UPSEE 2018

Solved Paper

Question 1

Physics

A very long solenoid is made out of a wire with n turns per unit length. The radius of the cylinder is a and is negligible compared to its length l . The interior of the cylinder is filled with materials such that the linear magnetic permeability varies with the distance r from axis according to

$$\mu(r) = \begin{cases} \mu_1 = \text{constant ; for } 0 < r < b \\ \mu_2 = \text{constant ;for } b < r < a \end{cases}$$

The self-inductance of the solenoid is

Options:

A. $\pi n^2 l\,[\mu_1 b^2 + \mu_2 b^2]$

B. $\pi n^2 l\,[\mu_1 + \mu_2]a^2$

C. $\pi n^2 l\,[\mu_1 b^2 + \mu_2(a^2 - b^2)]$

D. $\pi n^2 l\,([\mu_1 b^2 + (\mu_1 + \mu_2)a^2]$

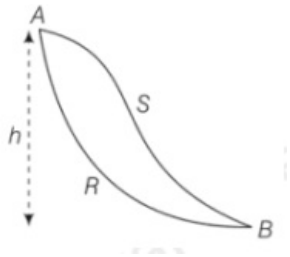
Answer: C

Solution:

Solution:
Self-inductance of the solenoid is given by
$L = \mu(r) n^2 \pi a^2 l$
Here, μ varies with distance r.
So, the self-inductance can be written as
$L = \mu_1 n^2 l\,\pi b^2|_{0<r<b} + \mu_2 n^2 l\,(\pi a^2 - \pi b^2)|_{b<r<a}$
$= \pi n^2 l\,[\mu_1 b^2 + \mu_2(a^2 - b^2)]$

Question 2

Two childrens Ramesh (on path ARB) and Sohan (on path ASB), travel down slides of identical height h but different shapes as shown in the figure. Assuming, they start down the frictionless slides at the same time with zero initial velocity, which of the following statement is true?



Options:

A. Ramesh reaches the bottom first with the same average velocity as Sohan.

B. Ramesh reaches the bottom first with a larger average acceleration than Sohan.

C. Ramesh reaches the bottom first with the same average acceleration as Sohan.

D. They reach the bottom at same time with the same average acceleration.

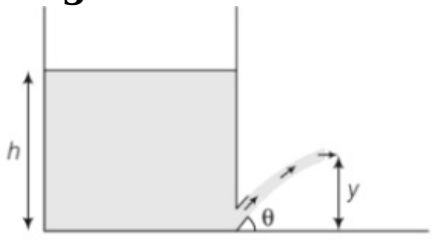
Answer: D

Solution:

Solution:
The acceleration of Ramesh and Sohan are same, i.e. acceleration due to gravity (g).
Value of g is independent of the shape of the slide because $g = \frac{GM}{R^2}$.
As, height is same = h.
∴ Both reach the bottom at same time.

Question 3

A stream of non-viscous liquid emerges from a very short outlet tube at the base of a large open tank, in which the depth of liquid is h. The tube is at a fixed angle θ to the ground as shown in the figure. The maximum height of the stream y is



Options:

A. $h\sin^2\theta$

B. $h\sin 2\theta$

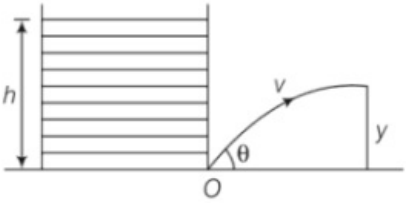
C. $\frac{1}{2}h\sin\theta$

D. $h\tan^2\theta$

Answer: A

Solution:

Solution:



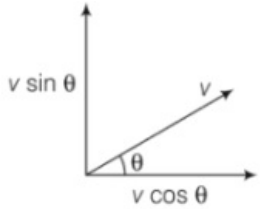
Let the speed of fluid at outlet is v.
Applying Bernoulli's theorem,

$p_1 + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

$p_0 + \rho gh + 0 = p_0 + 0 + \frac{1}{2}\rho v^2$

$\therefore v = \sqrt{2gh}$ .....(i)

At outlet,



$v \sin\theta$ vertical velocity of the fluid at outlet,

$v^2 = u^2 + 2gh_1$

For maximum height,

$0 = (v \sin\theta)^2 - 2gh_1$

$\therefore h_1 = \frac{v^2 \sin^2\theta}{2g}$

$= \frac{2gh\sin^2\theta}{2g}$ [{ from Eq. (i) }]

$= h\sin^2\theta$

$h_1 = h \sin^2\theta$

Question 4

An eagle flies at constant velocity horizontally across the sky, carrying a mouse and releases the mouse while in flight. From the eagle's perspective, the mouse falls vertically with speed v_1. From an observer on the ground's perspective, the mouse falls at an angle with speed v_2. What is the speed of the eagle with respect to the observer on the ground?

Options:

A. $v_1 + v_2$

B. $v_1 - v_2$

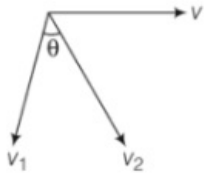
C. $\sqrt{{v_1}^2 - {v_2}^2}$

D. $\sqrt{{v_2}^2 - {v_1}^2}$

Answer: D

Solution:

Solution:



Let the speed of the eagle is v with respect to ground. Diagram of velocities, according to the question, where, v_1 is speed of mouse with respect to the eagle and v_2 is speed of mouse with respect to the ground. Thus, we have

$v_2 = \sqrt{v^2 + {v_1}^2 + 2vv_1 \cos 90°}$

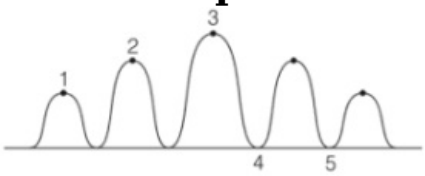
$\therefore v_2 = \sqrt{v^2 + {v_1}^2}$

${v_2}^2 = v^2 + {v_1}^2$

$\therefore v = \sqrt{{v_2}^2 - {v_1}^2}$

Question 5

The diffraction pattern of a single slit is shown in figure. The point at which the path difference of the extreme rays is two wavelengths is



Options:

A. point 1

B. point 2

C. point 4

D. point 5

Answer: D

Solution:

Solution:

Diffraction of light at a single slit, the pattern obtained consists of a central bright band having alternate dark and weak bands of decreasing intensity on both sides.

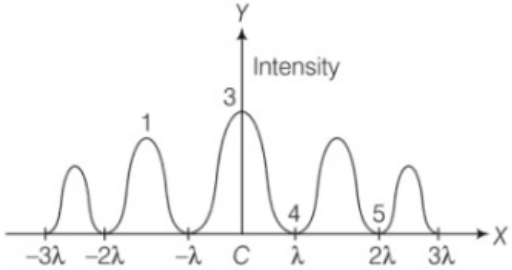
For nth minima (secondary),

$\sin\theta_n = \frac{n\lambda}{a}$

For nth maxima (secondary),

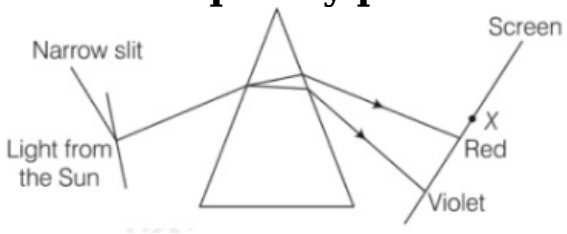
$\sin\theta_n = (2n + 1)\frac{\lambda}{2a}$

Thus, point 5 corresponds to path difference of extreme rays equal to 2λ.



Question 6

Which frequency produces a sound that can be heard by a person?



Options:

A. 100kH z

B. 40kH z

C. 2kH z

D. 30kH z

Answer: C

Solution:

Solution:
As we know, audible range of frequency of human being $20\mathrm{H\,z} - 20000\mathrm{H\,z}(20\mathrm{kH\,z})$.
Hence,$2\mathrm{kH\,z}$ is between $20\mathrm{H\,z}$ and $20\mathrm{kH\,z}$.
Hence, $2\mathrm{kH\,z}$ will be heard by the person. Rest options are all above $20\mathrm{kH\,z}$, hence known as ultrasonic which cannot be detected by human.

