Class XII Session 2023-24
Subject - Mathematics
Sample Question Paper - 2

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If (A - 2B) = $\begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$ and (2A - 3B) = $\begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$ then B = ? **[1]**

   a) $\begin{bmatrix} -4 & 6 \\ -3 & -3 \end{bmatrix}$ b) None of these

   c) $\begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix}$ d) $\begin{bmatrix} 6 & -4 \\ -3 & 3 \end{bmatrix}$

2. Let A = $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ where $0 \leq \theta \leq 2\pi$. Then **[1]**

   a) Det(A) = 0 b) Det(A) $\in$ [2, 4]

   c) Det(A) $\in$ (2, 4) d) Det(A) $\in (2, \infty)$

3. The system of equations x + 2y = 5, 4x + 8y =20 has **[1]**

   a) None of these b) no solution

   c) a unique solution d) infinitely many solutions

4. At x = 2, f(x) = [x] is **[1]**

   a) Continuous but not differentiable b) None of these

   c) Continuous as well as differentiable d) Differentiable but not continuous

5. The lines l_1 and l_2 intersect. The shortest distance between them is **[1]**

   a) infinity b) negative

   c) positive d) zero

6. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ is **[1]**

a) not defined

b) 1

c) 2

d) 3

7. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let Z = px + qy, where p, q > 0. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is **[1]**

a) q = 3p

b) q = 2p

c) p = q

d) p = 2q

8. If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ Then **[1]**

a) $\vec{a} \perp \vec{b}$

b) none of these

c) $|\vec{a}| = |\vec{b}|$

d) $\vec{a} || \vec{b}$

9. $\int \sqrt[3]{x} dx = ?$ **[1]**

a) $\frac{4}{3}x^{\frac{4}{3}} + C$

b) $\frac{3}{4}x^{\frac{4}{3}} + C$

c) $\frac{3}{2}x^{\frac{2}{3}} + C$

d) $\frac{4}{3}x^{\frac{3}{4}} + C$

10. Total number of possible matrices of order 3×3 with each entry 2 or 0 is **[1]**

a) 27

b) 81

c) 9

d) 512

11. Maximize Z = 5x+3y , subject to constraints x + y ≤ 300 , 2x + y ≤ 360, x ≥ 0, y ≥ 0. **[1]**

a) 1020

b) 1050

c) 1040

d) 1030

12. If $\vec{a} \cdot \vec{b}$ = 0 and $\vec{a} \times \vec{b}$ = 0, then which one of the following is correct? **[1]**

a) $\vec{a}$ is parallel to $\vec{b}$

b) $\vec{a}$ = 0 or $\vec{b}$ = 0

c) $\vec{a}$ is perpendicular to $\vec{b}$

d) None of these

13. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of |adj A| is **[1]**

a) a^2

b) a^6

c) a^9

d) a^{27}

14. If A and B are events such that P(A|B) = P(B|A), then **[1]**

a) A ⊂ B but A ≠ B

b) A = B

c) A ∩ B = ∅

d) P(A) = P(B)

15. Consider the following statements in respect of the differential equation $\frac{d^2y}{dx^2} + \cos\left(\frac{dy}{dx}\right) = 0$ **[1]**

i. The degree of the differential equation is not defined.

ii. The order of the differential equation is 2.

Which of the above statement(s) is/are correct?

a) Both (i) and (ii)

b) Only (ii)

c) Only (i)

d) Neither (i) nor (ii)

16. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, no two of which are collinear and the vector $\vec{a} + \vec{b}$ is collinear with $\vec{c}$, $\vec{b} + \vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + \vec{b} + \vec{c} =$ **[1]**

a) $\vec{a}$

b) $\vec{c}$

c) $\vec{b}$

d) None of these

17. If y = $\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{-1}{(1+x^2)}$

b) None of these

c) $\frac{2}{(1+x^2)}$

d) $\frac{-2}{(1+x^2)}$

18. The lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$ are **[1]**

a) parallel

b) intersecting

c) skew

d) coincident

19. **Assertion (A):** If x is real, then the minimum value of $x^2 - 8x + 17$ is 1. **[1]**

Reason (R): If f''(x) > 0 at a critical point, then the value of the function at the critical point will be the minimum value of the function.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Every function is invertible. **[1]**

Reason (R): Only bijective functions are invertible.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the principal value of $\text{cosec}^{-1}(-2)$. **[2]**

OR

Find the principal value of $\text{cosec}^{-1}(-\sqrt{2})$.

22. Find the intervals in function $f(x) = 2x^3 - 24x + 107$ is increasing or decreasing. **[2]**

23. The radius r of a right circular cone is decreasing at the rate of 3 cm/minute and the height h is increasing at the rate of 2 cm/minute. When r = 9 cm and h = 6 cm, find the rate of change of its volume. **[2]**

OR

A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which y-coordinates is changing 2 times as fast as x - coordinates.

24. Evaluate the integral: $\int_0^1 \log(1+x)dx$ **[2]**

25. Find the values of x for which the function , **[2]**

$f(x) = kx^3 - 9x^2 + 9x + 3$ is increasing in R

Section C

26. Evaluate: $\int \frac{dx}{(e^x-1)^2}$. **[3]**

27. In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII, what is the probability that a student chosen randomly studies in class XII, given that the chosen student is a girl? **[3]**

28. Evaluate $\int_0^{\pi} e^{2x} \cdot \sin\left(\frac{\pi}{4}+x\right) dx$. **[3]**

OR

Evaluate: $\frac{3x+1}{\sqrt{5-2x-x^2}} dx$

29. Solve the following differential equation: **[3]**

xdy - (y - x^3)dx = 0

OR

Find the particular solution of the differential equation $\left(1+x^2\right) \frac{dy}{dx}+2xy=\frac{1}{1+x^2}$, given that y = 0 when x = 1.

30. Solve the Linear Programming Problem graphically: **[3]**

Minimize Z = x - 5 y + 20 Subject to

x - y $\geq$ 0

- x + 2y $\geq$ 2

x $\geq$ 3

y $\leq$ 4

x, y $\geq$ 0

OR

Solve the Linear Programming Problem graphically:

Maximize Z = 50x + 30y Subject to

2x + y $\leq$ 18

3x + 2y $\leq$ 34

x, y $\geq$ 0

31. Differentiate the function with respect to x: $\tan^{-1}\left(\frac{a+b\tan x}{b-a\tan x}\right)$. **[3]**

Section D

32. Find the area enclosed by the parabola y^2 = 4ax and the line y = mx. **[5]**

33. Let n be a positive integer. Prove that the relation R on the set Z of all integers numbers defined by (x, y) $\in$ R $\Leftrightarrow$ x - y is divisible by n, is an equivalence relation on Z. **[5]**

OR

Let A = R - {3}, B = R - {1}. If $f : A \rightarrow B$ be defined by $f(x)=\frac{x-2}{x-3}$ $\forall x \in A$. Then, show that f is bijective.

