CHAPTER 4

Doubly Reinforced Rectangular Section

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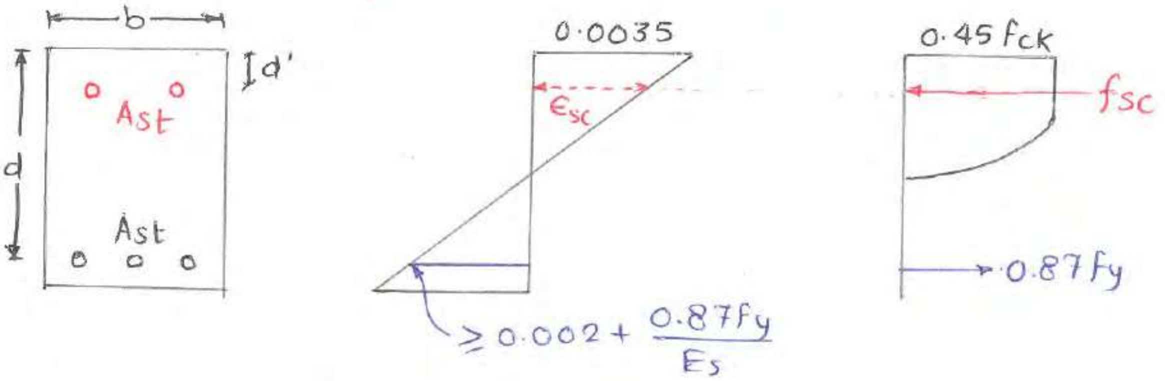
4 Doubly Reinforced Rectangular Sectio

4.1 Introduction:

If reinforcement is provided in tension and compression zone of section then such section is called as doubly reinforced. It is provided if section size is restricted (b & d both) and $BM_u > M_{u,lim}$

4.2 Analysis of Doubly Reinforced Section:



For position of N.A.

$$C = T$$

$$C_c + C_s = T$$

$$[0.36\, f_{ck} \cdot x_u \cdot b - A_{sc} \times 0.45 f_{ck}] + f_{sc} \cdot A_{sc} = 0.87 f_y \cdot A_{st}$$

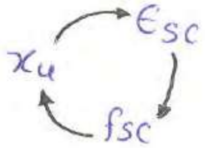
$$0.36\, f_{ck}\, x_u \cdot b + (f_{sc} - 0.45 f_{ck}) \cdot A_{sc} = 0.87 f_y \cdot A_{st}$$

From above expression x_u cannot be calculated without knowing f_{sc}. Since, f_{sc} depends on ϵ_{sc} so ϵ_{sc} is being calculated from strain diagram as follows.

$$\frac{\epsilon_{sc}}{x_u - d'} = \frac{0.0035}{x_u}$$

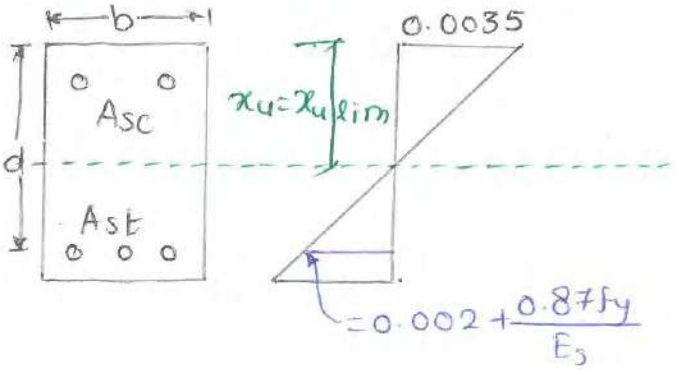
$$\epsilon_{sc} = \left(\frac{x_u - d'}{x_u}\right) \cdot 0.0035$$

In above calculation, x_u depends on f_{sc} and f_{sc} depends on ϵ_{sc}. Since ϵ_{sc} also depends on x_u so this is a cyclic problem which can be solved by trial and error only.



4.3 Position of Neutral Axis:

4.3.1 Balanced Section:



$x_u = x_{u,lim} = 0.53d$ (Fe 250)

$0.48d$ (Fe 415)

$0.46d$ (Fe 500)

4.3.2 Under Reinforced Section:

Step 1: Calculate $x_{u,lim}$ and assume $x_{u_1} = x_{u,lim}$

Step 2: Calculate ϵ_{sc}

$$\epsilon_{sc} = \left(\frac{x_{u_1} - d'}{x_{u_1}}\right) 0.0035$$

Step 3: Take f_{sc} from stress-strain diagram of steel under compression, corresponding to ϵ_{sc}

