Arithmetic Progressions

MATHEMATICAL REASONING

1. If 9th term of an A.P. is zero, then its 29th; term is _____ its 19th term.
(a) Thrice of (b) Twice of
(c) Half of (d) Equal to

2. Find the sum of first 20 terms of an A.P. whose nth term is given by $T_n = (7-3n).$;
(a) 382 (b) –490
(c) 420 (d) –382

3. In an A.P., the sum of first n terms is $\frac{3n^2}{2}+\frac{13n}{2}$. Find its 25th term.
(a) 80 (b) 120
(c) 60 (d) 78

4. Which term of the A.P. $5, 2, -1, ...$ is -22?
(a) 9 (b) 11
(c) 10 (d) 7

5. The sum of all terms of the arithmetic progression having ten terms except for the first term, is 99, and except for the sixth term, is 89. Find the 8th term of the progression if the sum of the first and the fifth term is equal to 10.
(a) 15 (b) 25
(c) 18 (d) 10

6. The 2nd, 31st and the last term of an AP are $7\frac{3}{4}, \frac{1}{2}$ and $-6\frac{1}{2}$, respectively. Find the number of terms.
(a) 60 (b) 59
(c) 65 (d) 45

7. In an A.P., if the p^{th} term is 'q' and the q^{th} term is 'p', then its n^{th} term is _____.
(a) $p+q-n$ (b) $p+q+n$
(c) $p-q+n$ (d) $p-q-n$

8. If in an A.P., $S_n = n^2 p$ and $S_m = m^2 p$, where S_r denotes the sum of r terms of the A.P., then S is equal to _____.
(a) $\frac{1}{2}p^3$ (b) mnp
(c) p^3 (d) $(m+n)p^2$

9. An A.P. consists of 21 terms. The sum of the three terms in the middle is 129 and of the last three is 237. Find the A.P.
(a) 4, 8, 12, 16 (b) 3, 6, 9, 12
(c) 4, 7, 10, 13 (d) 3, 7, 11, 15

10. If $x \neq y$ and the sequences x, a_1, a_2, y and x, b_1, b_2, y each are in A.P., then $\left(\frac{a_2-a_1}{b_2-b_1}\right)$ is _____.
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) 1 (d) $\frac{3}{4}$

11. The ratio of the sum of m and n terms of an A.P. is m2: n2, then find the ratio of m^{th} and n^{th} terms.
(a) $2m+1:2n+1$ (b) $2m-1:2n-1$
(c) $2m:n$ (d) $m:n$

12. Four numbers are inserted between the numbers 4 and 39 such that an A.P. results. Find the biggest of these four numbers.
(a) 33 (b) 31
(c) 32 (d) 30

13. If the m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then the sum of first mn terms is ___.
(a) $mn+1$ (b) $\frac{mn+1}{2}$
(c) $\frac{mn-1}{2}$ (d) $\frac{mn-1}{3}$

14. If 'a' is the A.M. of 3 numbers and 'b' is the A.M. of their squares, then the A.M. of their pair-wise products In terms of a and b is _____.

(a) $\frac{3a^2+b}{2}$ (b) $\frac{3a^2-b}{2}$

(c) $\frac{b-3a^2}{2}$ (d) $\frac{b^2-3a^2}{2}$

15. A circle with area A_1 is contained in the interior of a larger circle with area A_1+A_2. If the radius of the larger circle is 3 units and A_1, A_2, A_1+A_2 are in A.P., then the radius of the smaller circle is ___.

(a) $\sqrt{2}$ units (b) 1 unit

(c) 2 units (d) $\sqrt{3}$ units

EVERYDAY MATHEMATIC

16. The production of TV in a factory increases uniformly by a fixed number every year. It produced 8000 sets in 6th year and 11300 in 9th year. Find the production in the 6 years.

(a) 40500 (b) 20000

(c) 20500 (d) 31500

17. Deepak repays his total loan of Rs.1,18,000 by paying every month starting with the first instalment of Rs.1000. If he increases the instalment by Rs.100 every month, what amount will be paid as the last instalment of loan?