34. Solve the system of equations **[5]**

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$

$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$

$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

35. Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$. Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular. **[5]**

OR

Show that the lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})$ intersect Also, find the equation of the plane containing them.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(i) What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work.

(ii) An application selected at random is found to be under the below 18 category. Find the probability that it is under the folk genre. Show your work.

(iii) If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is equal to

OR

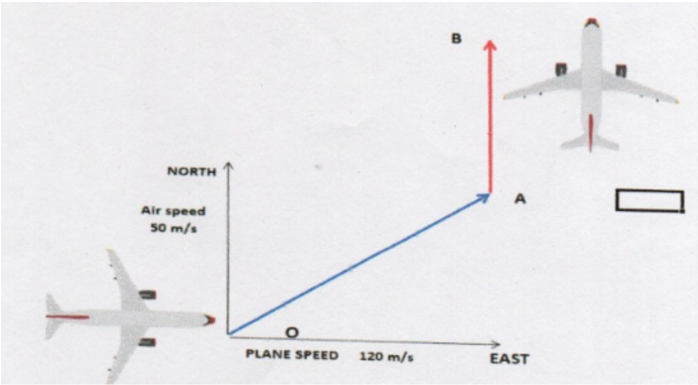
If A and B are two independent events with

$P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then find $P(A' \cap B')$.

37. **Read the text carefully and answer the questions:** **[4]**

A plane started from airport O with a velocity of 120 m/s towards east. Air is blowing at a velocity of 50 m/s towards the north As shown in the figure.

The plane travelled 1 hr in OA direction with the resultant velocity. From A and B travelled 1 hr with keeping velocity of 120 m/s and finally landed at B.



(i) What is the resultant velocity from O to A?

(ii) What is the direction of travel of plane O to A with east?

(iii) What is the total displacement from O to A?

OR

What is the resultant velocity from A to B?

38. **Read the text carefully and answer the questions:** **[4]**

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where f(x) is the temperature in ^oF at x days.

(i) Is the function differentiable in the interval (0, 12)? Justify your answer.

(ii) If 6 is the critical point of the function, then find the value of the constant m.

Solution

Section A

1. **(a)** $\begin{bmatrix}-4 & 6\\ -3 & -3\end{bmatrix}$

   Explanation: $(A-2B)=\begin{pmatrix}1 & -2\\ 3 & 0\end{pmatrix}$

   Multiplying equation by 2

   $2A-4B=\begin{pmatrix}2 & -4\\ 6 & 0\end{pmatrix}$ ...(i)

   $2A-3B=\begin{pmatrix}-2 & 2\\ 3 & -3\end{pmatrix}$ ...(ii)

   (ii) - (i)

   $B=\begin{pmatrix}-2 & 2\\ 3 & -3\end{pmatrix}-\begin{pmatrix}2 & -4\\ 6 & 0\end{pmatrix}$

   $=\begin{pmatrix}-4 & 6\\ -3 & -3\end{pmatrix}$

2. **(b)** Det(A) $\in$ [2, 4]

   Explanation: $A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$

   |A| = 1 (1 $\times$ 1 – sin θ $\times$ (-sin θ)) – sin θ (-sin θ+$sin\theta$) + 1 [(- sin θ) $\times(-sin\theta)-(-1)\times 1]$

   |A| = $1+sin^2\theta+sin^2\theta+1$

   |A| = $2+2\ sin^2\theta$

   |A| = $2(1+sin2\theta)$

   Now, $0\leq\theta\leq 2\pi$

   $\Rightarrow$ sin 0 $\leq$ sin θ $\leq$ sin 2π

   $\Rightarrow 0\leq\sin^2\theta\leq 1$

   $\Rightarrow$ 1 + 0 $\leq$ 1 + $\sin^2\theta\leq$ 1 + 1

   $\Rightarrow 2\leq 2(1+\sin^2\theta)\leq 4$

   $\therefore$ Det (A) $\in$ [2, 4]

3. **(d)** infinitely many solutions

   Explanation: x + 2y = 5,

   4x + 8y = 20

   $\Rightarrow$ A = $\begin{bmatrix}1 & 2\\ 4 & 8\end{bmatrix}$, X = $\begin{bmatrix}x\\ y\end{bmatrix}$, B = $\begin{bmatrix}5\\ 20\end{bmatrix}$

   |A| = 8 -8 = 0

   adjA = $\begin{bmatrix}8 & -2\\ -4 & 1\end{bmatrix}$

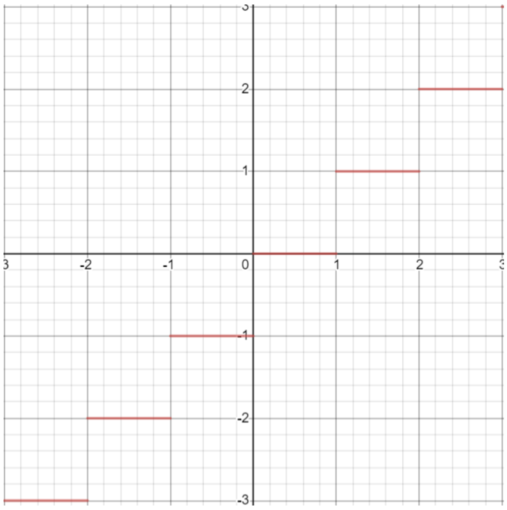
   now (adj A)B= $\begin{bmatrix}8 & -2\\ -4 & 1\end{bmatrix}\begin{bmatrix}5\\ 20\end{bmatrix}=\begin{bmatrix}40-40\\ -20+20\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$ $\Rightarrow$ (adj A)B = 0

   Since, |A|=0 and (adjA)B=0

   So, the pair of equation have infinitely many solutions

4. **(b)** None of these

   Explanation: Let us see that graph of the floor function, we get



We can see that f(x) = [x] is neither continuous and non differentiable at x = 2.

5.

(d) zero

Explanation: Since the lines intersect. Hence they have a common point in them. Hence the distance will be zero.

6. **(a)** not defined

Explanation: In general terms for a polynomial the degree is the highest power.

Degree of differential equation is defined as the highest integer power of highest order derivative in the equation

Here the differential equation is $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$

Now for degree to exist the given differential equation must be a polynomial in some differentials.

Here differentials mean $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ or .... $\frac{d^ny}{dx^n}$

The given differential equation is not polynomial because of the term $\sin\frac{dy}{dx}$ and hence degree of such a differential equation is not defined.