Stress level	Fe 415		Fe 500	
	Strain	Stress	Strain	Stress
$0.8 f_{yd}$	0.00144	288.7	0.00174	347.8
$0.85 f_{yd}$	0.00163	306.7	0.00195	369.6
$0.90 f_{yd}$	0.00192	324.8	0.00226	391.3
$0.95 f_{yd}$	0.00241	342.8	0.00277	413.0
$0.975 f_{yd}$	0.00276	351.8	0.00312	423.9
$1.0 f_{yd}$	0.00380	360.9	0.00417	434.8

* Note:-

- f_{yd} ⟶ Design yield stress = $0.87 f_y$

- If ϵ_{sc} lies between values of above table then f_{sc} is calculated by linear interpolation.

$$x_1 \longrightarrow y_1$$
$$\epsilon_{sc} \longrightarrow f_{sc}$$
$$x_2 \longrightarrow y_2$$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1)$$

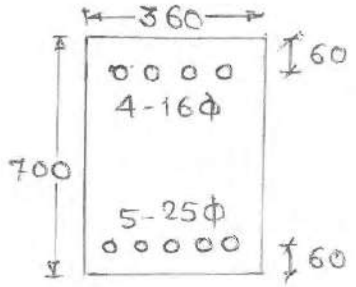
- If ϵ_{sc} is less than 0.00144 and 0.00174 for Fe415 and Fe 500 respectively then f_{sc} is calculated by directly multiplying ϵ_{sc} to E_s

Step 4: Equate net compressive force to net tensile force of section to calculate position of N.A.

$$C = T$$
$$C_c + C_s = T$$
$$0.36 f_{ck} x_u \cdot b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y \cdot A_{st}$$
$$x_u = ??$$

Step 5: If x_u of step 4 is not equal to x_{u_1} then assume $x_{u_2} = x_u$ of step 4 and repeat step 2 to step 4.

Ex. Calculate position of N.A.



M30

Fe415

⇒

Ist Trial:

$x_{u1} = x_{u,lim} = 0.48d$

$= 0.48 \times (700-60)$

$\Rightarrow x_{u1} = 307.2 mm$

$$\epsilon_{sc} = \frac{x_{u1} - d'}{x_{u1}} \times 0.0035$$

$$= \frac{307.2 - 60}{307.2} \times 0.0035$$

$\Rightarrow \epsilon_{sc} = 0.00281$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1)$$

$$= 351.8 + \left(\frac{360.9 - 351.8}{0.0038 - 0.00276}\right)(0.00281 - 0.00276)$$

$\Rightarrow f_{sc} = 352.23 \ N/mm^2$

$C = T$

$0.36 f_{ck} x_u \cdot b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y \cdot A_{st}$

$$0.36 \times 30 \times x_u \times 360 + (352.23 - 0.45 \times 30) \times 4 \times \frac{\pi}{4} \times 16^2$$

$$= 0.87 \times 415 \times 5 \times \frac{\pi}{4} \times 25^2$$

$\Rightarrow x_u = 157.85 mm \neq x_{u1} \ (307.2 mm)$

So, second Trial is required.

II^{nd} Trial:

$$\Rightarrow x_{u_2} = x_u = 157.85\,mm$$

$$\epsilon_{sc} = \left(\frac{x_{u_2} - d'}{x_{u_2}}\right) \times 0.0035$$

$$= \frac{157.85 - 60}{157.85} \times 0.0035$$

$$\Rightarrow \epsilon_{sc} = 0.00217$$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1)$$

$$= 324.8 + \left(\frac{342.8 - 324.8}{0.00241 - 0.00192}\right)(0.00217 - 0.00192)$$

