CBSE Class 09 Mathematics

Sample Paper 10 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part - A

1. Simplify: $(\sqrt{4})^{\frac{-3}{4}}$

OR

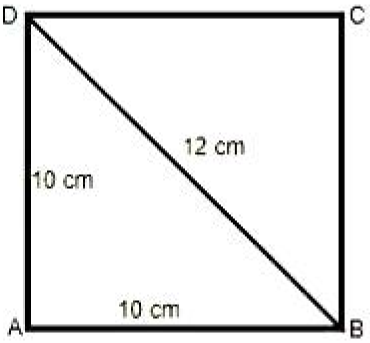
Prove that $\frac{2\sqrt{7}}{7\sqrt{7}}$ is a rational number.

2. Factorise: $(x^3 - x)$
3. The distance (in km) of 40 engineers from their residence to their place of work were found as follow :

5, 3, 10, 20, 25, 11, 13, 7, 12, 31, 19, 10, 12, 17, 18, 11, 32, 17, 16, 2, 7, 9, 7, 8, 3, 5, 12, 15, 18, 3, 12, 14, 2, 9, 6, 15, 15, 7, 6, 12

What is the empirical probability that an engineer lives less than 7 km from her place of work?

4. Construct a triangle ABC where BC = 5 cm, $\angle B = 30^0$ and AC - BC = 2 cm.
5. The perimeter of a rhombus ABCD is 40cm. find the area of rhombus if Its diagonals BD measures 12cm



OR

Each side of an equilateral triangle measures 8 cm. Find

1. the area of the triangle, correct to 2 places of decimal and
2. the height of the triangle, correct to 2 places of decimal. (Take $\sqrt{3} = 1.732$)

6. In a shower, 5 cm of rain falls. Find the volume of water that falls on 2 hectares of ground.
7. Simplify: $(25)^{\frac{-1}{3}} \times \sqrt[3]{16}$

OR

Classify the number $\frac{22}{7}$ as rational or irrational. Give reasons to support your answer.

8. Express of the equations in the form ax + by + c = 0 and indicate the values of a, b, c in case: $\frac{x}{5} - \frac{y}{6} = 1$
9. The surface area of a sphere is 5544 cm^2, find its diameter.

OR

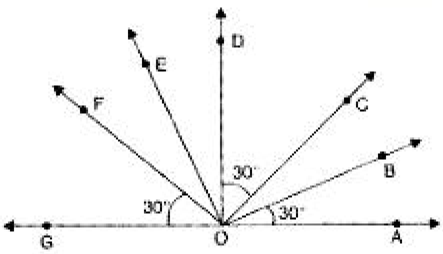
Find the volume of a sphere whose radius is 3.5 cm.

10. Simplify the following: $\frac{7.83\times7.83-1.17\times1.17}{6.66}$

11. Write the equation of the line that is parallel to y-axis and passing through the point (3, 5)

12. Is it polynomial? In case of a polynomial write its degree: $\frac{1}{3x}$

13. In figure, ∠AOF and ∠FOG form a linear pair, ∠EOB = ∠FOC = 90° and ∠DOC = ∠FOG = ∠AOB = 30°.Name three pairs of adjacent complementary angles.



14. 2x + y = 3 passes from origin. Is this statement true or false?

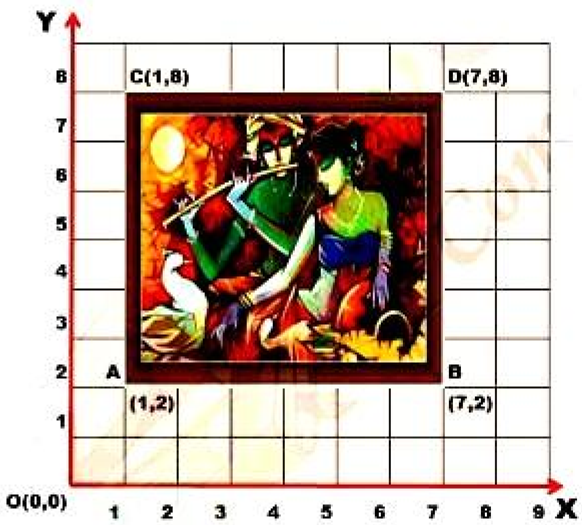
15. Check whether the point (a, -a) lies on y =x - a or not.

16. Classify the number 6.834834 ... as rational or irrational. Give reasons to support your answer.

OR

Examine the number rational or irrational: $\sqrt{8} \times \sqrt{2}$

17. **Read the Source/Text given below and answer any four questions:**

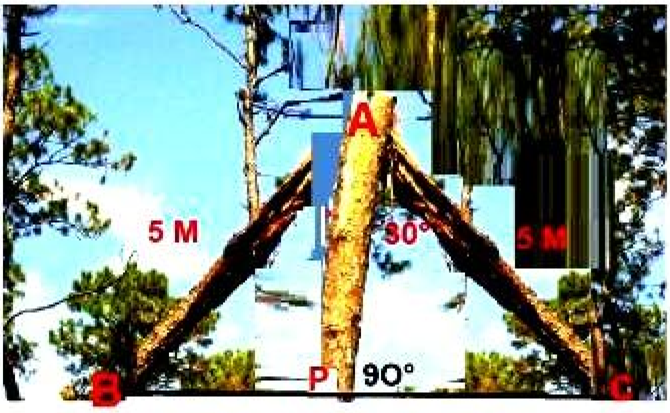


Rohit was putting up one of his paintings in his living room. Before this Rohit had put a grid on the wall where each unit measured equal to a foot. The upper-left corner of the frame is at point C (1, 8) and the upper-right corner at D (7, 8). The bottom-left corner is at A (1, 2) and the bottom-right corner at B (7, 2).