7. **(a)** q = 3p

Explanation: Since Z occurs maximum at (15, 15) and (0, 20), therefore, 15p + 15q = 0p + 20q ⇒ q = 3p.

8. **(a)** $\vec{a} \perp \vec{b}$

Explanation: Here $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

$\Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$

$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$

$\Rightarrow |a|^2 + 2\vec{a}\cdot\vec{b} + |b|^2 = |a|^2 - 2\vec{a}\cdot\vec{b} + |b|^2$

$\Rightarrow \quad 2\vec{a}\cdot\vec{b} = -2\vec{a}\cdot\vec{b}$

$\Rightarrow 4\vec{a}\cdot\vec{b} = 0$

$\Rightarrow \quad \vec{a}\cdot\vec{b} = 0$

$\Rightarrow \quad \vec{a} \perp \vec{b}$

9.

(b) $\frac{3}{4}x^{\frac{4}{3}} + C$

Explanation: Given integral is $\int \sqrt[3]{x}dx$

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\int \sqrt[3]{x}dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + c$

$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$

$= \frac{3}{4}x^{\frac{4}{3}} + c$

10.

(d) 512

Explanation: Since each element a_{ij} can be filled in two ways (with either '2' or ''0'), total number of possible matrices is 8x8x8 = 512

11. **(a)** 1020

Explanation: Here , Maximize Z = 5x+3y , subject to constraints x + y ≤ 300 , 2x + y ≤ 360, x ≥ 0, y ≥ 0.

Corner points	Z = 5x +3 y
P(0, 300)	900
Q(180, 0)	900
R(60, 240)	1020…………………..(Max.)
S(0, 0)	0

Hence, the maximum value is 1020

12.

(b) $\vec{a}$ = 0 or $\vec{b}$ = 0

Explanation: Given that, $\vec{a} \cdot \vec{b} = 0$,

i.e. $\vec{a}$ and $\vec{b}$ are perpendicular to each other and $\vec{a} \times \vec{b}$ = 0

i.e. $\vec{a}$ and $\vec{b}$ are parallel to each other. So, both conditions are possible iff $\vec{a}$ = 0 and $\vec{b}$ = 0

13.

(b) a^6

Explanation: $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

$|A| = a^3$

$|adj\ A| = |A|^{3-1} = |A|$

$|adj\ A| = (a^3)^2 = a^6$

14.

(d) P(A) = P(B)

Explanation: It is given that : P(A | B) = P(B | A)

$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$

$\Rightarrow \frac{1}{P(B)} = \frac{1}{P(A)} \Rightarrow P(A) = P(B)$

15. **(a)** Both (i) and (ii)

Explanation: Both (i) and (ii)

16.

(d) None of these

Explanation: Given that $\vec{a} + \vec{b}$ is collinear with $\vec{c}$

$\therefore \vec{a} + \vec{b} = x\vec{c}$ ...(i)

where x is scalar and $x \neq 0$

$\vec{b} + \vec{c}$ is collinear with $\vec{a}$

$\vec{b} + \vec{c} = y\vec{a}$ ...(ii)

y is scalar and $y \neq 0$

Subtracting (ii) from (i) we get

$\vec{a} - \vec{c} = x\vec{c} - y\vec{a}$

$\vec{a} + y\vec{a} = x\vec{c} + \vec{c}$

$\vec{a}(1 + y) = (1 + x)\vec{c}$

As given

$\vec{a}, \vec{c}$ are not collinear. (no two vecotors are collinear)

$\therefore$ 1 + y = 0 and 1 + x = 0

y = -1 and x = -1

Putting value of x in equation (i)

$\vec{a}+\vec{b}=-\vec{c}$

$\vec{a}+\vec{b}+\vec{c}=0$

17.

(d) $\frac{-2}{(1+x^2)}$

Explanation: Given that $y=\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

$\Rightarrow \sec y=\frac{x^2+1}{x^2-1}$

Since $\tan^2 x = \sec^2 x - 1$, thus

$\tan^2 y=\left(\frac{x^2+1}{x^2-1}\right)^2-1=\frac{4x^2}{(x^2-1)^2}$

Hence, $\tan y=-\frac{2x}{1-x^2}$ or $y=\tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

Hence, $y=\tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$, we obtain

$y=\tan^{-1}(-\tan 2\theta)$

Uisng -tan x = tan(-x), we obtain

$y = \tan^{-1}(\tan(-2\theta)) = -2\theta = -2\tan^{-1} x$

Differentiating with respect to x, we obtain

$\frac{dy}{dx}=\frac{-2}{1+x^2}$

18.

(d) coincident

Explanation: The equation of the given lines are

$\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ ....(i)

$\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$

$=\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ ....(ii)

Thus, the two lines are parallel to the vector $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ and pass through the points (0, 0, 0) and (1, 2, 3).

Now,

$(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b} = (\hat{i}+2\hat{j}+3\hat{k})\times(\hat{i}+2\hat{j}+3\hat{k})$

$= \vec{0}$ $[\because \vec{a}\times\vec{a}=\vec{0}]$

So, here the distance between the given two parallel lines is 0, the given lines are coincident.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Let $f(x) = x^2 - 8x + 17$

$\therefore$ f'(x) = 2x - 8

So, f'(x) = 0, gives x = 4

Here x = 4 is the critical number

Now, f''(x) = 2 > 0 , $\forall$ x

So, x = 4 is the point of local minima.

$\therefore$ Minimum value of f(x) at x = 4,

f(4) = 4 $\times$ 4 - 8 $\times$ 4 + 17 = 1

Hence, we can say that both Assertion and Reason are true and Reason is the correct explanation of the Assertion.

20.

(d) A is false but R is true.

Explanation: Assertion is false because every function is not invertible. The function which is one-one and onto i.e. bijective functions are invertible so reason is true.

Section B

21. $\operatorname{cosec}^{-1}x$ represents an angle in $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$ whose cosent is x.

Let $x = \operatorname{cosec}^{-1}(-2)$

$\Rightarrow cosec\ x = -2 = cosec\left(-\frac{\pi}{6}\right)$

$\Rightarrow x = -\frac{\pi}{6}$

$\therefore$ Principal value of $cosec^{-1}(-2)$ is $-\frac{\pi}{6}$.

OR

Let $cosec^{-1}(-\sqrt{2}) = y$. Then, cosec y = $-\sqrt{2} = -cosec\left(\frac{\pi}{4}\right) = cosec\left(-\frac{\pi}{4}\right)$.

We know that the range of the principal value branch of

$cosec^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and $cosec\left(-\frac{\pi}{4}\right) = -\sqrt{2}$.

Therefore, the principal value of $cosec^{-1}(-\sqrt{2})$ is $-\frac{\pi}{4}$.