Question 7

Light from the Sun passes through a prism and a spectrum is produced on a screen.
A thermometer placed at point X shows a temperature increase. For the given diagram, which type of radiation causes this temperature increase? Select the most appropriate option.

Options:

A. Infrared rays

B. X-rays

C. Ultraviolet rays

D. Visible light

Answer: A

Solution:

Solution:
According to the given set up point X, shows a temperature increase. Infrared radiation causes the temperature increase because these waves are produced from the heat radiating bodies and molecules. It is produced by vibration of atoms and molecules. They have moderate frequency range of $3 \times 10^{11}\mathrm{H\,z}$ to $4 \times 10^{14}\mathrm{H\,z}$.

Other options like ultraviolet rays are absorbed by ozone layer and can reach Earth. Only through ozone holes. Visible light do not produce heat. X-rays are high frequency waves produced by electrons in inner shells. Hence, options (b), (c), (d) are incorrect and option (a) is correct.

Question 8

Light has wavelength 600nm in free space. It passes into glass, which has an index of refraction of 1.50. What is the frequency of the light inside the glass?

Options:

A. $3.3 \times 10^{14} \mathrm{Hz}$

B. $5.0 \times 10^{14} \mathrm{Hz}$

C. $3.3 \times 10^{5} \mathrm{Hz}$

D. $5.0 \times 10^{5} \mathrm{Hz}$

Answer: B

Solution:

Solution:
Given, wavelength of light $= 600\mathrm{nm}$
$= 600 \times 10^{9}\mathrm{m}$
$n = 1.5$
As the frequency of light cannot change, $c = v\lambda$
Speed of light in free space $= 3 \times 10^{8}\mathrm{m/s}$
$\therefore 3 \times 10^{8} = v \times 600 \times 10^{9}$
$\Rightarrow v = \frac{3 \times 10^{8}}{600 \times 10^{-9}}$
$= 5 \times 10^{14}\mathrm{Hz}$

Question 9

Two SHM are represented by the equations

$X_1 = 20 \sin\left[5\pi t + \frac{\pi}{4}\right]$

and $x_2 = 10(\sin 5\pi t + \sqrt{3}\cos 5\pi t)$

The ratio of the amplitudes of the two motions is

Options:

A. 0.5

B. 1

C. 0.25

D. $\frac{\sqrt{3}}{2}$

Answer: B

Solution:

Solution:
For SHM (1),
$x_1 = 20\sin\left(5\pi t + \frac{\pi}{4}\right)$
Its amplitude, $A_1 = 20$
For SHM (2),
$x_2 = 10[\sin 5\pi t + \sqrt{3}\cos 5\pi t]$
$= 10 \times 2\left(\frac{\sin 5\pi t}{2} + \frac{\sqrt{3}}{2}\cos 5\pi t\right)$
$= 20\left(\sin 5\pi t \,.\, \cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos 5\pi t\right)$
$= 20\sin\left(5\pi t + \frac{\pi}{3}\right)$
It's amplitude, $A_2 = 20$
$\therefore \frac{A_1}{A_2} = \frac{20}{20} = 1$

Question 10

The rms speed of hydrogen molecule at a certain temperature is v. If the temperature is doubled and hydrogen gas and dissociates into atomic hydrogen, the rms speed will become

Options:

A. v

B. $\frac{v}{2}$

C. 2v

D. $\sqrt{2}v$

Answer: C

Solution:

Solution:
Let the molar mass of H_2 is M .
$\because$ Hydrogen gas is dissociated into atomic hydrogen.
$\therefore$ It molar mass is equal to $\frac{M}{2}$
$\because v_{rms} = \sqrt{\frac{3RT}{M}}$
$\therefore \frac{v_1}{v_2} = \sqrt{\frac{T_O}{M_{O_1}}} \times \frac{M_{O_2}}{T_2}$
$\because T_2 = 2T_1$ and $M_{O_2} = \frac{M_{O_1}}{2} = \frac{M}{2}$
$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{T_1}{M}} \cdot \frac{M/2}{2T_1} = \sqrt{\frac{1}{4}}$
$\therefore v_2 = 2v_1$

Question 11

The activity of a radioactive element decreases in 10 years to 1 / 5 of initial activity A_0. After further next 10 years, its activity will be

Options:

A. $\frac{A_0}{4}$

B. $\frac{A_0}{10}$

C. $\frac{A_0}{15}$

D. $\frac{A_0}{25}$

Answer: D

Solution:

Solution:
∵ Activity of a radioactive sample,

$$A = A_0\left(\frac{1}{2}\right)^{\frac{1}{t_1/2}}$$

where, A_0 = initial activity.
In first case,

$$\frac{A_0}{5} = A_0\left(\frac{1}{2}\right)^{\frac{10}{t_1/2}}$$

$$\frac{1}{5} = \left(\frac{1}{2}\right)^{\frac{10}{t_1/2}} \text{.....(i)}$$

In second case,

$$A = A_0\left(\frac{1}{2}\right)^{\frac{20}{t_1/2}} \text{.....(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1/5}{A} = \frac{\left(\frac{1}{2}\right)^{\frac{10}{t_1/2}}}{A_0\left(\frac{1}{2}\right)^{\frac{20}{t_1/2}}}$$

$$\Rightarrow \frac{1}{5A} = \frac{1}{A_0\left(\frac{1}{2}\right)^{\frac{10}{t_1/2}}}$$

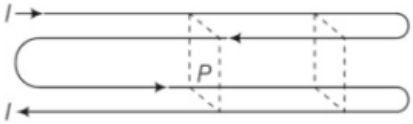
$$\Rightarrow \frac{1}{5A} = \frac{1}{\frac{A_0}{5}} \text{[from Eq. (i)]}$$

$$\therefore A = \frac{A_0}{5 \times 5} = \frac{A_0}{25}$$

Question 12

Four very long wires are arranged as shown in the figure, so that their cross-section forms a square, with connections at the ends, so that current I flows through all four wires as shown in the figure. Length of

each side of the formed such square is b. The magnetic field at the central point P (centre of the square) is



Options:

A. $\frac{\mu_0 l}{\pi b}$

B. $\frac{2\mu_0 l}{\pi b}$

C. 0

D. $\frac{\mu_0 l}{\sqrt{2}\pi b}$

Answer: C

Solution:

Solution:
Since, the current through two wires are from left to right, the magnetic field is directed outwards the plane of the paper. Also, current through the other two wires are from right to left, the magnetic field due to them is directed inwards the plane of the paper. Since, the value of current through all the wires is same and point P is equidistant from all the wires, the magnitude of magnetic fields are same. Hence, they are cancelled. So, the net magnetic field is zero.

Question 13

A non-conducting ring carries linear charge density λ. It is rotating with angular speed ω about its axis. The magnetic field at its centre is

Options:

A. $\frac{3\mu_0 \lambda \omega}{2\pi}$

B. $\frac{\mu_0 \lambda \omega}{2}$

C. $\frac{\mu_0 \lambda \omega}{\pi}$

D. $\mu_0 \lambda \omega$

Answer: B

Solution:

Solution:
Let the radius of the ring is r.
Charge on the ring,
$q = l\,\lambda = 2\,pi\,r\,\lambda [\because l = 2\pi r]$
Current due to the rotation of the ring,

$i = \frac{q}{t} = \frac{2\pi r\lambda}{\frac{2\pi}{\omega}} \left[\because t = \frac{2\pi}{\omega}\right]$

$= \omega\lambda r$

$i = \omega\lambda r$ .....(i)

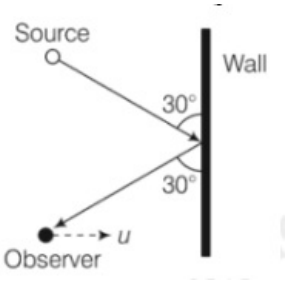
∴ Magnetic field at the centre of the ring due to the rotation of the charged ring,

$B = \frac{\mu_0}{2} \cdot \frac{i}{r} = \frac{\mu_0}{2} \cdot \frac{\omega\lambda r}{r}$

$= \frac{\mu_0}{2} \cdot \omega\lambda$[using Eq. (i)]