Please answer the following questions:

i. What is the width of the painting plus frame?
   a. 5 feet
   b. 8 feet
   c. 9 feet
   d. 6 feet
ii. What is the length of the painting plus frame?
   a. 9 feet
   b. 8 feet
   c. 6 feet
   d. 5 feet
iii. Which sides of the painting are parallel to x-axis?
   a. AB and CD
   b. AC and BD
   c. Diagonals AD and BC
   d. No one
iv. Which sides of the painting are parallel to y-axis?
   a. AB and CD
   b. AC and BD
   c. Diagonals AC and BD
   d. No one
v. Point A, B, C and D lie in which quadrant?
   a. I
   b. II
   c. III
   d. IV

18. **Read the Source/Text given below and answer any four questions:**
In a forest, a big tree got broken due to heavy rain and wind. Due to this rain the big branches AB and AC with lengths 5m fell down on the ground. Branch AC makes an angle of 30° with the main tree AP. The distance of Point B from P is 4 m. You can observe that ΔABP is congruent to ΔACP.



Now answer the following questions:

i. ΔACP and ΔABP are congruent by which criteria?

a. SSS
b. SAS
c. ASA
d. RHS

ii. What is the length of CP?

a. 4 m
b. 5m
c. 3m
d. 10 m

iii. What is the value of ∠BAP?

a. 40°
b. 50°
c. 30°
d. 60°

iv. What is the value of ∠APB?

a. 40°
b. 50°
c. 60°
d. 90°

v. What is the height of the remaining tree?

a. 4 m
b. 5m
c. 3m
d. 10m

19. **Read the Source/Text given below and answer any four questions:**

Four students of class IX B with names Ajay, Babloo, Charan and Deepak are playing a game in a circular playground.

All four students are holding radios with speaker and mic. These radios are connected by a wire of equal length that is 11 m (for each radio). Ajay Asks a question to Babloo. If Babloo gives the correct answer he gets 10 points and asks a new question to Charan, If he can not answer then he passes the same question to Charan and gets no points.

These conditions apply to all four players. After 10 rounds who gets maximum points, he becomes the winner.

i. What is the radius of the field?

a. 7 m

b. 14 m

c. 11 m

d. 22 m

ii. What is the area of the field?

a. 70 m^2

b. 154 m^2

c. 110 m^2

d. 220 m^2

iii. What is the area of the part marked with 1 on the field?

a. 50 m^2

b. 154 m^2

c. 76 m^2

d. 38.5 m^2

iv. What is the circumference of the field?

a. 22 m

b. 14 m

c. 44 m

d. 28 m

v. What is the direct distance from Ajay to Charan?

a. 7 m

b. 28 m

c. 15 m

d. 14 m

20. **Read the Source/Text given below and answer any four questions:**

There are 5 bags of seeds. If we select fifty seeds at random from each of 5 bags of seeds and sow them for germination. After 20 days, some of the seeds were germinated from each collection and were recorded as follows:

Bag	1	2	3	4	5
No. of seeds germinated	40	48	42	39	41

What is the probability of germination of

i. more than 40 seeds in a bag

a. 0.6

b. 0.1

c. 0.4

d. 0

ii. 49 seeds in a bag

a. 1

b. 0

c. 0.9

d. -1

iii. more than 35 seeds in a bag

a. 0

b. 3.5

c. 1

d. 0.35

iv. The sum of all probabilities equal to:

a. 4

b. 1

c. 3

d. 2

v. If P(E) = 0.44, then P(not E) will be:

a. 0.44

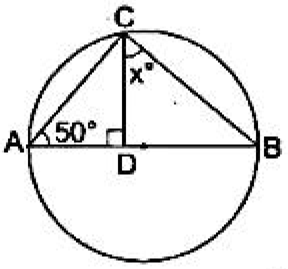
b. 0.55

c. 0.50

d. 0.56

Part - B

21. If O is the centre of the circle, find the value of x in given figure:



22. Find the value of $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{5}}$ if $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$

OR

Simplify $\frac{\sqrt{a^2-b^2}+a}{\sqrt{a^2+b^2}+b} \div \frac{\sqrt{a^2+b^2}-b}{a-\sqrt{a^2-b^2}}$.

23. Factorize: $x^4 + x^2 + 25$

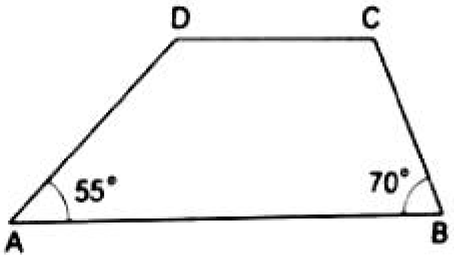
24. The radius and slant height of a cone are in the ratio 4 : 7. If its curved surface area is 792 cm^2, find its radius. (Use $\pi = \frac{22}{7}$).