22. Given: $f(x) = 2x^3 - 24x + 107$

$\Rightarrow f'(x) = \frac{d}{dx}\left(2x^3 - 24x + 107\right)$

$\Rightarrow f'(x) = 6x^2 - 24$

For f(x) lets find critical point, for this we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 - 24 = 0$

$\Rightarrow 6(x^2 - 4) = 0$

$\Rightarrow (x - 2)(x + 2) = 0$

$\Rightarrow x = -2, 2$

clearly, $f'(x) > 0$ if $x < -2$ and $x > 2$

and $f'(x) < 0$ if $-2 < x < 2$

Thus, the function f(x) increases on $(-\infty, -2) \cup (2, \infty)$ and f(x) is decreasing on interval $x \in (-2, 2)$.

23. We know that Volume of right circular cone $= \frac{\pi r^2 h}{3}$

$\frac{\partial V}{\partial t} = \frac{\pi}{3}\left(2rh\frac{\partial r}{\partial t} + r^2\frac{\partial h}{\partial t}\right)$

$\frac{\partial v}{\partial t} = \frac{\pi}{3}(108 \times -3 + 81 \times 2)$

$\frac{\partial v}{\partial t} = \frac{\pi}{3}(-162) = -54\pi \mathrm{cm}^2/\mathrm{min}$

Therefore Volume is decreasing at rate $54\pi cm^2$/min..

OR

Given curve is,

$6y = x^3 + 2$

$\Rightarrow 6\frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt}$ ... (i)

Given: $\frac{dy}{dt} = 2 \cdot \frac{dx}{dt}$ ... (ii)

from (i) and (ii), $2\left(2\frac{dx}{dt}\right) = x^2 \cdot \frac{dx}{dt}$

$\Rightarrow x = \pm 2$

when x = 2, y = $\frac{5}{3}$; when x = –2, y = -1

Therefore, Points are $\left(2, \frac{5}{3}\right)$ and (-2, -1)

24. Let I = $\int_0^1 \log(1+x)dx$, then

$I = \int_0^1 \log(1+x) \times 1dx$

$= [\log(1+x)x]_0^1 - \int_0^1 \frac{x}{1+x}dx$

$= [\log(1+x)x]_0^1 - \int_0^1 \left(1 - \frac{1}{1+x}\right)dx$

$= [x\log(1+x)]_0^1 - [x - \log(1+x)]_0^1$

= log 2 - 1 + log 2

= 2 log 2 - 1

= log 4 - log e

$= \log \frac{4}{e}$

25. we have, $f(x) = kx^3 - 9x^2 + 9x + 3$

$\Rightarrow f'(x) = 3kx^2 - 18x + 9$

Since f(x) is increasing on R, therefore , $f'(x) > 0\ \forall\ x \in R$

$\Rightarrow\ 3kx^2-18x+9>0, \forall\, x\in\ R$

$\Rightarrow\ kx^2-6x+3>0, \forall\, x\in\ R$

$\Rightarrow$ k>0 and 36-12k<0 $[\because\ ax^2+bx+c>0, \forall\, x\in\ R \Rightarrow\ a>0\, and\, discriminant<0]$

$\Rightarrow\ k>3$

Hence,f(x) is increasing on R, if k>3.

Section C

26. Putting t = e^x - 1

e^x = t + 1

dt = e^x dx

$\frac{dt}{e^x}=dx$

$\frac{dt}{t+1}=dx$

Putting above by have by partial fractions. $\frac{1}{(1+t)t^2}=\frac{A}{t+1}+\frac{Bt+C}{t^2}$ ... (1)

A(t^2) + (Bt + C) (t + 1) = 1

Put t + 1 = 0

t = -1

A = 1

Equating coefficients

A + B = 0

1 + B = 0

B = -1

C=1

From equation (1),we get,

$\frac{1}{(1+t)t^2}=\frac{1}{t+1}+\frac{-t+1}{t^2}$

$\int\frac{1}{(1+t)t^2}dt = \int\frac{1}{t+1}dt-\int\frac{t}{t^2}dt+\int\frac{1}{t^2}dt$

$=\log|t+1|-\int\frac{1}{t}dt+\int\frac{1}{t^2}dt$

$=\log|t+1|-\log|t|-\frac{1}{t}+c$

$\int\frac{1}{(e^x-1)^2}dx=\log|e^x|-\log|e^x-1|-\frac{1}{e^x-1}+c$

27. Let 'A' be the event that the chosen student studies in class XII and B be the event that the chosen student is a girl.

There are 430 girls out of 1000 students

So, P(B) = P (Chosen student is girl) $=\frac{430}{1000}=\frac{43}{100}$

Since, 10% of the girls studies in class XII

So, total number of girls studies in class XII

$=\frac{10}{100}\times 430=43$

Then, P(A ∩ B) = P (Chosen student is a girl of class XII)

$=\frac{43}{1000}$

$\therefore$ Required probability = P(A / B)

$=\frac{P(A\cap B)}{P(B)}\quad\left[\because P(A/B)=\frac{P(A\cap B)}{P(B)}\right]$

$=\frac{43/1000}{43/100}=\frac{1}{10}$

28. According to the question , $I=\int_0^\pi e^{2x}\cdot\sin\left(\frac{\pi}{4}+x\right)dx$ ...(i)

Consider , $I_1=\int \underset{II}{e^{2x}}\, \underset{I}{sin\left(\frac{\pi}{4}+x\right)}\,dx$ ....(ii)

By using integration bi parts, we get

$=\sin\left(\frac{\pi}{4}+x\right)\int e^{2x}dx-\int\left\{\frac{d}{dx}\sin\left(\frac{\pi}{4}+x\right)\int e^{2x}dx\right\}dx$

$=\sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}-\int\cos\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}dx$

$=\frac{e^{2x}}{2}\sin\left(\frac{\pi}{4}+x\right)-\frac{1}{2}\int \underset{II}{e^{2x}}\,\underset{I}{\cos\left(\frac{\pi}{4}+x\right)}\,dx$

By using integration by parts for second integral , we get

$=\frac{e^{2x}}{2}\sin\left(\frac{\pi}{4}+x\right)$- $\frac{1}{2}\left[\cos\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}-\int-\sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}dx\right]$

$= \frac{e^{2x}}{2}\sin\left(\frac{\pi}{4}+x\right) - \frac{e^{2x}}{4}\cos\left(\frac{\pi}{4}+x\right) - \frac{1}{4}\int e^{2x}\sin\left(\frac{\pi}{4}+x\right)dx$

$\Rightarrow I_1 = \frac{e^{2x}}{4}\left\{2\sin\left(\frac{\pi}{4}+x\right) - \cos\left(\frac{\pi}{4}+x\right)\right\} - \frac{1}{4}I_1$ [From eq.(ii)]