$$\Rightarrow f_{sc} = 333.97\ N/mm^2$$

Now, $C = T$

$$0.36 f_{ck} x_u b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$

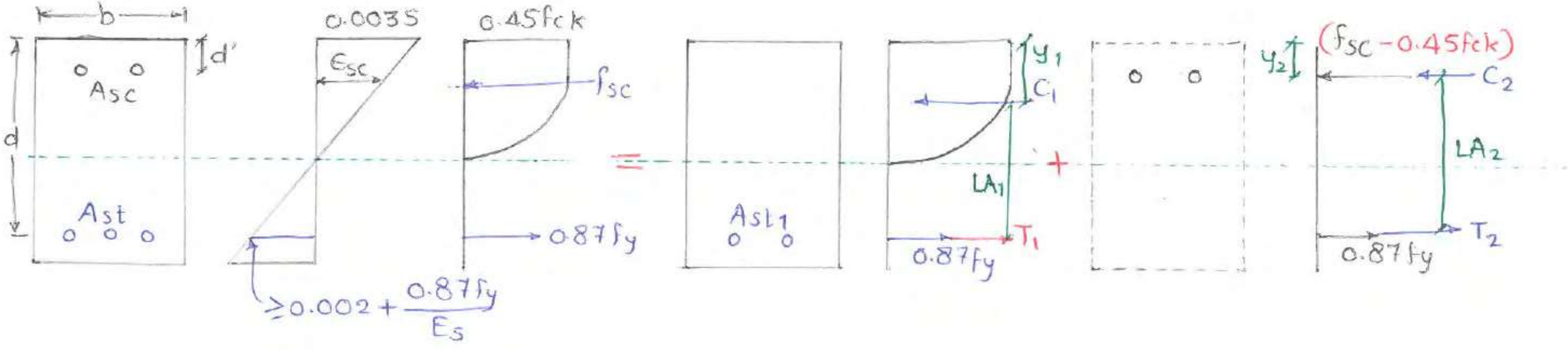
$$0.36 \times 30 \times x_u \times 360 + (333.97 - 0.45 \times 30) \times 4 \times \frac{\pi}{4} \times 16^2 = 0.87 \times 415 \times 5 \times \frac{\pi}{4} \times 25^2$$

$$\Rightarrow x_u = 161.67\ mm \neq x_{u_2}\ (157.85\,mm)$$

So, III^{rd} trial is required.

Few more trials are required for relatively closer to exact position of N.A. Assuming $x_u = 160\,mm$ as an approximate position of N.A. to reduce calculation effort.

4.4 Moment of Resistance of Section:



MR from compression Side

$MR = MR_1 + MR_2$

$= C_1 \times LA_1 + C_2 \times LA_2$

$MR = 0.36\, f_{ck}\, x_u\, b\,(d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} \cdot (d - d')$

MR from Tension Side.

Position of Net compressive force from top fibre $= \bar{y} = \dfrac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$

$y_1 = 0.42 x_u$

$y_2 = d'$

$MR = T \times LA$

$MR = 0.87 f_y\, A_{st}\, (d - \bar{y})$ (Not preferable)

Ex. Calculate MR of section given in previous example.

$x_u = 160$ mm

$\Rightarrow$

$\Rightarrow x_u = 160$ mm

$$\epsilon_{sc} = \left(\frac{x_u - d'}{x_u}\right) \cdot 0.0035$$

$$= \left(\frac{160-60}{160}\right) \times 0.0035$$

$\Rightarrow \epsilon_{sc} = 0.00218$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1)$$

$$= 324.8 + \left(\frac{342.8 - 324.8}{0.00241 - 0.00192}\right)(0.00218 - 0.00192)$$

$\Rightarrow f_{sc} = 334.62$ N/mm^2

$$MR = MR_1 + MR_2$$

$$= C_1 \times LA_1 + C_2 \times LA_2$$

$$= 0.36 f_{ck} x_u b (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d')$$

$$= 0.36 \times 30 \times 160 \times 360 \times (640 - 0.42 \times 160)$$

$$+ (334.62 - 0.45 \times 30) \times 4 \times \frac{\pi}{4} \times 16^2 \times (640 - 60)$$

$\Rightarrow$ M.R. $= 506.1$ kN m

4.5 Design of Doubly Reinforced Rectangular Section:

Doubly reinforced sections are designed if section size is restricted. It means, section size is known and steel is to be designed.

Step 1: Calculate design/factored/ultimate bending moment.

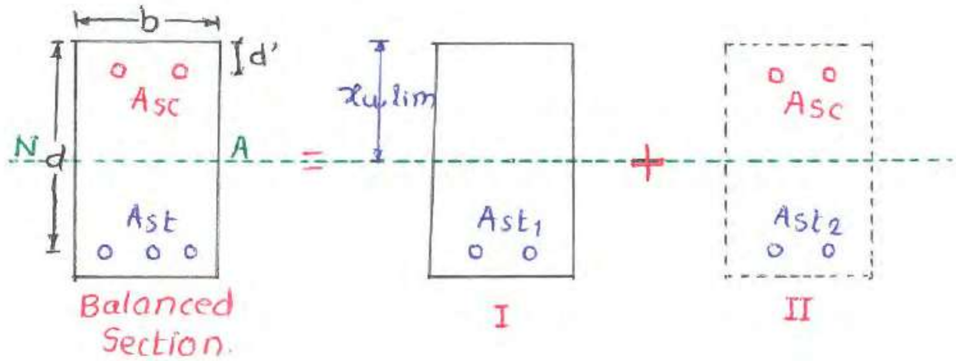
Step 2: Calculate $M_{u,lim}$ of given section.

$$M_{u,lim} = Q b d^2$$

Step 3: If $BM_u \leq M_{u,lim}$ then singly under reinforced section is designed.

$$A_{st} = \frac{0.5 f_{ck} bd}{f_y}\left[1 - \sqrt{1 - \frac{4.6 BM_u}{f_{ck} bd^2}}\right]$$

If $BM_u > M_{u,lim}$ then doubly reinforced balanced section is designed.