25. If the area of an equilateral triangle is $81\sqrt{3}$ cm^2, find its perimeter.

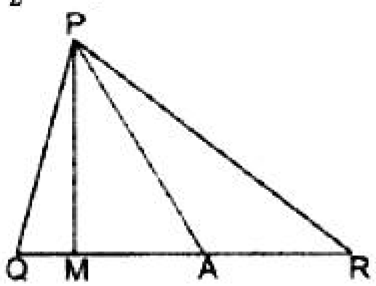
OR

Find the area of a triangle whose base is 25 cm long and the corresponding height is 10.8 cm.

26. In the adjoining figure, ABCD is a trapezium in which $AB||CD$. If $\angle A = 55^{\circ}$ and $\angle B = 70^{\circ}$ find $\angle C$ and $\angle D$



27. In given figure, $\angle Q > \angle R$, PA is the bisector of $\angle QPR$ and PM $\perp$ QR. Prove that $\angle APM = \frac{1}{2} (\angle Q - \angle R)$.



28. Construct perpendicular bisector of line segment 8cm.

OR

Construct an angle of 90° at the initial point of a given ray and justify the construction.

29. A juiceseller has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small cylindrical glasses of radius 3 cm up to a height of 8 cm, and sold for ₹ 15 each. How much money does he receive by selling the juice completely?

30. Factorize: $(x^2 - 4x)(x^2 - 4x - 1) - 20$

OR

By actual division, find the quotient and remainder when $3x^4 - 4x^3 - 3x - 1$ is divided by $x + 1$.

31. The sides of a triangle are in the ratio of 13 : 14 : 15 and its perimeter is 84 cm. Find the area of the triangle.

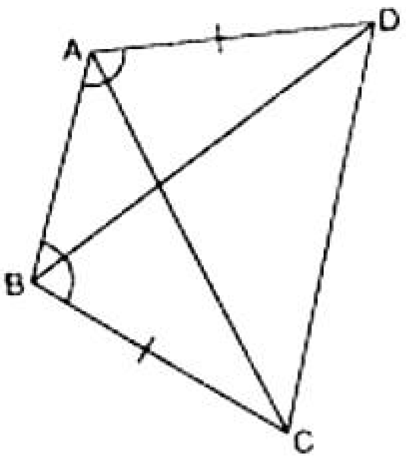
32. Simplify the following: $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$

33. ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA : Prove that:

  i. △ ABD ≅ △ BAC

  ii. BD = AC

  iii. ∠ DAB= ∠ CBA



34. The results of pass percentage of Class X and XII in C.B.S.E. examination for 5 years are given in the following table:

Year	1994-95	1995-96	1996-97	1997-98	1998-1999
X:	90	95	90	80	98
XII:	95	80	85	90	95

Draw bar graphs to represent the data.

OR

Draw the graph of the equation given below. Also, find the coordinates of the points where the graph cuts the coordinate axes.

$-x + 4y = 8$

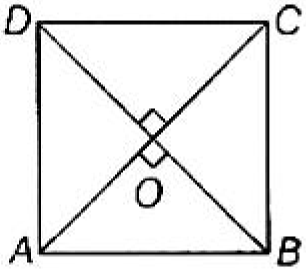
35. The percentage of marks obtained by a student in the monthly unit tests are given below:

Unit test	I	II	III	IV	V
Percentage of marks obtained	58	74	76	62	85

Find the probability that the student gets:

i. a first-class i.e. at least 60% marks
ii. marks between 70% and 80%
iii. a distinction i.e. 75% or above
iv. less than 65% marks

36. In the adjoining figure, ABCD is a quadrilateral such that AC = BD and AC and BD bisect each other at right angle. O is the intersection point of AC and BD. Show that the quadrilateral ABCD is a square.



CBSE Class 09 Mathematics

Sample Paper 10 (2020-21)

Solution

Part - A

1. $(\sqrt{4})^{\frac{-3}{4}} = 2^{\frac{-3}{4}} = \frac{1}{2^{\frac{3}{4}}}$

OR

We can cancel $\sqrt{7}$ in the numerator and denominator, as $\sqrt{7}$ is the common number in numerator as well as denominator, to get $\frac{2}{7}$

Therefore, we conclude that $\frac{2\sqrt{7}}{7\sqrt{7}}$ is a rational number.

2. We have,

$(x^3 - x) = x(x^2 - 1)$

$= (x)(x - 1)(x + 1)$ [$\because (a^2 - b^2) = (a - b)(a + b)$]

$\therefore (x^3 - x) = x(x - 1)(x + 1)$

3. Total number of engineers = 40

Number of engineers whose distance(in km) from their residence to their place of work is less than 7 km = 9

Therefore required Probability that an engineer lives less than 7 km from her place of work $= \frac{9}{40}$.

4. Given: In $\triangle$ABC, BC = 5 cm, B = 30^o and AC – BC = 2 cm.

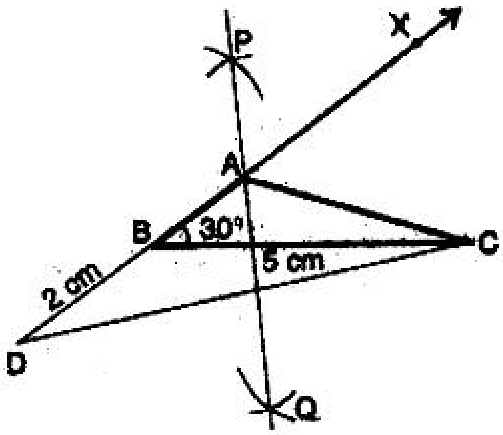
Required: To construct the $\triangle$ABC

Steps of construction :

i. Draw the base BC = 5 cm.

ii. At the point B construct an $\angle$XBC = 30^o

iii. Cut line segment BD equal to AC – AB = 2 cm. on the line XB extended or opposite side of line segment BC.



iv. Join DC and draw the perpendicular bisector, say PQ of DC.

v. Let PQ intersect BX at A.

vi. Join AC.