Question 14

A stationary source (see figure) emits sound waves of frequency f towards a stationary wall. If an observer moving with speed u in a direction perpendicular to the wall measures a frequency $f' = \frac{11}{8}f$ at the instant shown, then u is related to the speed of sound v_S as



Options:

A. $\frac{3}{4}v_S$

B. $\frac{3}{8}v_s$

C. $\frac{1}{4}v_S$

D. $\frac{8}{3}v_S$

Answer: B

Solution:

Solution:
This is the case of Doppler's effect in reflected sound.
The apparent frequency is given by

$f' = f\left(\frac{v + v_0}{v - v_s}\right)$

Given, $f' = \frac{11}{8}f$

$v_0 = u$

$v_s = 0$ (stationary source)

$v = v_s$

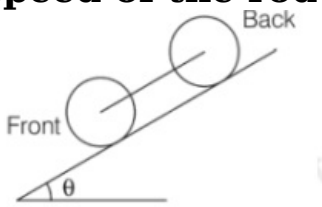
$\therefore \frac{11}{8}f = f\left(\frac{v_s + u}{v_s}\right)$

$\Rightarrow 11v_s = 8(v_s + u)$

$\Rightarrow u = \frac{3}{8}v_s$

Question 15

The front solid cylinder has mass $\frac{M}{3}$ while the back one solid cylinder has mass $\frac{2M}{3}$. The centres of these cylinders are connected by massless rod as shown. Both the cylinders have same radii R. The system is released from rest on the inclined plane. The cylinders roll down. The speed of the rod after system descending a vertical distance h is



Options:

A. $\sqrt{\frac{2gh}{3}}$

B. $\sqrt{2gh}$

C. $\sqrt{\frac{4gh}{3}}$

D. $\sqrt{\frac{3gh}{7}}$

Answer: C

Solution:

Solution:
Applying conservation of energy,
$E_1 = E_2$
or $\Delta U_1 + \Delta K E_1 = \Delta U_2 + \Delta K E_2$
$\Rightarrow (M_1 + M_2)gh + 0$
$= 0 + \frac{1}{2}(M_1 + M_2)v^2 + \frac{1}{2}(I_1 + I_2)\omega^2$
Given, $M_1 = \frac{M}{3}$ and $M_2 = \frac{2M}{3}$
$\because I = \frac{1}{2}MR^2$
In case of pure rolling,
$v_{CM} = R\omega$
$\therefore \left(\frac{M}{3} + \frac{2M}{3}\right)gh = \frac{1}{2}\left[\frac{M}{3} + \frac{2M}{3}\right]v^2 + \frac{1}{2}.\frac{1}{2}\left[\frac{M}{3} + \frac{2M}{3}\right]R^2\omega^2$
$\Rightarrow Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}.\frac{1}{2}Mv^2$ [$\because v_{CM} = R\omega$]
$\Rightarrow gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 \Rightarrow \frac{3v^2}{4} = gh$
$\therefore v = \sqrt{\frac{4gh}{3}}$

Question 16

Suppose a particle of mass m moving with potential energy $U = \frac{kx^2}{2} + Ae^{-ax^2}$ has velocity v_a when its position is x = a. Here, k, A and a are constants. The particle will be able to pass the origin, if

Options:

A. $A \geq \frac{mv_a^2 + ka^2}{2\left(1 - e^{-ax^2}\right)}$

B. $A \leq \frac{mv_a^2 + ka^2}{2\left(1 - e^{-\alpha a^2}\right)}$

C. $A \leq \frac{mv_a^2 + ka^2}{\left(1 - e^{-ax^2}\right)}$

D. $A \geq \frac{mv_a^2 + ka^2}{\left(1 - e^{-\alpha a^2}\right)}$

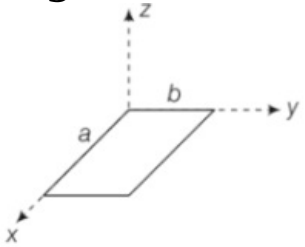
Answer: B

Solution:

Solution:
According to law of conservation of energy, the total energy possessed by the particle should be same everywhere.
Let origin be x = 0.
$\therefore T E_{(x=0)} = T E_{(x=a)}$
i.e. $PE_{(x=0)} + K E_{(x=0)} = PE_{(x=a)} + K E_{(x=a)}$
$\frac{K(0)^2}{2} + Ae^{-\alpha \times 0^2} + K E_{(x=0)}$
$= K a^2 2 + Ae^{-\alpha a^2} + \frac{1}{2}mv_a^2$
i.e. $A + K E_{(x=0)} = \frac{ka^2}{2} + Ae^{-\alpha a^2} + \frac{1}{2}mv_a^2$
So, $A \leq \frac{ka^2}{2} + Ae^{-\alpha a^2} + \frac{1}{2}mv_a^2$ [$\because K E_{(x=0)}$. is unknown, A < RH S],
if $K E_{(x=0)} = 0$, A = RH S]
i.e. $A\left(1 - e^{-\alpha a^2}\right) \leq \frac{ka^2 + mv_a^2}{2}$
i.e. $A \leq \frac{ka^2 + mv_a^2}{2\left(1 - e^{-\alpha a^2}\right)}$

Question 17

A rectangular wire loop with length a and width b lies in the xy -plane as shown. Within the loop, there is a time dependent magnetic field given by $B = c\left[(x\cos\omega t)\hat{i} + (y\sin\omega t)\hat{k}\right]$ Here, c and ω are constants. The magnitude of emf induced in the loop as a function of time is



Options:

A. $\left|\frac{ab^2c}{2}\omega\cos\omega t\right|$

B. $|ab^2c\omega\cos\omega t|$

C. $\left|\frac{a^2bc}{2}\omega\sin\omega t\right|$

D. None of these

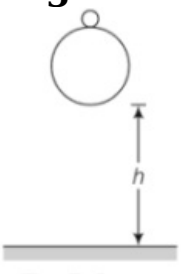
Answer: A

Solution:

Solution:
Area vector of the given loop,
$A = ab\hat{k}$
Time dependent magnetic field,
$B = c\left[(x\cos\omega t)\hat{i} + (y\sin\omega t)\hat{k}\right]$
∴ Magnetic flux through the loop,
$\phi_n = A \cdot B$
$= ab\hat{k}\left[c\left\{(x\cos\omega t)\hat{i} + (y\sin\omega t)\hat{k}\right\}\right]$
$= abcy\sin\omega t$ .....(i)
$\because 0 \le y \le b$
$\therefore y_{av} = \frac{0+b}{2} = \frac{b}{2}$
Putting the value of y in Eq. (i), we get
$\therefore \phi_n = abc \cdot \frac{b}{2} \cdot \sin\omega t = \frac{ab^2}{2}c\sin\omega t$
∴ Induced emf in the loop,
$\varepsilon = \frac{d}{dt}\phi_n = \frac{d}{dt} \cdot \frac{ab^2c}{2}\sin\omega t$
Hence, $|\varepsilon| = \left|\frac{ab^2c}{2}\omega\cos\omega t\right|$

Question 18

A tennis ball with (small) mass m_2, sits on the top of a basket ball with (large) mass m_1. The bottom of the basket ball is at a height h above the ground and the bottom of the tennis ball is at a height (h + d) above the ground. The balls are dropped from rest.
Here, all collisions are elastic and $m_1 >> m_2$. To what approximate height from the ground tennis ball bounce?



Options:

A. d + h

B. d + 2h

C. d + 3h

D. d + 9h

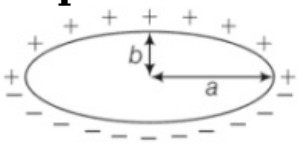
Answer: A

Solution:

Solution:
Since, the collisions are elastic, e = 1.
The height to which the basket ball bounces,
$H = he^{2n} = h(1)^2 = h$
The height to which the tennis ball bounces = h + d

Question 19

An ellipse has a uniform linear positive charge density in upper half (total upper half part charge +Q) and a uniform linear negative charge density in lower half (total lower half part charge −Q) as shown in figure. Here, semi-minor axis b< a semi-major axis. Select the correct statement about the magnitude of electric dipole moment p of the ellipse.



