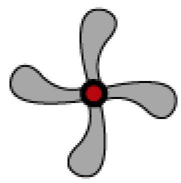
Symmetry

Concept Of Rotational Symmetry

Look at the following figure of a fan with four blades.

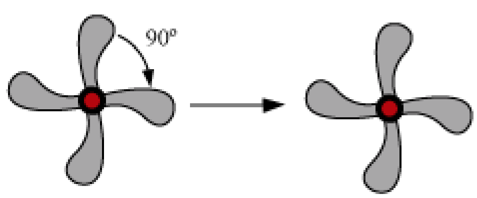


Does the fan have any line of symmetry?

A line which divides an object into two equal halves such that one half is the mirror image of the other is called the line of symmetry.

Therefore, we can see that the fan does not have any line of symmetry.

Now, rotate the fan through 90° about its centre.



What have you observed? Have you found any difference?

It looks exactly the same as the original figure after rotating it through 90°. Therefore, we can say that the fan has a **rotational symmetry**.

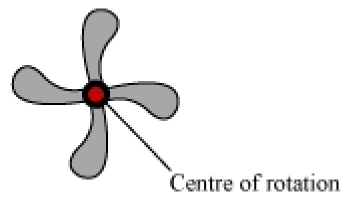
Thus, we can say that

"If an object remains identical after a rotation through certain degrees, then the object is said to possess rotational symmetry".

However, if we do not rotate the fan through the centre, then the identical structures would not be observed.

What is the centre of the fan called?

"The fixed point about which an object is rotated to attain the structure identical to the original one is called the centre of rotation."

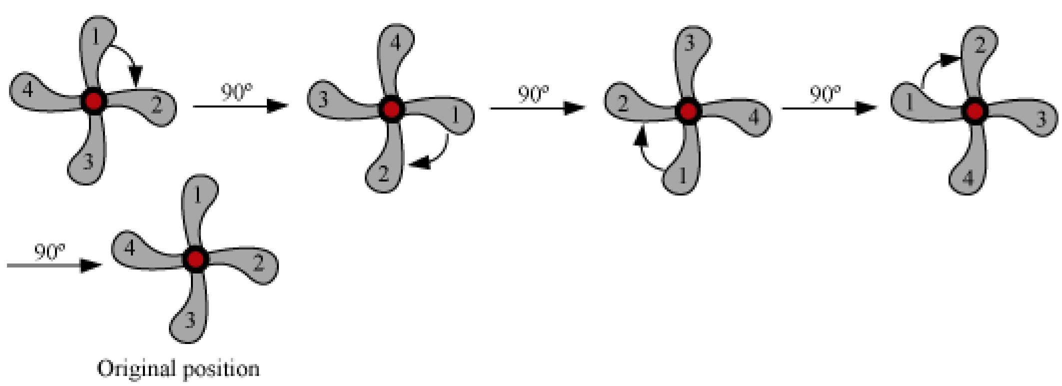


"The minimum angle through which an object is rotated to attain a structure identical to the original one is called angle of rotation."

In the above case, the fan is rotated through an angle of 90°. Therefore, the angle of rotation is 90°.

An object can be rotated in both the directions - clockwise and anti-clockwise.

Let us rotate the fan in clockwise direction.



After a rotation of 90°, the fan looks exactly the same as the original figure but it is not the same as the original one as the numbering on the blades is different.

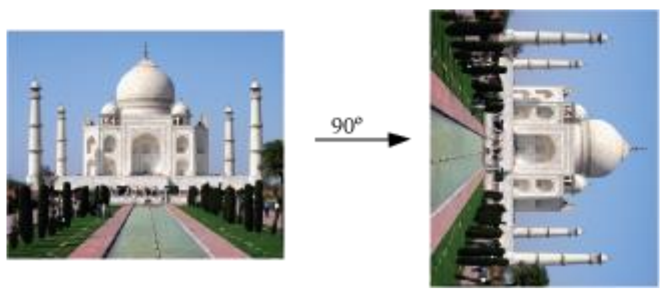
Only the last fan has the original position. It requires four rotations each of 90° to attain the original structure.

"The number of rotations (when rotated through the angle of rotation) required by an object to rotate about the centre of rotation to attain its original structure is known as the order of rotational symmetry."

