09

The materials having a definite shape and volume are known as solids or rigid bodies. They are of great importance for us as they are used in constructing buildings, bridges, railway tracks, automobiles, artificial limbs, electric poles, wires, ropes, etc. But in reality, these bodies can be deformed on applying force on them. Our aim is to study these materials under loads or forces and their mechanical properties. e.g. elasticity, plasticity, stress and strain etc., in this chapter.

MECHANICAL PROPERTIES OF SOLIDS

|TOPIC 1|

Elastic and Plastic Behaviour of Solids

CHAPTER CHECKLIST

- Elastic Body and Elasticity
- Plastic Body and Plasticity
- Stress
- Strain
- Hooke's Law
- Stress-strain Curve
- Elastic Modulus
- Energy Stored in a Deformed Body

ELASTIC BODY AND ELASTICITY

A body that returns to its original shape and size on the removal of the deforming force (when deformed within elastic limit) is called an **elastic body**. Quartz fibre, ivory ball and phosphor bronze are the elastic bodies.

The property of matter by virtue of which it regains its original shape and size, when the deforming forces have been removed is called **elasticity**. e.g. If we stretch a spring, and release, then it will regain its original size.

PLASTIC BODY AND PLASTICITY

A body that does not regain its original shape and size even after the removal of deforming force, is called a **plastic body**. Putty, paraffin wax, mud and quartz are nearly perfectly plastic bodies.

The property of a body by virtue of which it does not regain its original shape and size even after the removal of deforming force, is said to be a **plasticity**. e.g. If we stretch a piece of chewing gum and release, it will not regain its original shape and size.

Note

- All rigid bodies are elastic to some extent, which means we can change their dimensions slightly by pulling or pushing, them.
- No body is perfectly elastic or perfectly plastic. All the bodies found in nature lie between these two limits. When the elastic behaviour of a body decreases, its plastic behaviour increases.

Deforming Force

A force acting on a body, instead of producing a change in its state of rest or of uniform motion, produces a change in the shape of the body, such a force is called deforming force.

A rigid body can be noticeably stretched, compressed, bent or twisted by applying a suitable force. So, that a body can be deformed by a force. This can be easily shown by stretching a rubber band or by loading a spring.

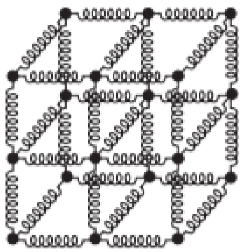
Elastic Behaviour of Solids

The atoms in a solid are held together by interatomic or intermolecular forces. These forces keep solid in a stable equilibrium position. When a solid is deformed, the atoms or molecules are displaced from their equilibrium positions. Thus, deformation causes change in interatomic or intermolecular distances.

On removing deforming forces, the interatomic forces tend to drive the displaced atoms or molecules to their original equilibrium positions.

Spring Ball Model of Solids

Atoms in a solid may be regarded as mass points or small balls connected in three-dimensional space through springs. Then, springs represent the interatomic forces of attraction between balls. This is called spring ball model of a solid as shown in figure.



Spring ball model for explaining elastic behaviour of solids

Normally, the balls occupy the positions of minimum potential energy or zero interatomic force. When any ball is displaced from its equilibrium or mean position, the various springs connected to it exert a restoring force on this ball. This force tends to bring the ball to its equilibrium position.

It shows the elastic behaviour of solid in terms of microscopic nature of the solid. Robert Hook, an English Physicist performed an experiment on springs and found that the elongation produced in a body is proportional to the applied force or load.

STRESS

When a deforming force is applied on a body, it changes the configuration of the body by changing the normal positions of the molecules or atoms of the body. As a result, an internal restoring force comes into play which tends to bring the body back to its initial configuration.

The internal restoring force acting per unit area of a deformed body is called **stress**.

i.e. $$\text{Stress} = \frac{\text{Restoring force}\,(F)}{\text{Area of cross-section}\,(A)}$$

If there is no permanent change in the configuration of the body i.e. in the absence of plastic behaviour of the body, the restoring force is equal and opposite to the external deforming force applied. Thus, quantitatively, stress can be given as

$$\text{Stress, } S = \frac{\text{External deforming force}}{\text{Area of cross-section}}$$

Units and Dimensional Formula of Stress

Its SI unit is N/m^2 or pascal (Pa) and in CGS system unit is $dyne/cm^2$. The dimensional formula of stress is $[ML^{-1}T^{-2}]$

Note

- Stress is not a vector quantity. Since, unlike a force, the stress cannot be assigned a specific direction.
- Breaking stress is constant for a material.

On the basis of applied forces on the body, the stress can be classified as

1. Normal Stress or Longitudinal Stress

It is defined as the restoring force per unit area, acts perpendicular to the surface of the body. It is of two types

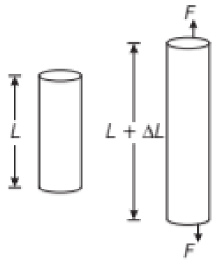
(i) Tensile Stress

When two equal and opposite forces are applied at the ends of a circular rod as shown in Fig. (a) to increase its length, then a restoring force equal to the applied force F normal to the cross-sectional area A of the rod comes into existence. This restoring force per unit area of cross-section is known as **tensile stress**.

$$\text{Tensile stress} = \frac{F}{A}$$

In case of tensile stress, there is increase in length of a body. Consider a rod of length L, the two equal forces F are applied in the direction as shown in figure, then the final length of the rod becomes $L + \Delta L$.

Thus, increase in length of the rod is ΔL.



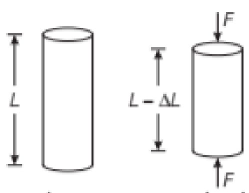
Tensile stress on a circular rod

(ii) Compressive Stress

When two equal and opposite forces are applied at the ends of a rod as shown in Fig. (b) to decrease its length or compress it, then again restoring force equal to the applied force comes into existence. This restoring force per unit area of cross-section of the rod is known as **compressive stress**.

$$\text{Compressive stress} = \frac{F}{A}$$

In case of compressive stress, there is decrease in length of a body. If a rod of length L, the two equal forces F are applied in the direction as shown in figure , then the final length of the rod becomes $L - \Delta L$. Thus, decrease in length of the rod is ΔL.



Compressive stress on a circular rod

Under tensile stress or compressive stress, the net force acting on an object is zero but the object is deformed.

Note

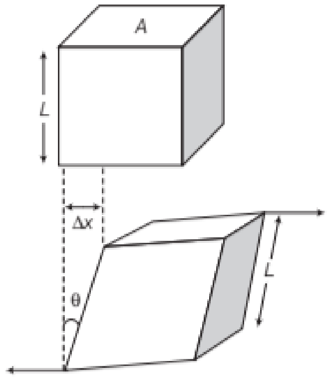
Tensile stress or compressive stress is also termed as **longitudinal stress.**

2. Tangential or Shearing Stress

When a deforming force acts tangentially to the surface of a body, it produces a change in the shape of the body without any change in volume. This tangential force applied per unit area of cross-section is known as **tangential stress.**

$$\text{Tangential stress} = \frac{F}{A}$$

In case of tangential stress, the deforming force F is applied on top surface of the cubical body in tangential direction due to which the upper face is deformed by an angle θ from its original position as shown in figure.



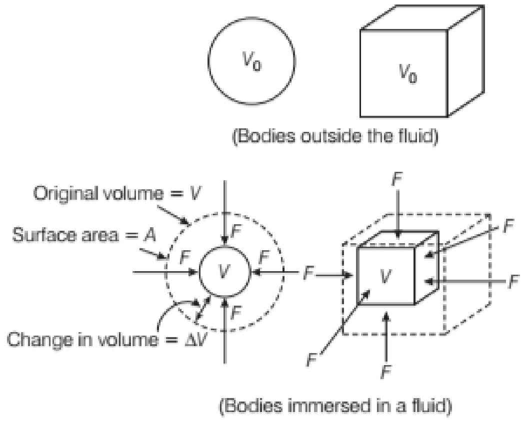
Deforming force on the surface of a body

3. Hydraulic or Bulk Stress

If a body is subjected to a uniform force from all sides, then the corresponding stress is called **hydraulic stress** or bulk stress. There is a change in volume of the body but not change in geometrical shape.

$$\text{Bulk stress} = \frac{F}{A}$$

In case of hydraulic stress, the force F is applied perpendicular to every point on the surface of body due to which the change in volume ΔV of a body occured.



Hydraulic stress on different surfaces

Note

- The hydraulic stress is also known as volumetric stress.
- The effect of stress is to produce distortion or change in size, volume and shape (i.e. configuration of the body).

EXAMPLE |1| Stress in a Wire

Calculate the value of stress in a wire of steel having radius of 2 mm of 10 kN of force is applied on it.

Sol. Force, $F = 10\,\text{kN} = 1\times10^4$ N

Radius, $r = 2\,\text{mm} = 2\times10^{-3}$ m

Area, $A = \pi r^2 = \pi\times(2\times10^{-3})^2$

$= 12.56\times10^{-6}\ \text{m}^2$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{1\times10^4\ \text{N}}{12.56\times10^{-6}\ \text{m}^2} = 0.0796\times10^{10} = 7.96\times10^8\ \text{N/m}^2$$

EXAMPLE |2| Flying tackle

A man carrying mass $M = 125$ kg makes a flying tackle at $v_1 = 4$ m/s on a stationary quarterback of mass $m = 85$ kg and his helmet makes solid contact with quarterback's femur.

(i) What is the final speed of two athletes immediately after contact and also determine the average force exerted on the quarterback's femur, when last collision occur at 0.100 s?

(ii) If area of cross-section of quarterback's femur is 5×10^{-4} m^2, then estimate the shear stress exerted on femur in the collision.

Sol.

(i) Here, $M = 125$ kg, $v_1 = 4$ m/s, $m = 85$ kg

Applying, conservation of linear momentum, we get

$p_{\text{initial}} = p_{\text{final}}$ i.e. $Mv_i = (M+m)v_f$

The value of final speed

$$v_f = \frac{Mv_i}{M+m} = \frac{125\times4}{(125+85)} = 2.38\ \text{m/s}$$

(ii) Average force exerted to the quarterback's femur

So, $F_{\text{av}}\times\Delta t = M(v_f - v_i)$

i.e. $$F_{\text{av}} = \frac{M(v_f-v_i)}{\Delta t} = \frac{125\,(4-2.38)}{0.1} = \frac{125\times1.62}{0.1} = 2.03\times10^3\ \text{N}$$

$$\text{Shearing stress} = \frac{F}{A} = \frac{2.03\times10^3}{5\times10^{-4}} = 4.06\times10^6\ \text{Pa}$$

STRAIN

When a deforming force acts on a body, the body undergoes a change in its shape and size. The ratio of the change in configuration of the body to the original configuration is called **strain.**

$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

If there is a change in any of the configuration of the body due to the applied deforming force on it, then the body is said to be strained or deformed.

Strain is the ratio of two like quantities, so it has no unit and dimension.

According to a change in configuration i.e. change in length, volume or shape of the body, the strain can be classified as

1. Longitudinal Strain

It is defined as the change in length per unit original length, when the body is deformed by external forces.

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta L}{L}$$

EXAMPLE |3| Percentage Strain in Rod

Consider a steel rod having radius of 8 mm and the length of 2m. If a force of 150 kN stretches it along its length, then calculate the stress, percentage strain in the rod if the elongation in length is 7.46 mm.

Sol. If the rod stretches along its length, then the stress produced is the tensile stress whereas the strain produced is longitudinal strain.

Radius, $r = 8\ \text{mm} = 8\times10^{-3}$ m, Length, $L = 2$ m

Force, $F = 150\,\text{kN} = 15\times10^4$ N

Area, $A = \pi r^2 = \pi\times(8\times10^{-3})^2 = 201\times10^{-6}\ \text{m}^2$

$\Delta L = 7.46\,\text{mm} = 7.46\times10^{-3}$ m, Percentage strain = ?

$$\text{Stress} = \frac{F}{A} = \frac{15\times10^4}{201\times10^{-6}} = 0.0746\times10^{10}\ \text{N/m}^2 = 7.46\times10^8\ \text{N/m}^2$$

$$\text{Longitudinal strain} = \frac{\Delta L}{L} = \frac{7.46\times10^{-3}}{2} = 3.73\times10^{-3}$$

Percentage strain $= 3.73\times10^{-3}\times100 = 0.37$ %

2. Volumetric Strain

It is defined as the change in volume per unit original volume, when the body is deformed by external forces.

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}$$

EXAMPLE |4| Volumetric Strain in a Cube

Consider a solid cube which is subjected to a pressure of 6×10^5 N/m^2. Due to this pressure, each side of the cube is shortened by 2%. Find out the volumetric strain of the cube.

Sol. Let L be the initial length of the each side of the cube.

Volume, $V = L \times L \times L = L^3$

$= \text{Initial volume } (V_i \text{ say})$

If the each side of the cube is shortened by 2%, then final length of the cube $= L - 2\%$ of L

$$= \left(L - \frac{2L}{100}\right) = L\left(1 - \frac{2}{100}\right)$$

$\therefore$ Final volume, $V_f = L^3\left(1 - \frac{2}{100}\right)^3 = V\left(1 - \frac{2}{100}\right)^3$

Change in volume, $\Delta V = V_f - V_i = V\left(1 - \frac{2}{100}\right)^3 - V$

$$= V\left[\left(1 - \frac{2}{100}\right)^3 - 1\right]$$

$$\frac{\Delta V}{V} = \left(1 - \frac{2}{100}\right)^3 - 1 \approx \left[1 - \frac{2 \times 3}{100}\right] - 1$$

$[\because (1 - x)^n \approx 1 - nx \text{ for } x << 1]$

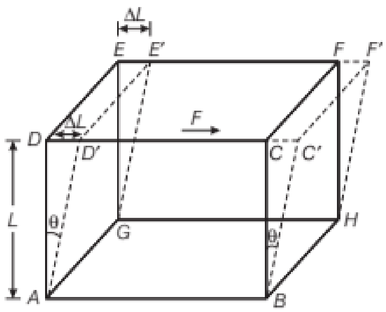
$\therefore$ Volumetric strain $= \frac{\Delta V}{V} = 1 - 0.06 - 1 = 0.06$

(take positive sign)

3. Shear Strain

If the deforming forces produce a change in the shape of the body, then the strain is called **shear strain**. Within elastic limit, it is measured by the ratio of the relative displacement of one plane to its distance from the fixed plane. It can also be measured by the angle through which a line originally perpendicular to the fixed plane as shown in figure.

Consider a cubical body which gets deformed under the effect of tangential force F. The vertical planes $ADEG$ and $BCFH$ are laterally shifted to positions $AD'E'G$ and $BC'F'H$ respectively through an angle θ. If $AD = L$ and $EE' = \Delta L$ (change in perpendicular distance of the displaced surface from the fixed surface $ADEG$)



Shear strain

The angle θ is called **angle of shear.**

$$\boxed{\text{Shear strain } (\theta) = \tan\theta = \frac{\Delta L}{L}}$$

If $L = 1$m, then shear strain $= \Delta L$

So, shear strain is the relative displacement between two parallel planes, a unit distance apart.

The strain is the ratio of two like quantities, i.e. the change in dimension to the original or initial dimension, it has no unit or dimensional formula.

Note

- The strain produced in a spring is longitudinal as well as shear if the spring is stretched by suspending a load at its free end.
- The strain persists even when the stress is removed and thus lags behind the stress.

EXAMPLE |5| A Cubical Body Gets Deformed

If the angle of shear is 30° for a cubical body and the change in length is 250 cm, then what must be the volume of this cubical body?