Options:

A. $p = \frac{Qb}{2}$

B. $0 < p < \frac{Qb}{2}$

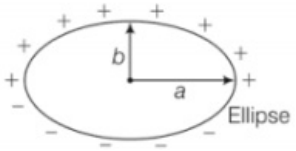
C. $\frac{Qb}{2} < p < 2Qb$

D. 2Qb

Answer: B

Solution:

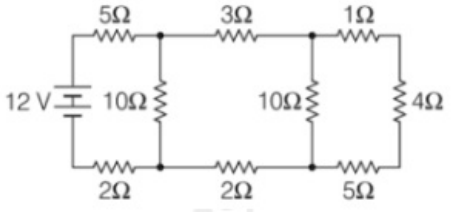
Solution:
As electric dipole moment p is a pair of equal and opposite charges +Q and −Q separated by some distance, hence
$p = q \times 2a$



For ellipse,
semi-major axis (b) < semi-minor axis (a), as b < a. Hence, magnitude of dipole moment p will be $0 < p < \frac{Qb}{2}$

Question 20

Consider the circuit shown below. The current in the 4Ω resistor is



Options:

A. 0.25A

B. 0.50A

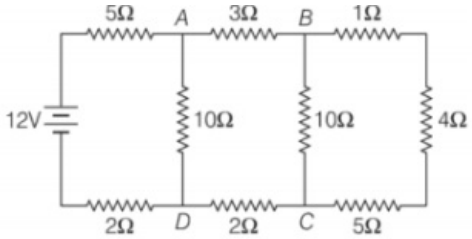
C. 0.75A

D. 1.00A

Answer: A

Solution:

Solution:
Given circuit below



Resistance between B and C on the right hand side of the circuit

$$= \frac{10 \times 10}{10 + 10} = \frac{100}{20} = 5\Omega$$

because $1\Omega + 4\Omega + 5\Omega = 10\Omega$ are in parallel with 10Ω.
Resistance between A and D on the right hand side of the circuit

$$= \frac{10 \times 10}{10 + 10} = \frac{100}{20} = 5\Omega$$

because 5Ω (equivalent resistance of 1Ω, 4Ω, 5Ω and 10Ω) is in series with 3Ω and 2Ω.
Hence, resistance between A and D will be 5Ω.
Equivalent resistance of the circuit,
$R = 5 + 2 + 5 = 12\Omega$
Current drawn from the battery,

$$I = \frac{12V}{12\Omega} = 1A$$

At the junction A, the current of $1A$ is divided equally between the 10Ω resistance and the remaining circuit of the 10Ω resistance.
At the junction B, the current of $0.5A$ is divided equally between the 10Ω resistor and remaining circuit of resistance 10Ω.
$\therefore$ Current through the $4\,\mathrm{Omega}$ resistor $= 0.25A$.

Question 21

A thin layer of oil (index of refraction 1.5 and thickness 500nm) floats on the surface of the liquid (index of refraction 1.24). Now, white light is normally incident on the thin film of oil. Of the following, the most reflected wavelength is

Options:

A. 500nm

B. 550nm

C. 600nm

D. 650nm

Answer: D

Solution:

Solution:
Given, $t = 500nm$
$\mu = 1.5$ (refractive index of oil)
$r = 0°$ (as white light incident normally)
$n = 1.24$ (refractive index of liquid)
Since,
$2\mu t \cos r = (n + 1) . \lambda$
$\therefore \lambda = \frac{2 \times 1.5 \times \cos 0° \times 500}{2.24} nm$
$= 669.64nm$
which is close to $650nm$, hence correct option is (d).

Question 22

Point charges 30C, −20C and 10C are located at (−1, 0, 2), (0, 0, 0) and (1, 5, −1),respectively. The total electric flux leaving cube of side 6m centred at the origin is

Options:

A. $\frac{-20}{\varepsilon_0}$

B. $\frac{10}{\varepsilon_0}$

C. $\frac{20}{\varepsilon_0}$

D. $\frac{30}{\varepsilon_0}$

Answer: B

Solution:

Solution:
Given, point charges $30C$, $-20C$ and $10C$ are located at $(-1, 0, 2)$, $(0, 0, 0)$ and $(1, 5, -1)$.
Side of the cube $= 6m$

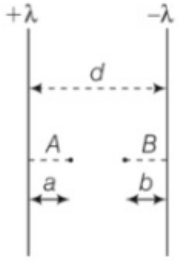
As per Gauss' theorem, net electric flux passing through any closed surface $= \frac{q_{in}}{\varepsilon_0}$

Here, $q_{in} = 30 - 20 + 10 = 20C$

$\therefore$ Net electric flux $= \frac{q}{\varepsilon_0} = \frac{10}{\varepsilon_0}$

Question 23

Two infinitely long wires carry linear charge densities +λ and −λ respectively, as shown. The potential difference between points A (at a distance a from the first wire) and B (at a distance b from the second wire) is



Options:

A. $\frac{\lambda}{3\pi\varepsilon_0}\ln\left(\frac{(d-a)(d-b)}{ab}\right)$

B. $\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{d^2}{ab}\right)$

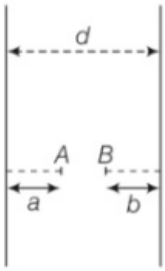
C. $\frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{(d-a)(d-b)}{ab}\right)$

D. $\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{(d-a)(d-b)}{ab}\right)$

Answer: D

Solution:

Solution:
Electric force at a point due to a linear charge distribution is given by



$F = \frac{q_0}{4\pi\varepsilon_0}\int\frac{\lambda d\,l}{r_0^2}r_0$

As potential difference between points A and $\mathrm{B} = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda d\,l}{r_0^2}r_0$

$= \frac{\lambda}{4\pi\varepsilon_0}\int_a^{d-a}\frac{d\,a}{a^2}$

$= \frac{\lambda}{4\pi\varepsilon_0}\int_b^{d-b}\frac{d\,b}{b^2}$

After adding $-b$, we get

Hence, together $\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{(d-a)(d-b)}{ab}\right)$

Question 24

A charged particle of mass 2kg and charge 3C starts with initial velocity $4\hat{i} + 3\hat{k}$ in an electric field $12\hat{i} + 10\hat{j}$. Here, all units are in SI. At time t = 1, its velocity becomes

Options:

A. $16\hat{i} + 19\hat{j} + 3\hat{k}$

B. $40\hat{i} + 20\hat{j} + \hat{k}$

C. $22\hat{i} + 15\hat{j} + 3\hat{k}$

D. $22\hat{i} + 18\hat{j}$

Answer: C

Solution:

Solution:
Given, m = 2kg, q = 3C

$u = 4\hat{i} + 3\hat{k}$

$E = 12\hat{i} + 10\hat{j}$

t = 1s

Force experienced by the charge in electric field,

$F = qE = 3(12\hat{i} + 10\hat{j})$

$= 36\hat{i} + 30\hat{j}$

Acceleration, $a = \frac{F}{m} = \frac{36\hat{i} + 30\hat{j}}{2}$

$= 18\hat{i} + 15\hat{j}$

∴ Final velocity, v = u + at

$= 4\hat{i} + 3\hat{k} + (18\hat{i} + 15\hat{j}) \times 1$

$= 22\hat{i} + 15\hat{j} + 3\hat{k}$

Question 25

Which of these materials requires the least value of magnetic field strength to magnetise it?

Options:

A. Nickel

B. Silver

C. Tungsten

D. Sodium chloride

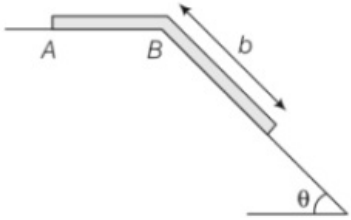
Answer: A

Solution:

Solution:
The least value of magnetic field strength required to magnetise is nickel because it is a ferromagnetic material. Ni is a ferromagnetic substance which get strongly magnetised when placed in an external magnetic field. When external magnetic field B_0, is applied the magnetic domains orient themselves in the direction of magnetic field and grow in size. Hence, requires least value of magnetic strength. $NaCl$ is diamagnetic material. Its field lines are repelled.

Question 26

A chain of length L and of mass m is placed upon a smooth surface. The length of BA is L − b. Now, the chain is released and it slides down. Calculate the speed of the chain when its end reaches B.