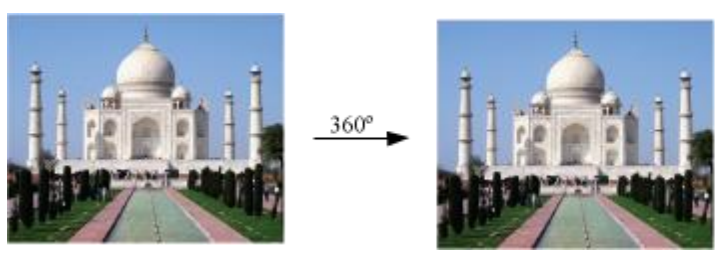
Therefore, the fan is said to have a rotational symmetry of order 4 about the centre of rotation.

Consider the following figure.

If we rotate this figure through 90°, would it coincide with the original figure? No, they will not coincide.



However, if we rotate this figure through an angle of 360°, then we will obtain the original figure.



Thus, we can say that this figure has a rotational symmetry of order one.

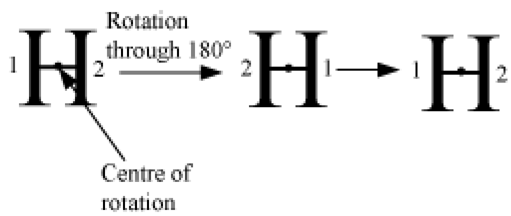
We can also say that **every object has a rotational symmetry of order at least one.**

Let us discuss this with the help of some examples. Consider the figures of a butterfly and a guitar.

If we rotate these figures by 360°, then they will retain their original figure.

This is true for every object in this world i.e., every object retains its original figure after rotating through 360°.

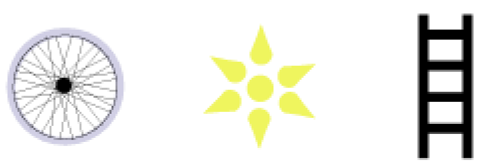
Some letters in the English alphabet also show rotational symmetries. Let us take the example of letter H.



Rotational symmetry

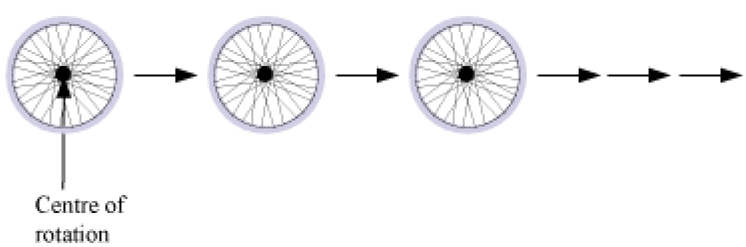
We can see that the letter H has rotational symmetry of order 2.

Let us consider the following objects one by one.



Wheel

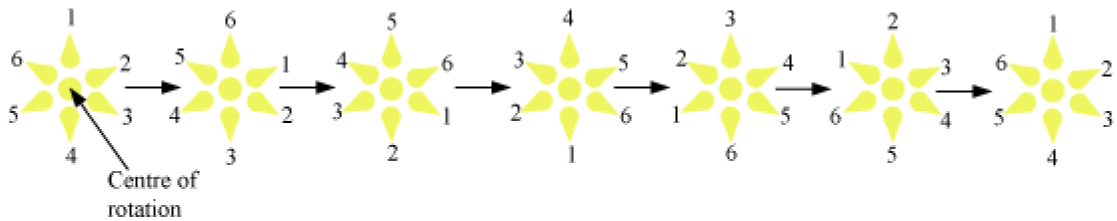
We can rotate the wheel through its centre at any angle to obtain the structure identical to the original wheel.



Therefore, we can say that the wheel has a rotational symmetry of infinite order.

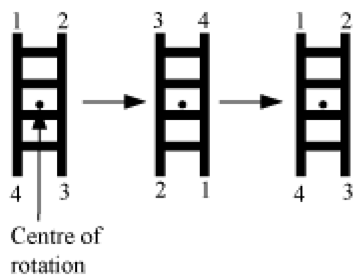
Flower

The flower shows a rotational symmetry of order 6.



Ladder

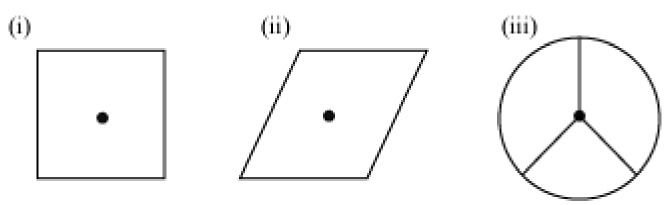
The ladder has a rotational symmetry of order 2.