Sol. Given, angle of shear = 30°

and change in length $\Delta L = 250 \text{ cm} = 2.5 \text{ m}$

$\therefore$ Shear strain, $\tan\theta = \frac{\Delta L}{L} \Rightarrow \tan 30° = \frac{2.5}{L}$

$$L = \frac{2.5}{\tan 30°} = \frac{2.5}{0.577} = 4.332 \text{ m}$$

Volume, $V = L^3 = 81.309 \text{ m}^3$

HOOKE'S LAW

From the experimental investigations, Robert Hooke, an English physicist (1635-1703 AD) in 1679 formulated a law known after him as Hooke's law which states that, the extension produced in the wire is directly proportional to the load applied within defined limit of elasticity.

So, $\boxed{\text{extension} \propto \text{load applied}}$

Later on, it was found that this law is applicable to all types of deformations such as compression, bending, twisting etc., and thus a modified form of Hooke's law was given as

Within elastic limit, the stress developed is directly proportional to the strain produced in a body.

i.e. $\text{Stress} \propto \text{Strain} \Rightarrow \text{Stress} = E \times \text{Strain}$

or $\boxed{E = \frac{\text{Stress}}{\text{Strain}}}$

where E is a constant and is known as **modulus of elasticity** of the material of the body.

EXAMPLE |6| Hooke's law

After a fall, a 95 kg rock climber finds himself dangling from the end of a rope that had been 15 m long and 9.6 mm in diameter but has stretched by 2.8 cm. For the rope, calculate.

(i) the strain, (ii) the stress and

(iii) the modulus of elasticity.

Sol. Here, $L = 1500$ cm is the unstretched length of the rope, $\Delta L = 2.8$ cm is the amount of length stretches.

$$\text{The strain} = \frac{\Delta L}{L} = \frac{2.8 \text{ cm}}{1500 \text{ cm}} = 1.9 \times 10^{-3}$$

$$\text{Stress} = \text{Force/Area}$$

Force, F = force of gravity on the rock climber

$$= mg = 95 \times 9.8 \text{N}$$

$$\text{Area, } A = \pi r^2 = \pi \times \left(\frac{D}{2}\right)^2 = \pi \times \left(\frac{9.6}{2} \times 10^{-3} \text{ m}\right)^2$$

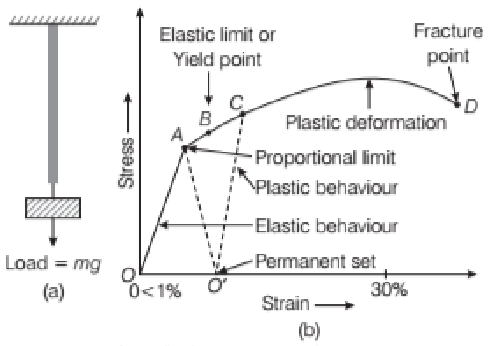
$$\text{Stress} = \frac{95 \times 9.8}{\pi \times (4.8)^2 \times 10^{-6}} \simeq 1.29 \times 10^7 \text{ N/m}^2$$

$$\text{Hence, } E = \frac{\text{Stress}}{\text{Strain}} = \frac{1.3 \times 10^7}{1.9 \times 10^{-3}} = 6.84 \times 10^9 \text{ N/m}^2$$

STRESS-STRAIN CURVE

When a wire is stretched by a load as in Fig. (*a*), it is seen that for small value of load, the extension produced in the wire is proportional to the load.

Hence, stress $\propto$ strain



A typical stress-strain curve

(i) In drawing, a stress *vs* strain graph, the stress is found to be proportional to strain (% elongation) up to point A. Thus, Hooke's law is fully obeyed in this region, the point A is known as point of **proportional limit**.

(ii) When stress is increased beyond A, then for small stress, there is a large strain in the wire upto point B.

(iii) When the load is gradually removed between points O to B, the wire return to its original length.

The wire regains its original dimension only when load applied is less than or equal to certain limit. This limit is called **elastic limit.** The point B on stress-strain curve is known as **elastic limit** or **yield point.**

The material of the wire in the region OB shows the elastic behaviour, hence known as **elastic region.**

Elastic Limit

Elastic limit is the upper limit of deforming force up to which, if deforming force is removed, the body regains its original form completely and beyond which if deforming force is increased, the body loses its property of elasticity and gets permanently deformed. Elastic limit is the property of a body whereas elasticity is the property of material of a body.

(iv) If the stress or load increases beyond point B, the strain further increases. This increase in strain represented by BC part of the curve. Now, if the load is removed, the wire does not regain its original length. But the increase in the length of the wire is permanent.

In other words, there is permanent strain equal to OO' in the wire even when the stress is zero.

This permanent strain in the wire is known as permanent set.

(v) Now, as the stress beyond C is increased, there is large strain in the wire. This large increase in the strain for small stress is represented by CD part of the curve. The wire breaks at point D which is also known as **fracture point.**

The material of the wire from point C to point D shows the **plastic behaviour** or **plastic deformation.** The stress needed to cause the actual fracture of the material is known as **breaking stress** or **ultimate tensile strength.**

Note

- Hooke's law is valid only in the linear portion of the stress-strain curve. The law is not valid for large values of strain.
- Elastic limit and limit of proportionality are very close to each other, so that Hooke's law is nearly applicable upto elastic limit.
- In the yield region, strain is 15 to 20 times those that takes place upto the proportional limit occur during yielding.

Breaking Stress of Some Materials

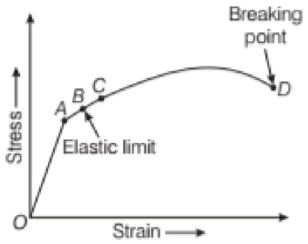
Material	Breaking stress (in Nm^{-2})
Aluminium	2.2×10^8
Iron	3.0×10^8
Brass	4.7×10^8
Phosphor bronze	5.6×10^8
Steel	5 to 20×10^8
Glass	10×10^8

On the basis of elastic and plastic properties, materials can be classified in two ways.

(i) Ductile Materials

The materials which have large plastic range of extension are called **ductile materials.** As shown in the stress-strain curve, the fracture point is widely separated from the elastic limit.

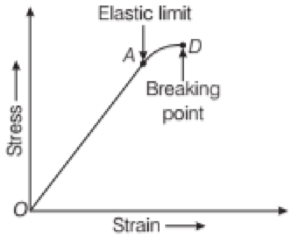
Such materials undergo an irreversible increase in length before snapping. So, they can be drawn into thin wires e.g. copper, silver, iron, aluminium, etc.



Stress-strain curve for a ductile material

(ii) Brittle Materials

The materials which have very small range of plastic extension are called **brittle materials**. Such materials break as soon as the stress is increased beyond the elastic limit.

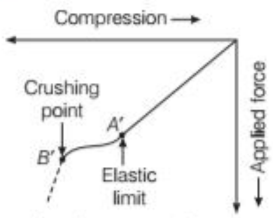


Stress-strain curve for a brittle material

Stress-strain curve for a brittle material

Their breaking point lies just close to their elastic limit, as shown in figure e.g. cast iron, glass, ceramics, etc.

Malleability



Load compression curve for a metal

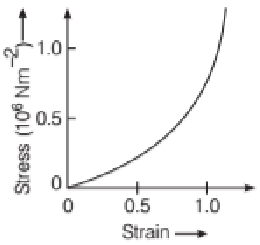
When a solid is compressed, a stage is reached beyond which it cannot regain its original shape after the deforming force is removed. This is the elastic limit point (A') for compression. The solid then behaves like a plastic body.

The yield point (B') obtained under compression is called **crushing point.** After this stage, metals are said to be **malleable** i.e. they can be hammered or rolled into thin sheets. e.g. gold, silver, lead, etc.

Elastomers

The materials which can be elastically stretched to large values of strain are called **elastomers**. e.g. rubber can be stretched to several times its original length but still it can regain its original length when the applied force is removed.

There is no well defined plastic region, rubber just breaks when pulled beyond a certain limit. Elastic region in such cases is very large, but the material does not obey Hooke's law. In our body, the elastic tissue of aorta (the large blood vessel carrying blood from the heart) is an elastomer, for which the stress-strain curve is shown in figure.



TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. The property of a body by virtue of which it tends to regain its original size and shape of a body when applied force is removed, is known as

(a) fluidity (b) elasticity
(c) plasticity (d) rigidity

Sol. (b) The property of a body, by virtue of which it tends to regain its original size and shape when the applied force is removed, is known as elasticity and the deformation caused is known as elastic deformation.

2. Elasticity is shown by materials because inter-atomic or inter-molecular forces

(a) increases when a body is deformed
(b) decreases when a body is deformed
(c) remains same when a body is deformed
(d) becomes non-zero when a body is deformed

Sol. (d) When a body is deformed, atoms/molecules are displaced from their equilibrium positions ($F = 0$). As a result, there is a force ($F \neq 0$) acts between them to restore their position.

3. The maximum load a wire can withstand without breaking, when its length is reduced to half of its original length, will **[NCERT Exemplar]**

(a) be double (b) be half
(c) be four times (d) remain same

Sol. (d) We know that,

$$\text{Maximum stress} = \frac{\text{Maximum force}}{\text{Area of cross-section}}$$

When length of the wire changes, area of cross-section remains same.

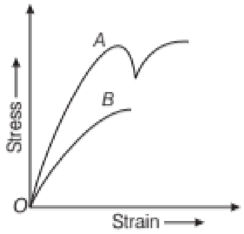
Hence, maximum force will be same when length changes.

4. A wire is stretched to double its length. The strain is

(a) 2 (b) 1 (c) zero (d) 0.5

Sol. (b) $\text{Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{2L - L}{L} = 1$

5. Stress-strain curves for the material A and B are shown below



Then,

(a) A is brittle material
(b) B is ductile material
(c) B is brittle material
(d) Both (a) and (b)

Sol. (c) B is brittle as there is no plastic region. However, A is ductile as it has large plastic range of extension.

VERY SHORT ANSWER Type Questions

6. Two identical solid balls, one of ivory and the other of wet-clay, are dropped from the same height on the floor. Which will rise to a greater height after striking the floor and why?

Sol. The ball of ivory will rise to a greater height because, ivory is more elastic than wet-clay.

7. Is stress a vector quantity? **[NCERT Exemplar]**

Sol. No, because stress is a scalar quantity, not a vector quantity.

$$\text{Stress} = \frac{\text{Magnitude of internal reaction force}}{\text{Area of cross-section}}$$

8. A thick wire is suspended from a rigid support, but no load is attached to its free end. Is this wire under stress?

Sol. Yes, the wire is under stress due to its own weight.

9. Stress and pressure are both forces per unit area. Then, in what respect does stress differ from pressure?

Sol. Pressure is an external force per unit area, while stress is the internal restoring force which comes into play in a deformed body acting transversely per unit area of a body.

10. Which type of strain is there, when a spiral spring is stretched by a force?

Sol. Longitudinal strain and shear strain.

11. What does the slope of stress *versus* strain graph indicate?

Sol. The slope of stress (on y-axis) and strain (on x-axis) gives modulus of elasticity.

The slope of stress (on x-axis) and strain (on y-axis) gives the reciprocal of modulus of elasticity.

12. Is it possible to double the length of a metallic wire by applying a force over it?

Sol. No, it is not possible because within elastic limit, strain is only order of 10^{-3}, wires actually break much before it is stretched to double the length.

SHORT ANSWER Type Questions

13. A steel cable with a radius of 1.5 cm supports a chair lift at a ski area. If the maximum stress is not to exceed 10^8 N/m^2, then what is the maximum load the cable can support? **[NCERT]**

Sol. Given, radius of steel cable $(r) = 1.5 \text{ cm} = 1.5 \times 10^{-2}$ m

$$\text{Maximum stress} = 10^8 \text{ N/m}^2$$

Area of cross-section of steel cable $(A) = \pi r^2$

$$= 3.14 \times (1.5 \times 10^{-2})^2 \text{ m}^2$$
$$= 3.14 \times 2.25 \times 10^{-4} \text{ m}^2$$

$$\text{Maximum stress} = \frac{\text{Maximum force}}{\text{Area of cross-section}}$$

or Maximum force = Maximum stress × Area of cross-section

$$= 10^8 \times (3.14 \times 2.25 \times 10^{-4}) \text{ N}$$
$$= 7.065 \times 10^4 = 7.1 \times 10^4 \text{ N}$$

14. A wire of length 2.5 m has a percentage strain of 0.012% under a tensile force. Determine the extension in the wire.

Sol. Here, original length, $L = 2.5$ m

$$\text{Strain} = \frac{\Delta L}{L} = 0.012\% = \frac{0.012}{100}$$

$$\Delta L = \text{Strain} \times L$$

or $$\Delta L = \text{extension} = \frac{0.012}{100} \times L$$

$$= \frac{0.012 \times 2.5}{100}$$
$$= 3 \times 10^{-4} \text{ m}$$
$$= 0.3 \text{ mm}$$

LONG ANSWER Type I Questions

15. Two strips of metal are riveted together at their ends by four rivets, each of diameter 6 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9×10^7 Pa? Assume that each rivet is to carry one-quarter of the load. **[NCERT]**

Sol. Diameter of each rivet, $D = 6$ mm

$\therefore$ Radius, $r = \frac{D}{2} = 3 \text{ mm} = 3 \times 10^{-3}$ m

Maximum shearing stress on each rivet $= 6.9 \times 10^7$ Pa

Let w be the maximum load that can be subjected to the riveted strip. As each rivet carry one-quarter of the load.

Therefore, load on each rivet $= \frac{w}{4}$

$$\text{Maximum shearing stress} = \frac{\text{Maximum shearing force}}{\text{Area}}$$

$$\therefore \quad 6.9 \times 10^7 = \frac{w/4}{\pi r^2}$$

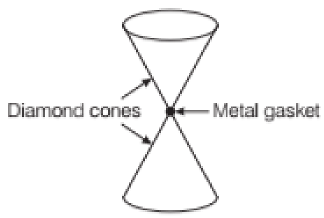
or $\quad w = 6.9 \times 10^7 \times 4\pi r^2$

or $\quad w = 6.9 \times 10^7 \times 4 \times 3.14 \times (3 \times 10^{-3})^2$

$= 6.9 \times 4 \times 3.14 \times 9 \times 10$

$= 7.8 \times 10^3$ N

16. Anvils made of single crystals of diamond, with the shape as shown in figure are used to investigate the behaviour of materials under very high pressure. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm and the wide ends are subjected to a compressional force of 50000 N. What is the pressure at the tip of the anvil? **[NCERT]**



Sol. Given, compressional force, $F = 50000$ N

Diameter, $D = 0.5 \text{ mm} = 5 \times 10^{-4}$ m

$\therefore \quad$ Radius, $r = \frac{D}{2} = 2.5 \times 10^{-4}$ m

$$\text{Pressure at the tip of the anvil } (p) = \frac{\text{Force}}{\text{Area}}$$

$$\therefore \quad p = \frac{F}{\pi r^2} = \frac{50000}{3.14 \times (2.5 \times 10^{-4})^2}$$

$$= 2.5 \times 10^{11} \text{ Pa}$$

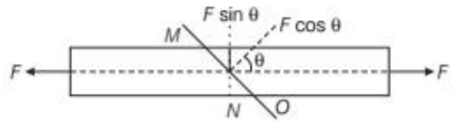
17. A bar of cross-section A is subjected to equal and opposite tensile forces at its ends. Consider a plane section of the bar whose normal makes an angle θ with the axis of the bar.

(i) What is the tensile stress on this plane?

(ii) What is the shearing stress on this plane?

(iii) For what value of θ is the tensile stress maximum?

(iv) For what value of θ is the shearing stress maximum?



Sol. (i) The resolved part of F along the normal is the tensile force on this plane and the resolved part parallel to the plane is the shearing force on the plane.