Options:

A. $\sqrt{\frac{g\sin\theta(L^2-b^2)}{L}}$

B. $\sqrt{2g\sin\theta(L-b)}$

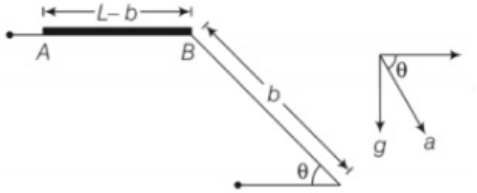
C. $\sqrt{2g\sin\theta(L-b)}$

D. $\sqrt{g\sin\theta(L-b)}$

Answer: B

Solution:

Solution:



Given, $u = 0$
Applying, $v^2 - u^2 = 2as$
$a = g\cos(90-\theta)$
$= g\sin\theta$
$v^2 = 2g\sin\theta(L-b)$
$\therefore v = \sqrt{2g\sin\theta(L-b)}$

Question 27

The basic idea of MASER was first given by

Options:

A. Max Planck

B. Einstein

C. Townes

D. Fresnel

Answer: C

Solution:

Solution:
The basic idea of MASER was given by Charles H Townes.
A MASER is a device that produces coherent electromagnetic waves through amplification.
The MASER is based on the principle of stimulated emission proposed by Albert Einstein.
Hence, option (c) is correct.

Question 28

The average value of rotational kinetic energy of one mole of oxygen gas at temperature T will be

Options:

A. RT

B. $\frac{3}{2}$RT

C. $\frac{5}{2}$RT

D. $\frac{1}{2}$RT

Answer: A

Solution:

Solution:
The average value of rotational kinetic energy of one mole of oxygen gas at temperature Twill be RT.
In case of diatomic gas, such as oxygen, each molecule has two rotational degrees of freedom.
Hence, the average value of rotational kinetic energy of one mole of oxygen is RT.

Question 29

If the coefficient of static friction between shoes of a runner and the track is 0.85, the greatest acceleration that can be generated by the runner is (here, g gravitational acceleration)

Options:

A. 1.85g

B. g

C. 0.85g

D. 0.15g

Answer: C

Solution:

Solution:
Given, μ (coefficient of static friction) $= 0.85$
As the runner is running, then force exerted by runner = frictional force (static friction) which is present between shoes and track.
If m is the mass of the runner, then
f = frictional force $= \mu_s mg$
Then,
$ma = \mu_s mg$
$a = 0.85g$

Question 30

Two planets (each having mass m) revolve around a stationary star (of mass M) in a circle of radius r. The two planets are always on opposite side of the star (i.e. they are diametrically opposite and always having separation 2r). The orbital period T of the planets is of the form $T = 2\pi\sqrt{\frac{r^3}{GM'}}$. What is the value of M′ ?

Options:

A. $M - \frac{m}{2}$

B. $M - \frac{m}{4}$

C. M

D. $M + \frac{m}{4}$

Answer: C

Solution:

Solution:
Mass of the star $= M$
∴ Time period of the planets revolving around the star in a circle of radius r is
$T = 2\pi\sqrt{\frac{r^3}{GM}}$
Given, $T = 2\pi\sqrt{\frac{r^3}{GM'}} \Rightarrow M' = M$

Question 31

An infinite number of capacitors 2.0μF , 4.0μF , 8.0μF , 16.0μF , ... are connected in series. The equivalent capacitance of the system is

Options:

A. ∞

B. 0.25μF

C. 0.5μF

D. 1.0μF

Answer: D

Solution:

Solution:
An infinite number of capacitors 2μF , 4μF , 8μF , 16μF , ..., are connected in series.
As we know, $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$
$C_{series} = \frac{C_1 C_2 C_3 ..... \infty}{C_1 + C_2 + C_3 + ... \infty}$
Equivalent capacitance of the system $= 1\mu F$

Question 32

A motion is described by $y = 4e^{x}(e^{-5t})$, where y, x are in metres and t is in second.

Options:

A. This represents progressive wave propagating along $-x$ -direction with $5 m / s$

B. This represents progressive wave propagating along $+x$ -direction with $5 m / s$

C. This does not represent progressive wave

D. This represents standing wave

Answer: B

Solution:

Solution:
$\because y = 4e^{x}(e^{-5t})$ (given)
$= 4e^{x-5t}$
$\because$ It is a form of $y = f(ax \pm bt)$ So, it represent a progressive wave, propagating in positive X-axis direction because coefficient of x and coefficient of t have opposite sign.
Speed of a wave, $v = \frac{\text{Coefficient of t}}{\text{Coefficient of x}} = \frac{5}{1} = 5 m / s$

Question 33

A ray of light is incident on the plane mirror at rest. The mirror starts turning at a uniform angular acceleration of $\pi\,\text{rad}\,\text{s}^{-2}$. The reflected ray at the end of $\frac{1}{4}$ s must have turned through

Options:

A. 9°

B. 45°

C. 22.5°

D. 11.25°

Answer: C

Solution:

Solution:
As we know, if a mirror is rotated.
Given, angular acceleration, $\alpha = \pi \text{rad} / s^2$
Initial angular velocity $= \omega_0 = 0$
Time, $t = \frac{1}{4}s$
Now, as $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$= 0 \times \frac{1}{4} + \frac{1}{2} \times \pi \times \left(\frac{1}{4}\right)^2$
$= \frac{180}{2 \times 16} = 5.625°$
Since, mirror is turned by 5.625°, therefore reflected ray will be turned by $2 \times 5.625 = 11.25°$.
Hence, option (c) is correct.

Question 34

In n -p-n transistor circuit, the collector current is 20mA. If 90% of the electrons emitted reach the collector, then the

Options:

A. emitter current will be about 16mA

B. emitter current will be 19mA

C. base current will be about 2mA

D. base current will be about 10mA

Answer: C

Solution:

Solution:
Given, I_C = 90% of I_E, I_E = 20mA.
In n -p-n transistor, as we know,
$I_E = I_B + I_C$
where, I_C = collector current,

I_B = base current
and I_E = emitter current.
As, $I_C = 90\% I_E$
$= \frac{90}{100} \times 20$
$= 18mA$
$\therefore I_B = I_E - I_C$
$= 20 - 18 = 2mA$

Question 35

A wheel of radius 2m rolls on the ground with a uniform velocity 4m / s. The relative acceleration of the top most point of the wheel with respect to the bottom most point of the wheel is

Options:

A. $8m / s^2$

B. $16m / s^2$

C. $4m / s^2$

D. $32m / s^2$

Answer: B

Solution:

Solution:
Given, the wheel's radius = 2m,
velocity = 4m / s
As velocity is a uniform, therefore acceleration experienced by the wheel in circular motion,
$a = \frac{v^2}{r} = \frac{(4)^2}{2} = 8m / s$
$\therefore$ Relative acceleration of the top most point of the wheel with respect to bottom most point of the wheel
$= V_T - v_B$
$= 8 - (-8) = 16m / s^2$

Question 36

A stone is released from an aeroplane which is rising with upward acceleration $5m / s^2$. Here, $g = 10m / s^2$. Two seconds after the release, separation between stone and aeroplane will be

Options:

A. 10m

B. 20m

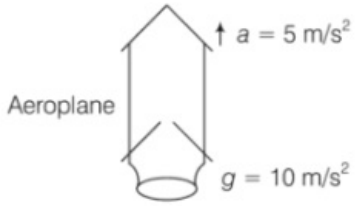
C. 30m

D. 25m

Answer: C

Solution:

Solution:



∴ Acceleration of aeroplane w.r.t. the earth,
$a_{A \to E} = 5m / s^2$
Acceleration of stone w.r.t. the earth,
$a_{S \to E} = -10m / s^2$
or $a_{E \to S} = -a_{S \to E} = 10m / s^2$
Thus, acceleration of aeroplane w.r.t. the stons,
$a_{A \to S} = a_{A \to E} + a_{E \to S} = 5 + 10 = 15m / s^2$
∴ Relative separation between the stone and aeroplane in 2s,
$s = \frac{1}{2}(a_{A \to S})t^2 = \frac{1}{2} \times 15 \times (2)^2 = 30m$

Question 37

A solid body rotates an angle θ about a stationary axis according to the law $\theta = 6t - 2t^3$. What is the mean value of angular velocity over the time interval between t = 0 and the time when the body comes to rest?