(a) Rs. 4900 (b) Rs. 5400

(c) Rs. 3500 (d) Rs. 4500

18. A manufacturer of laptop produced 6000 units in 3rd year and 7000 units in the 7th year. Assuming that production increases uniformly by a fixed number every year, find the production in the 5th year.

(a) 6500 units

(b) 5000 units

(c) 6000 units

(d) 8000 units

19. Raghav buys a shop for Rs. 120000. He pays half of the amount in cash and agrees to pay the balance in 12 annual instalments of ^ 5000 each. If the rate of interest Is 12% and he pays the interest due on the unpaid amount with the instalment. Find the total cost of the shop.

(a) Rs. 156800 (b) Rs. 156700

(c) Rs. 165200 (d) Rs. 166800

20. There are 25 trees at equal distances of 5 metres in a line with a well, the distance of the well from the nearest tree being 10 metres. A gardener waters all the trees separately starting from the well and he returns to the well after watering each tree to get water for the next. Find the total distance the gardener will cover in order to water all the trees.

(a) 3000 m (b) 3500 m

(c) 3800 m (d) 4000 m

ACHIEVERS SECTION (HOTS)

21. Which of the following statements is INCORRECT?

(a) Sum of n terms of the list of numbers $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$ is $\frac{n(n+1)}{\sqrt{2}}$

(b) The common difference of the A.P. given by is 3.

(c) The sum of the A.P. $(-5), (-8), (-11), \ldots\ldots (-230)$ is -8930.

(a) Only (a) (b) Only (b)

(c) Both (a) and (b) (d) (a), (b) and (c)

22. If there are $(2n+1)$ terms in A. P., then find the ratio of the sum of odd terms and the sum of even terms.

(a) $n:(n+1)$ (b) $(n+1):n$

(c) $n:(n+2)$ (d) $(n+2):n$

23. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. between a and b, then find the value of n.

(a) 0 (b) 1

(c) 2 (d) 3

24. The sum of the third and seventh terms; of an A.P. is 6 and their product is 8. Find the sum of first sixteen terms of the A. P.

(a) 86
(b) 90
(c) Both (A) and (B)
(d) None of these

25. Fill in the blanks.

(i) If the ratio of sum of n terms of two A.P is $(7n+1):(4n+27)$, then ratio of their m^{th} terms is **P**.

(ii) Sum of n odd natural numbers is **Q**

(iii) If sum of n terms of three A.P. are S_1, S_2, S_3. The first term of each is 1 and common differance are 1, 2 and 3 respectively, then $\frac{S_1+S_3}{S_2} = \underline{R}$

	P	Q	R
(a)	$\frac{14m-6}{8m+23}$	n^2	2
(b)	$\frac{13m+6}{7m+9}$	n^2	5
(c)	$\frac{14m+6}{8m+23}$	$2n+1$	1
(d)	$\frac{7m+1}{4m+27}$	$2n-1$	3

ANSWER KEY

1. B	**2.** B	**3.** A	**4.** C	**5.** A
6. B	**7.** A	**8.** C	**9.** D	**10.** C
11. B	**12.** C	**13.** B	**14.** B	**15.** D
16. D	**17.** A	**18.** A	**19.** D	**20.** B
21. D	**22.** B	**23.** A	**24.** D	**25.** A

HINTS AND SOLUTION

1. (b): Let 1st term of A.P. be a and common difference be d.