$\because$ Area of MO plane section $= A \sec\theta$

$$\text{Tensile stress} = \frac{\text{Force}}{\text{Area}} = \frac{F\cos\theta}{A\sec\theta}$$

$$= \frac{F}{A}\cos^2\theta \qquad \left[\because \sec\theta = \frac{1}{\cos\theta}\right]$$

(ii) Shearing stress applied on the top face

So, $\quad F = F\sin\theta$

$$\text{Shearing stress} = \frac{\text{Force}}{\text{Area}} = \frac{F\sin\theta}{A\sec\theta} = \frac{F}{A}\sin\theta\cos\theta$$

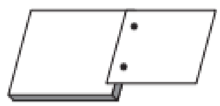
$$= \frac{F}{2A}\sin 2\theta \qquad [\because \sin 2\theta = 2\sin\theta\cos\theta]$$

(iii) Tensile stress will be maximum when $\cos^2\theta$ is maximum i.e. $\cos\theta = 1$ or $\theta = 0°$.

(iv) Shearing stress will be maximum when $\sin 2\theta$ is maximum i.e. $\sin 2\theta = 1$ or $2\theta = 90°$ or $\theta = 45°$.

LONG ANSWER Type II Questions

18. Two long metallic strips are joined together by two rivets each of radius 0.1 cm (see Fig.). Each rivet can withstand a maximum shearing stress of $3.0 \times 10^8 \text{ Nm}^{-2}$. Calculate the maximum tangential force a strip can exert.



Sol. Let F be the tensile force applied. Since, each rivet shares the stretching force equally, so the shearing force on each rivet $= F/2$.

If A is the area of each rivet, then shearing stress on each rivet $= \frac{F}{2A}$. Now, maximum shearing stress on each

strip $\quad = 3.0 \times 10^8 \text{ Nm}^{-2}$

i.e. $\quad \frac{F_{max}}{2A} = 3.0 \times 10^8 \text{ Nm}^{-2}$

where, F_{max} is maximum tangential force.

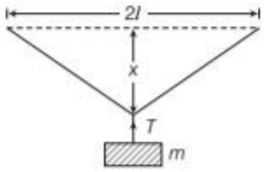
or $\quad F_{max} = 3.0\times10^8 \times 2A = 6.0\times10^8\times\pi r^2$

$\therefore \quad r = 0.1 \text{ cm} \Rightarrow 0.1\times10^{-2}$

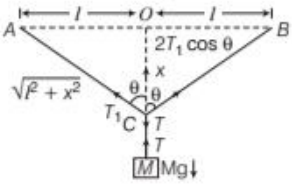
$r = 1\times10^{-3}$ m

$\Rightarrow \quad F_{max} = 6.0\times10^8\times\frac{22}{7}\times(1\times10^{-3})^2 = 1885\text{ N}$

19. A steel wire of length $2l$ and cross-sectional area A is stretched within elastic limit as shown in figure. Calculate the strain and stress in the wire.



Sol. Total length $L = 2l$. Increase in length of the wire, when it is stretched from its mid-point.



From Pythagoras theorem, $BC^2 = l^2 + x^2$

$$BC = \sqrt{l^2+x^2}$$

Similarly, $\quad AC = \sqrt{l^2+x^2}$

Change in length of the wire

$$\Delta L = (AC+CB) - AB = (\sqrt{l^2+x^2}+\sqrt{l^2+x^2}) - 2l$$

$$= 2(l^2+x^2)^{1/2} - 2l = 2l\left(1+\frac{x^2}{l^2}\right)^{1/2} - 2l \quad \text{...(i)}$$

Since $x << l$, so using Binomial expansion, we have

$$\left(1+\frac{x^2}{l^2}\right)^{1/2} = \left(1+\frac{x^2}{2l^2}\right)$$

[neglecting terms containing higher powers of x]

$$\therefore \quad \Delta L = 2l\left(1+\frac{x^2}{2l^2}\right) - 2l = \frac{x^2}{l}$$

Hence, strain $= \frac{\Delta L}{L} = \frac{x^2}{l\times 2l} = \frac{x^2}{2l^2}$

$$\because \quad T = 2T_1\cos\theta \;\therefore\; T_1 = \frac{Mg}{2\cos\theta} \quad [\because T = Mg]$$

Putting $\cos\theta = \frac{x}{\sqrt{l^2+x^2}}$

$$T_1 = \frac{Mg}{2x}(\sqrt{l^2+x^2}) = \frac{Mgl}{2x}\left(1+\frac{x^2}{l^2}\right)^{1/2}$$

$$= \frac{Mgl}{2x}\left(1+\frac{x^2}{2l^2}\right) \quad [\text{using } (1+x)^x = 1+xx]$$

$$\because \quad x << l \;\therefore\; \frac{x^2}{2l^2}\to 0 \quad \text{Thus, } 1+\frac{x^2}{2l^2} = 1$$

$$\therefore \quad T_1 = \frac{Mgl}{2x}$$

Stress in the wire $= \frac{T_1}{A} = \frac{Mgl}{2xA}$

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. Elasticity is due to

(a) decrease of PE with separation between atoms/molecules
(b) increase of PE with separation between atoms/molecules
(c) asymmetric nature of PE curve
(d) None of the above

2. A uniform bar of square cross-section is lying along a frictionless horizontal surface. A horizontal force is applied to pull it from one of its ends, then

(a) the bar is under same stress throughout its length
(b) the bar is not under any stress because force has been applied only at one end
(c) the bar simply moves without any stress in it
(d) the stress developed gradually reduces to zero at the end of the bar where no force is applied

3. A spring is stretched by applying a load to its free end. The strain produced in the spring is

(a) volumetric **[NCERT Exemplar]**
(b) shear
(c) longitudinal and shear
(d) longitudinal

4. A wire of diameter 1 mm breaks under a tension of 1000 N. Another wire of same material as that of the first one, but of diameter 2 mm breaks under a tension of

(a) 500 N (b) 1000 N (c) 10000 N (d) 4000 N

5. The length of a wire increases by 1% by a load of 2 kg-wt. The linear strain produced in the wire will be

(a) 0.02 (b) 0.001 (c) 0.01 (d) 0.002

6. A uniform cube is subjected to volume compression. If each side is decreased by 1%, then bulk strain is

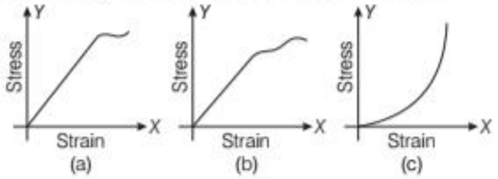
(a) 0.01 (b) 0.06 (c) 0.02 (d) 0.03

Answers

1. (c)	2. (d)	3. (b)	4. (d)	5. (c)
6. (d)				

VERY SHORT ANSWER Type Questions

7. Following are the graphs of elastic materials. Which one correspond to that of brittle material?



8. Metal wires after being heavily loaded do not regain their lengths completely. Explain, why?

9. The breaking force for a wire is *F*. What will be the breaking forces for two parallel wires of this size?

SHORT ANSWER Type Questions

10. A wire fixed at the upper end stretches by length l by applying a force F. What is the work done in stretching the wire?

11. The ratio stress/strain remains constant for a small deformation. What happens to this ratio, if deformation is made very large?

12. A wire is replaced by another wire of same length and material but of twice diameter.

(i) What will be the effect on the increase in its length under a given load?

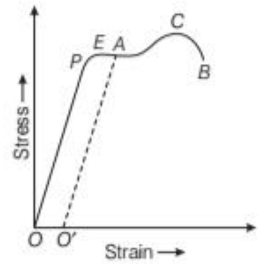
(ii) What will be the effect on the maximum load which it can bear?

LONG ANSWER Type I Question

13. Show graphically the change of potential energy and kinetic energy of a block attached to a spring which obeys Hooke's law.

LONG ANSWER Type II Question

14. The stress-strain graph for a metal wire is shown in figure. Up to the point *E*, the wire returns to its original state 0 along the curve *EPO* when it is gradually unloaded. Point *B* corresponds to the fracture of the wire.



(i) Up to which point on the curve is Hooke's law obeyed? This point is sometimes called proportionality limit.

(ii) Which point on the curve corresponds to elastic limit and yield point of the wire?

(iii) Indicate the elastic and plastic regions of the stress-strain graph.

(iv) Describe what happens when the wire is loaded up to a stress corresponding to the point *A* on the graph and then unloaded gradually. In particular, explain the dotted curve.

(v) What is peculiar about the portion of the stress-strain graph from *C* to *B* ? Up to what stress can the wire be subjected without causing fracture?

|TOPIC 2|
Modulus of Elasticity or Elastic Modulus

ELASTIC MODULUS

The modulus of elasticity or coefficient of elasticity of a body is defined as the ratio of stress to the corresponding strain within the elastic limit.

$$\boxed{\text{Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}}}$$

Its SI unit is Nm^{-2} or Pascal (Pa) and its dimensional formula is $[ML^{-1}T^{-2}]$.

- Modulus of elasticity depends on the nature of the material of the body.
- Modulus of elasticity of a body is independent of its dimensions (i.e. length, area, volume etc.)
- Dimensional formula of modulus of elasticity is same as that of the stress or pressure.

There are three types of modulus of elasticity

1. Young's Modulus of Elasticity

Within the elastic limit, the ratio of longitudinal stress to the longitudinal strain is called Young's modulus of the material of the wire.

i.e. $$\text{Young's modulus } (Y) = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$\boxed{\text{Young's modulus, } Y = \frac{\text{Tensile (or compressive) stress } (\sigma)}{\text{Longitudinal strain } (\varepsilon)}}$$

Consider a wire of radius r and length L. Let a force F be applied on the wire along its length i.e. normal to the surface of the wire as shown in figure.

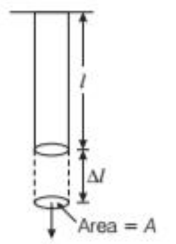
If ΔL be the change in length of the wire,

Longitudinal stress = F/A, where A is the area of cross-section of the wire.

Longitudinal strain = $\Delta L/L$

∴ Young's modulus,

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L} \quad \text{...(i)}$$



Young's modulus of elasticity

If the extension is produced by the load of mass m, then $F = mg$

Here, wire has a circular elasticity. So, area of cross-section of the wire,

$$A = \pi r^2$$

From Eq. (i), it can be written as

$$Y = \frac{mgL}{\pi r^2 \Delta L}$$

or $$\boxed{\text{Young's modulus, } Y = \frac{FL}{\pi r^2 \Delta L}}$$

If $L = 1\text{m}$, $A = 1\text{m}^2$ and $\Delta L = 1\text{m}$, then $Y = F$

Thus, **Young's modulus of elasticity is equal to the force required to extend a wire of unit length and unit area of cross-section by unit amount**

Extension is proportional to the deforming force and length of rod. Greater the force, larger the deformation and longer the rod larger, the extension.

EXAMPLE |1| An Elongated Wire

If a wire of length 4 m and cross-sectional area of 2m^2 is stretched by a force of 3 kN, then determine the change in length due to this force. Given Young's modulus of material of wire is 110×10^9 N / m^2.

Sol. Given, area of cross-section, $A = 2\text{ m}^2$

Force, $F = 3\text{ kN} = 3 \times 10^3$ N

Length, $L = 4$ m

Young's modulus, $Y = 110 \times 10^9$ N / m^2

Change in length, $\Delta L = ?$

Apply, $$Y = \frac{FL}{A\Delta L}$$

$$\Rightarrow \quad \Delta L = \frac{FL}{AY} = \frac{3 \times 10^3 \times 4}{2 \times 110 \times 10^9} = 0.0545 \times 10^{-6} \text{ m}$$

$$\Delta L = 54.5 \times 10^{-3} \text{ mm}$$

EXAMPLE |2| Foucault Pendulum

In a physics department, a Foucault pendulum consists of a 130 kg steel ball which swings at the end of a 8.0 m long steel cable having the diameter of 3.0 mm. If the ball was first hung from the cable, then determine how much did the cable stretch?

The amount by which the cable stretches depends on the elasticity of the steel cable. Young's Modulus for steel is given as $Y = 20 \times 10^8$ N/m^2

Sol. Given, Diameter, $D = 3.0\text{ mm} = 3.0 \times 10^{-3}$ m

Length, $L = 8.0$ m; Mass, $m = 130$ kg

Radius, $r = \dfrac{D}{2} = \dfrac{3.0 \times 10^{-3}}{2} = 1.5 \times 10^{-3}$ m

The area of cross-section of the cable

$$A = \pi r^2 = \pi \times (1.5 \times 10^{-3})^2 = 7.065 \times 10^{-6}\text{ m}^2$$

Thus, $F = w = mg = 130 \times 9.8$

$$F = 1274\text{ N}$$

The change in length

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L} \Rightarrow \Delta L = \frac{LF}{AY}$$

$$\Delta L = \frac{8.0 \times 1274}{7.065 \times 10^{-6} \times 20 \times 10^{8}} = 0.72\text{ m} = 720\text{ mm}$$

EXAMPLE |3| Finding Young's Modulus

The ball of 200 g is attached to the end of a string of an elastic material (say rubber) and having length and cross-sectional area of 51 cm and 22 mm^2, respectively. Find the Young's modulus of this material if string is whirled round, horizontally at a uniform speed of 50 rpm in a circle of diameter 104 cm.

Sol. Mass of the ball, $M = 200\text{ g} = 0.2$ kg

Area of cross-section, $A = 22\text{ mm}^2 = 22 \times 10^{-6}\text{ m}^2$

Radius of the circle, $r = \dfrac{D}{2} = \dfrac{104}{2} = 52\text{ cm} = 0.52$ m

Length of the string, $l = 51\text{ cm} = 0.51$ m

Revolution per second, $= 50 \times 60\text{ rps} = 3000$ rps

Certain petal force, $F = mr\omega^2 = 0.2 \times 0.52 \times (2\pi \times N)^2$

$$F = 36.95 \times 10^6\text{ N}$$

The change in length Δl

$$\Delta l = \text{radius of the circle} - \text{length of the string}$$
$$= 0.52 - 0.51$$
$$\Delta l = 0.01\text{ m}$$

Young's modulus of the material

$$Y = \frac{F}{A}\frac{l}{\Delta l} = \frac{36.95 \times 10^6}{22 \times 10^{-6}} \times \frac{0.51}{0.01} = 85.67 \times 10^{12}\text{ Nm}^{-2}$$

Determination of Young's Modulus of the Material of a Wire

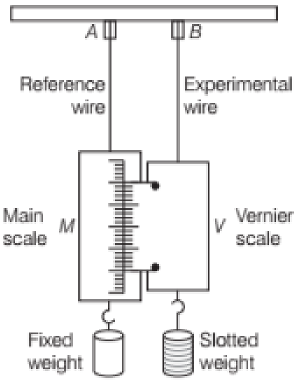
A simple experimental arrangement used for the determination of Young's modulus of the material of a wire is shown in Fig. (a).

Constructional Details

The experiment consists of two long straight wires of same length and equal radius suspended side by side from a fixed rigid support. The wire A, called the **reference wire**, carries a main milliammeter scale M and below it a heavy fixed load. This load keeps the wire tight and free from kinks.

The wire B, called the **experimental wire**, carries a Vernier scale at its bottom.

The Vernier scale can slide against the main scale attached to the reference wire. A hanger is attached at the lower end of the Vernier scale. Slotted half kg weights can be slipped into this hanger.



(a)

Experimental arrangement for the determination of Young's modulus

Observations

With the help of a **screw gauge**, the radius of the experimental wire is measured at several places. Let r be the initial average radius and L be the initial length of the experimental wire.

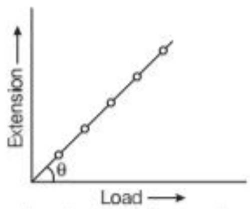
A small initial load, say 1 kg, is put on the hanger. This keeps the experimental wire straight and kink free. The Vernier scale reading is noted. A half kg weight is added to the hanger. The wire is allowed to elongate for a minute. The Vernier scale reading is again noted.