ABC is the required triangle.

5. $\therefore$ AB = BC = CD = DA = $\frac{40}{4}$ cm 10cm

now in $\triangle$ABD,

AB = 10cm, BD = 12cm and DA = 10cm

$\therefore$ S$=\frac{10+12+10}{2} cm =$16cm

Area of

$\triangle$ABD $=\sqrt{16(16-10)\,(16-12)\,(16-10)}$

$=\sqrt{16 \times 6 \times 4 \times 6} =$48sq cm

$\therefore$ area of rhombus ABCD

$=2 \times$ area of $\triangle$ABD

$= 2 \times 48$sq cm

= 96 sq cm

OR

i. Side of the equilateral triangle = 8 cm

Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{ Side })^2$

$= \frac{\sqrt{3}}{4} \times (8)^2$

$= \frac{1.732 \times 64}{4}$

Area of equilateral triangle = 27.71 cm^2

ii. Side of the equilateral triangle = 8 cm

Height $= \frac{\sqrt{3}}{2} \times$ Side

$= \frac{\sqrt{3}}{2} \times 8$

$= \frac{1.732 \times 8}{2}$

Height = 6.93 cm

6. Given,

Area of field = 2 hectare = 20000 m^2 [1 hectare = 10000 m^2]

Height of rainfall = 5 cm = 0.05 m {1m = 100cm}

Volume of water that falls = Area $\times$ height

= 20000 $\times$ 0.05 = 1000 m^3

7. We have ,

$(25)^{\frac{-1}{3}} \times \sqrt[3]{16} = (25)^{\frac{-1}{3}} \times (16)^{\frac{1}{3}} = \frac{(16)^{\frac{1}{3}}}{(25)^{\frac{1}{3}}} = \left(\frac{16}{25}\right)^{\frac{1}{3}}$

$= \left(\frac{2^3 \times 2}{5^2}\right)^{\frac{1}{3}} = \left(\frac{2^3 \times 2 \times 5}{5^2 \times 5}\right)^{\frac{1}{3}} = \left(\frac{2^3}{5^3} \times 10\right)^{\frac{1}{3}}$

$= \left(\frac{2^3}{5^3}\right)^{\frac{1}{3}} \times 10^{\frac{1}{3}} = \left\{\left(\frac{2}{5}\right)^3\right\}^{\frac{1}{3}} \times 10^{\frac{1}{3}} = \frac{2}{5} \times 10^{\frac{1}{3}}$

OR

The number $\frac{22}{7} = 3.\overline{142857}$,

$\frac{22}{7}$ is a non-terminating decimal and is considered as a rational number.

8. We have $\frac{x}{5} - \frac{y}{6}$ = 1

$\Rightarrow \frac{6x-5y}{30}$ = 1

$\Rightarrow$ 6x - 5y = 30

$\Rightarrow$ 6x - 5y - 30 = 0

On comparing this equation with ax + by + c = 0, we obtain

a = 6, b = -5 and c = -30

9. $\therefore$ Surface area = 5544 cm^2

$\Rightarrow 4\pi r^2$ = 5544

$\Rightarrow r^2 = \frac{5544}{4\pi}$

$\Rightarrow$ r = 21 cm

$\therefore$ Diameter = 2r = 42 cm

OR

r = 3.5 cm

$\therefore$ volume = $\frac{4}{3}\pi r^3$

= $\frac{4}{3} \times \frac{22}{7} \times (3.5)^3$

= 179.66 cm^3

10. We have,

$\frac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66}$

= $\frac{(7.83+1.17)(7.83-1.17)}{6.66}$ [$\because a^2 - b^2 = (a + b)(a-b)$]

= $\frac{(9.00)(6.66)}{6.66}$

= 9

11. The equation of the line that is parallel to the y-axis and passing through (3, 5) will be x = 3

12. $\frac{1}{3x}$ may be written as $\frac{1}{3} x^{-1}$, which is not a polynomial, as exponent of x is -1, which is a negative integer.

13. ∠GOF, ∠FOD, ∠FOE, ∠EOC, ∠EOD, ∠DOB.

14. When the line passes through origin, co-ordinates will be $(0, 0)$

$\therefore 2(0) + 0 = 3$

$0 \neq 3$

Since $LHS \neq RHS$,

$\therefore$ the given statement is false.

15. Given equation is y =x - a

RHS = x - a = a - a = 0 $\neq$ y

$\therefore$ (a,- a) does not lie on y = x - a

16. $6.834834\ldots = 6.\overline{834}$

The given number 6.834834 ... is a non-terminating recurring decimal and hence taken as a rational number....

OR

We have.

$\sqrt{8} \times \sqrt{2} = \sqrt{4 \times 2} \times \sqrt{2} = 2\sqrt{2} \times \sqrt{2} = 4$

Hence, it is a rational number.