Let us go through some more examples to understand the concept better.

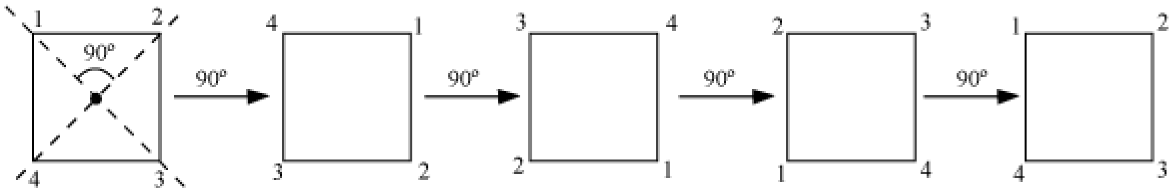
Example 1:

What is the order of rotational symmetry in the following figures about the centre of rotation marked as dot (•)?

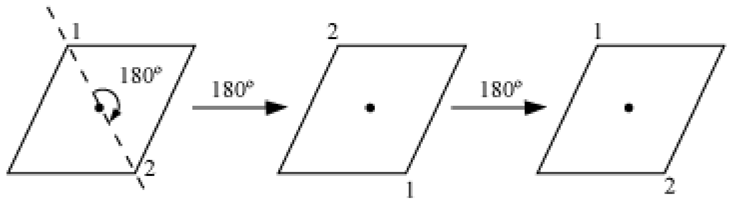


Solution:

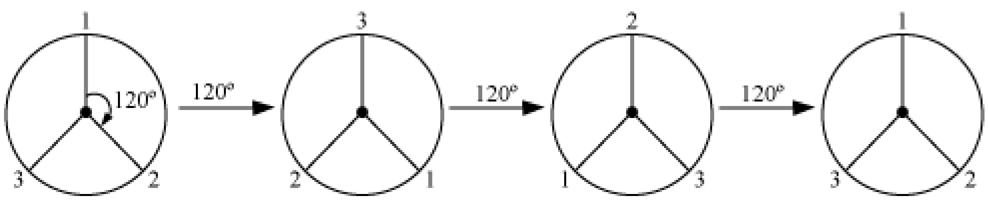
(i) The given figure shows a square. It has to be rotated through 90° four times to attain its original position. Therefore, it has a rotational symmetry of order 4.



(ii) For the given figure of parallelogram, the order of rotational symmetry is 2.

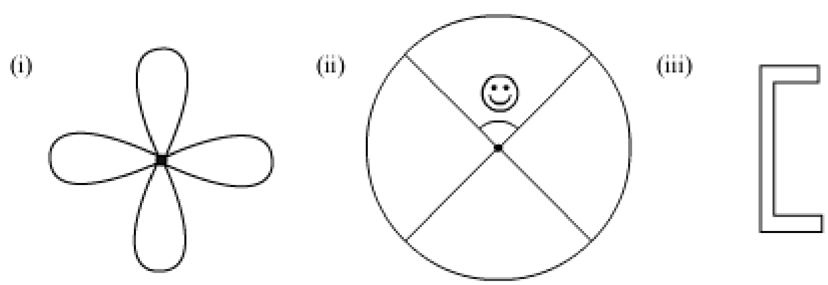


(iii) Here, the order of rotational symmetry is 3, as after rotating the figure through 120° three times, the figure comes back to its original position.



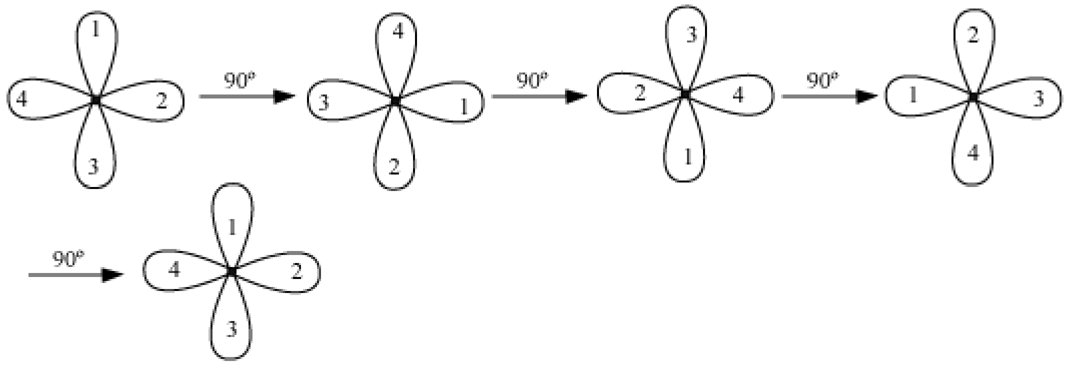
Example 2:

Which of the following figures have rotational symmetry of order more than 1?