The difference between the two vernier readings gives the extension produced due to the extra weight added. The weight is gradually increased in few steps and every time we note the extension produced.

Area of cross-section of the wire $B = \pi r^2$

and stretching force $= mg$

A graph is plotted between the load applied and extension produced. It will be a straight line passing through the origin, as shown in Fig. (b).



Load-extension graph

(b)

Mathematical Interpretation

Let M be the mass that produced an elongation ΔL in the wire. Thus, the applied force is equal to Mg, where g is the acceleration due to gravity. The slope of the load-extension line $= \tan\theta = \dfrac{\Delta L}{Mg}$

Longitudinal stress $= \dfrac{Mg}{\pi r^2}$ and longitudinal strain $= \dfrac{\Delta L}{L}$

The Young's modulus of the material of the experimental wire will be

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{Mg}{\pi r^2} \cdot \frac{L}{\Delta L} = \frac{L}{\pi r^2 \tan\theta}$$

Thus, Y can be determined.

$$\text{Young's modulus, } Y = \frac{L}{\pi r^2} \times \frac{1}{\text{Slope of the graph}}$$

EXAMPLE |4| Elongation of Copper Wire

A copper wire is stretched by 10 N force. If radius of wire decreases by 2%. How will Young's modulus of wire be affected?

Sol. Since, Young's modulus depends only on the nature of material and not on its dimensions. So, the value of Young's modulus of the copper wire is not changed when its radius decreases. Thus, Young's modulus of the wire remains the same.

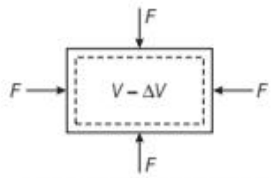
2. Bulk Modulus of Elasticity

Within the elastic limit, the ratio of normal stress to the volumetric strain is called bulk modulus of elasticity. In other words, the ratio of hydraulic stress to the hydraulic strain is called bulk modulus.

Consider a body of volume V and surface area A. Suppose a force F acts uniformly over the whole surface of the body and it decreases the volume by ΔV as shown in figure.

The Bulk modulus of elasticity is given by

$$B = \frac{\text{Normal stress}}{\text{Volumetric strain}} = \frac{F/A}{\Delta V/V}$$



Bulk modulus of elasticity

$\therefore$ Bulk modulus, $B = \dfrac{-F}{A}\dfrac{V}{\Delta V}$

$$\Rightarrow \quad \text{Bulk modulus, } B = -\frac{pV}{\Delta V}$$

where, $p = \dfrac{F}{A}$ is the normal pressure.

Note

- Negative sign shows that the volume decreases with the increase in stress. But for a system in equilibrium, the value of bulk modulus is always positive.
- Bulk modulus for a perfect rigid body and ideal liquid is infinite.

Compressibility

The reciprocal of the Bulk modulus of a material is called its compressibility.

Compressibility, $k = \dfrac{1}{B} = \dfrac{-\Delta V}{pV}$

SI unit of compressibility $= N^{-1} m^2$

CGS unit of compressibility $= \text{dyne}^{-1} \text{cm}^2$

The dimensional formula of compressibility is $[M^{-1}LT^2]$.

EXAMPLE |5| Volumetric Analysis

What will be the decrease in volume of 100 cm^3 of water under pressure of 100 atm if the compressibility of water is 4×10^{-5} per unit atmospheric pressure?

Sol. Bulk modulus $(B) = \dfrac{1}{\text{Compressibility}} = \dfrac{1}{k}$

$$= \frac{1}{4 \times 10^{-5}} = 0.25 \times 10^5 \text{ atm}$$
$$= 0.25 \times 10^5 \times 1.013 \times 10^5 \text{ N/m}^2$$
$$= 2.533 \times 10^9 \text{ N/m}^2$$

Volume, $V = 100 \text{ cm}^3 = 10^{-4} \text{ m}^3$

Pressure, $p = 100 \text{ atm} = 100 \times 1.013 \times 10^5 \text{ N/m}^2$

$$= 1.013 \times 10^7 \text{ N/m}^{-2}$$

Now, apply $\dfrac{1}{B} = k = \dfrac{\Delta V}{pV}$

$$\therefore \quad \Delta V = \frac{pV}{B} = \frac{1.013 \times 10^7 \times 10^{-4}}{2.533 \times 10^9}$$

$$\Delta V = 0.4 \times 10^{-6} \text{ m}^3 = 0.4 \text{ cm}^3$$

3. Modulus of Rigidity or Shear Modulus

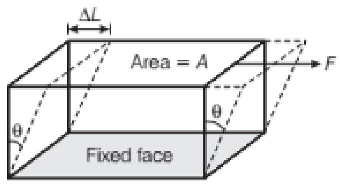
Within the elastic limit, the ratio of tangential stress (shear stress) to shear strain is called modulus of rigidity. It is denoted as G or η. Let us consider a cube whose lower face is fixed and a tangent force F acts on the upper face whose area is A, as shown in figure.

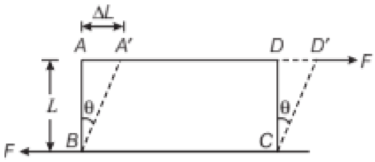
$\therefore$ $\boxed{\text{Tangential stress (shear stress)} = \dfrac{F}{A}}$

Let the vertical sides of the cube shifts through an angle θ, called shear strain.

$\therefore$ Modulus of rigidity is given by

$$\eta \text{ or } G = \frac{\text{Tangential stress (shear stress)}}{\text{Shear strain}}$$





According to diagram, by displacing its upper face through distance $AA' = \Delta L$

Let $AB = DC = L$ and $\angle ABA' = \theta$,

$$\eta = \frac{F/A}{\theta} = \frac{F}{A\theta}$$

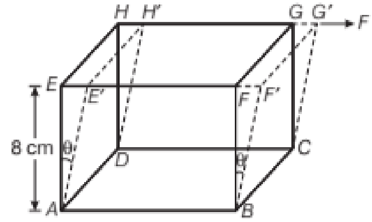
Shear strain, $\theta \approx \tan\theta = \dfrac{AA'}{AB} = \dfrac{\Delta L}{L}$

$$\boxed{\text{Shear modulus, } \eta = \frac{F}{A}\cdot\frac{L}{\Delta L}}$$

EXAMPLE |6| Shear Deformation Under Action of Tangential Force

Consider an Indian rubber cube having modulus of rigidity of 2×10^7 dyne/cm^2 and of side 8 cm. If one side of the rubber is fixed, while a tangential force equal to the weight of 300 kg is applied to the opposite face, then find out the shearing strain produced and distance through which the strain side moves.

Sol. Given, modulus of rigidity, $\eta = 2\times10^7$ dyne/cm^2



Side of the cube, $l = 8\,\text{cm}$

Area, $A = l^2 = 64\,\text{cm}^2$

Force or load, $F = 300\,\text{kgf} = 300\times1000\times981$ dyne

As, $\eta = \dfrac{F}{A\theta}$

$\Rightarrow$ $\theta = \dfrac{F}{A\eta}$

$\theta = \dfrac{300\times1000\times981}{64\times2\times10^7} \simeq 0.23$ rad

As, $\eta = \dfrac{F}{A}\dfrac{l}{\Delta l}$

$\Rightarrow$ $\dfrac{\Delta l}{l} = \theta$

$\Rightarrow$ $\Delta l = l\theta = 8\times0.23$

$\Delta l = 1.84$ cm

EXAMPLE |7| Shear Modulus is Less than Young's Modulus

The shear modulus of a material is always considerably smaller than the Young's modulus for it. What does it signify?

Sol. **η of a material is smaller than its Y.** As, we know that it is easier to slide layers of atoms of solids over one another than to pull them apart or to squeeze them to close together.

Note

- Shear modulus for ideal liquid is zero.
- A solid has all types of moduli of elasticity i.e. Y, B and η. But fluids (i.e. liquids and gases) has only Bulk modulus of elasticity.

Stress, Strain and Various Elastic Moduli

S.No.	Types of stress	Stress	Strain	Change in Shape	Change in Volume	Elastic modulus	Name of modulus	State of Matter
1.	Tensile or compressive	Two equal and opposite forces perpendicular to opposite faces ($\sigma = F/A$)	Elongation or compression parallel to force direction ($\Delta L/L$) (longitudinal strain)	Yes	No	$Y = (F \times L)/(A \times \Delta L)$	Young's modulus	Solid
2.	Shearing	Two equal and opposite forces parallel to opposite surfaces (forces in each case) such that total forces and total torque on the body vanishes ($\sigma_s = F/A$)	Pure shear, θ	Yes	No	$G = (F \times \theta)/A$	Shear modulus	Solid
3.	Hydraulic	Forces perpendicular everywhere to the surface, force per unit area (pressure) same everywhere	Volume change (compression or elongation) ($\Delta V/V$)	No	Yes	$B = -p(\Delta V/V)$	Bulk modulus	Solid, liquid and gas

POISSON'S RATIO

When a wire is loaded, its length increases but its diameter decreases. The strain produced in the direction of applied force is called **longitudinal strain** and strain produced in the perpendicular direction is called **lateral strain**.

Within the elastic limit, the ratio of lateral strain to the longitudinal strain is called **Poisson's ratio**.

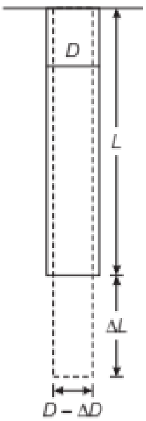
Let the length of the loaded wire increases from L to $L + \Delta L$ and its diameter decreases from D to $D - \Delta D$.

$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

$$\text{Lateral strain} = \frac{-\Delta D}{D}$$

$$\text{Poisson's ratio } \sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-\Delta D/D}{\Delta L/L}$$

$$\boxed{\text{Poisson's ratio, } \sigma = \frac{-L}{D} \cdot \frac{\Delta D}{\Delta L}}$$



Poisson's ratio

The negative sign indicates that longitudinal and lateral strain are in opposite sense, it has no unit and dimensions.

$$\begin{cases} -1 < \sigma < 0.5 & \text{for theoretical purpose} \\ 0 < \sigma < 0.5 & \text{for practical purpose} \end{cases}$$

Poisson's ratio is a constant provided a material remains elastic, homogeneous and isotropic in nature.

EXAMPLE |8| Poisson's Ratio

A material having Poisson's ratio 0.2. A load is applied on it, due to which it suffers the longitudinal strain 3.0×10^{-3}, then find out the percentage change in its volume.

Sol. Given, Poisson's ratio, $\sigma = 0.2$

Longitudinal strain $= \frac{\Delta L}{L} = 3.0 \times 10^{-3}$

As, $\sigma = -\frac{\Delta D/D}{\Delta L/L}$ or $\sigma = \frac{-\Delta R/R}{\Delta L/L}$

$\Rightarrow \frac{\Delta R}{R} = -\sigma \times \frac{\Delta L}{L} = -0.6 \times 10^{-3}$

Volume, $V = \pi R^2 L$

Then, percentage change in volume,

$$\left(\frac{\Delta V}{V} \times 100\right) = \left(\frac{2\Delta R}{R} + \frac{\Delta L}{L}\right) \times 100$$

$$= [2 \times (-0.6 \times 10^{-3}) + 3.0 \times 10^{-3}] \times 100$$

$$= 0.18\ \%$$

Relation between Y, B, η and σ

The different elastic constants exhibit the following relationship.

(i) $Y = 3B(1 - 2\sigma)$ (ii) $Y = 2\eta(1 + \sigma)$

(iii) $\sigma = \frac{3B - 2\eta}{2\eta + 6B}$ (iv) $\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$

FACTORS AFFECTING ELASTICITY OF MATERIAL

The following factors affect the elasticity of a material

(i) **Hammering and rolling** In both of these processes, the crystal grains are broken into small units and the elasticity of the material increases.

(ii) **Annealing** This process result in the formation of larger crystal grains and elasticity of the material decreases.

(iii) **Presence of impurities** Depending on the nature of impurity, the elasticity of material can be increased or decreased.

(iv) **Temperature** Elasticity of most of the materials decreases with increase in the temperature but elasticity of invor steel (alloy) does not change with change of temperature.

Elastic After Effect

When the deforming force is removed from the elastic bodies, the bodies tend to return to their respective original state. It has been found that some bodies return to its original state immediately, others take appreciably long time to do so. The delay in regaining the original position is known as elastic after effect.

Note

A quartz fibre returns immediately to its normal state, when the twisting torque acting on it ceases to act. On the other hand, a glass fibre will take hours to return to its original state.

Elastic Fatigue

The elastic fatigue is defined as the loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

e.g. A hard wire can be broken by bending it repeatedly in opposite directions, as it loses strength due to elastic fatigue. For the same reason, the railway bridges are declared unsafe after a reasonably long period to avoid the risk of a mishappening.

ENERGY STORED IN A DEFORMED BODY

When a wire is stretched, interatomic forces come into play which oppose the change in configuration of the wire. Hence, work has to be done against these restoring forces. The work done in stretching the wire is stored in it as its elastic potential energy.

Let a force F applied on a wire of length L increases its length by ΔL. Initially, the internal restoring force in the wire is zero. When the length is increased by ΔL, the internal force increases from 0 to F. (applied force)

$\therefore$ Average internal force for an increase in length

$$\Delta L \text{ of wire} = \frac{0+F}{2} = \frac{F}{2}$$

Work done on the wire,

$$W = \text{Average force} \times \text{Increase in length} = \frac{F}{2} \times \Delta L$$

This work done is stored as elastic potential energy U in the wire,

$$U = \frac{1}{2} F \times \Delta L = \frac{1}{2} \text{Stretching force} \times \text{Increase in length}$$

Let A be the area of cross-section of the wire.

Then, $$U = \frac{1}{2} \frac{F}{A} \times \frac{\Delta L}{L} \times AL$$

Elastic potential energy,

$$\boxed{U = \frac{1}{2} \text{Stress} \times \text{Strain} \times \text{Volume of wire}}$$

Elastic potential energy per unit volume of the wire or elastic energy density is

$$\mu = \frac{U}{\text{Volume}} = \frac{1}{2} \text{Stress} \times \text{Strain}$$

But stress = Young's modulus × Strain

Elastic energy density,

$$\boxed{\mu = \frac{1}{2} \text{Young's modulus} \times (\text{Strain})^2}$$

EXAMPLE |9| Elastic Potential Energy of a Wire

When the load of a wire is increased from 3 kg wt to 5 kg wt, the length of that wire changes from 0.61 mm to 1.02 mm. Calculate the change in the elastic potential energy of the wire.

Sol. Here, $F_1 = 3 \text{ kg-f} = 3 \times 9.8 \text{N} = 29.4 \text{N}$

$$\Delta l_1 = 0.61 \text{ mm} = 6.1 \times 10^{-4} \text{m}$$

$$F_2 = 5 \text{ kg-f} = 5 \times 9.8 = 49 \text{N}$$

$$\Delta l_2 = 1.02 \text{ mm} = 1.02 \times 10^{-3} \text{m}$$

$$\therefore \quad U_1 = \frac{1}{2} F_1 \cdot \Delta l_1 = \frac{29.4 \times 6.1 \times 10^{-4}}{2} = 8.96 \times 10^{-3} \text{J}$$

and $$U_2 = \frac{1}{2} F_2 \cdot \Delta l_2 = \frac{49 \times 1.02 \times 10^{-3}}{2} = 24.99 \times 10^{-3} \text{J}$$

$\therefore$ Change in elastic potential energy of the wire,

$$\Delta U = U_2 - U_1 = 24.99 \times 10^{-3} - 8.96 \times 10^{-3} = 16.03 \times 10^{-3} \text{J}$$

APPLICATIONS OF ELASTIC BEHAVIOUR OF SOLIDS

(i) Any metallic part of a machinery is never subjected to a stress beyond the elastic limit of the material.

