\sin A}=$

1. 2 tan 2A 2. 2 cosec 2A
3. 2 cot 2A 4. 2 sec 2A

26. $\frac{\sin 12A}{\sin 4A}-\frac{\cos 12A}{\cos 4A}=$

1. 6 2. 4 3. 2 4. 1

27. $\frac{3\cos\theta+\cos 3\theta}{3\sin\theta-\sin 3\theta}=$

1. $\operatorname{cosec}^2\theta$ 2. $\cot^4\theta$
3. $\cot^3\theta$ 4. $2\cot\theta$

28. $\theta<\frac{\pi}{16}, \sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=k\cos\theta$

$\Rightarrow k=$

1. 2 2. 4 3. 8 4. 16

29. $8\sin\theta\cos\theta.\cos 2\theta\cos 4\theta=\sin x\Rightarrow x=$

1. 16θ 2. 8θ 3. 4θ 4. 32θ

30. $\frac{\cos^3 40^0+\cos^3 20^0}{\cos 40^0+\cos 20^0}=$

1. $\frac{1}{4}$ 2. $\frac{1}{2}$ 3. $\frac{3}{4}$ 4. 1

31. $\begin{vmatrix}\sin^2 13^0 & \sin^2 77 & \tan 135^0\\ \sin^2 77^0 & \tan 135^0 & \sin^2 13^0\\ \tan 135^0 & \sin^2 13^0 & \sin^2 77^0\end{vmatrix}=$

1. 1 2. -1 3. 0 4. 2

32. $\cos\theta=\frac{1}{2}\left(a+\frac{1}{a}\right)$ then $\cos 3\theta=K\left(a^3+\frac{1}{a^3}\right)$ where K is equal to

1. $\frac{1}{2}$ 2. $-\frac{1}{2}$ 3. 1 4. $\frac{3}{2}$

33. which of the following is rational number

1. $\sin 15^0$ 2. $\cos 15^0$
3. $\sin 15^0\cos 15^0$ 4. $\sin 15^0\cos 75^0$

34. 3 sin 10^0 - 4 $\sin^3 10^0$ =

1. 0 2. $\frac{1}{\sqrt{2}}$ 3. $\frac{\sqrt{3}}{2}$ 4. $\frac{1}{2}$

35. 3 sin 6^0 - 4 $\sin^3 6^0$ =

1. $\frac{\sqrt{5}-1}{4}$ 2. $\frac{\sqrt{5}+1}{4}$
3. $\frac{\sqrt{3}}{2}$ 4. $\frac{\sqrt{10-2\sqrt{5}}}{4}$

36. 4 $\cos^3 40^0$ - 3 sin 50^0 =

1. $\frac{1}{2}$ 2. $\frac{1}{\sqrt{2}}$ 3. $\frac{-\sqrt{3}}{2}$ 4. $\frac{-1}{2}$

37. 4 $\cos^3 15^0$ - 3 cos 15^0 =

1. $\frac{1}{2}$ 2. $\frac{1}{\sqrt{2}}$ 3. $\frac{1}{\sqrt{3}}$ 4. $\sqrt{3}$

38. $\sin^2 24^0$ - $\sin^2 6^0$ =

1. $\frac{\sqrt{5}+1}{8}$ 2. $\frac{\sqrt{5}-1}{8}$ 3. $\frac{\sqrt{5}+2}{8}$ 4. $\frac{\sqrt{5}-2}{8}$

39. $\cos^2 72^0$ - $\sin^2 54^0$ =

1. $\frac{\sqrt{5}}{2}$ 2. $\frac{-\sqrt{3}}{4}$ 3. $\frac{-\sqrt{5}}{4}$ 4. $\frac{\sqrt{5}}{8}$

40. $\cos\frac{\pi}{5}+\cos\frac{3\pi}{5}=$

1. $\sqrt{5}$ 2. 2 3. $\frac{1}{2}$ 4. $\frac{1}{4}$

41. $\sin\frac{\pi}{10}\sin\frac{13\pi}{10}=$

1. $\frac{1}{2}$ 2. $\frac{-1}{2}$ 3. $\frac{-1}{4}$ 4. 1

42. sec 72^0 - sec 36^0 =

1. 2 2. $\frac{1}{2}$ 3. 4 4. $\frac{1}{4}$

43. $\cos^2\frac{\pi}{12}+\cos^2\frac{\pi}{4}+\cos^2\frac{5\pi}{12}=$

1. 1 2. $\frac{1}{2}$ 3. $\frac{3}{2}$ 4. $\frac{1}{4}$

44. If $\tan\beta=2\sin\alpha\sin\gamma\operatorname{cosec}(\alpha+\gamma)$, then $\cot\alpha,\cot\beta,\cot\gamma$ are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