$\Rightarrow I_1 + \frac{1}{4}I_1 = \frac{e^{2x}}{4}\left\{2\sin\left(\frac{\pi}{4}+x\right) - \cos\left(\frac{\pi}{4}+x\right)\right\}$

$\Rightarrow \quad \frac{5}{4}I_1 = \frac{e^{2x}}{4}\left\{2\sin\left(\frac{\pi}{4}+x\right) - \cos\left(\frac{\pi}{4}+x\right)\right\}$

$\Rightarrow \quad I_1 = \frac{e^{2x}}{5}\left\{2\sin\left(\frac{\pi}{4}+x\right) - \cos\left(\frac{\pi}{4}+x\right)\right\}$

From (i) and (ii) ,we get

$\therefore \quad I = [I_1]_0^{\pi}$

$= \left[\frac{e^{2x}}{5}\left\{2\sin\left(\frac{\pi}{4}+x\right) - \cos\left(\frac{\pi}{4}+x\right)\right\}\right]_0^{\pi}$

$= \frac{1}{5}\left[e^{2\pi}\left\{2\sin\left(\frac{\pi}{4}+\pi\right) - \cos\left(\frac{\pi}{4}+\pi\right)\right\} - e^0\left\{2\sin\left(\frac{\pi}{4}+0\right) - \cos\left(\frac{\pi}{4}+0\right)\right\}\right]$

$= \frac{1}{5}\left[e^{2\pi}\left\{-2\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right\} - e^0\left\{2\sin\frac{\pi}{4} - \cos\frac{\pi}{4}\right\}\right]$

$= \frac{1}{5}\left[e^{2\pi}\left\{-2\times\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right\} - 1\left\{2\times\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right\}\right]$

$= \frac{1}{5}\left[e^{2\pi}\left\{-\frac{1}{\sqrt{2}}\right\} - \frac{1}{\sqrt{2}}\right]$

$= -\frac{1}{5\sqrt{2}}\left[e^{2\pi}+1\right]$

$\therefore I = -\frac{1}{5\sqrt{2}}\left[e^{2\pi}+1\right]$ sq units.

OR

Let the given integral be,

I = $\frac{3x+1}{\sqrt{5-2x-x^2}}dx$

Let 3x + 1 = $\lambda\frac{d}{dx}$ (5 - 2x + x^2) + μ

= λ(- 2 - 2x) + μ

3x + 1 = (-2λ)x + 2λ + μ

Comparing the coefficients of like powers of x,

-2λ = 3 $\Rightarrow \lambda = -\frac{3}{2}$

-2λ + μ = 1

$\Rightarrow -2\left(-\frac{3}{2}\right) + \mu = 1$

μ = -2

So, $I = \int \frac{-\frac{3}{2}(-2-2x)-2}{\sqrt{5-2x-x^2}}dx$

$= -\frac{3}{2}\int\frac{(-2-2x)}{\sqrt{5-2x-x^2}}dx - 2\int\frac{1}{\sqrt{-[x^2+2x-5]}}dx$

$I = -\frac{3}{2}\int\frac{(-2-2x)}{\sqrt{5-2x-x^2}}dx - 2\int\frac{1}{\sqrt{-\left[x^2+2x+(1)^2-(1)^2-5\right]}}dx$

$I = -\frac{3}{2}\int\frac{(-2-2x)}{\sqrt{5-2x-x^2}}dx - 2\int\frac{1}{\sqrt{-\left[(x+1)^2-(\sqrt{6})^2\right]}}dx$

$I = -\frac{3}{2}\int\frac{(-2-2x)}{\sqrt{5-2x-x^2}}dx - 2\int\frac{1}{\sqrt{(\sqrt{6})^2-(x+1)^2}}dx$

$I = -\frac{3}{2}\times 2\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c$ [since, $\int\frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c, \int\frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)$ + c]

$I = -3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c$

29. xdy - (y - x^3)dx = 0

This can be written as

xdy = (y - x^3)dx

Divide throughout by x,

$\frac{dy}{dx} = \frac{y}{x} - x^2$

$\frac{dy}{dx} - \frac{y}{x} = -x^2$

This is a linear differential equation of the form,

$\frac{dy}{dx}$ + Py = Q

The integrating factor I.F is

$e^{\int Pdx} = e^{\int \frac{-1}{x} dx}$ dx = $e^{-\log x}$ = $e^{\log\left(\frac{1}{x}\right)} = \frac{1}{x}$

The required solution is

$ye^{\int Pdx} = \int Qe^{\int Pdx}.dx + c$

$y \cdot \left(\frac{1}{x}\right) = -\int x^2 \times \frac{1}{x} dx + c$

$\frac{y}{x}$ = -$\int x dx + c$

$\frac{y}{x} = \frac{-x^2}{2}$ + c

$\frac{y}{x} + \frac{x^2}{2}$ = c

$2y + x^3 = 2cx$

$\Rightarrow x^3 - 2cx + 2y = 0$ is the required solution.

OR

$\left(1+x^2\right) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$

Divide both sides by $1 + x^2$

$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)\cdot(1+x^2)}$

$\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right) y = \frac{1}{\left(1+x^2\right)^2}$

Comparing with $\frac{dy}{dx}$ + Py = Q,

P = $\frac{2x}{1+x^2}$ & Q = $\frac{1}{\left(1+x^2\right)^2}$

Finding Integrating factor:

IF = $e^{\int Pdx}$

IF = $e^{\int \frac{2x}{1+x^2} dx}$

Let $1 + x^2 = t$

Diff. w.r.t. x

2x = $\frac{dt}{dx}$

dx = $\frac{dt}{2x}$

Thus, IF = $e^{\int \frac{2x}{t} \frac{dt}{2x}}$

IF = $e^{\int \frac{dt}{t}}$

IF = $e^{\log |t|}$

IF = t

IF = $1 + x^2$

Solution of the differential equation:

y × I.F. = $\int Q \times I.F\, dx$

Putting values,

y × $(1 + x^2) = \int \frac{1}{\left(1+x^2\right)^2}(1+x^2)dx$

y $(1 + x^2) = \int \frac{1}{(1+x^2)}$ dx

y $(1 + x^2) = \tan^{-1}x + C$ ...(1)