In case, the metallic part of the machinery is subjected to a stress beyond the elastic limit, it will get permanently deformed and hamper its working.

(ii) The thickness of metallic ropes used in cranes to lift and move heavy weights is decided on the basis of the elastic limit of the rope and the factor of safety.

Note

Factor of safety also known as Safety Factor (SF), is a term describing the structural capacity of a system beyond the expected loads or actual loads.

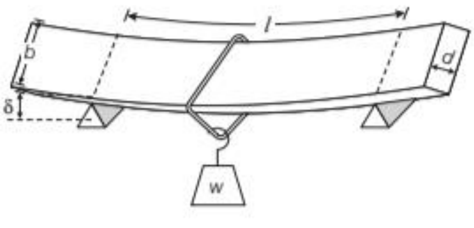
(iii) In designing a bridge or beam that has to be designed such that it can withstand the load of the following traffic, the force of winds and its own weight. In both cases, the over coming of the problem of bending of a bridge or beam under a load is of prime importance. The bridge or beam should not bend too much or break.

Consider a beam of length l and of rectangular cross-section having breadth b and depth d. When the beam is loaded in the middle with a load w, then it gets depressed by an amount δ given by

$$\text{Depression its beam, } \delta = \frac{wl^3}{4\,Ybd^3}$$

where, $Y =$ Young's modulus of elasticity.

Since, the value of depression δ varies inversely as its breadth (b) and as the cube of its depth (d), the depression can be reduced more effectively by increasing the depth of the beam rather than increasing its breadth.



Bending Resistance and Elastic Modulus

A beam can support load by producing resistance against bending. Larger the bending resistance, greater the load bearing capacity. So, the bending resistance is directly proportional to elastic modulus of the material, shape of the cross-section of beam and ultimate stress of the material.

(iv) Maximum height of a mountain on earth (~10 km) can be estimated from the elastic behaviour of earth. A mountain base is not under uniform compression and this provides some shearing stress to the rocks under which they can flow. To illustrate this, consider a mountain of height h, the force per unit area due to the weight of the mountain at its base is $h\rho g$, where ρ is density of material of mountain and g is the acceleration due to gravity. The material at the bottom experiences this force in the vertical direction. The elastic limit for a typical rock is $30 \times 10^7\ \text{Nm}^{-2}$. Equating this to $h\rho g$.

$$h\rho g = 30 \times 10^7$$

$$h \times 3 \times 10^3 \times 10 = 30 \times 10^7$$

$$h = 10\ \text{km}$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. Which of the following statements is incorrect?

(a) Young's modulus and shear modulus are relevant only for solids.

(b) Bulk modulus is relevant for liquids and gases.

(c) Metals have larger values of Young's modulus than elastomers.

(d) Alloys have larger values of Young's modulus than metals.

Sol. (d) Metals have larger values of Young's modulus than alloy and elastomers.

2. When a pressure of 100 atmosphere is applied on a spherical ball of rubber, then its volume reduces to 0.01%. The bulk modulus of the material of the rubber in dyne cm^{-2} is

(a) 10×10^{12} (b) 100×10^{12} (c) 1×10^{12} (d) 20×10^{12}

Sol. (c) $1\ \text{atm} = 10^5\ \text{Nm}^{-2}$

$\therefore$ $100\ \text{atm} = 10^7\ \text{Nm}^{-2}$ and $\Delta V = 0.01\%\ V$

$$\therefore \quad \frac{\Delta V}{V} = 0.0001$$

$$B = \frac{p}{\Delta V / V} = \frac{10^7}{0.0001} = 1 \times 10^{11}\ \text{Nm}^{-2} = 1 \times 10^{12}\ \frac{\text{dyne}}{\text{cm}^2}$$

3. Modulus of rigidity of ideal liquids is **[NCERT Exemplar]**

(a) infinity

(b) zero

(c) unity

(d) some finite small non-zero constant value

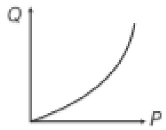
Sol. (b) No frictional (viscous) force exists in case of ideal fluid, hence, tangential forces are zero, so there is no stress developed.

4. A material has Poisson's ratio 0.5. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, then the percentage change in volume is

(a) 0.6 (b) 0.4
(d) 0.2 (d) zero

Sol. (d) As, the Poisson's ratio of material is 0.5, so there is no change in volume.

5. The graph shows the behaviour of a length of wire in the region for which the substance obeys Hooke's law. P and Q represent



(a) P = applied force, Q = extension
(b) P = extension, Q = applied force
(c) P = extension, Q = stored elastic energy
(d) P = stored elastic energy, Q = extension

Sol. (c) The graph between applied force and extension will be straight line because in elastic range,

Applied force $\propto$ extension

But the graph between extension and stored elastic energy will be parabolic in nature.

As, $U = \frac{1}{2}kx^2$ or $U \propto x^2$

VERY SHORT ANSWER Type Questions

6. What is the Young's modulus for a perfect rigid body?

Sol. Young's modulus $(Y) = \frac{F}{A}\times\frac{l}{\Delta l}$

For a perfectly rigid body, change in length $\Delta l = 0$

$\therefore$ $Y = \frac{F}{A}\times\frac{l}{0} = \infty$

Therefore, Young's modulus for a perfectly rigid body is ∞.

7. A metal bar of length L, area of cross-section A, Young's modulus Y and coefficient of linear expansion α, is clamped between two stout pillars. What is the force exerted by the bar when it is heated through t°C?

Sol. $Y = \frac{FL}{Al}$, where $l = L\alpha\Delta t$ and l = change in length.

$$Y = \frac{FL}{AL\alpha\Delta t} = \frac{F}{A\alpha\Delta t} = \frac{F}{A\alpha\Delta t}$$

$\therefore$ $F = YA\alpha\cdot\Delta t$

8. A wire increases by 10^{-3} of its length when a stress of $10^8\ \text{Nm}^{-2}$ is applied to it. What is the Young's modulus of the material of the wire?

Sol. Given, $\Delta L = 10^{-3}L$, with L as the original length

$$\text{Strain} = \frac{\Delta L}{L} = 10^{-3} \text{ and Stress} = \frac{F}{A} = 10^8\ \text{N/m}^2$$

$\because$ $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L}$

$$Y = \frac{1\times10^8}{10^{-3}} = 10^{11}\ \text{N/m}^2$$

9. What is Bulk modulus for a perfectly rigid body?

Sol. Bulk modulus $(B) = \frac{p}{\Delta V/V} = \frac{pV}{\Delta V}$

For perfectly rigid body, change in volume $\Delta V = 0$

$\therefore$ $B = \frac{pV}{0} = \infty$

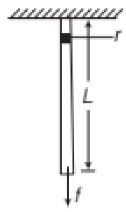
Therefore, Bulk modulus for a perfectly rigid body is ∞.

SHORT ANSWER Type Questions

10. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force f, its length increases by l. Another wire of the same material of length $2L$ and radius $2r$, is pulled by a force $2f$. Find the increase in length of this wire.

In this problem, we have to apply Hooke's law and then elongation in each wire will be compared.

Sol. The situation is shown in the diagram.



Now, Young's modulus $(Y) = \frac{f}{A}\times\frac{L}{l}$

For first wire, $Y = \frac{f}{\pi r^2}\times\frac{L}{l}$ ...(i)

For second wire, $Y = \frac{2f}{\pi(2r)^2}\times\frac{2L}{l'}$

$= \frac{f}{\pi r^2}\times\frac{L}{l'}$ ...(ii)

For Eqs. (i) and (ii), $\frac{f}{\pi r^2}\times\frac{L}{l} = \frac{f}{\pi r^2}\times\frac{L}{l'}$

$\therefore$ $l = l'$

[$\because$ both wires are of same material, hence, Young's modulus will be same].

11. Two wires made of same material are subjected to forces in the ratio 1 : 4. Their lengths are in the ratio 2 : 1 and diameters in the ratio 1 : 3. What is the ratio of their extensions?

Sol. According to Hooke's law,

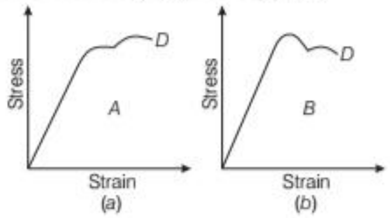
Modulus of elasticity, $E = \frac{F}{\pi r^2}\times\frac{l}{\Delta l}$ or $\Delta l = \frac{Fl}{\pi r^2 E}$

or $\Delta l \propto \frac{Fl}{r^2}$ [$\because$ E is same for two wires]

$$\therefore \quad \frac{\Delta l_1}{\Delta l_2} = \frac{F_1}{F_2} \times \frac{l_1}{l_2} \times \frac{r_2^2}{r_1^2} = \frac{1}{4} \times \frac{2}{1} \times \left(\frac{3}{1}\right)^2 = \frac{9}{2}$$

So, $\Delta l_1 : \Delta l_2 = 9 : 2$

12. **The stress-strain graphs for materials *A* and *B* are shown in Fig. (a) and Fig. (b).**



The graphs are drawn to the same scale.

(i) Which of the materials has greater Young's modulus?

(ii) Which of the two is the stronger material?

[NCERT]

Sol. (i) In the two graphs, the slope of graph in Fig. (a) is greater than the slope of graph in Fig. (b), so material *A* has greater Young's modulus.

(ii) Material *A* is stronger than material *B* because it can withstand more load without breaking. For material *A*, the break even point (*D*) is higher.

13. **A wire elongates by *l* mm when a load *W* is hanged from it. If the wire goes over a pulley and two weights *W* each are hung at the two ends, then what will be the elongation of the wire in mm?**

Sol. According to Hooke's law,

Modulus of elasticity, $E = \frac{W}{A} \times \frac{L}{l}$

where, L = original length of the wire

A = cross-sectional area of the wire

$\therefore$ Elongation, $\Delta l = \frac{WL}{E}$ ...(i)

On either side of the wire, tension is W and length is $l/2$.

$$\Delta l = \frac{W L/2}{AE} = \frac{WL}{2AE} = \frac{l}{2} \quad \text{[from Eq. (i)]}$$

$\therefore$ Total elongation in the wire $= \frac{l}{2} + \frac{l}{2} = l$

14. **Two wires *A* and *B* are of the same material. Their lengths are in the ratio 1:2 and the diameters in the ratio 2:1. If they are pulled by the same force, then what will be the ratio of their increase in lengths?**

Sol. We know, $\Delta L = \frac{FL}{AY}$, $\frac{L_A}{L_B} = \frac{1}{2}$ and $\frac{r_A}{r_B} = \frac{2}{1}$ (given)

[$\because$ the wires A and B are pulled by the same force and they are made up of same material, hence, $F_A = F_B = F$, $Y_A = Y_B = Y$]

$$\frac{\Delta L_A}{\Delta L_B} = \frac{L_A}{\pi r_A^2} \times \frac{\pi r_B^2}{L_B}$$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{L_A}{L_B} \times \left(\frac{r_B}{r_A}\right)^2 \Rightarrow \frac{\Delta L_A}{\Delta L_B} = \frac{1}{2} \times \left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{1}{8}$$

15. **The Young's modulus for steel is much more than that for rubber. For the same longitudinal strain, which one will have greater tensile stress?** **[NCERT Exemplar]**

Sol. Young's modulus $(Y) = \frac{\text{Stress}}{\text{Longitudinal strain}}$

For same longitudinal strain, $Y \propto$ stress

$$\therefore \quad \frac{Y_{\text{steel}}}{Y_{\text{rubber}}} = \frac{(\text{stress})_{\text{steel}}}{(\text{stress})_{\text{rubber}}} \quad \text{...(i)}$$

But $Y_{\text{steel}} > Y_{\text{rubber}}$

$$\therefore \quad \frac{Y_{\text{steel}}}{Y_{\text{rubber}}} > 1$$

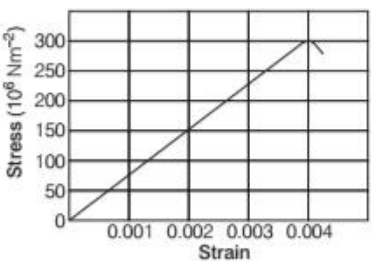
Therefore, from Eq. (i), we get

$$\frac{(\text{stress})_{\text{steel}}}{(\text{stress})_{\text{rubber}}} > 1$$

or $(\text{stress})_{\text{steel}} > (\text{stress})_{\text{rubber}}$

16. **Figure shows the strain-stress curve for a given material. What are (i) Young's modulus and (ii) approximate yield strength for this material?**

[NCERT]



The slope of straight line portion of strain-stress curve for a given material represents its Young's modulus. The maximum stress that a material can sustain is called its yield strength.

Sol. (i) Young's modulus of the given material (Y)

= Slope of strain-stress curve

$$Y = \frac{150 \times 10^6}{0.002} = 75 \times 10^9 \text{ N/m}^2$$

$$= 7.5 \times 10^{10} \text{ N/m}^2$$

(ii) Yield strength of the given material

= Maximum stress, the material can sustain

$= 300 \times 10^6$ N/m^2

$= 3 \times 10^8$ N/m^2

17. Calculate the percentage increase in length of a wire of diameter 2.5 mm stretched by a force of 100 kg weight. Young's modulus of elasticity of wire is 12.5×10^{11} dyne/sq cm.

Sol. Here, $2r = 2.5\text{mm} = 0.25\text{cm}$ or $r = 0.125$ cm

$\therefore \quad a = \pi r^2 = \frac{22}{7}\times(0.125)^2$ sq. cm

$F = 100 \text{ kg} = 100\times1000\text{ g}$

$F = 10\times1000\times980$ dyne

$Y = 12.5\times10^{11}$ dyne/sq. cm

As $\quad Y = \frac{F\times l}{a\times\Delta l}$

$\therefore \quad \frac{\Delta l}{l} = \frac{F}{aY}$

Hence, % increase in length

$$= \frac{\Delta l}{l}\times100 = \frac{F}{aY}\times100$$

$$= \frac{(100\times1000\times980)\times7\times100}{22\times(0.125)^2\times12.5\times10^{11}}$$

$$= 0.1812\%$$

18. A solid sphere of radius R made of a material of bulk modulus B is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid.

When a mass M is placed on the piston to compress the liquid, find fractional change in the radius of the sphere?

Sol. When mass M is placed on the piston, the excess pressure, $p = Mg/A$. As this pressure is equally applicable from all the direction on the sphere, hence there will be decrease in volume due to decrease in radius of sphere. Volume of the sphere, $V = \frac{4}{3}\pi R^3$.

Differentiating it, we get,

$$\Delta V = \frac{4}{3}\pi(3R^2)\Delta R = 4\pi R^2\Delta R$$

$$\therefore \quad \frac{\Delta V}{V} = \frac{4\pi R^2\Delta R}{\frac{4}{3}\pi R^3} = \frac{3\Delta R}{R}$$

We know that, $B = \frac{P}{dV/V} = \frac{Mg}{A}\Big/\frac{3\Delta R}{R}$

or $\quad \frac{\Delta R}{R} = \frac{Mg}{3BA}$

19. The Marina trench is located in the Pacific ocean and at one place, it is nearly 11 km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of trench. What is the change in the volume of the ball when it reaches to the bottom if the Bulk modulus of steel is $1.6\times10^{11}\text{N/m}^2$? **[NCERT]**

Sol. Depth $(h) = 11 \text{ km} = 11\times10^3$ m

Pressure at the bottom of the trench $(p) = 1.1\times10^8$ Pa

Initial volume of the ball $(V) = 0.32\text{ m}^3$

Bulk modulus of steel $(B) = 1.6\times10^{11}\text{ N/m}^2$

Bulk modulus of steel $(B) = \frac{p}{(\Delta V/V)} = \frac{pV}{\Delta V}$

$$\Delta V = \frac{pV}{B} = \frac{1.1\times10^8\times0.32}{1.6\times10^{11}}$$

$$\Delta V = 2.2\times10^{-4}\text{ m}^3$$

20. To what depth must a rubber ball be taken in deep sea so that its volume is decreased by 0.1%? (The Bulk modulus of rubber is 9.8×10^8 N/m^2; and the density of seawater is 10^3 kg/m^3.)