Options:

A. $1rad / s$

B. $2rad / s$

C. $3rad / s$

D. $4rad / s$

Answer: D

Solution:

Solution:
Given, $\theta = 6t - 2t^3$.....(i)
$\therefore \frac{d\theta}{dt} = \omega = 6 - 6t^2$
As,$\omega = 0$ (given)
$6 - 6t^2 = 0$
$t^2 = \pm 1$
As, $t = -1s$ is not possible, hence $t = 1s$.
Substituting $t = 1s$ in Eq. (i), we get
$\theta_1 = 6 - 2 = 4rad$
and $\theta_0 = 6 \times 0 - 2 \times 0^2 (t = 0) = 0$
$\therefore$ Mean value of omega $= \frac{\theta_1 + \theta_0}{t_1 + t_0} = \frac{4 + 0}{1 + 0} = 4rad\, s^{-1}$
Hence, option (d) is correct.

Question 38

For sodium light, the two yellow lines occur at λ_1 and λ_2 wavelengths. If the mean of these two is 6000Å and $|\lambda_2 - \lambda_1| = 6$Å, then the approximate energy difference between the two levels corresponding to λ_1 and λ_2 is

Options:

A. 2×10^{-3}eV

B. 2eV

C. 2000eV

D. 2×10^{-6}eV

Answer: A

Solution:

Solution:
Given, mean of λ_1 and λ_2 = 6000Å
$|\lambda_2 - \lambda_1| = 6$Å
i.e. $\frac{\lambda_1 + \lambda_2}{2} = 6000$Å
$\lambda_1 + \lambda_2 = 12000$Å .....(i)
$\lambda_2 - \lambda_1 = 6$Å .....(i)
Equating Eqs. (i) and (ii), we get
$2\lambda_1 = 12006$
$\lambda_2 = \frac{12006}{2} = 6003$Å
and $\lambda_1 = 5997$Å
Now, the energy difference is
$\Delta E = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$
$\Delta E = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$
$\Delta E = 6.6 \times 10^{-34} \times 3 \times 10^8 \times \left(\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right)$
$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 6 \times 10^{-10}}{5997 \times 10^{-10} \times 6003 \times 10^{-10} \times 1.6 \times 10^{-19}}$eV
$= 2.12 \times 10^{-3}$eV $\simeq 2 \times 10^{-3}$eV

Question 39

During the adiabatic expansion of 2mol of an ideal gas, the increase in internal energy was found to be equal to $(-200J)$. The work done by the gas during the process will be equal to

Options:

A. 0

B. 400J

C. $-200J$

D. $200J$

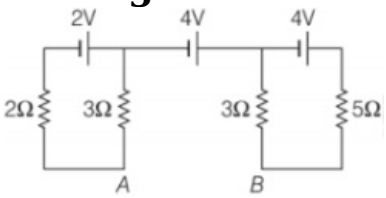
Answer: D

Solution:

Solution:
According to first law of thermodynamics,
$\Delta Q = \Delta U + \Delta W$ .....(i)
Given, $\Delta U = -200J$
$\Delta Q = 0$ (due to adiabatic process)
Substituting in Eq. (i), we get
$0 = -200 + \Delta W$
$\Rightarrow \Delta W = 200J$

Question 40

In the given circuit, potential difference between points A and B is



Options:

A. 6.7V

B. 3.7V

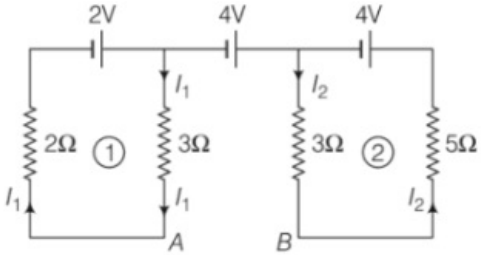
C. 4V

D. 10V

Answer: A

Solution:

Solution:
Let I_1 is current flowing through first loop.
$\therefore I_1 = \frac{2}{3+2} = \frac{2}{5}A$



and I_2 is current flowing through the second loop.
$\therefore I_2 = \frac{4}{3+5} = \frac{1}{2}A$
Let V_A is potential of point A and V_B is potential of point B.
Now, we apply Kirchhoff's law between points A and B,
$\therefore V_A - 3I_1 - 4 + 3I_2 = V_B$

$\Rightarrow V_A - V_B = 3 \times \frac{2}{5} + 4 - 3 \times \frac{1}{2}$

$= \frac{12 + 40 - 15}{10} = \frac{37}{10}$

$\Rightarrow V_A - V_B = 3.7V$

Hence, (a) is the correct option.

Question 41

The angular momentum of an electron in hydrogen atom is $\frac{h}{\pi}$. The kinetic energy of theelectron is

Options:

A. 13.6eV

B. 3.4eV

C. 1.51eV

D. 0.85eV

Answer: B

Solution:

Solution:

The angular momentum of an electron in hydrogen atom $= \frac{h}{\pi}$

As we know, from Bohr's second postulate,

$mvr = \frac{h}{\pi} \Rightarrow v = \frac{h}{\pi mr}$ .....(i)

Now, kinetic energy of electron

$= \frac{1}{2}mv^2 = \frac{1}{2}m \times \left(\frac{h}{\pi mr}\right)^2 \left[\right.$ from Eq. (i) $]$

$= \frac{h^2}{2\pi^2 mr^2}$

$= \frac{(6.63 \times 10^{-34})^2}{2 \times (3.14)^2 \times 9.1 \times 10^{-31} \times (5.3 \times 10^{-11})^2 \times 1.6 \times 10^{-19}}$

$= 3.4eV$

Question 42

Which of the following is not the unit of surface tension?

Options:

A. $\frac{N}{m}$

B. $\frac{J}{m^2}$

C. $\frac{kg}{s^2}$

D. $\frac{W}{m}$

Answer: D

Solution:

Solution:

As we know, surface tension, $S = \frac{\text{force}}{\text{length}} = \frac{F}{l}$

Hence, its SI unit is $\frac{N}{m}$.

So, option (a) is unit of surface tension.

Again, option (b) $\frac{\text{joule}}{(\text{metre})^2} \cong \frac{\text{energy}}{(\text{length})^2}$

$= \frac{\text{force} \times \text{length}}{(\text{length})^2} \frac{\text{force}}{\text{length}} = \frac{F}{l}$

In terms of SI unit, it will be $\frac{N}{m}$, hence it is also equal to surface tension.

Again, option (c) $\frac{kg}{s^2} \approx \frac{\text{mass}}{(\text{time})^2}$

But $\frac{\text{energy}}{(\text{length})^2} = \frac{F \times s}{s^2} = \frac{m \times a \times s}{s^2}$

$= m \times \frac{\text{distance}}{(\text{time})^2} \times \frac{1}{\text{distance}} = \frac{kg}{s^2} = \frac{F}{1}$

Hence, option (c) also equivalent to surface tension.

But, option (d) $= \frac{\text{watt}}{\text{metre}} = \frac{\text{power}}{\text{mass}}$

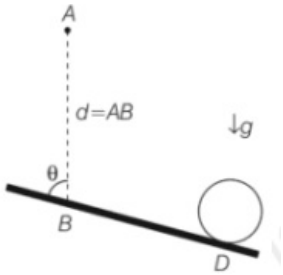
$= \frac{\text{force} \times \text{distance}}{\text{mass}} = \frac{N - m}{kg}$

cannot be equal to surface tension.

Hence, option (d) is correct.