17. i. (d) 6 feet

ii. (c) 6 feet

iii. (a) AB band CD

iv. (b) AC and BD

v. (a) I

18. i. (d) RHS

ii. (a) 4 m

iii. (c) 30°

iv. (d) 90°

v. (c) 3 m

19. i. (a) 7 m

ii. (b) 154 m^2

iii. (d) 38.5 m^2

iv. (c) 44 m

v. (d) 14 m

20. i. (a) 0.6

ii. (b) 0

iii. (c) 1

iv. (b) 1

v. (d) 0.56

Part - B

21. In $\triangle$ADC, we have

$$\angle CAD + \angle ADC + \angle ACD = 180^\circ$$
$$\Rightarrow 50^\circ + 90^\circ + \angle ACD = 180^\circ$$
$$\Rightarrow \angle ACD = 40^\circ$$

But, $\angle ACB = 90^\circ$ [angle in a semicircle].

$$\therefore \quad \angle ACD + x^\circ = 90^\circ$$
$$\Rightarrow \quad 40^\circ + x^\circ = 90^\circ$$

=> $x^o = 50^o$.

22. $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}+5}{5}$

$= \frac{3.162+5}{5} = \frac{8.162}{5} = 1.6324$

OR

We have

$\frac{\sqrt{a^2-b^2}+a}{\sqrt{a^2+b^2}+b} \div \frac{\sqrt{a^2+b^2}-b}{a-\sqrt{a^2-b^2}}$

$= = \frac{a+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}+b} \times \frac{a-\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-b}$

$= \frac{(a)^2-(\sqrt{a^2-b^2})^2}{(\sqrt{a^2+b^2})^2-(b)^2}$

$= \frac{a^2-(a^2-b^2)}{(a^2+b^2)-b^2}$

$= \frac{a^2-a^2+b^2}{a^2+b^2-b^2}$

$= \frac{b^2}{a^2}$

23. We have,

$x^4 + x^2 + 25 = (x^2)^2 + 2.x^2.5 + 5^2 - 9x^2$

$= \{(x^2)^2 + 2.x^2.5 + 5^2\} - (3x)^2$

[using $a^2 + 2ab + b^2 = (a + b)^2$]

$= \{x^2 + 5\}^2 - (3x)^2$

[By using $a^2 - b^2 = (a + b)(a - b)$]

$= (x^2 + 5 + 3x)(x^2 + 5 - 3x)$

24. Let the radius of cone $(r) = 4x\ cm$ and the slant height of the cone $(l) = 7x\ cm$

Curved surface area of cone $= \pi rl$

$\therefore \pi rl = 792 cm^2$

$\Rightarrow \frac{22}{7} \times 4x \times 7x = 792$

$\Rightarrow x^2 = \frac{792}{22 \times 4} = 9$

$\Rightarrow x = 3cm$

$\therefore$ Radius of the cone $= 4 \times 3 = 12cm$

25. Let the side of an equilateral triangle is a cm.

Area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$\frac{\sqrt{3}}{4}a^2 = 81\sqrt{3}$

$\Rightarrow a^2 = 81 \times 4$

$\Rightarrow a = 18$ cm

$\therefore$ Perimeter of equilateral triangle = 3a = 3 $\times$ 18 = 54 cm.

OR

Here, base = 25 cm & height = 10.8 cm.

Area of the triangle $= (\frac{1}{2} \times$ base $\times$ height) sq units

Area of the triangle$= \left(\frac{1}{2} \times 25 \times 10.8\right) \text{cm}^2$ = 135 cm^2

Hence, the area of the given triangle is 135 cm^2

26. Here given that ABCD is trapezium where $AB||CD$.

We observe that $\angle A$ and $\angle D$ are the interior angles on the same side of transversal line AD, whereas $\angle B$ and $\angle C$ are the interior angles on the same side of transversal line BC.

As $\angle A$ and $\angle D$ are interior angles, we have,

$\angle A + \angle D = 180^\circ$

$\therefore \angle D = 180^\circ - \angle A$

$\therefore \angle D = 180^\circ - 55^\circ = 125^\circ$

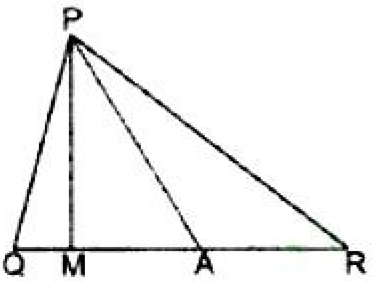
Similarly for $\angle B$ and $\angle C$,

$\angle B + \angle C = 180^\circ$

$\therefore \angle C = 180^\circ - \angle B$

$\therefore \angle C = 180^\circ - 70^\circ = 110^\circ$

Hence, measure of $\angle D$ and $\angle C$ are 125° and 110° respectively.

27. 

Since PA is the bisector of $\angle$QPR

$\therefore \angle$QPA = $\angle$APR ...(i) [see figure]

In $\triangle$PQM, we have

PQM + $\angle$PMQ + $\angle$QPM = 180° (Angle sum property of triangle)

$\Rightarrow \angle$PQM + 90° + $\angle$QPM = 180° [Since, PM is perpendicular to QR]

$\Rightarrow \angle$PQM = 90° - $\angle$QPM

$\Rightarrow \angle Q = 90^\circ - \angle QPM$ ....(ii)

In $\triangle$PMR, we have

$\angle$PMR + $\angle$PRM + $\angle$RPM = 180° (Angle sum property of triangle)

$\Rightarrow$ 90°+ $\angle$PRM + $\angle$RPM = 180° [Since, PM is perpendicular to QR]