Sol. Bulk modulus of rubber $(B) = 9.8\times10^8\text{ N/m}^2$

Density of seawater $(\rho) = 10^3\text{ kg/m}^3$

Percentage decrease in volume,

$$\left(\frac{\Delta V}{V}\times100\right) = 0.1 \text{ or } \frac{\Delta V}{V} = \frac{0.1}{100}$$

or $\quad \frac{\Delta V}{V} = \frac{1}{1000}$

Let the rubber ball be taken up to depth h.

$\because$ Change in pressure $(p) = h\rho g$

$\therefore$ Bulk modulus $(B) = \frac{p}{(\Delta V/V)} = \frac{h\rho g}{(\Delta V/V)}$

or $\quad h = \frac{B\times(\Delta V/V)}{\rho g} = \frac{9.8\times10^8\times\frac{1}{1000}}{10^3\times9.8} = 100$ m

21. The maximum stress that can be applied to the material of a wire used to suspend on elevator is $1.3\times10^8\text{ Nm}^{-2}$. If the mass of the elevator is 900 kg and it moves up with an accleration of 2.2 ms^{-2}, then what is the minimum diameter of the wire?

Sol. As the elevator moves up, the tension in the wire,

$$F = mg + ma = m(g + a) = 900\times(9.8 + 2.2) = 10800\text{N}$$

Stress in the wire $= \frac{F}{A} = \frac{F}{\pi r^2}$

Clearly, when the stress is maximum, r is minimum.

$\therefore$ Maximum stress $= \frac{F}{\pi r_{\min}^2}$

or $\quad r_{\min}^2 = \frac{F}{\pi\times\text{Maximum stress}}$

$$= \frac{10800}{3.14 \times 1.3 \times 10^8} = 0.2645 \times 10^{-4} \text{ m}$$

or $$r_{min} = 0.5142 \times 10^{-2} \text{ m}$$

Minimum diameter

$$= 2r_{min} = 2 \times 0.5142 \times 10^{-2}$$

$$= 1.0284 \times 10^{-2} \text{ m}$$

LONG ANSWER Type I Questions

22. A steel wire of length 4.7 m and cross-sectional area $3.0 \times 10^{-5}\text{ m}^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area $4.0 \times 10^{-5}\text{ m}^2$ under a given load. What is the ratio of the Young's modulus of steel to that of copper? **[NCERT]**

Sol. Given, **for steel wire**, length $(l_1) = 4.7$ m

Area of cross-section $(A_1) = 3.0 \times 10^{-5}\text{ m}^2$

For copper wire

Length $(l_2) = 3.5$ m

Area of cross-section $(A_2) = 4.0 \times 10^{-5}\text{ m}^2$

Let F be the given load under which steel and copper wires be stretched by the same amount Δl.

Young's modulus $(Y) = \frac{F/A}{\Delta l / l} = \frac{F \times l}{A \times \Delta l}$

For steel, $$Y_s = \frac{F \times l_1}{A_1 \times \Delta l} \quad \text{...(i)}$$

For copper, $$Y_c = \frac{F \times l_2}{A_2 \times \Delta l} \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{Y_s}{Y_c} = \frac{F \times l_1}{A_1 \times \Delta l} \times \frac{A_2 \times \Delta l}{F \times l_2}$$

$$= \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \frac{4.7}{3.5} \times \frac{4.0 \times 10^{-5}}{3.0 \times 10^{-5}}$$

$$\frac{Y_s}{Y_c} = \frac{18.8}{10.5} = 1.79 = 1.8$$

23. Identical springs of steel and copper are equally stretched. On which, more work will have to be done? **[NCERT Exemplar]**

Sol. Work done in stretching a wire is given by

$$W = \frac{1}{2} F \times \Delta l$$

As springs of steel and copper are equally stretched. Therefore, for same force (F),

$$W \propto \Delta l \quad \text{...(i)}$$

Young's modulus $$(Y) = \frac{F}{A} \times \frac{l}{\Delta l}$$

or $$\Delta l = \frac{F}{A} \times \frac{l}{Y}$$

As both springs are identical,

$\therefore$ $$\Delta l \propto \frac{1}{Y} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $W \propto \frac{1}{Y}$

$\therefore$ $$\frac{W_{steel}}{W_{copper}} = \frac{Y_{copper}}{Y_{steel}} < 1 \quad [\text{as } Y_{steel} > Y_{copper}]$$

or $$W_{steel} < W_{copper}$$

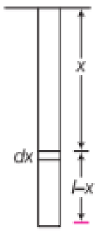
Therefore, more work will be done for stretching copper spring.

24. A uniform heavy rod of weight W, cross-sectional area A and length l is hanging from a fixed support. Young's modulus of the material of the rod is Y. Neglecting the lateral contraction, find the elongation produced in the rod.

Sol. As shown in figure, consider a small element of thickness dx at distance x from the fixed support. Force acting on the element dx is

$$F = \text{Weight of length } (l - x) \text{ of the rod}$$

$$= \frac{W}{l}(l - x)$$



Elongation of the element

$$= \text{Original length} \times \frac{\text{Stress}}{Y}$$

$$= dx \times \frac{F/A}{Y} = \frac{W}{l\,Ay}(l - x)dx$$

Total elongation produced in the rod

$$= \frac{W}{l\,AY}\int_0^l (l - x)dx = \frac{W}{l\,Ay}\left[lx - \frac{x^2}{2}\right]_0^l$$

$$= \frac{W}{l\,Ay}\left[l^2 - \frac{l^2}{2}\right] = \frac{Wl}{2Ay}$$

25. One end of a nylon rope, of length 4.5 cm and diameter 6 mm, is fixed to a free limb. A monkey, weighing 100 N, jumps to catch the free end and stays there. Find the elongation of the rope and the corresponding change in the diameter. Given Young's modulus of nylon $= 4.8 \times 10^{11}\text{ N/m}^2$ and Poisson's ratio of nylon $= 0.2$.

Sol. Here, $l = 4.5\text{ m}$; $D = 6\text{ mm} = 6 \times 10^{-3}\text{ m}$

$F = Mg = 100\text{ N}$,

$Y = 4.8 \times 10^{11}\text{ N/m}^2$; $\Delta l = ?$; $\Delta D = ?$

$$Y = \frac{F}{(\pi D^2/4)} \times \frac{l}{\Delta l}$$

or $\Delta l = \frac{4Fl}{\pi D^2 Y} = \frac{4 \times 100 \times 4.5}{3.14 \times (6 \times 10^{-3})^2 \times (4.8 \times 10^{11})}$

$= 3.315 \times 10^{-5}$ m

$\sigma = \frac{\Delta D/D}{\Delta l/l}$ (in magnitude)

or $\Delta D = \sigma \frac{\Delta l}{l} \times D$

$= 0.2 \times \frac{(3.315 \times 10^{-5})}{4.5} \times (6 \times 10^{-3})$

$= 8.84 \times 10^{-9}$ m

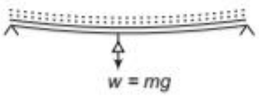
26. A mild steel wire of length 1 m and cross-sectional area 0.5×10^{-2} cm^2 is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid-point of the wire. Calculate the depression at the mid-point. Given Young's modulus for steel $(Y_S) = 2 \times 10^{11}$ Pa. **[NCERT]**

When a load w is suspended from a stretched wire of length l, then depression at the mid-point is given by

$$\delta = \frac{wl^3}{12\pi r^4 Y}$$

where, r = radius of the wire,

Y = Young's modulus of the material of the wire.



Sol. Given, length $(l) = 1$ m

Area of cross-section $(A) = 0.5 \times 10^{-2}$ cm^2

$= 0.5 \times 10^{-6}$ m^2

Mass $(m) = 100$ g $= 0.1$ kg

$\therefore$ Load $(w) = mg = 0.1 \times 9.8$ N $= 0.98$ N

Young's modulus for steel $(Y) = 2 \times 10^{11}$ Pa

Area $(A) = \pi r^2$

or $r^2 = \frac{A}{\pi}$

$= \frac{0.5 \times 10^{-6}}{\pi}$

Depression in a wire when a load is suspended at its centre

$$\delta = \frac{wl^3}{12\pi r^4 Y} = \frac{0.98 \times (1)^3}{12\pi \times \left(\frac{0.5 \times 10^{-6}}{\pi}\right)^2 \times 2 \times 10^{11}}$$

$$\delta = \frac{0.98 \times \pi}{12 \times 0.25 \times 2 \times 10^{-1}} = 5.12 \text{ m}$$

27. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7×10^6 Pa. Bulk modulus for copper $= 140 \times 10^9$ Pa. **[NCERT]**

Sol. Given, each side of cube (l) = 10 cm = 0.1 m

Hydraulic pressure $(p) = 7 \times 10^6$ Pa

Bulk modulus for copper $(B) = 140 \times 10^9$ Pa

Volume contraction $(\Delta V) = ?$

Volume of the cube $(V) = l^3 = (0.1)^3 = 1 \times 10^{-3}$ m^3

Bulk modulus for copper $(B) = \frac{p}{\Delta V/V}$

$= \frac{pV}{\Delta V}$

or $\Delta V = \frac{pV}{B}$

$\Delta V = \frac{7 \times 10^6 \times 1 \times 10^{-3}}{140 \times 10^9} = \frac{1}{20} \times 10^{-6}$ m^3

$= 0.05 \times 10^{-6}$ m^3 $= 5 \times 10^{-8}$ m^3

LONG ANSWER Type II Questions

28. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kg/m^3, compressibility of water is 45.8×10^{-11} Pa^{-1}. **[NCERT]**

Sol. Density of water at the surface $(\rho_0) = 1.03 \times 10^3$ kg/m^3

Pressure $(p) = 80.0$ atm $= 80.0 \times 1.013 \times 10^5$ Pa

[$\because$ 1 atm $= 1.013 \times 10^5$ Pa]

Compressibility of water $\left(\frac{1}{B}\right) = 45.8 \times 10^{-11}$ Pa^{-1}

Let V and V' be the volumes of certain mass of water at the surface and at a given depth. The density of water at the given depth be ρ'.

Volume of water at the surface, $V = \frac{m}{\rho}$

At the given depth, $V' = \frac{m}{\rho'}$

$\therefore$ Change in volume, $\Delta V = V - V' = m\left(\frac{1}{\rho} - \frac{1}{\rho'}\right)$

Volumetric strain $= \frac{\Delta V}{V}$

$= m\left(\frac{1}{\rho} - \frac{1}{\rho'}\right) \times \frac{\rho}{m}$

$= \left(1 - \frac{\rho}{\rho'}\right)$

$$\text{Compressibility} = \frac{1}{\text{Bulk modulus }(B)}$$

$$= \frac{1}{\frac{\Delta p}{(\Delta V/V)}} = \frac{\Delta V}{\Delta p V}$$

$$45.8\times10^{-11} = \left(1-\frac{\rho}{\rho'}\right)\times\frac{1}{80\times1.013\times10^5}$$

$$45.8\times10^{-11}\times80\times1.013\times10^5 = 1-\frac{1.03\times10^3}{\rho'}$$

$$3.712\times10^{-3} = 1-\frac{1.03\times10^3}{\rho'}$$

$$\frac{1.03\times10^3}{\rho'} = 1-3.712\times10^{-3}$$

or $$\rho' = \frac{1.03\times10^3}{1-0.003712} = 1.034\times10^3 \text{kg/m}^3$$

29. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2 m long. These at each end are of copper and middle one is of iron. Determine the ratio of their diameters if each is to have the same tension. Young's modulus of elasticity for copper and steel are $110\times10^9 \text{N/m}^2$ and $190\times10^9\text{N/m}^2$, respectively. **[NCERT]**

Sol. Young's modulus of copper $(Y_1) = 110\times10^9 \text{N/m}^2$

Young's modulus of steel $(Y_2) = 190\times10^9 \text{N/m}^2$

Let d_1 and d_2 be the diameters of copper and steel wires. Since, tension in each wire is same, therefore each wire has same extension. As each wire is of same length, hence each wire has same strain.

$$\text{Young's modulus }(Y) = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\text{Strain}}$$

or $$Y = \frac{F}{\left(\frac{\pi d^2}{4}\right)\times\text{Strain}} = \frac{4F}{\pi d^2\times\text{Strain}}$$

$\because$ $$Y \propto \frac{1}{d^2} \Rightarrow d^2 \propto \frac{1}{Y}$$

$\therefore$ $$\frac{d_1^2}{d_2^2} = \frac{Y_2}{Y_1}$$

or $$\frac{d_1}{d_2} = \sqrt{\frac{Y_2}{Y_1}} = \sqrt{\frac{190\times10^9}{110\times10^9}}$$

$$= \sqrt{\frac{19}{11}} = \sqrt{1.73} = 1.31$$

$\therefore$ $d_1 : d_2 = 1.31 : 1$

30. A 14.5 kg mass, fastened to one end of a steel wire of unstretched length 1 m is whirled in a vertical circle with an angular frequency of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path. **[NCERT]**

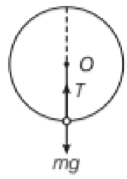
Sol. Given, mass $(m) = 14.5$ kg

Length of wire $(l) = 1$ m

Angular frequency $(\nu) = 2$ rev/s

Angular velocity $(\omega) = 2\pi\nu$

$$= 2\pi\times2 \text{ rad/s} = 4\pi \text{ rad/s}$$



Area of cross-section of wire $(A) = 0.065 \text{ cm}^2$

$$= 6.5\times10^{-6} \text{ m}^2$$

Young's modulus for steel $(Y) = 2\times10^{11} \text{ N/m}^2$.