Now, $a_9 = 0 \Rightarrow a+8d = 0 \Rightarrow a = -8d$ ... (i)

Now, $a_{29} = a+28d = -8d+28d$

$\Rightarrow \quad a_{29} = 20d$ of ....(ii)

Also, $a_{19} = a+18d = -8d+18d = 10d$

$\Rightarrow \quad 2\times a_{19} = 2\times 10d = 20d$ .... (iii)

From (ii) and (iii), we have

$a_{29} = 2\, a_{19}$

2. (b): We have. $T_n = (7-3n)$

First term, $T_1 = (7-3\times 1) = 4$

Second term, $T_2 = 7-3\times 2 = 1$

Third term, $T_3 = 7-3\times 2 = -2$

$\therefore$ Series is $4, 1, -2, \ldots\ldots\ldots$

and common difference $= -3$

Sum of first 20 terms (S_{20})

$= \frac{20}{2}[2\times 4+(20-1)(-3)] = 10[8-57] = -490$

3. (a): We have given that $S_n = \frac{3n^2}{2} + \frac{13n}{2}$

25th term = Sum of 25 terms - Sum of 24 terms

$= S_{25} - S_{24}$

Now, $S_{25} = 1100$ and $S_{24} = 1020$

$\therefore$ 25th term $= 1100 - 1020 = 80$

4. (c): Given A.R is $5, 2, -1, \ldots\ldots$

$\Rightarrow \quad a = 5,\ d = 2-5 = -3$

$T_n = -22 \Rightarrow a+(n-1)d = -22$

$\Rightarrow 5+(n-1)(-3) = -22 \Rightarrow n = 10$

Hence, 10th term of the given A.P. is -22.

5. (a): According to the question, we have

$a_2 + \ldots\ldots + a_{10} = 99$ ......(i) and

$a_1 + \ldots\ldots . a_5 + a_7 + \ldots\ldots . a_{10} = 89$ ... (ii)

Subtracting (ii) from (i), we get

$\Rightarrow a_6 - a_1 = 10 \Rightarrow a_1 + 5d - a_1 = 10$

$\Rightarrow \quad 5d = 10 \Rightarrow d = 2$

Also, $a_1 + a_5 = 10 \Rightarrow a_1 + a_1 + 4d = 10$

$\Rightarrow \quad 2a_1 + 8 = 10 \Rightarrow a_1 = 1$

$\therefore$ 8th term $= a_1 + 7d = 1+14 = 15$

6. (b)

7. (a): We have given that $a_p = q$ and $a_q = p$

$\Rightarrow \quad q = a+(p-1)d$ and ... (i)

$p = a+(q-1)d$ ... (ii)

Subtracting (ii) from (i), we get
$q-p=d(p-q) \Rightarrow d=-1$
Now, $q=a+1-p$ [From (i)];
$\Rightarrow \quad a=q+p-1$
$\therefore \ a_n=a+(n-1)d=q+p-1+(n-1)(-1)$
$=q+p-1+1-n=q+p-n$

8. (c): We have given that,
$S_n=n^2p$ and $S_m=m^2p$
Thus, $n^2p=\frac{n}{2}[2a+(n-1)d]$
$\Rightarrow \quad 2np=2a+(n-1)d$ ....(i)
Similarly, $2np=2a+(n-1)d$ ....(ii)
Subtracting (ii) from (i), we get
$2p(n-m)=(n-m)d \Rightarrow d=2p$
Now, $2np=2a+(n-1)\times 2p$ [From (i)]
$\Rightarrow \ np=a+pn-p \ \Rightarrow \ a=p$
$\therefore \ S_p=\frac{p}{2}[2p+(p-1)\times 2p]=p[p+p^2-p]=p^3$

9. (d): Let 1st term of A.P. be a and common difference be d.
Now, three middle terms of this A.P. are a_{10}, a_{11} and a_{12}.
According to the question, $a_{10}+a_{11}+a_{12}=129$
$\Rightarrow \ (a+9d)+(a+10d)+(a+11d)=129$
$\Rightarrow \ 3a+30d=129 \Rightarrow a+10d=43$
$\Rightarrow \quad a=43-10d$ ... (i)
Also, last three terms are a_{19}, a_{20} and a_{21}.
$\therefore \quad a_{19}+a_{20}+a_{21}=237$
$\Rightarrow \ (a+18d)+(a+19d)+(a+20d)=237$
$\Rightarrow \ 3a+57d=237 \Rightarrow a+19d=79$
$\Rightarrow \ 43-10d+19d=79$ (Using (i))
$\Rightarrow \ 9d=36 \ \Rightarrow \ d=4$ and $a=3$
$\therefore$ A.P. is 3, 7, 11, 15....