Question 43

A disk of radius R and mass M is at equilibrium at position D on the smooth inclined plane which makes an angle θ with the vertical as shown. The disk's centre is attracted to a point A located at a vertical distance d above the surface as shown. Assume that the force of attraction is proportional to the distance from the disk's centre of mass to point A, i.e. assume that F = −kr, where r is the distance from the point A to the disk's centre of mass and k is constant. Then, distance BD is



Options:

A. $\left(\frac{Mg}{k} - d\right) \cos\theta$

B. $\left(\frac{Mg}{k} - d\right) \sin\theta$

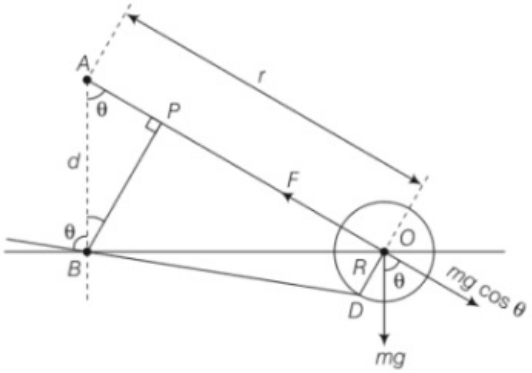
C. $\left(\frac{Mg}{k} + d\right) \cos\theta$

D. $\left(\frac{M g}{k} - d\right) \tan\theta$

Answer: A

Solution:

Solution:



In ΔABP, $AP = d\ \cos\theta$
For equilibrium, $|F| = M g \cos\theta$

$kr = M g \cos\theta$
$r = \frac{M g \cos\theta}{k}$
From the figure, $BD = OP = r - AP$
$= \frac{M g \cos\theta}{k} - d\ \cos\theta\ = \left(\frac{M g}{k} - d\right) \cos\theta$

Question 44

One milligram of matter converted into energy will give

Options:

A. 90J

B. 9×10^3J

C. 9×10^{10}J

D. 9×10^5J

Answer: C

Solution:

Solution:
Given, mass of the matter $= 1\text{mg} = 1 \times 10^{-6}\text{kg}$
As we know, $c = 3 \times 10^8 \text{m / s}$
Matter will be converted into energy by the formula, $E = mc^2$
$= 1 \times 10^{-6} \times (3 \times 10^8)^2$
$= 1 \times 10^{-6} \times 9 \times 10^{16}$
$= 9 \times 10^{10}$J

Question 45

An object of mass 10 kg falls from rest through a vertical distance of 10m and acquires a velocity of 10m / s. The work done by the push of air on the object is ($g = 10m / s^2$)

Options:

A. 500J

B. $-500J$

C. 250J

D. $-250J$

Answer: B

Solution:

Solution:
Given, mass of object $= 10kg$
Initial velocity (rest), $u = 0$
Final velocity, $v = 10m / s$
$g = 10m / s^2$
Work done by air $= -$(Work done by the object to cover the vertical distance)

$$= -F \times s = -mgh \left[\begin{array}{c} as, v^2 - u^2 = 2gh \\ \Rightarrow gh = \frac{v^2 - u^2}{2} \end{array} \right]$$

$$= -10 \times (10^2 - 0^2)2 = -\frac{10 \times 10 \times 10}{2} = -500J$$

Question 46

What type of waves are produced in a sitar wire?

Options:

A. Transverse progressive

B. Longitudinal progressive

C. Transverse stationary

D. Longitudinal stationary

Answer: C

Solution:

Solution:
A stationary wave is produced by the superposition of waves moving to and fro reflecting back and forth between two fixed points. Interference effect produces both nodes, where the two waves cancel by destructive interference and anti-nodes, where the waves reinforce by constructive interference.
In sitar wire, transverse stationary waves are produced.

Question 47

A long rigid wire lies along the X-axis and carries a current of 10A in the positive x -direction. Round the wire external magnetic field is $B = \hat{i} + 2x^2\hat{j}$ with x in metres and B in tesla. The magnetic force (in SI units) on the segment of the wire betweenx = 1m and x = 4m is

Options:

A. 1260

B. $\frac{1280}{3}$

C. 1310

D. 420

Answer: D

Solution:

Solution:
Given, i = 10A,
length of the wire between, x = 4m and x = 1m, i = 4 − 1 = 3m
can be written as
at x = 1m, $B_1 = \hat{i} + 2\hat{j}$
x = 4m, $B_2 = \hat{i} + 2 \times 4^2\hat{j} = \hat{i} + 32\hat{j}$
As we know, magnetic force.
$F = \hat{i}(l + B)$
After solving, $F = 420N$

Question 48

A train travels east towards Hubli at 80km / h. A man on the train runs from the front of the train toward the rear of the train at 10km / h with respect to train. As he runs, he carries a plate of fruit with him. He notices a giant spider on the plate and throws the plate away from him (toward the rear of the train) at 20km / h with respect to him. Just after that instant, the startled spider jumps towards the man at 5km / h with respect to plate. The instant after the spider jumps toward the man, how fast is the spider approaching Hubli?

Options:

A. 45km / h

B. 115km / h

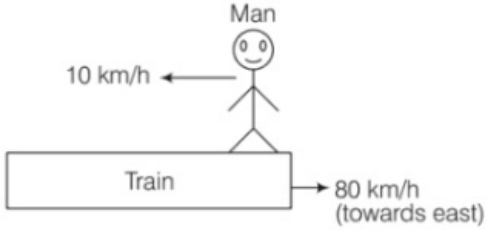
C. 55km / h

D. 95km / h

Answer: C

Solution:

Solution:
Given, velocity of train w.r.t. the earth,



$v_{T \to E} = 80km / h$ .....(i)
Velocity of the man w.r.t. the train,
$v_{M \to T} = -10km / h$ .....(ii)
Velocity of the plate w.r.t. the man,
$v_{P \to M} = -20km / h$ .....(iii)
Velocity of the spider w.r.t. the plate,
$v_{S \to P} = 5km / h$ .....(iv)
From Eqs. (i), (ii), (iii) and (iv), we get
∴ The velocity of the spider w.r.t. the earth,
$v_{S \to E} = V_{S \to P} + V_{P \to M} + V_{M \to T} + V_{T \to E}$
$= 5 - 20 - 10 + 80$
$= 55km / h$ (towards Hubli)

Question 49

The ratio of magnetic field at the centre of a current carrying circular coil to its magnetic moment is x. If the current and radius each of them are made three times, the new ratio will become

Options:

A. 3x

B. 9x

C. x / 9

D. x / 27

Answer: D

Solution:

Solution:
The magnetic field, $B = \frac{\mu_0 I}{2R}$ therefore Magnetic moment, $M = I A$ given, $R_2 = 3R$, $I_\alpha = 3I_1$)
$= I \pi R^2$
$\frac{B}{\mu} = x$, after taking ratio

$$\frac{\frac{B_1}{\mu_1}}{\frac{B_2}{\mu_2}} = \frac{\frac{\frac{\mu_0 I_1}{2R_1}}{I\pi R_1^2}}{\frac{\frac{\mu_0 3I_1}{2\times 3R_1}}{3[\pi 9R]_2^2.}} = \frac{x}{27} = \frac{\frac{\mu_0 I_1}{2\times I_1 \pi R_1^3}}{\frac{\mu_0 3I_1}{6R_1 \times 27 . I\pi R_1^3}} = \frac{x}{27}$$

Question 50

If λ_1 and λ_2 denote the de-Broglie wavelengths of two particles with same masses but charges in the ratio of 1 : 2 after they are accelerated from rest through the same potential difference, then

Options:

A. $\lambda_1 = \lambda_2$

B. $\lambda_1 < \lambda_2$

C. $\lambda_1 > \lambda_2$

D. None of these

Answer: C

Solution:

Solution:
As, the expression for the de-Broglie wavelength associated with a charged particle having charge Q and mass m is
$\lambda = \frac{h}{\sqrt{2mqV}}$
where, V = potential through which the particle is accelerated.
Now, given
$m_1 = m_2 = m$
and $\frac{q_1}{q_2} = \frac{1}{2}$
$\Rightarrow q_2 = 2q_1$
Then, $\frac{\lambda_1}{\lambda_2} = \frac{\frac{h}{\sqrt{2mqv}}}{\frac{h}{\sqrt{4mqV}}} = \frac{\sqrt{2}}{1}$
Hence, $\lambda_1 = \sqrt{2}\lambda_2$
$\Rightarrow \lambda_1 > \lambda_2$

Question 51

Chemistry
If a mixture of FeO and Fe_3O_4 contains 75% Fe, what will be the percentage amount of each oxide in the mixture?