At lowest point of the vertical circle,

$$T - mg = ml\omega^2$$

or $$T = mg + ml\omega^2$$

$$= (14.5\times9.8) + 14.5\times1\times(4\pi)^2$$

$$= 14.5(9.8 + 16\pi^2)$$

$$= 14.5(9.8 + 16\times9.87) \qquad [\because \pi^2 = 9.87]$$

$$= 14.5\times167.72 \text{ N} = 2431.94 \text{ N}$$

$$\text{Young's modulus }(Y) = \frac{\text{Stress}}{\text{Strain}} = \frac{(T/A)}{\Delta l/l} = \frac{Tl}{A\cdot\Delta l}$$

$\therefore$ $$\Delta l = \frac{T\cdot l}{A\cdot Y} = \frac{2431.94\times1}{6.5\times10^{-6}\times2\times10^{11}}$$

$$= 1.87\times10^{-3} \text{ m} = 1.87 \text{ mm}$$

31. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50000 kg. The inner and outer radii of each column are 30 cm and 60 cm, respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. Young's modulus, $Y = 2.0\times10^{11}$ Pa. **[NCERT]**

Sol. Given, total mass supported by cylindrical columns (m)

$$= 50000 \text{ kg}$$

$\because$ Total weight supported by cylindrical columns

$$= mg = 50000\times9.8 = 490000 \text{ N}$$

$\therefore$ Load acting on each cylindrical support

$$F = \frac{mg}{4} = \frac{490000}{4} \text{ N} = 122500 \text{ N}$$

Inner radius of each column $(r_1) = 30 \text{ cm} = 0.3$ m

Outer radius of each column $(r_2) = 60 \text{ cm} = 0.6$ m

$\therefore$ Area of cross-section of each cylindrical column

$$A = \pi r_2^2 - \pi r_1^2 = \pi(r_2^2 - r_1^2)$$

$$= 3.14[(0.6)^2 - (0.3)^2] = 3.14\times0.27 \text{ m}^2$$

Young's modulus $(Y) = 2 \times 10^{11}$ Pa

Compressional strain of each column = ?

$$\text{Young's modulus } (Y) = \frac{\text{Compressional stress}}{\text{Compressional strain}}$$

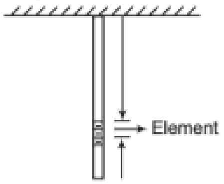
$$\text{or Compressional strain } = \frac{\text{Compressional stress}}{\text{Young's modulus}}$$

$$= \frac{F/A}{Y} = \frac{F}{AY}$$

$$= \frac{122500}{(3.14 \times 0.27) \times 2 \times 10^{11}}$$

$$= 0.722 \times 10^{-6} = 7.22 \times 10^{-7}$$

32. Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in Fig. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0×10^{11} Pa. Compute the elongations of steel and brass wires. $(1 \text{ Pa} = 1 \text{ N/m}^2)$. **[NCERT]**



Sol. Given, diameter of wires $(2r) = 0.25$ cm

$\therefore \quad r = 0.125 \text{ cm} = 1.25 \times 10^{-3}$ m

For steel wire,

$$\text{Load } (F_1) = (4 + 6) \text{ kgf}$$
$$= 10 \times 9.8 \text{ N} = 98 \text{ N}$$

Length of steel wire $(l_1) = 1.5$ m

Young's modulus $(Y_1) = 2.0 \times 10^{11}$ Pa

$$\text{Young's modulus } (Y) = \frac{F_1 \times l_1}{A_1 \times \Delta l_1}$$

$$\therefore \text{ Change in length } (\Delta l_1) = \frac{F_1 \times l_1}{A_1 \times Y_1} = \frac{F_1 \times l_1}{\pi r_1^2 \times Y_1}$$

$$= \frac{98 \times 1.5}{3.14 \times (1.25 \times 10^{-3})^2 \times 2.0 \times 10^{11}}$$

$$= 1.5 \times 10^{-4} \text{ m}$$

For brass wire,

Load $(F_2) = 6 \text{ kgf} = 6 \times 9.8 \text{ N} = 58.8$ N

Length of brass wire $(l_2) = 1.0$ m

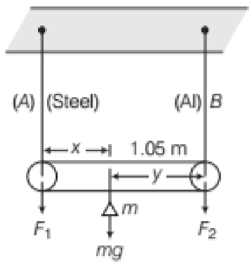
Young's modulus $(Y_2) = 0.91 \times 10^{11}$ Pa

$$\text{Change in length } (\Delta l_2) = \frac{F_2 \times l_2}{\pi r_2^2 \times Y_2}$$

$$= \frac{58.8 \times 1.0}{3.14 \times (1.25 \times 10^{-3})^2 \times 0.91 \times 10^{11}}$$

$$= 1.3 \times 10^{-4} \text{ m}$$

33. A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in Fig. The cross-sectional areas of wires A and B are 1 mm^2 and 2 mm^2, respectively. Young's modulus of elasticity for steel and aluminium are 2×10^{11} and $7 \times 10^{11} \text{ N/m}^2$, respectively.



At what point along the rod should a mass m be suspended in order to produce (i) equal stresses and (ii) equal strains in both steel and aluminium wires. **[NCERT]**

Sol. Let the length of wires A and B is equal to L and their area of cross-section be A_1 and A_2, respectively.

Given, $\quad A_1 = 1 \text{ mm}^2 = 1 \times 10^{-6} \text{ m}^2$

$$A_2 = 2 \text{ mm}^2 = 2 \times 10^{-6} \text{ m}^2$$
$$Y_{\text{steel}} = 2 \times 10^{11} \text{ N/m}^2$$
$$Y_{\text{Al}} = 7.0 \times 10^{10} \text{ N/m}^2$$

Let F_1 and F_2 be the tensions in the two wires, respectively.

(i) When equal stresses are produced, then

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\text{or} \quad \frac{F_1}{F_2} = \frac{A_1}{A_2} = \frac{1 \times 10^{-6}}{2 \times 10^{-6}} \quad \text{or} \quad \frac{F_1}{F_2} = \frac{1}{2} \quad \text{...(i)}$$

Let mass m be suspended at distance x from steel wire A.

Taking moment of forces about the point of suspension of mass from the rod, we get

$$F_1 \times x = F_2 \times (1.05 - x)$$

$$\text{or} \quad \frac{F_1}{F_2} = \frac{(1.05 - x)}{x} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2} = \frac{(1.05 - x)}{x}$$

$$x = 2.10 - 2x$$

or $\quad 3x = 2.10$

or $\quad x = 0.70$m

$\therefore$ The mass m must be suspended at a distance 0.70 m from steel wire A.

(ii) $$\text{Young's modulus } (Y) = \frac{\text{Stress}}{\text{Strain}}$$

$$\therefore \quad \text{Strain} = \frac{\text{Stress}}{Y} = \frac{F/A}{Y}$$

For steel wire A, $$(\text{Strain})_{\text{steel}} = \frac{F_1}{A_1 Y_1}$$

For aluminium wire B,

$$(\text{Strain})_{Al} = \frac{F_2}{A_2 Y_2}$$

When equal strains are produced in both wires,

then $$\frac{F_1}{A_1 Y_1} = \frac{F_2}{A_2 Y_2}$$

or $$\frac{F_1}{F_2} = \frac{A_1 Y_1}{A_2 Y_2} \quad \text{...(iii)}$$

$\therefore$ From Eqs. (ii) and (iii), we get

$$\frac{(1.05 - x)}{x} = \frac{A_1 Y_2}{A_2 Y_2}$$

$$= \frac{1 \times 10^{-6}}{2 \times 10^{-6}} \times \frac{2 \times 10^{11}}{7 \times 10^{10}}$$

$$\frac{(1.05 - x)}{x} = \frac{10}{7}$$

$$10x = 7.35 - 7x$$

$$\Rightarrow \quad 17x = 7.35 \text{ or } x = \frac{7.35}{17}$$

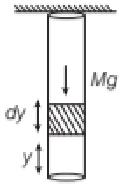
$$x = 0.43 \text{ m}$$

$\therefore$ The mass m must be suspended at a distance 0.43 m from the steel with A.

34. A rubber string 10 m long is suspended from a rigid support at its one end. Calculate the extension in the string due to its own weight. The density of rubber is 1.5×10^3 kg/m^3 and Young's modulus for the rubber is 5×10^6 N/m^2. The breaking stress for a metal is 7.8×10^9 N/m^2. Calculate the maximum length of the wire made of this metal which may be suspended without breaking. The density of metal $= 7.8 \times 10^3$ kg/m^3.

Sol. $l = 10$ m, $\rho = 1.5 \times 10^3$ kg/m^3

$Y = 5 \times 10^6$ N/m^2



We know, $Y = \dfrac{F\,l}{A\,\Delta l}$

Efficient force $= Mg$

Consider a small length dy at a distance y from free end.

The length above this, $(l - y)$ will experience a force of

$$F_{dy} = \frac{M}{l}(dy).gdy$$

$\therefore$ Extension $dl = \dfrac{F\,l}{AY}$

$$\Rightarrow \quad dl = \frac{(l-y)}{AY} \cdot \frac{M}{l} gdy = \frac{Mg}{lAY}(l-y)dy$$

$$\left.\begin{array}{l}\text{Net extension due}\\ \text{to its own weight}\end{array}\right\} = \int dl$$

$$= \frac{Mg}{AYl}\int_0^l (l-y)dy = \frac{Mg}{lAY}\left[ly - \frac{y^2}{2}\right]_0^l = \frac{Mgl}{2AY}$$

Net extension $= \dfrac{Mgl}{2AY} = \dfrac{Mgl^2}{2YV} = \dfrac{\rho g l^2}{2Y}$

Extension of rubber string

$$= \frac{1.5 \times 10^3 \times 10 \times 10^2}{2 \times 5 \times 10^6} = 0.15 \text{ m}$$

Breaking stress for a metal $= 7.8 \times 10^9$ N/m^2

Density $= 7.8 \times 10^3$ kg/m^3.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{Mg}{A} = \frac{Mgl}{Al} = \frac{Mgl}{\text{Volume}} = \rho g l$$

If $\rho gl >$ Breaking stress, the wire will break.

$$\therefore \quad l \le \frac{7.8 \times 10^9}{\rho g}, \; l \le \frac{7.8 \times 10^9}{7.8 \times 10^3 \times 10}$$

i.e. $\quad l \le 10^5$ m

Maximum length of wire $= 10^5$ m

35. Compute the Bulk modulus of water from the following data; initial volume = 100.0 L, pressure increase = 100.0 atm (1 atm = 1.013 $\times 10^5$ Pa), final volume = 100.5 L. Compare the Bulk modulus of water with that of air (at constant temperature). Explain in simple terms, why the ratio is so large? **[NCERT]**

Sol. Given, initial volume $(V_1) = 100.0$ L

Final volume $(V_2) = 100.5$ L

$\therefore$ Increase in volume $(\Delta V) = V_2 - V_1$

$= 100.5 - 100.0 = 0.5$ L

$= 0.5 \times 10^{-3}$ m^3 $\quad [\because 1 \text{ L} = 10^{-3} \text{ m}^3]$

Increase in pressure,

$(\Delta p) = 100.0$ atm

$= 100.0 \times 1.013 \times 10^5$ Pa

$[\because 1 \text{ atm} = 1.013 \times 10^5 \text{Pa}]$

$= 1.013 \times 10^7$ Pa

Bulk modulus of water

$$(B_w) = \frac{\Delta p}{(\Delta V / V)}$$

$$= \frac{\Delta p V}{\Delta V} = \frac{1.013 \times 10^7 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}}$$

$$= \frac{10.13}{5} \times 10^9 \text{ Pa}$$

$$= 2.026 \times 10^9 \text{ Pa}$$

Bulk modulus of air $(B_a) = 1.0 \times 10^5$ Pa

$$\therefore \quad \frac{\text{Bulk modulus of water } (B_w)}{\text{Bulk modulus of air } (B_a)} = \frac{2.026 \times 10^9}{1.0 \times 10^5}$$

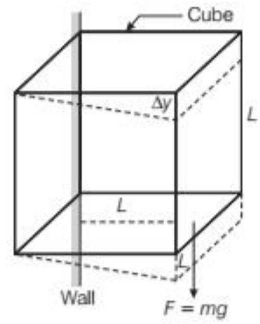
$$= 2.026 \times 10^4$$

This ratio is too large as gases are more compressible than those of liquids. In liquids, interatomic forces are more strong than that for gases.

36. The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube.

The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face? **[NCERT]**

Sol.



Given, side of a cube $(l) = 10 \text{ cm} = 0.1 \text{ m}$

Area of its each face $(A) = l^2 = (0.1)^2 = 0.01 \text{ m}^2$

Load $(m) = 100 \text{ kg}$

Tangential force acting on one face of the cube,

$$F = mg = 100 \times 9.8 = 980 \text{ N}$$

Shear stress acting on this face $= \dfrac{F}{A} = \dfrac{980}{0.01} \text{ N/m}^2$

$= 9.8 \times 10^4 \text{ N/m}^2$

Shear modulus of aluminium $(\eta) = 25 \text{ GPa}$

$= 25 \times 10^9 \text{ N/m}^2$

Shear modulus $(\eta) = \dfrac{\text{Shearing stress}}{\text{Shearing strain}}$

or shearing strain $\left(\dfrac{\Delta L}{L}\right) = \dfrac{\text{Shearing stress}}{\text{Shear modulus}}$

or $\Delta L = \dfrac{\text{Shearing stress}}{\text{Shear modulus}} \times L = \dfrac{9.8 \times 10^4}{25 \times 10^9} \times 0.1$

$= 0.0392 \times 10^{-5} \text{ m}$

$= 3.92 \times 10^{-7} \text{ m}$

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. On applying a stress of $20 \times 10^8 \text{ Nm}^{-2}$, the length of a perfectly elastic wire is doubled. Its Young's modulus will be

(a) $40 \times 10^8 \text{ Nm}^{-2}$ (b) $20 \times 10^8 \text{ Nm}^{-2}$

(c) $10 \times 10^8 \text{ Nm}^{-2}$ (d) $5 \times 10^8 \text{ Nm}^{-2}$

2. A wire of length 2 m is made from 10 cm^3 of copper. A force F is applied so that its length increases by 2 mm. Another wire of length 8 m is made from the same volume of copper. If the force F is applied to it, its length will increase by

(a) 0.8 cm (b) 1.6 cm

(c) 2.4 cm (d) 3.2 cm

3. In steel, the Young's modulus and the strain at the breaking point are $2 \times 10^{11} \text{ Nm}^{-2}$ and 0.15, respectively. The stress at the breaking point for steel is therefore

(a) $1.33 \times 10^{11} \text{ Nm}^{-2}$ (b) $1.33 \times 10^{12} \text{ Nm}^{-2}$

(c) $7.5 \times 10^{-13} \text{ Nm}^{-2}$ (d) $3 \times 10^{10} \text{ Nm}^{-2}$

4. Elasticity of a material can be altered by

(a) annealing (b) hammering

(c) adding impurities (d) All of these

5. Two wires of the same material and length but diameter in the ratio 1 : 2 are stretched by the same load. The ratio of elastic potential energy per unit volume for the two wires is

(a) 1 : 1 (b) 2 : 1 (c) 4 : 1 (d) 16 : 1

Answers

1. (b)	2. (d)	3. (d)	4. (d)	5. (d)

VERY SHORT ANSWER Type Questions

6. What are the factors on which the modulus of elasticity depends?

7. What is the value of bulk modulus for an incompressible liquid?

SHORT ANSWER Type Questions

8. A wire elongates by 8 mm when a load of 9 kg is suspended from it. What is the elongation when its radius is doubled, if all other quantities are the same as before? [**Ans.** 2 mm]

9. Find the change in volume which 1cc of water at the surface will undergo, when it is taken to the bottom of the lake 100m deep, given that volume elasticity is 22000 atmospheres [**Ans.** 0.00044 cc]

10. A square lead slab of side 50 cm and thickness 10 cm is subjected to a shearing force (on its narrow edge) of 9.0×10^4 N. The lower edge is reveted to the floor. How much will the upper edge be displaced? Modulus of rigidity of lead $= 5.6 \times 10^9$ N/m^2

[**Ans.** 0.16 mm]

LONG ANSWER Type I Questions

11. A cube of aluminium of each side 4 cm is subjected to a tangential (shearing) force. The top face of the cube is sheared through 0.012 cm with respect to the bottom face.

Find (i) shearing strain (ii) shearing stress and (iii) the shearing force. Given, $\eta = 2.08 \times 10^{11}$ dyne cm^{-2}.

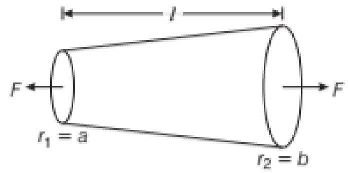
12. Two cylinders A and B of radii r and $2r$ are soldered co-axially. The free end of A is clamped and the free end of B is twisted by an angle ϕ. Find twist at the junction taking the material of two cylinders to be same and of equal length.

13. A metal bar of length L and area of cross-section A, is rigidly clamped between two walls. The Young's modulus of its material is Y and the coefficient of linear expansion is α. The bar is heated so that its temperature is increased from 0 to θ°C. Find the force exerted at the ends of the bar.

14. A wire of length l and area of cross-section A is stretched by the application of a force. If the Young's modulus is Y, then what is the work done per unit volume?