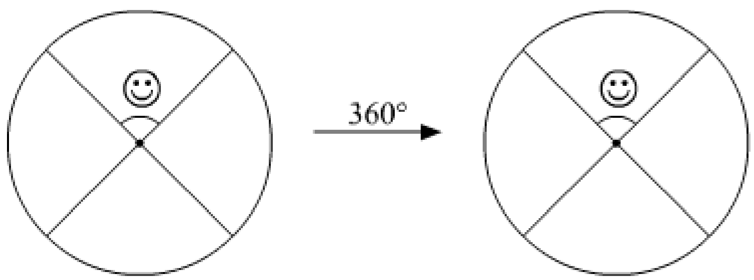


Solution:

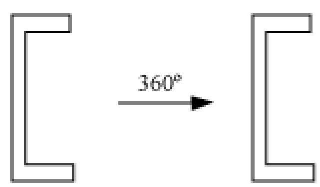
(i) The given figure has a rotational symmetry of order 4.



(ii) It has a rotational symmetry of order 1. It can attain the original and identical structure only by rotation through 360°.



(iii) It has a rotational symmetry of order 1.



Thus, figure **(i)** has rotational symmetry of order more than 1.

Example 3:

Which of the following letters of English alphabet have rotational symmetry? What are their orders?

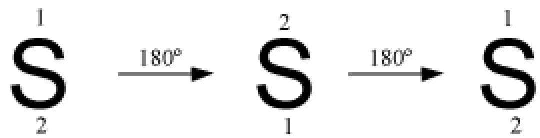
(i) M (ii) S

Solution:

1. The letter M does not have rotational symmetry. It can attain the original and identical structure only by rotation through 360°.

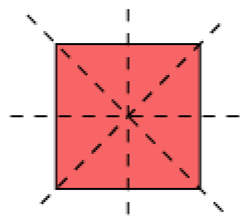
M

2. The letter S has a rotational symmetry of order 2. It has been represented in the following figure.



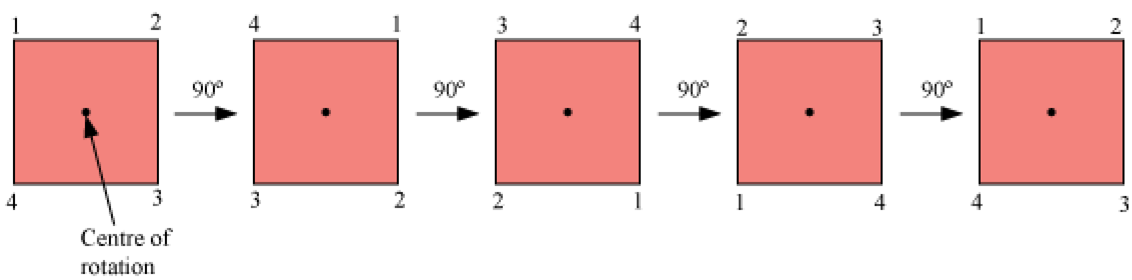
Lines Of Symmetry And Rotational Symmetry

We know that a square has four lines of symmetry as shown in the following figure.



4 line symmetry

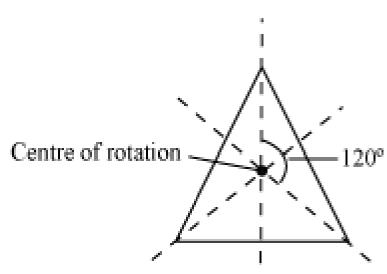
If we rotate the square about the point of intersection of its diagonals through an angle of 90° four times then an original figure will be obtained as shown in the following figure.