45. If $180^0<\theta<270^0, \cot\theta=\frac{4}{3}$, then $\sin\frac{\theta}{2}=$

1) $\frac{3}{\sqrt{10}}$ 2) $-\frac{2}{\sqrt{5}}$ 3) $-\frac{1}{\sqrt{5}}$ 4) $\frac{1}{\sqrt{5}}$

46. If $180^0 < \theta < 270^0, \cos\theta = -\frac{2}{3}$, then $\tan\frac{\theta}{2} =$

1) $\frac{\sqrt{5}}{6}$ 2) $-\sqrt{5}$ 3) $-\frac{1}{\sqrt{6}}$ 4) $\frac{1}{\sqrt{6}}$

47. If $180^0 < \theta < 270^0, \sin\theta = -\frac{3}{5}$, then $\cos\frac{\theta}{2} =$

1) $\frac{-1}{\sqrt{10}}$ 2) $\frac{1}{\sqrt{10}}$ 3) $\frac{1}{10}$ 4) 10

48. If $90^0 < \theta < 180^0, \sin\theta = \frac{3}{5}$, then $\sin 3\theta =$

1) $\frac{117}{125}$ 2) $\frac{-117}{125}$ 3) $\frac{-125}{117}$ 4) $\frac{125}{117}$

49. If $90^0 < \theta < 180^0, \cos\theta = -\frac{12}{13}$, then $\sin 2\theta =$

1) $\frac{120}{169}$ 2) $\frac{-120}{169}$ 3) $\frac{169}{120}$ 4) $\frac{-169}{120}$

50. If $\frac{\sin\alpha}{a} = \frac{\cos\alpha}{b}$, then $a\sin 2\alpha + b\cos 2\alpha =$

1) a 2) b 3) a + b 4) 0

51. $\cos\left(\frac{2\pi}{2n+1}\right)+\cos\left(\frac{4\pi}{2n+1}\right)+\cos\left(\frac{6\pi}{2n+1}\right)+....+\cos\left(\frac{2n\pi}{2n+1}\right)=$

1. $\frac{1}{2}$ 2. $-\frac{1}{2}$ 3. $\frac{3}{2}$ 4. $-\frac{3}{2}$

52. $\cos\left(\frac{\pi}{2n+1}\right)+\cos\left(\frac{3\pi}{2n+1}\right)+\cos\left(\frac{5\pi}{2n+1}\right)+....+\cos\left(\frac{(2n-1)\pi}{2n+1}\right)=$

1. $\frac{1}{2}$ 2. $-\frac{1}{2}$ 3. $\frac{3}{2}$ 4. $-\frac{3}{2}$

53. $4\sin 23^0 \sin 37^0 \sin 83^0 =$

1) $\cos 21^0$ 2) $\sin 21^0$
3) $\sin 79^0$ 4) $\cos 79^0$

KEY

1. 1	2. 3	3. 4	4. 2
5. 3	6. 4	7. 1	8. 3
9. 2	10. 3	11. 2	12. 1
13. 1	14. 1	15. 1	16.2
17. 3	18. 3	19. 3	20. 2
21. 4	22. 4	23. 2	24. 3
25. 1	26. 3	27. 3	28. 1
29. 2	30. 3	31. 3	32. 1
33. 3	34. 4	35. 1	36. 4
37. 2	38. 2	39. 3	40. 3
41. 3	42. 1	43. 3	44. 1
45. 1	46. 2	47. 1	48. 1
49. 2	50. 2	51. 2	52. 2
53. 1			

HINTS & SOLUTIONS

7. $A = \frac{B}{2}$

15. $\tan x = \tan 4A \Rightarrow x = 4A$

19. Apply formula $\cos^3\theta = \frac{\cos 3\theta + 3\cos\theta}{4}$

22. $\operatorname{cosec} 2B = \sec 2A \Rightarrow A + B = \frac{\pi}{4}$

31. $C_1 \to C_1 + C_2 + C_3$

32. $a = \cos\theta + i\sin\theta$

44. $\cot\beta = \frac{\sin(\alpha+\gamma)}{2\sin\alpha\sin\gamma} \Rightarrow 2\cot\beta = \cot\gamma + \cot\alpha$

53. Apply formula

$\sin\theta\sin\left(60^0+\theta\right)\sin\left(60^0-\theta\right) = \frac{1}{4}\sin 3\theta$