LONG ANSWER Type II Questions

15. A slightly tapering wire of length l and end radii a and b on both sides is subjected to the stretching forces F on both sides as shown in figure. If Y is the Young's modulus of the wire, then calculate the extension produced in the wire.



16. Four identical cylindrical columns of steel support a big structure of mass 50000 kg. The inner and outer radii of each column are 30 cm and 40 cm, respectively. Assume the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11} Pa.

17. A composite wire of diameter 1 cm consisting of copper and steel wire of lengths 2.2 m and 2.0 m, respectively.

Total extension of the wire, when stretched by a force is 1.2 mm. Calculate the force, given that Young's modulus for copper is 1.1×10^{11} Pa and for steel is 2.0×10^{11} Pa.

18. A light rod of length 2m is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends.

One of the wires is made of steel and is of cross-sectional area 0.1 sq cm and the other is of brass of cross-sectional area 0.2 sq cm. Find out the position along the rod at which a weight may be hung to produce

(i) equal stress in both wires,
(ii) equal strain in both wires.

Given, Y for steel $= 20 \times 10^{11}$ dyne cm^{-2} and Y for brass $= 10 \times 10^{11}$ dyne cm^{-2}.

SUMMARY

- Materials having a definite shape and volume are known as **solids** or rigid bodies.
- A body that returns to its original shape and size on the removal of deforming force is called elastic body and this property is called **elasticity**.
- If a body does not show any tendency to regain its original shape and size even after the removal of deforming force are called plastic body and this property is called **plasticity**.
- The internal restoring force acting per unit area of a deformed force is called **stress**.

  i.e $$\text{Stress} = \frac{F}{A}$$

  SI unit of stress is N/m^2 or pascal
- The restoring forces per unit area act perpendicular to the surface of the body are called **normal forces**.
- When there is an increase in the length or the extension of the body in the direction of the force applied then it is called **tensile stress**.
- When there is a decrease in the length or the compression of the body due to the deforming force applied normally on the body then it is called **compressive stress.**
- If a body is subjected to a uniform force from all sides, then the corresponding stress is called **bulk stress** or **hydraulic stress** or volumetric stress.
- **Strain** is defined as the ratio of change in configuration to the original configuration.
- The materials which can be elastically stretched to large values of strain are called **elastomers**.
- **Longitudinal strain** (ε) is the change in length per unit original length, when the body is deformed by external forces,

  i.e. $$\varepsilon = \frac{\Delta L}{L}$$
- **Volumetric strain** is the change in volume per unit original volume, when the body is deformed by external forces. i.e. $\varepsilon = \frac{\Delta V}{V}$.
- **Shearing strain** is the deforming forces produce a change in the shape of the body, i.e. $\theta = \tan\theta = \frac{\Delta L}{L}$.
- The material which have large plastic range of extension are called **ductile material.**
- The materials which have very small range of plastic extension are called **brittle material.**
- Within elastic limit, the stress developed is directly proportional to the strain produced in a body. This is **Hooke's law.**
- From **Hooke's law**, stress $\propto$ strain.

  $\Rightarrow$ Stress $= E \times$ Strain, here E is modulus of elasticity.
- The ratio of longitudinal stress (σ) to the longitudinal strain (ε) within the elastic limit is called **Young's modulus** (Y).

  i.e. $$Y = \frac{\sigma}{\varepsilon}$$
- Ratio of normal stress to the volumetric strain within the elastic limit is called **Bulk Modulus** (B)

  i.e. $B = \frac{F}{A} \Big/ \frac{\Delta v}{v} = \frac{1}{k}$, where k = compressibility
- The ratio of tangential stress to the shear strain within elastic limit is called shear modulus or **modulus of rigidity** (η).

  i.e. $$\eta = \frac{F}{A} \cdot \frac{L}{\Delta L}$$
- The ratio of lateral strain to the longitudinal strains within the elastic limit it is called **Poisson's ratio.**
- Potential energy stored in a deformed body.

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. In solids, inter-atomic forces are
(a) totally repulsive
(b) totally attractive
(c) combination of (a) and (b)
(d) None of the above

2. The nature of molecular forces resembles with the nature of the
(a) gravitational force (b) nuclear force
(c) electromagnetic force (d) weak force

3. A and B are two wires. The radius of A is twice that of B. They are stretched by the same load. Then, the stress on B is
(a) equal to that on A (b) four times that on A
(c) two times that on A (d) half that on A

4. On suspending a weight Mg, the length l of elastic wire having area of cross-section A, becomes double the initial length. The instantaneous stress action on the wire is
(a) Mg/A (b) $Mg/2A$
(c) $2Mg/A$ (d) $4Mg/A$

5. A cube of aluminium of side 0.1 m is subjected to a shearing force of 100 N. The top face of the cube is displaced through 0.02 cm with respect to the bottom face. The shearing strain would be
(a) 0.02 (b) 0.1 (c) 0.005 (d) 0.002

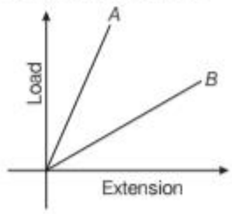
6. A steel rod of length 1 m and radius 10 mm is stretched by a force 100 kN along its length. The stress produced in the rod is $Y_{Steel} = 2\times10^{11}\ Nm^{-2}$.
(a) $3.18\times10^{6}\ Nm^{-2}$ (b) $3.18\times10^{7}\ Nm^{-2}$
(c) $3.18\times10^{8}\ Nm^{-2}$ (d) $3.18\times10^{9}\ Nm^{-2}$

7. The upper end of a wire of radius 4 mm and length 100 cm is clamped and its other end is twisted through an angle of 30°. Then, angle of shear is
(a) 12° (b) 0.12° (c) 1.2° (d) 0.012°

8. A copper and a steel wire of the same diameter are connected end to end. A deforming force F is applied to this composite wire which causes a total elongation of 1 cm. The two wires will have
(a) the same stress and strain
(b) the same stress but different strain
(c) the same strain but different stress
(d) different strains and stress

9. A wire of length L and radius r is rigidly fixed at one end. On stretching the other end of the wire with a force F, the increase in its length is l. If another wire of same material but of length $2L$ and radius $2r$ is stretched with a force of $2F$, the increase in its length will be
(a) l (b) $2l$
(c) $l/2$ (d) $l/4$

10. In the given figure, if the dimension of the wire are the same and materials are different, Young's modulus is more for



(a) A (b) B (c) Both (d) None of these

ASSERTION AND REASON

Direction (Q.Nos. 11-16) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*
(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

11. **Assertion** Stress is the internal force per unit area of a body.
Reason Rubber is less elastic than steel.

12. **Assertion** When a solid is deformed, the atoms or molecules are displaced from their equilibrium position.
Reason Due to change in inter-atomic spacing, intermolecular/interatomic forces arise.

13. **Assertion** The restoring force F, on a stretched string for extension x is related to potential energy U as

$$F = \frac{-dU}{dx}$$

Reason $F = -kx$ and $U = (1/2)kx^2$, where k is a spring constant for the given stretched string.

14. **Assertion** The strain produced by a hydraulic pressure is volumetric in nature.
Reason It is a ratio of change in volume (ΔV) of the original volume (V).

15. **Assertion** Young's modulus for a perfectly plastic body is zero.
Reason For a perfectly plastic body, restoring force is zero.

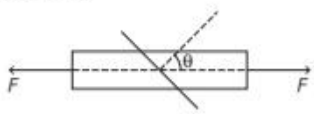
16. **Assertion** Ropes are always made of a number of thin wires braided together.
Reason It helps to ease in manufacture, flexibility and strength.

CASE BASED QUESTIONS

Direction (Q. Nos. 17-18) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

17. **Restoring Force due to Stress**

When a bar of cross-section A is subjected to equal and opposite tensile forces at its ends, then a restoring force equal to the applied force normal to its cross-section comes into existence. This restoring force per unit area of cross-section is known as tensile stress. While when the deforming force acts tangentially to the surface, then this tangential force applied per unit area of cross-section is known as tangential stress. Consider a plane section of the bar whose normal makes an angle θ with the axis of the bar.



(i) Which of the following property of the bar does not change due to this force?
(a) Area (b) Volume
(c) Shape (d) Size

(ii) What is the tensile stress on this plane?
(a) $(F/A)\cos^2\theta$ (b) F/A
(c) $(F/A)\tan\theta$ (d) $(F/A)\sec^2\theta$

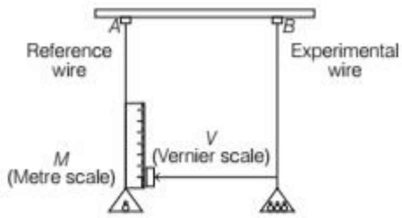
(iii) What is the shearing stress on this plane?
(a) $\frac{F}{2A}\sin 2\theta$ (b) $\frac{F}{A}\cos 2\theta$
(c) $\frac{F}{2A}\cos^2\theta$ (d) $\frac{F}{4A^2}$

(iv) For what value of θ is the tensile stress maximum?
(a) 0° (b) 90°
(c) 45° (d) 30°

(v) For what value of θ is the shearing stress maximum?
(a) 45° (b) 30°
(c) 90° (d) 60°

18. **Young's Modulus Experiment**

A typical experimental arrangement to determine the Young's modulus of a material of wire under tension is shown in figure. It consists of two long straight wires of same length and equal radii suspended side-by-side from a fixed rigid support. The wire A (called the reference wire) carries a millimeter main scale M and a pan to place a weight.



The wire B (called the experimental wire) of uniform area of cross-section also carries a pan in which known weights can be placed, vernier scale is attached to a pointer at the bottom of experimental wire B and main scale is fixed to the reference wire A.

(i) When a weight is placed in the pan, which type of stress is produced in it,
(a) Tensile
(b) Tangential
(c) Bulk
(d) Compressive

(ii) The reference wire is used to compensate for any change in length due to change in
  (a) length of experimental wire
  (b) volume of experimental wire
  (c) room temperature
  (d) weight of pan

(iii) The difference between which two readings gives the elongation produced in the wire.
  (a) Main (b) Vernier
  (c) Reference (d) Original wire

(iv) Suppose M be the mass of wire that produced an elongation ΔL in the wire, then the applied force is equal to
  (a) Mg (b) Ma
  (c) Mv (d) Mv^2

(v) The Young's modulus of an experimental wire is
  (a) $Mg \times L/(\pi r^2 \times \Delta L)$
  (b) $Mg \times (\pi r^2 \times \Delta L)/L$
  (c) $(\Delta L \times \pi r^2)/Mg \times L$
  (d) $Mg \times \pi r^2 \times L/(\Delta L)^2$

Answers

1. (c)	2. (c)	3. (b)	4. (c)	5. (d)
6. (c)	7. (b)	8. (b)	9. (a)	10. (a)
11. (b)	12. (a)	13. (a)	14. (b)	15. (a)
16. (a)				
17. (i) (b)	(ii) (a)	(iii) (a)	(iv) (a)	(v) (a)
18. (i) (a)	(ii) (c)	(iii) (b)	(iv) (a)	(v) (a)

VERY SHORT ANSWER Type Questions

19. A wire 50 cm long and 1 sq mm in cross-section has the Young's modulus, $Y = 2\times10^{10}$ Nm^{-2}. How much work is done in stretching the wire through 1 mm?

[**Ans.** 2×10^{-2} J]

20. The star Sirius has a mass of 7×10^{30} kg, its distance from the earth is 8×10^{16} m and the mass of the earth is 6×10^{24} kg. Calculate the cross-section of a steel cable that can withstand the gravitational pull between the Sirius and the earth. Given, $G = 6.67\times10^{-11}$Nm^{2}kg^{-2} and breaking stress $= 10^{10}$Nm^{-2}. [**Ans.** 44m^2]

21. A solid sphere of radius R made of a material of Bulk modulus B is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, find fractional change in the radius of the sphere. $\left[\textbf{Ans.} \left(\frac{\Delta R}{R} = \frac{Mg}{3AB}\right)\right]$

22. A uniform pressure p is exerted on all sides of a solid cube at temperature $t°$ C. By what amount should the temperature of the cube be raised in order to bring its volume back to the volume it had before the pressure was applied, if the Bulk modulus and coefficient of volume expansion of the material are B and γ, respectively?

$\left[\textbf{Ans.} \left(\frac{p}{\gamma B}\right)\right]$

SHORT ANSWER Type Questions

23. Determine the fractional change in volume as the pressure of the atmosphere 1.0×10^5 Pa around a metal block is reduced to zero by placing the block in vacuum. The Bulk modulus for the block is 1.25×10^{11} Nm^{-2}. [**Ans.** 8×10^{-7}]

24. (a) Which is more elastic, rubber or glass? Why?
(b) Identical springs of steel and copper are equally stretched. On which spring, more work will have to be done?

25. A lift is tied with thick iron wires and its mass is 1000 kg. If the maximum acceleration of lift is 1.2 ms^{-2} and the maximum safe stress is 1.4×10^8 Nm^{-2}, then find the minimum diameter of the wire. Take, $g = 9.8$ ms^{-2}. [**Ans.** 0.01 m]

LONG ANSWER Type I Questions

26. Two parallel steel wires A and B are fixed to rigid support at the upper ends and subjected to the same load at the lower ends. The lengths of the wires are in the ratio 4:5 and their radii are in the ratio 4:3. The increase in the length of the wire A is 1 mm. Calculate the increase in the length of the wire B. [**Ans.** 2.22 mm]

27. Assume that if the shear stress in steel exceeds about 4×10^8 N/m^2, the steel reptures.
Determine the shearing force necessary to (a) shear a steel bolt 1.00 cm in diameter and (b) punch a 1 cm diameter hole in a steel plate 0.500 cm thick. [**Ans.** 3.14×10^4 N, 6.28×10^4 N]

28. A uniform heavy rod of weight w, cross-sectional area A and length l is hanging from a fixed support. Young's modulus of the material of the rod is Y. Neglecting the lateral contraction, find the elongation produced in the rod. $\left[\textbf{Ans. } \frac{wl}{2AY}\right]$

LONG ANSWER Type II Questions

29. What is the length of a wire that breaks under its own weight when suspended vertically? Breaking stress = 5×10^7 Nm^{-2} and density of the material of the wire = 3×10^3 kg/m^3. [**Ans.** 1.67 km]

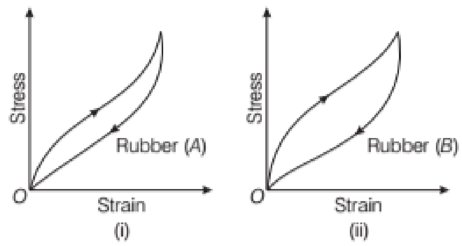
30. A silica glass rod has a diameter of 1 cm and is 10 cm long. The ultimate strength of glass is $50 \times 10^6 Nm^{-2}$. Estimate the largest mass that can be hung from it without breaking it. Take, $g = 10 Nkg^{-1}$. [**Ans.** 392.5 kg]

31. Two different types of rubber are found to have the stress-strain curves shown below in figure.

(i) In which significant ways do these curves differ from the stress-strain curve of a metal wire.

(ii) A heavy machine is to be installed in a factory. To absorb vibrations of the machine, a block of rubber is placed between the machinery and the floor. Which of the two rubbers A and B would you prefer to use for this purpose? Why?

(iii) Which of the two rubber materials would you choose for a car tyre?



32. Two wires of equal cross-section but one made of steel and the other copper are joined end to end. When the combination is kept under tension, the elongation in the two wires is found to be equal. Given Young's moduli of steel and copper are 2.0×10^{11} Nm^{-2} and 1.1×10^{11} Nm^{-2}. Find the ratio between the lengths of steel and copper wires. [**Ans.** 20 : 11]