Putting that y = 0 and x = 1,

$0(1 + 1^2) = \tan^{-1}(1) + C$

0 = $\frac{\pi}{4}$ + C

C = -$\frac{\pi}{4}$

Putting value of C in eq(1),

$y(1 + x^2) = \tan^{-1}x + C$

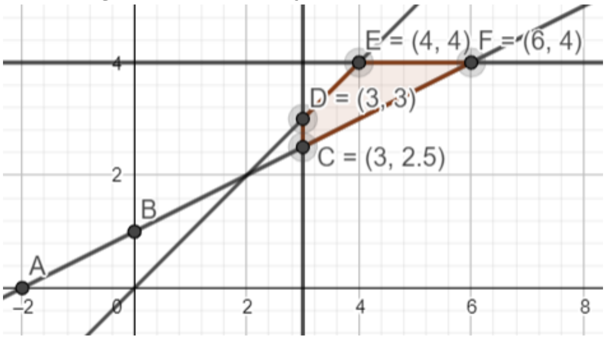
$y(1 + x^2) = \tan^{-1}x - \frac{\pi}{4}$

30. First, we will convert the given inequations into equations, we obtain the following equations:

x - y = 0, - x + 2 y = 2, x = 3, y = 4, x = 0 and y = 0

Region represented by $x - y \geq 0$ or $x \geq y$ The line x - y = 0 or x = y passes through the origin. The region to the right of line x = y will satisfy the given inequation. Check by taking an example like if we take a point (4,3) to the right of the line x = y. Here $x \geq y$. So, it satisfies the given inequation. Take a point (4,5) to the left of the line x = y. Here, $x \leq y$. That means it does not satisfy the given inequation. Region represented by - x + 2 y $\geq$ 2 The line - x + 2 y = 2 meets the coordinate axes at A(- 2,0) and B(0,1) respectively. By joining these points we obtain the line - x + 2 y = 2. Clearly (0,0) does not satisfies the inequation - x + 2 y $\geq$ 2 . So, the region in x y plane which does not contain the origin represents the solution set of the inequation - x + 2 y $\geq$ 2 The line x = 3 is the line that passes through the point (3,0) and is parallel to Y-axis. x $\geq$ 3 is the region to the right of line x = 3 The line y = 4 is the line that passes through the point (0,4) and is parallel to X-axis. y $\leq$ 4 is the region below the line y = 4 Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0 and y $\geq$ 0 The feasible region determined by subject to the constraints are x - y $\geq$ 0, - x + 2 y $\geq$ 2, x $\geq$ 3, y $\leq$ 4, non negative ,x $\geq$ 0 and y $\geq$ 0 are as follows.



The corner points of the feasible region are

C(3, $\frac{5}{2}$), D(3,3), E(4,4) and F(6,4)

The values of objective function at the corner points are as follows:

Corner point: z = x - 5 y + 20

$C\left(3, \frac{5}{2}\right) : 3 - 5 \times \frac{5}{2} + 20 = \frac{21}{2}$

D(3, 3) : $3 - 5 \times 3 + 20 = 8$

$E(4, 4) : 4 - 5 \times 4 + 20 = 4$

F(6, 4) : $6 - 5 \times 4 + 20 = 6$

Therefore, the minimum value of objective function Z is 4 at the point E(4,4) . Hence, x = 4 and y = 4 is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is 4.

OR

First, we will convert the given inequations into equations, we obtain the following equations:

2x + y = 18, 3x + 2y = 34

Region represented by 2x + y $\geq$ 18 :

The line 2x + y = 18 meets the coordinate axes at A(9,0) and B(0,18) respectively. By joining these points we obtain the line 2x + y = 18 Clearly (0,0) does not satisfies the inequation 2x + y $\geq$ 18 . So, the region in xy plane which does not contain the origin represents the solution set of the inequation 2x + y $\geq$ 18.

Region represented by 3x + 2y $\leq$ 34 :

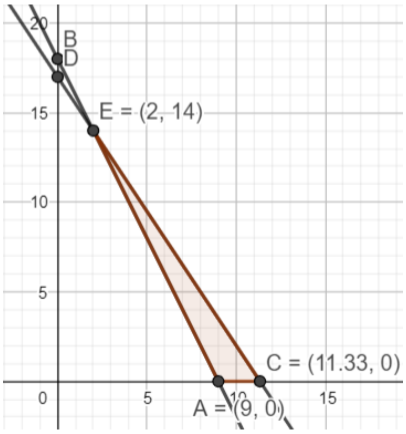
The line 3x + 2y = 34 meets the coordinate axes at

$C\left(\frac{34}{3}, 0\right)$ and D(0,17) respectively.

By joining these points we obtain the line 3x + 2y = 34 Clearly (0,0) satisfies the inequation 3x + 2y $\leq$ 34 . So, the region containing the origin represents the solution set of the inequation 3x + 2y $\leq$ 34

The corner points of the feasible region are A(9,0)

$C\left(\frac{34}{3}, 0\right)$ and E(2,14) and feasible region is bounded



The values of Z objective function at these corner points are as follows.

Corner point	Z = 50x + 30y
A(9, 0)	$50 \times 9 + 3 \times 0 = 450$
$C\left(\frac{34}{3}, 0\right)$	$50 \times \frac{34}{3} + 30 \times 0 = \frac{1700}{3}$
E(2, 14)	$50 \times 2 + 30 \times 14 = 520$

Therefore, the maximum value of objective function Z is

$\frac{1700}{3}$ at the point $\left(\frac{34}{3}, 0\right)$ Hence, x = $\frac{34}{3}$ and y = 0 is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is $\frac{1700}{3}$.

31. Let, $y = \tan^{-1}\left[\frac{a+b\tan x}{b-a\tan x}\right]$

$\Rightarrow y = \tan^{-1}\left[\frac{\frac{a+b\tan x}{b}}{\frac{b-a\tan x}{b}}\right]$

$\Rightarrow y = \tan^{-1}\left[\frac{\frac{a}{b}+\tan x}{1-\frac{a}{b}\tan x}\right]$

$\Rightarrow y = \tan^{-1}\left[\frac{\tan\left(\tan^{-1}\frac{a}{b}\right)+\tan x}{1-\tan\left(\tan^{-1}\frac{a}{b}\right)\times\tan x}\right]$

$\Rightarrow y = \tan^{-1}\left[\tan\left(\tan^{-1}\frac{a}{b}+x\right)\right]$

$\Rightarrow y = \tan^{-1}\left(\frac{a}{b}\right) + x$

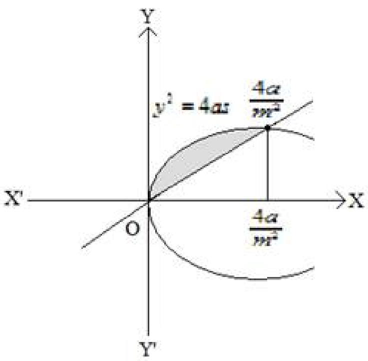
Differentiate it with respect to x,

$\frac{dy}{dx} = 0 + 1$

$\therefore \frac{dy}{dx} = 1$

Hence the derivative is equal to 1 for the given function .

Section D

32. 