LEVEL - II

1. $\tan 9^0 - \tan 27^0 - \tan 63^0 + \tan 81^0 =$

1. 0 2. 2 3. 1 4. 4

2. $\tan\theta = \frac{b}{a} \Rightarrow \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} =$

1. $\frac{2\sin\theta}{\sqrt{\sin 2\theta}}$ 2. $\frac{2\cos\theta}{\sqrt{\cos 2\theta}}$

3. $\frac{2\cos\theta}{\sqrt{\sin 2\theta}}$ 4. $\frac{2\sin\theta}{\sqrt{\cos 2\theta}}$

3. If $\alpha, \beta, \gamma, \delta$ are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive quantity K the values of

$4\sin\left(\frac{\alpha}{2}\right) + 3\sin\left(\frac{\beta}{2}\right) + 2\sin\left(\frac{\gamma}{2}\right) + \sin\left(\frac{\delta}{2}\right) =$

1. $2\sqrt{1-K}$ 2. $2\sqrt{1+K}$
3. $2\sqrt{K}$ 4. $\sqrt{K+1}$

4. $x = \frac{\sin^3 p}{\cos^2 p}, y = \frac{\cos^3 p}{\sin^2 p}$ and $\sin p + \cos p = 1/2$

then $x + y =$

1. $\frac{75}{18}$ 2. $\frac{44}{9}$ 3. $\frac{79}{18}$ 4. $\frac{48}{9}$

5. $\cos\alpha = \dfrac{2\cos\beta - 1}{2 - \cos\beta}(0 < \alpha, \beta < \pi)$, $\alpha + \beta = \pi$, then $\tan\dfrac{\alpha}{2} =$

1) $3^{1/4}$ 2) $3^{1/2}$ 3) 3 4) 3^2

6. If $\dfrac{x}{\tan(\theta+\alpha)} = \dfrac{y}{\tan(\theta+\beta)} = \dfrac{z}{\tan(\theta+\gamma)}$, then $\sum \dfrac{x+y}{x-y}\sin^2(\alpha-\beta) =$

1) 1 2) -1 3) 0 4) $\dfrac{1}{2}$

7. The quadratic equation whose roots are $\sin^2 18^0, \cos^2 36^0$

1) $16x^2 - 12x + 1 = 0$ 2) $x^2 - 12x + 1 = 0$
3) $16x^2 - 12x - 1 = 0$ 4) $16x^2 + 12x + 1 = 0$

8. $0 < \theta < \dfrac{\pi}{2}$ and $x = \sum_{n=0}^{\infty} \cos^{2n}\theta, y = \sum_{n=0}^{\infty} \sin^{2n}\theta,$

$Z = \sum_{n=0}^{\infty} \cos^{2n}\theta.\sin^{2n}\theta$ then the value of xyz is

1. $x + y + z$ 2. $xz + y$
3. $yz + x$ 4. 1

9. $\cos 6^0 \sin 24^0 \cos 72^0 =$

1. $\dfrac{-1}{8}$ 2. $\dfrac{-1}{4}$ 3. $\dfrac{1}{8}$ 4. $\dfrac{1}{4}$

10. $4\cos 6^0 \cos 42^0 \cos 60^0 \cos 66^0 \cos 78^0$

1. $\dfrac{1}{32}$ 2. $\dfrac{1}{16}$ 3. $\dfrac{1}{8}$ 4. $\dfrac{1}{4}$

11. $\cos 12^0 \cos 24^0 \cos 48^0 \cos 84^0 =$

1. $\dfrac{1}{8}$ 2. $\dfrac{1}{16}$ 3. $\dfrac{-1}{16}$ 4. $\dfrac{-1}{8}$

12. $\cos 12^0 \cos 24^0 \cos 36^0 \cos 48^0 \cos 72^0 \cos 84^0 =$

1. $\dfrac{1}{64}$ 2. $\dfrac{1}{32}$ 3. $\dfrac{1}{16}$ 4. $\dfrac{1}{4}$

13. $\cos\dfrac{\pi}{11}\cos\dfrac{2\pi}{11}\cos\dfrac{3\pi}{11}\cos\dfrac{4\pi}{11}\cos\dfrac{5\pi}{11} =$

1. $\dfrac{1}{4}$ 2. $\dfrac{1}{8}$ 3. $\dfrac{1}{16}$ 4. $\dfrac{1}{32}$

14. $\cos\dfrac{2\pi}{15}\cos\dfrac{4\pi}{15}\cos\dfrac{8\pi}{15}\cos\dfrac{14\pi}{15} =$