Step 4: Calculate A_{st_1}

$$A_{st_1} = A_{st,lim} = 0.414\left(\frac{f_{ck}}{f_y}\right) x_{u,lim} \cdot b$$

Step 5: Calculate Moment resisted by Section II

$$BM_u = MR = MR_1 + MR_2$$

$$MR_2 = BM_u - M_{u,lim} \quad \cdots\cdots \quad (\because MR_1 = M_{u,lim})$$

Step 6: Calculate A_{st_2}

$$MR_2 = T_2 \times LA_2$$

$$BMu - Mu,lim = 0.87 f_y A_{st_2} \cdot (d-d')$$

$$A_{st_2} = ??$$

Step 7: Calculate A_{sc}

$$C_2 = T_2$$

$$(f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st_2}$$

$$A_{sc} = ??$$

Step 8: A_{sc} and A_{st} calculated above should be within permissible limits.

$A_{sc,min}$ = No value

$A_{sc,max} = 0.04 bD$

$A_{st,min}$ = No need to check

$A_{st,max} = 0.04 bD$

Ex. Design a section for factored BM 110 kN·m if size is restricted to 250 x 380 mm. Effective cover 50 mm, M20, Fe415.

⇒

Step 1: ⇒ $BM_u = 110 \text{ kN·m}$

Step 2: $M_{u,lim} = 0.138 f_{ck} b d^2$

$= 0.138 \times 20 \times 250 \times (380-50)^2$

⇒ $M_{u,lim} = 75.14 \text{ kN·m}$

Since, $BM_u > M_{u,lim}$ so doubly reinforced balanced section is designed.

Step 3: $A_{st_1} = A_{st,lim} = 0.414 \left(\frac{f_{ck}}{f_y}\right) x_{u,lim} \cdot b$

$= 0.414 \times \left(\frac{20}{415}\right) \times 0.48 \times 330 \times 250$

⇒ $A_{st_1} = 790.1 \text{ mm}^2$

Step 4: $MR_2 = BM_u - M_{u,lim}$

$= 110 - 75.14$

⇒ $MR_2 = 34.86 \text{ kN·m}$

Step 5: $MR_2 = T_2 \times LA_2$

$= 0.87 f_y A_{st_2} \times (d-d')$

$MR_2 = 0.87 \times 415 \times A_{st_2} \times (330-50) = 34.86 \times 10^6$

⇒ $A_{st_2} = 344.8 \text{ mm}^2$

Step 6: $x_{u,lim} = 0.48d = 0.48 \times 330$

$x_{u,lim} = 158.4 \text{ mm}$

$\epsilon_{sc} = \left(\frac{x_{u,lim} - d'}{x_{u,lim}}\right) \times 0.0035 = \left(\frac{158.4-50}{158.4}\right) \times 0.0035$

$\epsilon_{sc} = 0.00239$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1)$$

$$= 324.8 + \left(\frac{342.8 - 324.8}{0.00241 - 0.00192}\right)(0.00239 - 0.00192)$$

$f_{sc} = 342.06\ N/mm^2$

Now, $C_2 = T_2$

$(f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y \cdot A_{st_2}$

$(342.06 - 0.45 \times 20) \times A_{sc} = 0.87 \times 415 \times 344.8$

$\Rightarrow$ $\boxed{A_{sc} = 373.77\ mm^2}$

Step 7: $A_{st} = A_{st_1} + A_{st_2} = 790.1 + 344.8$

$\boxed{A_{st} = 1134.9\ mm^2}$

$\boxed{A_{sc} = 373.77\ mm^2}$

$A_{sc,max} = A_{st,max} = 0.04bD = 0.04 \times 250 \times 380$

$= 3800\ mm^2$

$\Rightarrow$ Providing $A_{st} = 2 - 25\phi + 1 - 16\phi$

$A_{sc} =$ ~~$2 - 16\phi$~~ $\Rightarrow 2 - 20\phi$

Compression steel is increased by more amount than increase in tension steel to make section under-reinforced because $f_{sc} < 0.87 f_y$

*** Note :-**

- If reduction of concrete area due to presence of steel is not considered then term $0.45 f_{ck} A_{sc}$ is removed from all expressions.
- If table of f_{sc} is not given in examination then assume f_{sc} between $0.75 f_y$ to $0.8 f_y$ for HYSD and $0.87 f_y$ for mild Steel.

.... Chapter 4 Ends Here ...