$\Rightarrow \angle PRM = 180° - 90° - \angle RPM$

$\Rightarrow \angle R = 90° - \angle RPM$ ...(iii)

Subtracting (iii) from (ii), we get

$\angle Q - \angle R = (90° - \angle QPM) - (90° - \angle RPM)$

$\Rightarrow \angle Q - \angle R = \angle RPM - \angle QPM$

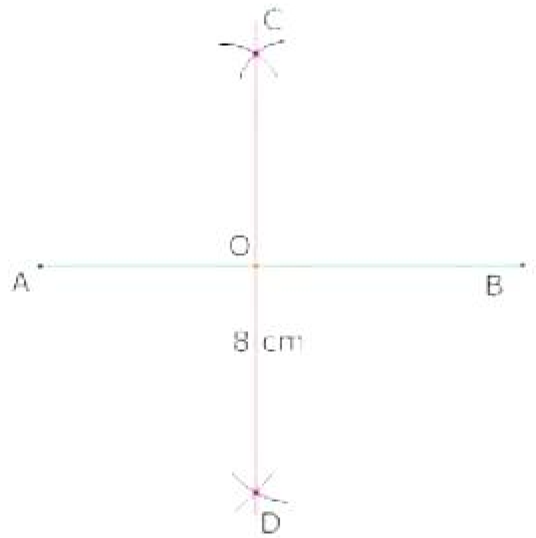
$\Rightarrow \angle Q - \angle R = (\angle RPA + \angle APM) - (\angle QPA - \angle APM)$ [from figure]

$\Rightarrow \angle Q - \angle R = \angle QPA + \angle APM - \angle QPA + \angle APM$. [Using (i)]

$\Rightarrow \angle Q - \angle R = 2\angle APM$

Hence, $\angle APM = \frac{1}{2}(\angle Q - \angle R)$. Proved.

28. Steps of construction



i. Draw a line segment AB = 8cm
ii. Taking A as a centre draw arcs of radius more $\frac{1}{2}AB$ on both side of AB.
iii. Taking B as a centre draw arcs of same radius on both sides of AB which intersect previous arcs at point C and D.
iv. Join CD which intersect AB at point O
v. OA = OB = 4cm

OR

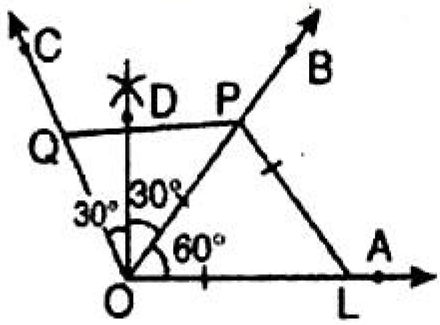
Steps of construction:

a. Draw a ray OA.
b. With O as centre and convenient radius, draw an arc LM cutting OA at L.
c. Now with L as centre and radius OL, draw an arc cutting the arc LM at P.
d. Then taking P as centre and radius OL, draw an arc cutting arc PM at the point Q.

e. Join OP to draw the ray OB. Also, join O and Q to draw the OC. We observe that: $\angle AOB = \angle BOC = 60°$

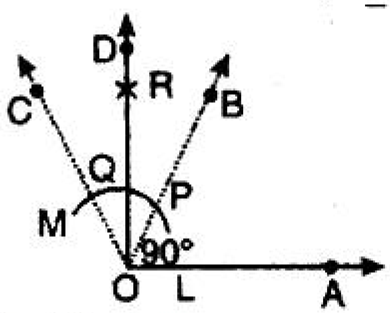
f. Now we have to bisect BOC. For this, with P as centre and radius greater than $\frac{1}{2}$ PQ draw an arc.

g. Now with Q as centre and the same radius as in step 6, draw another arc cutting the arc drawn in step 6 at R.



h. Join O and R and draw ray OD.

Then $\angle AOD$ is the required angle of 90°.



Justification:

Join PL, then OL = OP = PL [by construction]

Therefore ΔOLP is an equilateral triangle and $\angle POL$ which is same as $\angle BOA$ is equal to 60°.

Now join QP, then OP = OQ = PQ [by construction]

Therefore ΔOQP is an equilateral triangle.

$\therefore \angle POQ$ which is same as $\angle BOC$ is equal to 60°.

By construction, OD is the bisector of $\angle BOC$.

$\therefore DOC = \angle DOB = \frac{1}{2}\angle BOC = \frac{1}{2} \times 60° = 30°$

Now, $\angle DOA = \angle BOA + \angle DOB$

$\Rightarrow \angle DOA = 60° + 30°$

$\Rightarrow \angle DOA = 90°$

29. We are given that,

Radius (r) of cylindrical vessel = 15 cm

Height (h) of cylindrical vessel = 32 m

Therefore, Volume of cylindrical vesel = $\pi r^2 h$

$= \left(\frac{22}{7} \times 15 \times 15 \times 32\right) \text{cm}^3$

$= \frac{158400}{7} \text{cm}^3$

Radius of small cylindrical glass = 3 cm

Height of a small cylindrical glass = 8 cm

Volume of each small cylindrical glass

= $\left(\frac{22}{7} \times 3 \times 3 \times 8\right) \text{cm}^3 = \frac{1584}{7} \text{cm}^3$