1. $\dfrac{1}{16}$ 2. $\dfrac{1}{8}$ 3. $\dfrac{3}{4}$ 4. $\dfrac{1}{4}$

15. $\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7} =$

1. $\dfrac{1}{8}$ 2. $\dfrac{-1}{8}$ 3. $\dfrac{1}{4}$ 4. $\dfrac{-1}{4}$

16. $\tan 6^0 \tan 42^0 \tan 66^0 \tan 78^0 =$

1. $\dfrac{1}{16}$ 2. $\dfrac{1}{4}$ 3. $\dfrac{1}{8}$ 4. 1

17. $\cos^2\dfrac{\pi}{10} + \cos^2\dfrac{2\pi}{5} + \cos^2\dfrac{3\pi}{5} + \cos^2\dfrac{9\pi}{10} =$

1. 1 2. $\dfrac{1}{2}$ 3. $\dfrac{3}{2}$ 4. 2

18. $\sin^4\dfrac{\pi}{8} + \sin^4\dfrac{3\pi}{8} + \sin^4\dfrac{5\pi}{8} + \sin^4\dfrac{7\pi}{8} =$

1. $\dfrac{1}{2}$ 2. $\dfrac{1}{4}$ 3. $\dfrac{3}{2}$ 4. $\dfrac{3}{4}$

19. $\cos^4\dfrac{\pi}{8} + \cos^4\dfrac{3\pi}{8} + \cos^4\dfrac{5\pi}{8} + \cos^4\dfrac{7\pi}{8} =$

1. $\dfrac{1}{2}$ 2. $\dfrac{3}{2}$ 3. $\dfrac{1}{4}$ 4. $\dfrac{3}{4}$

20. $\left(1+\cos\dfrac{\pi}{8}\right)\left(1+\cos\dfrac{3\pi}{8}\right)\left(1+\cos\dfrac{5\pi}{8}\right)\left(1+\cos\dfrac{7\pi}{8}\right) =$

1. $\dfrac{-1}{8}$ 2. $\dfrac{1}{8}$ 3. $\dfrac{3}{2}$ 4. $\dfrac{3}{4}$

21. If $\sin\beta$ is geometric mean between $\sin\alpha$ and $\cos\alpha$, then $\cos 2\beta =$

1) $2\sin^2\left(\dfrac{\pi}{4} - \alpha\right)$ *or* $2\cos^2\left(\dfrac{\pi}{4} + \alpha\right)$

2) $2\sin^2\left(\dfrac{\pi}{3} - \alpha\right)$ *or* $2\cos^2\left(\dfrac{\pi}{3} + \alpha\right)$

3) $\sin^2\left(\dfrac{\pi}{4} - \alpha\right)$ *or* $\cos^2\left(\dfrac{\pi}{4} + \alpha\right)$

4) $\sin^2\left(\dfrac{\pi}{3} - \alpha\right)$ *or* $\cos^2\left(\dfrac{\pi}{3} + \alpha\right)$

22. If $2\cos\dfrac{A}{2} = \sqrt{1+\sin A} - \sqrt{1-\sin A}$, then

1) $2n\pi + \dfrac{\pi}{4} < \dfrac{A}{2} < 2n\pi + \dfrac{3\pi}{4}$

2) $2n\pi - \dfrac{\pi}{4} < \dfrac{A}{2} < 2n\pi - \dfrac{3\pi}{4}$

3) $2n\pi - \dfrac{3\pi}{4} < \dfrac{A}{2} < 2n\pi + \dfrac{5\pi}{4}$

4) $n\pi + \dfrac{\pi}{4} < \dfrac{A}{2} < n\pi + \dfrac{3\pi}{4}$

23. If $2\sin\frac{A}{2}=\sqrt{1+\sin A}+\sqrt{1-\sin A}$, then $\frac{A}{2}$ lies between

1) $2n\pi+\frac{\pi}{4}$ and $2n\pi+\frac{3\pi}{4}, n\in z$

2) $2n\pi-\frac{\pi}{4}$ and $2n\pi+\frac{\pi}{4}, n\in z$

3) $2n\pi-\frac{3\pi}{4}$ and $2n\pi-\frac{\pi}{4}, n\in z$

4) $-\infty$ and ∞

24. If $2\cos\frac{A}{2}=\sqrt{1+\sin A}+\sqrt{1-\sin A}$, then $\frac{A}{2}$ lies between

1) $2n\pi+\frac{\pi}{4}$ and $2n\pi+\frac{3\pi}{4}$

2) $2n\pi-\frac{\pi}{4}$ and $2n\pi+\frac{\pi}{4}$

3) $2n\pi-\frac{3\pi}{4}$ and $2n\pi-\frac{\pi}{4}$

4) $-\infty$ and ∞

25. $2\sin^2\beta+4\cos(\alpha+\beta)\sin\alpha\sin\beta+\cos 2(\alpha+\beta)=$

1) $\sin 2\alpha$ 2) $\cos 2\alpha$

3) $\tan 2\alpha$ 4) $\cot 2\alpha$

26. If θ is in III quadrant, then $\sqrt{4\sin^4\theta+\sin^2 2\theta}+4\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)=$