10. (c): For sequence, x, a_1, a_2, y
$y==x+3d \ \Rightarrow \ d=\frac{y-x}{3}$
$\Rightarrow \ a_1=x+\frac{y-x}{3}, a_2=x+2\left[\frac{y-x}{3}\right]$ and
Similarly, $a_2-a_1=\left[\frac{y-x}{3}\right]$
For sequence, x, b_1, b_2, y
$d'=\frac{y-x}{3}$ and $b_2-b_1=\frac{y-x}{3} \Rightarrow$
$\frac{a_2-a_1}{b_2-b_1}=1$

11. (b): We have, given that, $\frac{S_m}{S_n}=\frac{m^2}{n^2}$
$$\Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]}=\frac{m^2}{n^2}$$
$$\Rightarrow \quad \frac{2a+(m-1)d}{2a+(n-1)d}=\frac{m}{n}$$
Replacing m with $2m-1$ and n with $2n-1$, we get
$$\Rightarrow \frac{2a+2(m-1)d}{2a+2(n-1)d}=\frac{2m-1}{2n-1}$$
$$\Rightarrow \frac{a+(m-1)d}{a+(n-1)d}=\frac{2m-1}{2n-1} \Rightarrow \frac{a_m}{a_n}=\frac{2m-1}{2n-1}$$

12. (c) : Series after the insertion of terms between 4 and 39 is 4, $a_1, a_2, a_3, a_4, 39$
Now, $39=4+5d \ \Rightarrow \ 35=5d \ \Rightarrow \ d=7$
$\therefore \ a_4=4+4\times 7=32$

13. (b) : We have given, $a_m=\frac{1}{n}$ and $a_n=\frac{1}{m}$
Then, $a+(m-1)d=\frac{1}{n}$ .....(i)
And, $a+(n-1)d=\frac{1}{m}$ ......(ii)
Subtracting (ii) from (i), we get,
$\Rightarrow \frac{1}{n}-\frac{1}{m}=(m-n)d \ \Rightarrow d=\frac{1}{mn}$
$\therefore \quad \frac{1}{n}=a+(m-1)\frac{1}{mn}$ [From (i)]
$\Rightarrow \frac{1}{n}-\frac{m-1}{mn}=a \ \Rightarrow \ a=\frac{1}{mn}$
Now, $S_{mn}=\frac{mn}{2}[2a+(mn-1)d]$

$=\frac{1}{2}[mn+1]=\frac{mn+1}{2}$

14. (b): Let the three terms be α,β and γ.
According to question,
$a=\frac{\alpha+\beta+\gamma}{3}$ .....(i) and
$b=\frac{\alpha^2+\beta^2+\gamma^2}{3}$ .....(ii)
We have to find $\frac{\alpha\beta+\beta\gamma+\alpha\gamma}{3}$
Squaring (i), we get
$9a^2=\alpha^2+\beta^2+\gamma^2+2[\alpha\beta+\alpha\gamma+\beta\gamma]$
Using (ii), $9a^2=3b+2[\alpha\beta+\alpha\gamma+\beta\gamma]$
$\Rightarrow \frac{9a^2-3b}{2}=\alpha\beta+\alpha\gamma+\beta\gamma$
Thus, $\frac{\alpha\beta+\alpha\gamma+\beta\gamma}{3}=\frac{3a^2-b}{2}$

15. (d): We have given, A_1, A_2, A_1+A_2 are in A.P.
$\Rightarrow 2A_2=A_1+A_1+A_2 \Rightarrow A_2=2A_1$ ..... (i)
And $A_2+A_1=\pi(3)^2 \Rightarrow 3A_1=9\pi$ ...(using (i)
$\Rightarrow A_1=3\pi \Rightarrow \pi r_1^2=3\pi$
[Here y, is the radius of smaller circle.]
$\Rightarrow r_1=\sqrt{3}$ units