Therefore, Number of small glasses filled

= $\frac{\text{Volume of cylindrical vessel}}{\text{Volume of each glass}}$

$= \frac{\frac{158400}{7}}{\frac{1584}{7}}$

= 100

cost of 1 glass = Rs. 15

$\Rightarrow$ Cost of 100 glasses = Rs. (15 $\times$ 100) = Rs.1500

30. We have,

$(x^2 - 4x)(x^2 - 4x - 1) - 20$

$= (x^2 - 4x)^2 - (x^2 - 4x) - 20$

$= y^2 - y - 20$, where $y = x^2 - 4x$

$= y^2 - 5y + 4y - 20$

$= (y^2 - 5y) + (4y - 20)$

$= y(y - 5) + 4(y - 5)$

$= (y - 5)(y + 4)$

$= (x^2 - 4x - 5)(x^2 - 4x + 4)$ [Replacing y by $x^2 - 4x$]

$= (x^2 - 5x + x - 5)(x - 2)^2$

$= \{x(x - 5) + (x - 5)\}(x - 2)^2 = (x - 5)(x + 1)(x - 2)^2$

OR

$$
\begin{array}{r|l}
 & 3x^3 - 7x^2 + 7x - 10 \\
\hline
x+1 & 3x^4 - 4x^3 - 3x - 1 \\
 & \underline{{}_{-}3x^4 \pm 3x^3} \\
 & -7x^3 - 3x - 1 \\
 & \underline{\mp 7x^3 \quad \mp 7x^2} \\
 & 7x^2 - 3x - 1 \\
 & \underline{{}_{-}7x^2 \pm 7x} \\
 & -10x - 1 \\
 & \underline{\mp 10x \mp 10} \\
 & 9
\end{array}
$$

Quotient = $3x^3 - 7x^2 + 7x - 10$, Remainder $= 9$

31. Perimeter = 84 cm.

Ratio of sides = 13 : 14 : 15

Sum of the ratios = 13 + 14 + 15 = 42

$\therefore$ One side (a) = $\frac{13}{42} \times 84$ = 26 cm.

Second side (b) = $\frac{14}{42} \times 84$ = 28 cm.

Third side (c) = $\frac{15}{42} \times 84$ = 30 cm

$\therefore$ s = $\frac{a+b+c}{2}$

= $\frac{26+28+30}{2}$ = $\frac{84}{2}$ = 42 cm

$\therefore$ Area of the triangle

= $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{42(42-26)(42-28)(42-30)}$

= $\sqrt{42(16)(14)(12)}$

= $\sqrt{42(16)(14)(4\times 3)}$

= (42)(4)(2) = 336 cm^2

32. We have,

$\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$

= $\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{1}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}}$

= $\frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} + \frac{2\sqrt{5}+2\sqrt{3}}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} + \frac{2+\sqrt{5}}{(2-\sqrt{5})(2+\sqrt{5})}$

= $\frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} + \frac{2\sqrt{5}+2\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{2+\sqrt{5}}{(2)^2-(\sqrt{5})^2}$

$= \frac{2-\sqrt{3}}{1} + \frac{2\sqrt{5}+2\sqrt{3}}{2} + \frac{2+\sqrt{5}}{-1}$

$= 2 - \sqrt{3} + \sqrt{5} + \sqrt{3} - 2 - \sqrt{5}$

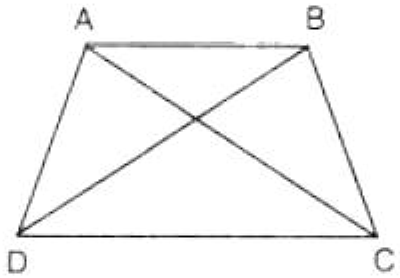
$= 0$

$\therefore \frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}} = 0$

33. i. $\triangle ABD \cong \triangle BAC$ ...[From (i)]

ABD = BAC ...[c.p.c.t.]

ii.



Given: In Fig., AD = BC and BD = CA.

To Prove: $\angle DAB = \angle CBA$.

Proof: In ΔABD and ΔABC, we have

AD = BC [Given]

BD = CA [Given]

and, AB = AB [Common]

$\triangle ABD \cong \triangle CBA$ [by SSS congruence criterion]

$\Rightarrow \angle DAB = \angle ABC$ [CPCT]

$\Rightarrow \angle DAB = \angle CBA$

Hence proved.

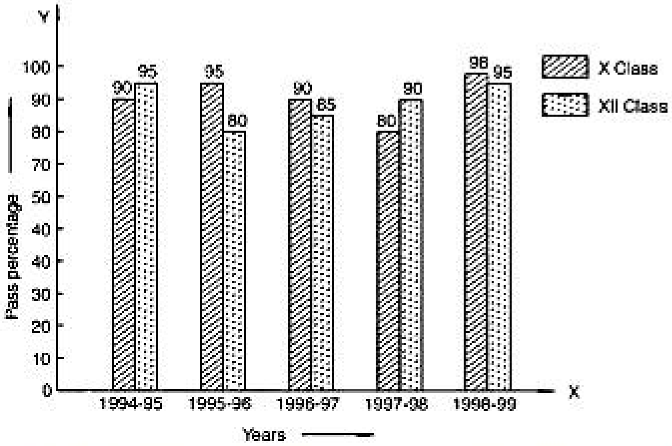
iii. Since

$\triangle ABD \cong \triangle BAC$

$\therefore$ BD = AC [By C.P.C.T.]