1) 2 2) - 2 3) 0 4) 1

27. If $x\cos\alpha=y\cos\left(\frac{2\pi}{3}+\alpha\right)=z\cos\left(\frac{4\pi}{3}+\alpha\right)$, then $xy+yz+zx=$

1) 0 2) 1 3) - 1 4) 2

28. If $A=340^0$, then $\sqrt{1-\sin A}-\sqrt{1+\sin A}=$

1) $2\cos\frac{A}{2}$ 2) $2\sin\frac{A}{2}$

3) $-2\cos\frac{A}{2}$ 4) $-2\sin\frac{A}{2}$

29. $4\sin 27^0=$

1) $\sqrt{5+\sqrt{5}}+\sqrt{3-\sqrt{5}}$

2) $\sqrt{5-\sqrt{5}}+\sqrt{3+\sqrt{5}}$

3) $\sqrt{5+\sqrt{5}}-\sqrt{3-\sqrt{5}}$

4) $\sqrt{5+\sqrt{5}}+\sqrt{3+\sqrt{5}}$

30. If $\sin^2 A=x$, then $\sin A\sin 2A\sin 3A\sin 4A$ is a polynomial in x, the sum of whose coefficients is

1) 0 2) 40 3) 168 4) 336

31. $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}=$

1) $\frac{5}{16}$ 2) $\frac{3}{8}$ 3) $\frac{1}{16}$ 4) $\frac{1}{8}$

32. If $x=\cot 6^0\cot 42^0, y=\tan 66^0\tan 78^0$ then

1) $2x=y$ 2) $x=2y$

3) $x=y$ 4) $2x=3y$

33. $\sin 20^0\sin 40^0\sin 80^0=$

1) $\frac{\sqrt{3}}{4}$ 2) $\frac{\sqrt{3}}{8}$ 3) $\frac{3}{4}$ 4) $\frac{\sqrt{3}}{2}$

34. $\sin 10^0\sin 30^0\sin 50^0\sin 70^0\sin 90^0=$

1) $\frac{1}{8}$ 2) $\frac{1}{32}$ 3) $\frac{1}{16}$ 4) $\frac{1}{4}$

35. If $0<\theta<\frac{\pi}{2}$, $\sin 2\theta=\cos 3\theta$, then $\sin\theta=$

1) $\frac{\sqrt{5}-1}{4}$ 2) $\frac{\sqrt{5}+1}{4}$

3) 0 4) $\frac{\sqrt{10-2\sqrt{5}}}{4}$

36. $\sqrt{3}\operatorname{cosec}20^0-\sec 20^0=$

1) 2 2) 3 3) 1 4) 4

37. $\frac{1}{\sin 10^0}-\frac{\sqrt{3}}{\cos 10^0}=$

1) 4 2) 3 3) 2 4) - 1

KEY

1. 4	2. 2	3. 2	4. 3
5. 1	6. 3	7.1	8.1
9.3	10.3	11.2	12.1
13.4	14.1	15.2	16.4
17.4	18.3	19.2	20.2
21.1	22.1	23.1	24.2
25.2	26.1	27.1	28.2
29.3	30.1	31.4	32.3
33.2	34.3	35.1	36.4
37.1			

HINTS AND SOLUTIONS

1. $\left(\tan 9^0 + \cot 9^0\right) - \left(\tan 27^0 + \cot 27^0\right) =$

$\frac{2}{\sin 18^0} - \frac{2}{\cos 36^0} = 4$

3. $\beta = 180^0 - \alpha$, $\gamma = 360^0 + \alpha$, $\delta = 540^0 - \alpha$

then $4\sin\frac{\alpha}{2} + 3\sin\frac{\beta}{2} + 2\sin\frac{\gamma}{2} + \sin\frac{\delta}{2}$

$= 4\sin\frac{\alpha}{2} + 3\cos\frac{\alpha}{2} - 2\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}$