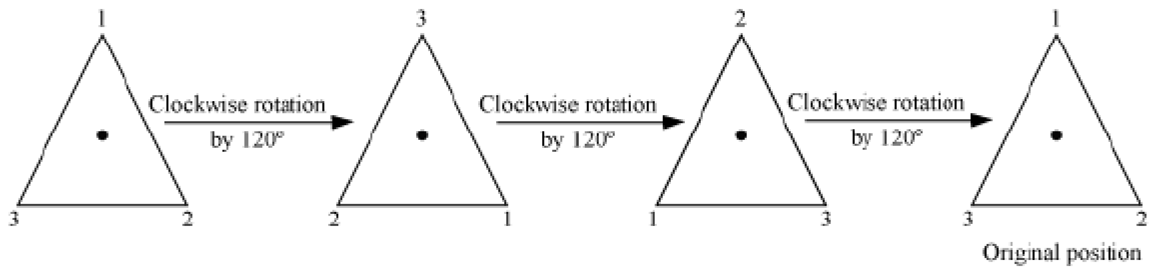
Rotational symmetry

Thus, we can say that a square has a rotational symmetry of order 4.

Now, let us look at the following figure of an equilateral triangle.



An equilateral triangle has three lines of symmetry. If we rotate the triangle about the centre of rotation through an angle of 120° then a similar shape will be obtained. Here, we have to rotate the triangle through 120° three times to take the triangle to its original position.



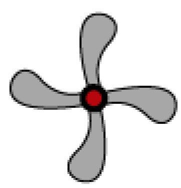
Therefore, the equilateral triangle is said to have a **rotational symmetry of order 3,** about the centre of rotation.

From the above examples, we can observe that

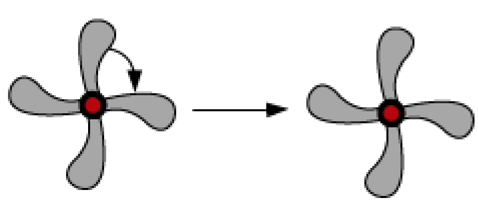
"***If a figure has more than one line of symmetry then it has rotational symmetry of order equal to the number of its lines of symmetry".***

What happens when the lines of symmetry do not exist? Can we say anything about the figure then?

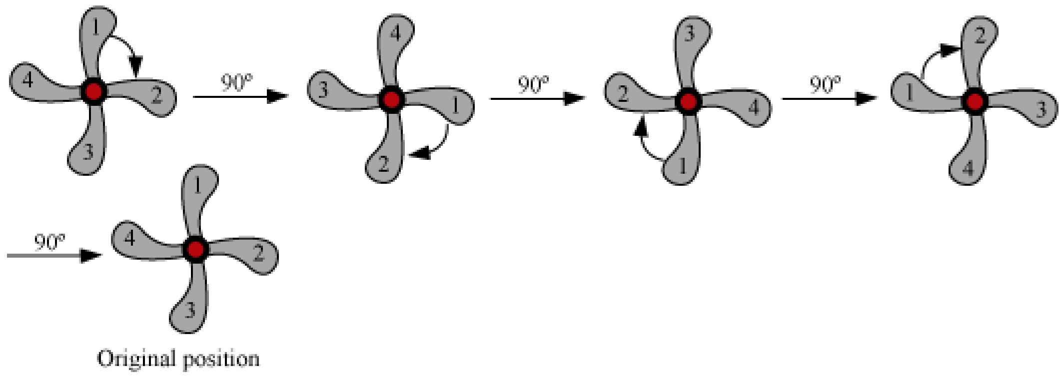
Consider the figure of a fan with four blades.



It does not have any line of symmetry but still it has rotational symmetry of order 4. If we rotate the fan about its centre through an angle of 90°, then a similar shape will be obtained.



We have to rotate the fan through 90° fourtimes to take the fan to its original position.



Therefore, the fan is said to have a **rotational symmetry of order 4,** about the centre of rotation.

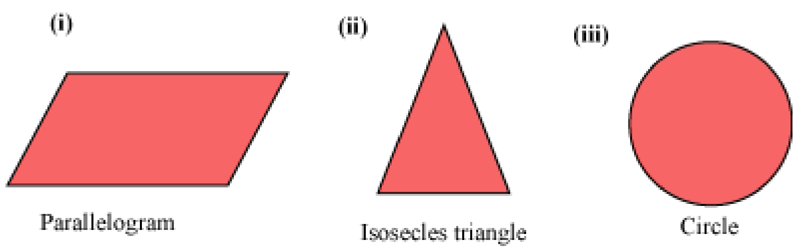
Thus, we can say that

"***When the lines of symmetry do not exist for a figure, the figure can still have rotational symmetry".***

Let us solve some more examples to understand the concept better.