16. (d): Since, production of TV in 6th year = 8000
$\Rightarrow a_6=8000 \Rightarrow a+5d=8000$ ....(i)
Also, production of TV in 9th year = 11300
$\Rightarrow a_9=11300$ or $a+8d=11300$ ....(ii)
Subtracting (i) from (ii), we get
$3d=3300 \Rightarrow d=1100$
From (i), a = 2500
$\therefore$ Production in 6 years, i.e.,
$S_6=\frac{6}{2}\left[2(2500)+(6-1)(1100)\right]=31500$

17. (a): 1st instalment =Rs. 1000
2nd instalment
$=Rs.1000+Rs.100=Rs.1100$
3rd instalment
$=Rs.1100+Rs.100=Rs.1200$ and so on
Let number of instalments = n
$\therefore$ $1000+1100+1200+\ldots$ up ton terms $=118000$
$\Rightarrow \frac{n}{2}\left[2\times1000+(n-1)100\right]=118000$
$\Rightarrow 100n^2+1900n-236000=0$
$\Rightarrow n^2+19n-2360=0 \Rightarrow (n+59)(n-40)=0$
$\Rightarrow n=40$ $(\because n\neq -59)$
$\therefore$ Total no. of instalments = 40
Now, last instalment = 40th instalment
$\therefore$ $a_{40}=a+39d=Rs.4900$

18. (a): Since, production of Laptops in 3rd year = 6000
$\Rightarrow a_3=6000 \Rightarrow a+2d=6000$ .... (I)
Also production of laptops in 7th year = 7000
$a_7=7000 \Rightarrow a+6d=7000$ ..... (ii)
Subtracting (i) from (ii), we get
$4d=1000 \Rightarrow d=250$
From (i), $a+2(250)=6000 \Rightarrow a=5500$
Hence, production in fifth year
$a_5=a+4d=(5500+4(250))=6500$ units

19. (d): Amount paid in cash
$=Rs.\left(\frac{1}{2}\times120000\right)=Rs.\ 60000$
Remaining amount
= Rs.$(120000-60000)$ =Rs. 60000
Amount of 1st instalment
$=Rs.(5000+\frac{12}{100}\times60000)=Rs.12200$
Amount of 2nd instalment
$=Rs.(5000+\frac{12}{100}\times55000)=Rs.11,600$
Amount of 3rd instalment
$=Rs.\left(5000+\frac{12}{100}\times50000\right)=Rs.11,000$
Total amount paid
$=Rs.12200+Rs.11600+Rs.11000+\ldots$ (12 instalments)
which form an A.P. with number of terms, $n=12,\ a=12200$ and $d=-600$

$\therefore$ Sum $= n\frac{n}{2}[2a+(n-1)d]$

$= \frac{12}{2}\left[2\times 12200+11\times(-600)\right]=106800$

$\therefore$ Total cost of the shop

$= Rs.60000+Rs.106800=Rs.166800$

20. (b): Since, distance of nearest tree from the well = 10 m
Also, each tree is at equal distance of 5 m from the next tree.
$\therefore$ A.P. formed is 10, 15, 20, ...
Here, $a=10$, $d=5$ and $n=25$

$S_{25}=\frac{25}{2}\left[2(10)+(25-1)5\right]=1750$

Hence, the total distance the gardener will cover in order to water all the trees
$=2\times 1750=3500\ m$

21. (d): (a) Given A.P. is

$\sqrt{2},2\sqrt{2},3\sqrt{2},4\sqrt{2},......$

$S_n=\frac{n}{2}\left[2\left(\sqrt{2}\right)+(n-1)\left(\sqrt{2}\right)\right]$

$=\frac{n}{2}\times\sqrt{2}[2+n-1]=\frac{n}{\sqrt{2}}[n+1]$

(b) Since, $a_n=3n+2$

Here, $a_1=3(1)+2=5$

$a_2=3(2)+2=8$

$\therefore$ Common difference $=a_2-a_1=3$

(c) Given A.P. is

$(-5),(-8),(-11),......(-230)$

$\because\ a_n=a+(n-1)(-3)$

$\Rightarrow\ -230=-5+(n-1)(-3)$

$\Rightarrow\ \frac{-225}{-3}=(n-1)\Rightarrow n=75+1=76$

$\Rightarrow\ S_n=\frac{76}{2}\left((-5)+(-230)\right)=-8930$

22. (b): Let a and d be the first term and common difference respectively of the given A.P.
Now, S_1 = Sum of odd terms