34. We use the following steps:

Step I: We draw two lines perpendicular to each other on a graph paper and call them horizontal and vertical axes as shown in Figure



Step II: Along the horizontal axis, we mark the 'years' and along the vertical axis, we mark the 'pass percentage'.

Step III: We choose a suitable scale to determine the heights of bars.

Here, we choose the scale as 1 big division to represent 10.1.

Step IV: First we draw the bars for Class X results and then bars for Class XII results for different years.

Bars for X and XII class results are shaded separately and the shading is shown in the top-right comer of the graph paper.

OR

We have,

$-x + 4y = 8$

$\Rightarrow 4y - 8 = x$

$\Rightarrow x = 4y - 8$ .....(i)

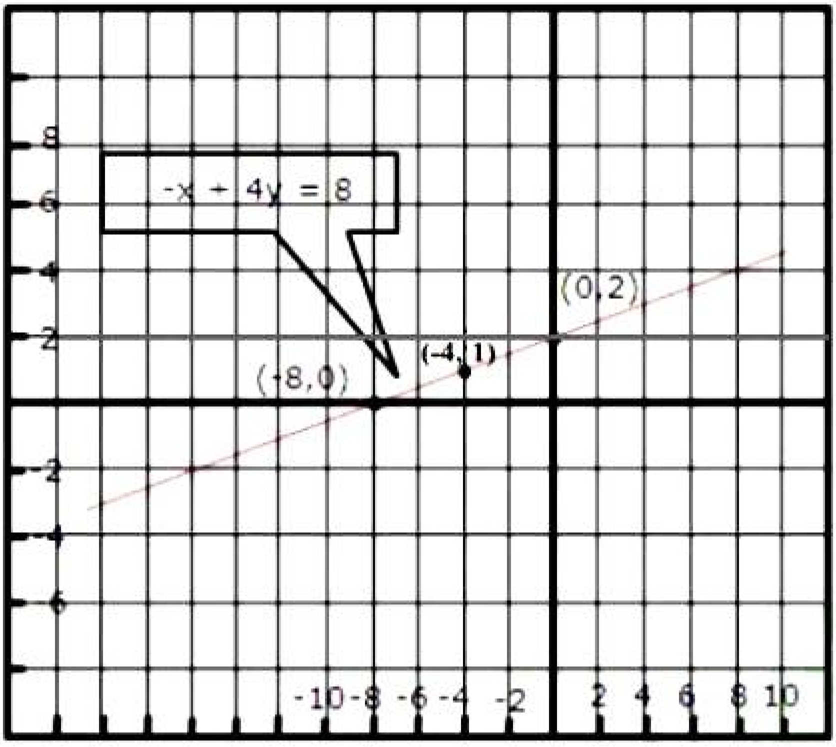
Putting y = 1 in (i), we get $x = 4 \times 1 - 8 = -4$

Putting y = 2 in (i), we get $x = 4 \times 2 - 8 = 0$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $-x + 4y = 8$.

x	-4	0
y	1	2

The graph of line $-x + 4y = 8$:



Clearly, the line intersects with the coordinate axes at (-8, 0) and (0, 2).

35. Total number of unit tests held = 5

   i. Number of unit tests in which the student gets a first class i.e. at least 60% marks = 4

   ∴ Probability that the student gets a first class = $\frac{4}{5}$ = 0.8

   ii. Number of tests in which the student gets marks between 70% and 80% = 2

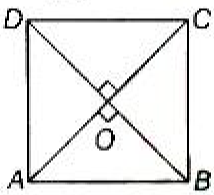
   ∴ Probability that a student gets marks between 70% and 80% = $\frac{2}{5}$ = 0.4

   iii. Number of tests in which the student gets distinction = 2

   ∴ Probability that the student gets distinction = $\frac{2}{5}$ = 0.4

   iv. Number of tests in which the student gets less than 65% marks = 2

   ∴ Probability that a student gets less than 65% marks = $\frac{2}{5}$ = 0.4

36. 

Given :- ABCD is a quadrilateral such that AC = BD and AC and BD bisect each other at right angle at point O.

∴ OA = OB = OC = OD [∵ AC = BD] ...(i)

and ∠AOB = ∠BOC = ∠COD = ∠DOA = 90° ...(ii)

To prove :- Quadrilateral ABCD is a square.

Consider, In △AOB, OA = OB [given]

and $\triangle AOB = 90°$ [given]

$\angle OAB = \angle OBA = 45°$ [angles opposite to equal sides are also equal]

Similarly, we can write that,

$\angle OAD = \angle ODA = 45°$

$\angle OBC = \angle OCB = 45°$

Then, $\angle A = \angle BAD = \angle BAO + \angle OAD = 45° + 45° = 90°$

Similarly, we can also write that

$\angle B = \angle ABC = \angle OBA + \angle OBC = 45° + 45° = 90°$

Also, $\angle C = 90°$ and $\angle D = 90°$

In $\triangle AOB$ and $\triangle COB$, OA = OC [given]

OB = OB [common side]

and $\angle AOB = \angle BOC$ [given]

$\triangle AOB \cong \triangle COB$ [by SAS congruence rule]

Then, AB = BC [As corresponding parts of the congruent triangles are equal]

Similarly, we can write that,

BC = CD and CD = AD

Thus, we get

AB = BC = CD = AD and also we have that

$\angle A = \angle B = \angle C = \angle D = 90°$

Hence, as all the sides of the given quadrilateral are equal and all the angle are of 90°, we can write that, the given quadrilateral ABCD is a square.