Example 1:

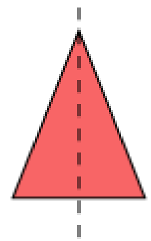
Which of the following figures has only line symmetry, only rotational symmetry, and both line and rotational symmetries?



Solution:

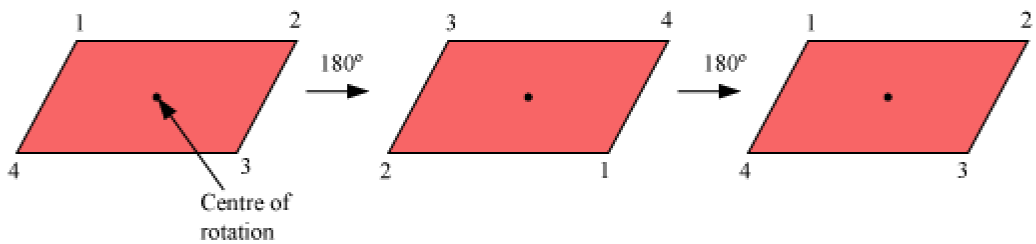
Here, we do not consider the rotational symmetry of order 1 as every object has rotational symmetry of order 1.

The figure (**ii**), an isosceles triangle, has only one of line symmetry. It does not have any rotational symmetry.

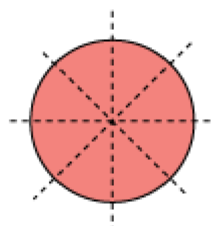


One line of symmetry

The figure (**i**), a parallelogram, has rotational symmetry of order 2 only, and does not possess any line of symmetry.



The figure **(iii)**, a circle, has infinite lines of symmetry and rotational symmetry of order infinite.



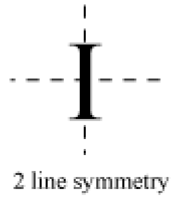
Example 2:

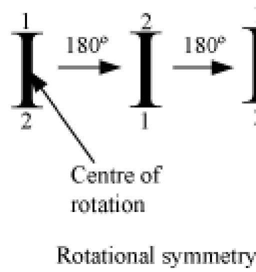
Which of the following letters have both line and rotational symmetry?

(i) C (ii) W (iii) I (iv) X

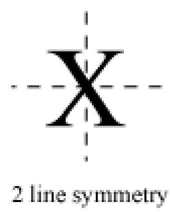
Solution:

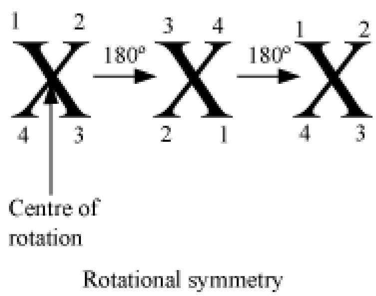
The letters I and X have both line symmetry and rotational symmetry.





The letter I has two lines of symmetry and rotational symmetry of order 2 about its centre.





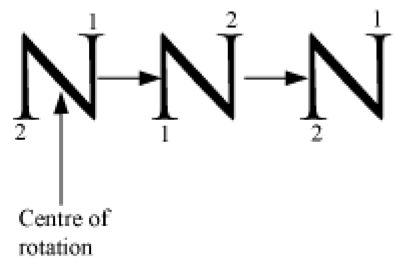
The letter X has two lines of symmetry and rotational symmetry of order 2 about its centre.

Example 3:

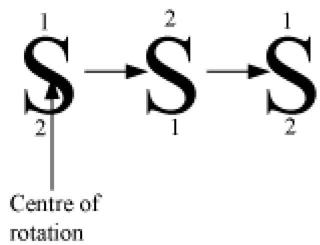
Which of the letters in English alphabets have rotational symmetry but no line of symmetry?

Solution:

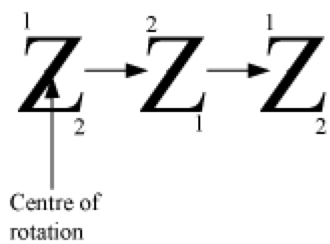
The letters N, S, and Z have rotational symmetry of order 2 but no line symmetry.



N N N
1
2
Centre of rotation



S S S
1
2
Centre of rotation



Z Z Z
1
2
Centre of rotation