$= 2\sqrt{1+\sin\alpha}$ $= 2\sqrt{1+k}$

4. $\left(\sin p + \cos p\right) = \frac{1}{2} \Rightarrow 1 + 2\sin p\cos p = \frac{1}{4}$

$\Rightarrow \sin p\cos p = \frac{-3}{8}$

$x + y = \frac{\sin^5 p + \cos^5 p}{\sin^2 p\cos^2 p}$

$= \frac{(\sin p+\cos p)(1-\sin p\cos p) - \sin^2 p\cos^2 p(\sin p+\cos p)}{\sin^2 p\cos^2 p}$

5. Apply componendo and dividendo

$\frac{\cos\alpha + 1}{\cos\alpha - 1} = \frac{\cot^2\frac{\beta}{2}}{3}$

6. Put $\theta = 0$ in given equation

$\frac{x}{\tan\alpha} = \frac{y}{\tan\beta} = \frac{z}{\tan\gamma} = k$

$= \sum\frac{x+y}{x-y}\sin^2\left(\alpha-\beta\right) = \sum\left(\sin^2\alpha - \sin^2\beta\right) = 0$

10. Apply the formula

$\cos\theta\cos\left(60^0+\theta\right)\cos\left(60^0-\theta\right) = \frac{1}{4}\cos 3\theta$

Put $\theta = 6$ and $\theta = 18^0$

18. Given expression $= 2\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}\right) = \frac{3}{2}$

19. $\sin^2\beta = \sin\alpha\cos\alpha$

$\cos 2\beta = 1 - 2\sin\alpha\cos\alpha = \left(\sin\alpha - \cos\alpha\right)^2$

22. $S + C - \left(S - C\right) = \sqrt{1+\sin A} - \sqrt{1-\sin A}$

Q $S + C > 0, S - C > 0$

25. $2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + 2\cos^2(\alpha+\beta) - 1$

$= 2\sin^2\beta + 2\cos\left(\alpha+\beta\right)\cos\left(\alpha-\beta\right) - 1$

30. Let G. E.be $f(\sin A) = 2\sin A\left(3\sin A - 4\sin^3 A\right)$

$\left(1-2\sin^2 A\right)4\sin^2 A\left(1-\sin^2 A\right)$

Put $\sin A = 1$

sum of the coefficients $= f\left(1\right) = 0$

32. Apply formula

$\tan\theta\tan\left(60^0+\theta\right)\tan\left(60^0-\theta\right) = \tan 3\theta$

LEVEL - III

1. $\cos\theta = \frac{a\cos\varphi + b}{a + b\cos\varphi} \Rightarrow \frac{\tan\frac{\theta}{2}}{\tan\frac{\varphi}{2}}$

1.$\frac{a-b}{a+b}$ 2.$\frac{a+b}{a-b}$ 3.$\sqrt{\frac{a+b}{a-b}}$ 4.$\sqrt{\frac{a-b}{a+b}}$

2. If $\cos\left(\theta-\alpha\right), \cos\theta, \cos\left(\theta+\alpha\right)$ are in H.P, then $\cos\theta\sec\frac{\alpha}{2} =$

1) $\pm\frac{1}{\sqrt{2}}$ 2) $\pm\sqrt{2}$ 3) ± 1 4) $\pm\frac{1}{2}$

3. If $\tan\frac{\theta}{2} = \cos ec\theta - \sin\theta$, then the numerical value of $\cos^2\frac{\theta}{2}$ is

1) $\frac{\sqrt{3}-1}{4}$ 2) $\frac{\sqrt{5}+1}{4}$

3) $\frac{\sqrt{3}+1}{4}$ 4) $\frac{\sqrt{5}-1}{4}$

4. If $\tan\frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\alpha}{2}$, then $\cos\alpha =$

1) $\frac{1-e\cos\theta}{\cos\theta+e}$ 2) $\frac{1+e\cos\theta}{\cos\theta+e}$

3) $\frac{1-e\cos\theta}{\cos\theta-e}$ 4) $\frac{\cos\theta-e}{1-e\cos\theta}$

5. $4\cos 36^0 + \cot 7\frac{1}{2}^0 =$

1) $1+\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{5}+\sqrt{6}$

2) $1+\sqrt{2}-\sqrt{3}+\sqrt{4}-\sqrt{5}+\sqrt{6}$

3) $1-\sqrt{2}+\sqrt{3}-\sqrt{4}+\sqrt{5}-\sqrt{6}$

4) $1+\sqrt{2}-\sqrt{3}+\sqrt{4}+\sqrt{5}-\sqrt{6}$

6. The value of the expression $\tan^6 20^0 - 33\tan^4 20^0 + 27\tan^2 20^0$ is
1) 0 2) 1 3) 2 4) 3

KEY

1. 4 2. 2 3. 2 4. 4 5. 1 6. 2

HINTS AND SOLUTIONS

1. Apply componendo and devidendo

2. $\frac{2}{\cos\theta} = \frac{1}{\cos(\theta-\alpha)} + \frac{1}{\cos(\theta+\alpha)}$