$y^2 = 4ax$ ....(1)

y = mx........(2)

Using (2) in (1), we get,

$(mx)^2 = 4ax$

$\Rightarrow m^2x^2 = 4ax$

$x(m^2x - 4a) = 0$

$\Rightarrow x = 0, \frac{4a}{m^2}$

From (2),

When x = 0, y = m(0) = 0

When $x = \frac{4a}{m^2}, y = m \times \frac{4a}{m^2} = \frac{4a}{m}$

$\therefore$ points of intersection are (0, 0) and $(\frac{4a}{m^2}, \frac{4a}{m})$

Area $= \int_0^{4a/m^2} \sqrt{4ax}dx - \int_0^{\frac{4a}{m^2}} mx\,dx$

$= \sqrt{4a}\int_0^{\frac{4a}{m^2}} \sqrt{x}dx - m\int_0^{\frac{4a}{m^2}} xdx$

$= \sqrt{4a}[\frac{2}{3}x^{\frac{3}{2}}]_0^{\frac{4a}{m^2}} - m[\frac{x^2}{2}]_0^{\frac{4a}{m^2}}$

$= \sqrt{4a}[\frac{2}{3}(\frac{4a}{m^2})^{\frac{3}{2}} - 0] - \frac{m}{2}[(\frac{4a}{m^2})^2 - 0]$

$= \frac{2}{3m^3}(4a)^2 - \frac{1}{2m^3}(4a)^2$

$= \frac{(4a)^2}{m^3}[\frac{2}{3} - \frac{1}{2}]$

$= \frac{8a^2}{3m^3} sq\,unit.$

33. We observe the following properties of relation R.

Reflexivity: For any a ∈N

a - a = 0 = 0 × n

⇒ a - a is divisible by n

⇒ (a, a) ∈ R

Thus, (a, a) ∈ for all a ∈ Z. So, R is reflexive on Z

Symmetry: Let (a, b) ∈R. Then,

(a, b) ∈ R

⇒ (a - b) is divisible by n

⇒ (a - b) = np for some p ∈ Z

⇒ b - a = n (-p)

⇒ b - a is divisible by n $[\because p \in Z \Rightarrow -p \in Z]$

⇒ (b, a) ∈ R

Thus, (a, b) ∈ R ⇒ (b, a) ∈R for all a, b ∈ Z.

So, R is symmetric on Z.

Transitivity: Let a, b, c ∈ Z such that (a, b) ∈R and (b, c) ∈ R. Then,

(a, b) ∈ R

⇒ (a - b) is divisible by n

⇒ a - b = np for some p ∈ Z

and, (b, c) ∈ R

⇒ (b - c) is divisible by n

⇒ b - c = nq for some q ∈ Z

∴ (a, b) ∈ R and (b, c) ∈ R

⇒ a - b = np and b - c = nq

⇒ (a - b) + (b - c) = np + nq

⇒ a - c = n (p + q)

⇒ a - c is divisible by n $[\because p, q \in Z \Rightarrow p + q \in Z]$

⇒ (a, c) ∈ R

Thus, (a, b) ∈ R and (b, c) ∈ R ⇒(a, c) ∈ R for all a, b, c ∈ Z.

OR

Given that, A = R - {3}, B = R - {1}.

$f : A \rightarrow B$ is defined by $f(x) = \frac{x-2}{x-3}\ \forall x \in A$

For injectivity

Let $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

⇒ $(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

⇒ $x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$

⇒ $-3x_1 - 2x_2 = -3x_2 - 2x_1$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

So, f(x) is an injective function

For surjectivity

Let $y = \frac{x-2}{x-3} \Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = 2 - 3y $\Rightarrow x = \frac{2-3y}{1-y}$

$\Rightarrow x = \frac{3y-2}{y-1} \in A,\ \forall y \in B$ [codomain]

So, f(x) is surjective function.

Hence, f(x) is a bijective function.

34. Let $\frac{1}{x} = u, \frac{1}{y} = v$ and $\frac{1}{z} = w$

2u + 3v + 10w = 4

4u – 6v+ 5w = 1

6u + 9v – 20w = 2

$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} u \\ v \\ w \end{bmatrix}\ B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$

= 2[120 - 45] -3[-80 - 30] +10[36 + 36]

= 150 + 330 + 720 = 1200$\neq$ 0

$\Rightarrow$A is non-singular and hence A^{-1} exists.

Now, A_{11} = 75, A_{12} = 110, A_{13} = 72

A_{21} = 150, A_{22} = -100, A_{23} = 0

A_{31} = 75, A_{32} = 30, A_{33} = -2

$\therefore adjA = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

$X = A^{-1}B$

$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$

$\begin{bmatrix} y \\ v \\ w \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$

$u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5}$

$\frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$

x = 2, y = 3, z = 5

35. Suppose the point (1, 0, 0) be P and the point through which the line passes be Q(1,-1,-10). The line is parallel to the vector $\vec{b} = 2\hat{i} - 3\hat{j} + 8\hat{k}$

Now,

$\overrightarrow{PQ} = 0\hat{i} - \hat{j} - 10\hat{k}$

$\therefore \vec{b} \times \overrightarrow{PQ}$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 8 \\ 0 & -1 & -10 \end{vmatrix}$

$= 38\hat{i} + 20\hat{j} - 2\hat{k}$

$\Rightarrow |\vec{b} \times \overrightarrow{PQ}| = \sqrt{38^2 + 20^2 + 2^2}$

$= \sqrt{1444 + 400 + 4}$

$= \sqrt{1848}$

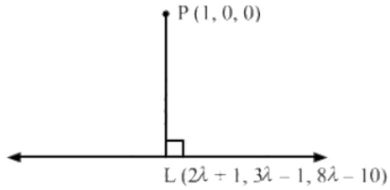
d = $\frac{|\vec{b} \times \overrightarrow{PQ}|}{|\vec{b}|}$

$= \frac{\sqrt{1848}}{\sqrt{77}}$

$= \sqrt{24}$

$= 2\sqrt{6}$

Suppose L be the foot of the perpendicular drawn from the point P(1,0,0) to the given line-



The coordinates of a general point on the line

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ are given by

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda$

$\Rightarrow x = 2\lambda + 1$

$y = -3\lambda - 1$

$z = 8\lambda - 10$

Suppose the coordinates of L be

$(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

Since,The direction ratios of PL are proportional to,

$2\lambda + 1 - 1, -3\lambda - 1 - 0, 8\lambda - 10 - 0$, i.e., $2\lambda, -3\lambda - 1, 8\lambda - 10$

Since,The direction ratios of the given line are proportional to 2, -3, 8, but PL is perpendicular to the given line.