$\Rightarrow S_1=a_1+a_3+a_5+....+a_{2n+1}$

$\Rightarrow S_1=\frac{n+1}{2}\{a_1+a_{2n+1}\}$

$\Rightarrow S_2=\frac{n}{2}[a_2+a_{2n}]$

$\Rightarrow S_2=\frac{n}{2}\left[(a+d)+\{a+(2n-1)d\}\right]$

$\Rightarrow S_2=n(a+nd)$

$\therefore S_1:S_2=(\mathrm{n}+1)\ (\mathrm{a}+\mathrm{nd}):\mathrm{n}(\mathrm{a}+\mathrm{nd})$

$=(n+1):\mathrm{n}$

23. (a): A.M. between a and $b=\frac{a+b}{2}$

According to question, $\frac{a+b}{2}=\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$\Rightarrow aa^n+ba^n+ab^n+bb^n=2a^{n+1}+2b^{n+1}$

$\Rightarrow 2a^{n+1}+2b^{n+1}-a^{n+1}-ba^n-ab^n-b^{n+1}=0$

$\Rightarrow a^{n+1}+b^{n+1}-ba^n-ab^n=0$

$\Rightarrow a^n(a-b)-b^n(a-b)=0$

$\Rightarrow\ (a-b)(a^n-b^n)=0$

But $a-b\neq 0\ \Rightarrow\ a^n-b^n=0\Rightarrow a^n=b^n$

$\Rightarrow\quad \left(\frac{a}{b}\right)^n=1=\left(\frac{a}{b}\right)^0\Rightarrow n=0$

24. (d): Let a be the first term and d be the common difference of the A.P.
We have, $a_3+a_7=6$ and $a_3a_7=8$

$\Rightarrow (a+2d)+(a+6d)=6$ and

$(a+2d)(a+6d)=8$

$\Rightarrow 2a+8d=6$ and $(a+2d)(a+6d)=8$

$\Rightarrow d=\pm\frac{1}{2}$

Case-I: When $d=\frac{1}{2};\ a=1$

Case-II: When $d=-\frac{1}{2};a=5\Rightarrow S_{16}=20$

25. (a): (i) Let S_n be the sum of n terms of 1st A.P. and S'_n be sum of n terms of 2nd A.P.

According to question, $\frac{S_n}{S'_n}=\frac{7n+1}{4n+27}$

$$\frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]}=\frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{2a_1+(n-1)d_1}{2a_2(n-1)d_2}=\frac{7n+1}{4n+27}$$

Put $n=(2m-1)$ in above equation, we get

$$\frac{2a_1+(2m-1-1)d_1}{2a_2+(2m-1-1)d_2}=\frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\Rightarrow \quad \frac{[a_1+(m-1)d_1]}{[a_2+(m-1)d_2]}=\frac{14m-6}{8m+23}$$

$$\Rightarrow \quad \frac{a_m}{a_m}=\frac{14m-6}{8m+23}$$

(ii) A.P. of odd n natural numbers is 1, 3,5,.....

$$S_n=\frac{n}{2}[2(1)+(n-1)(2)]=\frac{n}{2}\times 2n=n^2$$

(iii) We have,

$$S_1=\frac{n}{2}[2(1)+(n-1)(1)]=\frac{n}{2}[n+1]$$

$$S_2=\frac{n}{2}[2(1)+(n-1)(2)]=\frac{n}{2}\times 2n=n^2$$

$$S_3=\frac{n}{2}[2(1)+(n-1)(3)]=\frac{n}{2}[3n-1]$$

Now, $\frac{S_1+S_3}{S_2}=2$