$\Rightarrow \cos\theta \sec\frac{\alpha}{2} = \pm\sqrt{2}$

3. $\tan\frac{\theta}{2} = \frac{\cos^2\theta}{\sin\theta} \Rightarrow 4\cos^4\frac{\theta}{2} - 2\cos^2\frac{\theta}{2} - 1 = 0$

$\Rightarrow \cos^2\frac{\theta}{2} = \frac{\sqrt{5}+1}{4}$

6. Apply the formula $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Put $\theta = 20^0$

LEVEL - IV
NEW PATTERN QUESTIONS

1. Assertion (A): $\tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\tan 8\theta + 16\cot 6\theta = \cot\theta$
Reason (R): $\cot\alpha - \tan\alpha = 2\cot 2\alpha$
1. A is true, R is true and R is correct explanation of A
2. A is true, R is true and R is not correct explanation of A
3. A is true, R is false 4. A is false, R is true

2. Assertion (A): $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}} = 2\cos\left(\frac{\pi}{32}\right)$
Reason (R): $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+.....\text{n terms}}}}} = 2\cos\left(\frac{\pi}{2^n}\right)$
1. A is true, R is true and R is correct explanation of A
2. A is true, R is true and R is not correct explanation of A
3. A is true, R is false 4. A is false, R is true

3. Assertion (A): If $x\cos\alpha + y\sin\alpha = 2a$, $x\cos\beta + y\sin\beta = 2a$ $(\alpha \neq \beta)$ then $\cos\alpha.\cos\beta = \frac{4a^2 - y^2}{x^2 + y^2}$
Reason (R): $a\alpha^2 + b\alpha + c = 0$, $a\beta^2 + b\beta + c = 0$ $(\alpha \neq \beta)$ then α, β are the roots of $ax^2 + bx + c = 0$
1. A is true, R is true and R is correct explanation of A
2. A is true, R is true and R is not correct explanation of A
3. A is true, R is false 4. A is false, R is true.

4. Assertion(A): If $\sin\alpha + \cos\alpha = m$ then $\sin^6\alpha + \cos^6\alpha = \frac{4 - 3(m^2-1)^2}{4}$
Reason(R): $0 \leq \sin 2\alpha \leq 1$, $\forall \alpha \in i$
1. Both A and R are true but R is not the correct explanation of A.
2. Both A and R are true but R is the correct explanation of A.
3. Both A and R are false
4. A is true but R is false

5. Observe the following lists.

List I	List II
A) $\frac{1-\cos x}{\sin x}$	1) $\cot^2\frac{x}{2}$
B) $\frac{1+\cos x}{1-\cos x}$	2) $\cot\frac{x}{2}$
C) $\sqrt{1+\sin 2x}$	3) $\cos x + \sin x$
D) $\frac{1+\cos x}{\sin x}$	4) $\tan\frac{x}{2}$
	5) $\sin\frac{x}{2}$

The correct match for list I from list II is

	A	B	C	D
1.	4	3	1	5
2.	4	1	3	2
3.	1	3	4	5
4.	2	4	5	3

6. Observe the following lists.

List I	List II
A) $\sin^2\frac{3\pi}{5} + \sin^2\frac{4\pi}{5}$	1. $\frac{3}{4}$
B) $\cos^2\frac{3\pi}{5} + \cos^2\frac{4\pi}{5}$	2. $\frac{\sqrt{5}}{4}$
C) $\cos^2\frac{\pi}{5} - \cos^2\frac{2\pi}{5}$	3. $\frac{5}{4}$
D) $\cos^2\left(\frac{\pi}{12}\right) + \cos^2\left(\frac{3\pi}{12}\right) + \cos^2\left(\frac{5\pi}{12}\right)$	4. $\frac{3}{2}$
	5. $\frac{1}{4}$