$\therefore 2(2\lambda) - 3(-3\lambda - 1) + 8(8\lambda - 10)$ = 0

$\Rightarrow \lambda$ = 1 Substituting λ = 1 in $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$ we get the coordinates of L as (3, -4, -2). Equation of the line PL is given by

$\frac{x-1}{3-1} = \frac{y-0}{-4-0} = \frac{z-0}{-2-0}$

$= \frac{x-1}{1} = \frac{y}{-2} = \frac{z}{-1}$

$\Rightarrow \vec{r} = \hat{i} + \lambda(\hat{i} - 2\hat{j} - \hat{k})$

OR

Given lines are $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$

and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

On comparing both equations of lines with

$\vec{r} = \vec{a} + \lambda\vec{b}$ respectively, we get ,

$\overrightarrow{a_1} = \hat{i} + \hat{j} - \hat{k}, \overrightarrow{b_1} = 3\hat{i} - \hat{j}$

and $\overrightarrow{a_2} = 4\hat{i} - \hat{k}, \overrightarrow{b_2} = 2\hat{i} + 3\hat{k}$

Now $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$

$= \hat{i}(-3 - 0 - \hat{j}(9 - 0) + \hat{k}(0 + 2)$

$= -3\hat{i} - 9\hat{j} + 2\hat{k}$

and $\overrightarrow{a_2} - \overrightarrow{a_1} = \left(4\hat{i} - \hat{k}\right) - (\hat{i} + \hat{j} - \hat{k}) = 3\hat{i} - \hat{j}$

Now, $(\vec{a}_2 - \vec{a}_1) \cdot \left(\vec{b}_1 \times \vec{b}_2\right) = (3\hat{i} - \hat{j})$.$(-3\hat{i} - 9\hat{j} + 2\hat{k})$

= - 9 + 9 = 0

Hence, given lines are coplanar.

Now, cartesian equations of given lines are

$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$

and $\frac{x-4}{2}=\frac{y-0}{0}=\frac{z+1}{3}$

Then, equation of plane containing them is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix} x-1 & y-1 & z+1 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}=0$$

(x - 1) (-3- 0) - (y - 1) (9- 0)+(z+1)(0+2)=0

- 3x + 3 - 9y + 9 + 2z + 2 = 0

3x + 9y - 2z = 14

Section E

36. **Read the text carefully and answer the questions:**

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(i) According to given information, we construct the following table.

Given, total applications = 2000

	Folk Genre	**Classical Genre**
	960 (given)	2000 - 960 = 1040
Below 18	192 (given)	1040 - 104 = 936
18 or Above 18	960 - 192 = 768	104 (given)

Let E_1 = Event that application for folk genre

E_2 = Event that application for classical genre

A = Event that application for below 18

B = Event that application for 18 or above 18

$\therefore$ P(E_2) = $\frac{1040}{2000}$

and P(B ∩ E_2) = $\frac{104}{2000}$

Required Probability = $\frac{P(B\cap E_2)}{P(E_2)}$

= $\frac{\frac{104}{2000}}{\frac{1040}{200}}$ = $\frac{1}{10}$

(ii) Required probability = $P\left(\frac{\text{folk}}{\text{below 18}}\right)$

= $P\left(\frac{E_1}{A}\right)$

= $\frac{P(E_1\cap A)}{P(A)}$

Now, P(E_1 ∩ A) = $\frac{192}{2000}$

and P(A) = $\frac{192+936}{2000}$ = $\frac{1128}{2000}$

$\therefore$ Required probability = $\frac{\frac{192}{2000}}{\frac{1128}{2000}}$ = $\frac{192}{1128}$ = $\frac{8}{47}$

(iii) Here,

P(A) = 0.4, P(B) = 0.8 and P(B|A) = 0.6

$\because$ P(B|A) = $\frac{P(B\cap A)}{P(A)}$

$\Rightarrow$ P(B ∩ A) = P(B|A).P(A)

= 0.6 $\times$ 0.4 = 0.24

$\because$ P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

$= 0.4 + 0.8 - 0.24$
$= 1.2 - 0.24 = 0.96$

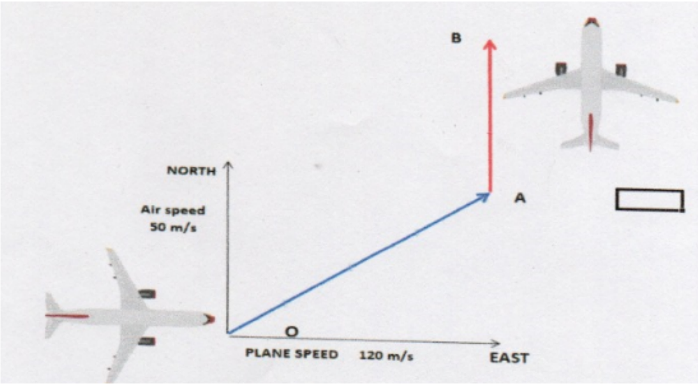
OR

Since, A and B are independent events, A' and B' are also independent. Therefore,
$P(A' \cap B') = P(A') \cdot P(B')$
$= (1 - P(A)(1 - P(B))$
$= \left(1 - \frac{3}{5}\right)\left(1 - \frac{4}{9}\right)$
$= \frac{2}{5} \cdot \frac{5}{9}$
$= \frac{2}{9}$

37. **Read the text carefully and answer the questions:**

A plane started from airport O with a velocity of 120 m/s towards east. Air is blowing at a velocity of 50 m/s towards the north As shown in the figure.

The plane travelled 1 hr in OA direction with the resultant velocity. From A and B travelled 1 hr with keeping velocity of 120 m/s and finally landed at B.



(i) Resultant velocity from O to A
$= \sqrt{(V_{\text{Plane}})^2 + (V_{\text{wind}})^2}$
$= \sqrt{(120)^2 + (50)^2}$
$= \sqrt{14400 + 2500}$
$= \sqrt{16900}$
$= 130$ m/s

(ii) $\tan\theta = \frac{V_{\text{wind}}}{V_{\text{aeroplane}}}$
$\tan\theta = \frac{50}{120}$
$\tan\theta = \frac{5}{12}$
$\theta = \tan^{-1}\left(\frac{5}{12}\right)$

(iii) Displacement from O to A = Resultant velocity $\times$ time
$|\overrightarrow{OA}| = |\vec{V}| \times t$
$= 130 \times \frac{18}{5} \times 1$
$= 468$ km

OR

Since, from A to B both Aeroplane and wind have velocity in North direction.
So,
$\vec{V}_{plane,AtoB} = 120 + 50$
$= 170$ m/s

38. **Read the text carefully and answer the questions:**

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where f(x) is the temperature in ^{o}F at x days.

(i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12).

(ii) $f(x) = -0.2x + m$

At Critical point

$0 = -0.2 \times 6 + m$

$m = 1.2$