The correct match for list I from list II is

	A	B	C	D
1.	2	4	5	3
2.	3	1	2	4
3.	2	1	3	4
4.	3	4	2	1

7. Observe the following lists.

List I	List II
A) $\sin 10^0 \sin 30^0 \sin 50^0 \sin 70^0$	1) 3
B) $\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0$	2) $\frac{1}{8}$
C) $\tan 20^0 \tan 40^0 \tan 60^0 \tan 80^0$	3) $\frac{3}{16}$
D) $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}$	4) $\frac{1}{16}$
	5) $\frac{1}{4}$

The correct match for list I from list II is

	A	B	C	D
1.	4	3	2	5
2.	3	4	1	2
3.	4	3	1	2
4.	3	4	2	5

8. Statement I: If $m\cos(\theta+\alpha) = n\cos(\theta-\alpha)$ then $Tan\theta.Tan\alpha = \frac{m+n}{m-n}$

Statement -II : If $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{a+b}{a-b}$ then $Tan\alpha.\cot\beta = \frac{a}{b}$

Which of the above statements is correct

1. Only I 2. Only II
3. Both I & II 4. Neither I nor II

9. Arrange the values of the following expressions in descending order

A) $\sin 75^0 + \cos 75^0 =$

B) $\sin 12^0 \sin 48^0 \sin 54^0 =$

C) $e^{\log_{10} tan 1^0 + \log_{10} \tan 2^0 + \log_{10} \tan_{10} 3^0 + ... + \log_{10} \tan 89^0} =$

D) $\tan 9^0 - Tan 27^0 - \tan 63^0 + \tan 81^0 =$

1. D,A,B,C 2. A,D,B,C
3. D,A,C,B 4. A,C,D,B

KEY

1.1	2.3	3.1	4.4
5.2	6.2	7.3	8.2
9.3			

LEVEL -V

Let $S = \sin A/2;\ C = \cos A/2$ then

$S + C = \pm\sqrt{1+\sin A},\ S - C = \pm\sqrt{1-\sin A}$

and $S + C = \pm\sqrt{1+\sin A},\ S - C = \pm\sqrt{1-\sin A}$

$S + C > 0$ for $-\frac{\pi}{4} < \frac{A}{2} < \frac{3\pi}{4}$

$S + C < 0$ for $\frac{3\pi}{4} < \frac{\pi}{2} < \frac{7\pi}{4}$

$S - C > 0$ for $\frac{\pi}{4} < \frac{A}{2} < \frac{5\pi}{4}$

$S - C < 0$ for $\frac{-\pi}{4} < \frac{A}{2} < \frac{3\pi}{4}$

1) $2\sin\frac{A}{2} = \sqrt{1+\sin A} - \sqrt{1-\sin A}$ then

1) $2n\pi - \frac{\pi}{4} < \frac{A}{2} < 2n\pi + \frac{\pi}{4}, n \in z$

2) $2n\pi + \frac{\pi}{4} < \frac{A}{2} < 2n\pi + \frac{3\pi}{4}, n \in z$

3) $2n\pi + \frac{3\pi}{4} < \frac{A}{2} < 2n\pi + \frac{5\pi}{4}, n \in z$

4) $n\pi - \frac{\pi}{4} < \frac{A}{2} < n\pi + \frac{\pi}{4}, n \in z$

2) If $A = 340^0$ then $\sqrt{1-\sin A} - \sqrt{1+\sin A} =$

1) $2\cos\frac{A}{2}$ 2) $2\sin\frac{A}{2}$

3) $-2\cos\frac{A}{2}$ 4) $-2\sin\frac{A}{2}$

3) $4\sin 27^0 =$

1) $\sqrt{5+\sqrt{5}} + \sqrt{3-\sqrt{5}}$

2) $\sqrt{5-\sqrt{5}} + \sqrt{3+\sqrt{5}}$

3) $\sqrt{5+\sqrt{5}} - \sqrt{3-\sqrt{5}}$

4) $\sqrt{5-2\sqrt{5}} + \sqrt{3-2\sqrt{5}}$

4) The range of all valuesof 'θ' such that $\sin 3\theta - \cos 3\theta > 0$

1) $2n\pi + \frac{\pi}{6} < \theta < 2n\pi + \frac{5\pi}{6}$

2) $\frac{2n\pi}{3} + \frac{\pi}{12} < \theta < \frac{2n\pi}{3} + \frac{5\pi}{12}$

3) $2n\pi - \frac{\pi}{6} < \theta < 2n\pi + \frac{\pi}{6}$

4) $\frac{n\pi}{3} - \frac{\pi}{6} < \theta < \frac{n\pi}{3} + \frac{\pi}{6}$

KEY

1. 1 2. 2 3. 3 4. 2