Options:

A. 64.10%FeO and 35.90%Fe_2O_3

B. 50%F eO and 50%Fe_2O_3

C. 75%F eO and 25%Fe_2O_4

D. 35.90%F eO and 64.10%Fe_2O_3

Answer: A

Solution:

Solution:
A mixture of F eO and Fe_3O_4 (i.e. F eO . Fe_2O_3) actually consists of 2 units of F eO and 1 unit of Fe_2O_3. In case of F eO,
The percentage of F e in F eO

$$= \frac{\text{Atomic weight of F e}}{\text{Formula weight of F eO}} \times 100$$

$$= \frac{56}{72} \times 100 = 77.78\%$$

The percentage of F e in Fe_2O_3

$$= \frac{(\text{Atomic weight of F e}) \times 2}{\text{Formula weight of } Fe_2O_3} \times 100$$

$$= \frac{112}{160} \times 100 = 70.00\%$$

Suppose, 'x'g of FeO (in total) and 'y' g of Fe_2O_3 form the mixture that contains 75% Fe.
Hence mathematically,
77.78%of x + 70.00%y = 75%of (x + y)
or 77.78% of x + 70.00%y = 75.00 of x +75.00% of y
or (77.78 − 75.00%) of x = (75.00 − 70.00)% of y
or 2.77% of x = 5.00% of y

$$\text{or } \frac{x}{y} = \frac{5.00\%}{2.78} = \frac{500}{278} = \frac{250}{139}$$

or x : y = 250 : 139
Now, percent amount of F eO in mixture (x)

$$= \frac{250}{(250 + 139)} \times 100 = \frac{25000}{389}\% = 64.3\%$$

Hence, percent amount of Fe_2O_3 in mixture (y)
= (100 − 64.3)% = 35.7%

Question 52

What will be the correct number of total electrons in 1.6g methane?

Options:

A. 6.02×10^{24}

B. 6.02×10^{23}

C. 6.02×10^{22}

D. 9.632×10^{23}

Answer: B

Solution:

Solution:
Given, mass = 1.6g of methane

$$\text{Therefore, moles of methane} = \frac{\text{given mass}}{\text{molecular mass}}$$

$= \frac{1.6g}{16gmol^{-1}} = 0.1mol$
1 mole of methane has 6.022×10^{23} molecules.
$\therefore$0.1 mole of methane has
$6.022 \times 10^{23} \times 0.1$ molecules
$= 6.022 \times 10^{22}$ molecules
Now, 1 mole of methane has 10 electrons
$\therefore$0.1 mole of methane has
$= 10 \times 6.022 \times 10^{22}$ electrons
$= 6.022 \times 10^{23}$ electrons

Question 53

Which of the following pair of salt produces odourless gas with dil H_2SO_4 ?

Options:

A. HCO_3^- and HSO_3^-

B. HCO_3 and CO_3^-

C. $S_2O_3^{2-}$ and $CH_3CO_2^-$

D. CO_3^{2-} and $CH_3CO_2^-$

Answer: B

Solution:

Solution:
HCO_3^- and CO_3^- will produce odourless gas i.e. carbon dioxide with dil. H_2SO_4.
$NaHCO_3(aq) + H_2SO_4(aq) \rightarrow Na_2SO_4(aq) + 2CO_2(g) + 2H_2O(l)$
Baking soda
$Na_2CO_3 + H_2SO_4(aq) \rightarrow Na_2SO_4(aq) + H_2O(I) + CO_2(g)$
Washing soda

Question 54

One litre of an acidified solution of $KMnO_4$ containing 15.8g$KMnO_4$ is decolorised by passing sufficient amount of SO_2. If SO_2 is produced by roasting of iron pyrite (FeS_2). The amount of pyrite required to produce the necessary amount of SO_2 will be

Options:

A. 15.8gFeS_2

B. 15.0gFeS_2

C. 7.5gFeS_2

D. 7.9gF eS_2

Answer: B

Solution:

Solution:
The chemical reaction between $KMnO_4$ and SO_2 is represented by the following equation,
$2KMnO_4 + 5SO_2 + 2H_2O \rightarrow K_2SO_4 + 2MnSO_4 + 2H_2SO_4$
Molar mass of $KMnO_4$ is $158.034 g / mol$
$\therefore$ We have number of moles of $KMnO_4$
$= \frac{15.8g}{158.034g / mol}$
$= 0.10 moles$
According to given equation, 2 moles of $KMnO_4$ reacts with 5 moles of SO_2.
$\therefore 0.10$ moles of $KMnO_4$ will react with $\frac{5}{2} \times 0.10$moles of SO_2 i.e. 0.25 moles of SO_2 .
Given, SO_2 is produced by roasting of iron pyrite (FeS_2) as:
$4FeS_2 + 11O_2 \rightarrow 2Fe_2O_3 + 8SO_2$
So, 8 moles of SO_2 will be formed by 4 moles of FeS_2.
$\therefore 0.25$ moles of SO_2 will be formed by
$\frac{4}{8} times 0.25$ moles of FeS_2 = 0.125 moles of FeS_2
Molecular mass of FeS_2 = $119.98 gmol^{-1}$
$\therefore$ Amount of FeS_2 required to give necessary SO_2
$= 119.98 gmol^{-1} \times 0.125 mol$
$= 14.99g \approx 15g$ of FeS_2

Question 55

If an e^- is revolving in the first bohr orbit of a H -atom with a velocity of $2.19 \times 10^8 cms^{-1}$,what will be the velocity of the e^- in the third orbit of H -atom?

Options:

A. $2.19 \times 10^8 cms^{-1}$

B. $7.3 \times 10^7 cms^{-1}$

C. $6.57 \times 10^8 cms^{-1}$

D. $1.09 \times 10^8 cms^{-1}$

Answer: B

Solution:

Solution:
The velocity of electron in different orbits is given by the following expression:
$V_n = V_o \times \frac{Z}{n}$
Given, for first Bohr orbital of H -atom
$V_0 = 2.19 \times 10^8 cms^{-1}$
$\therefore$ The velocity of e^- in third orbit of H -atom is
($\because n = 3$)

$V_n = 2.19 \times 10^8 \times \frac{1}{3}$
$V_n = 0.73 \times 10^8 cms^{-1}$
or $= 7.3 \times 10^7 cms^{-1}$

Question 56

Which is the correct formula of forsterite?

Options:

A. Mg_2SiO_4

B. $Al_2(OH)_4(Si_2O_5)$

C. $ZrSiO_4$

D. $Ca_3Si_3O_9$

Answer: A

Solution:

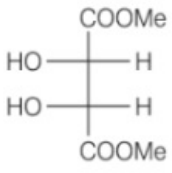
Solution:
Forsterite (Mg_2SiO_4) is magnesium rich end member of olivine solid solution series.
Ca_3SiO_9 is walstromite.
$ZrSiO_4$ is zircon.
$Al_2(OH)_4(Si_2O_5)$ is kaolinite, a clay mineral.

Question 57

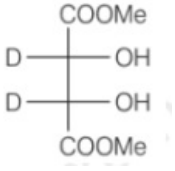
Which of the following is optically active molecule?

Options:

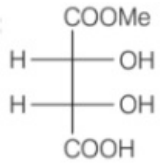
A.



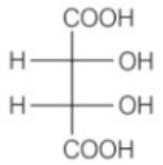
B.



C.



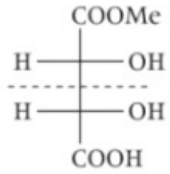
D.



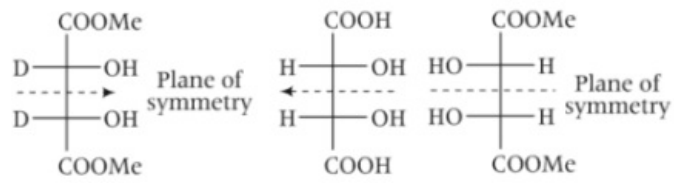
Answer: C

Solution:

Solution:



is optically active molecule.
An optically active molecule is the one that does not have either plane of symmetry or center of symmetry. Compound given in option (c) does not have plane of symmetry, hence it is optically active, whereas all other compounds given possess plane of symmetry.



Question 58

Which of the following does not show a resonance effect?

Options:

A. C_6H_5OH

B. C_6H_5Cl

C. $C_6H_5NH_2$

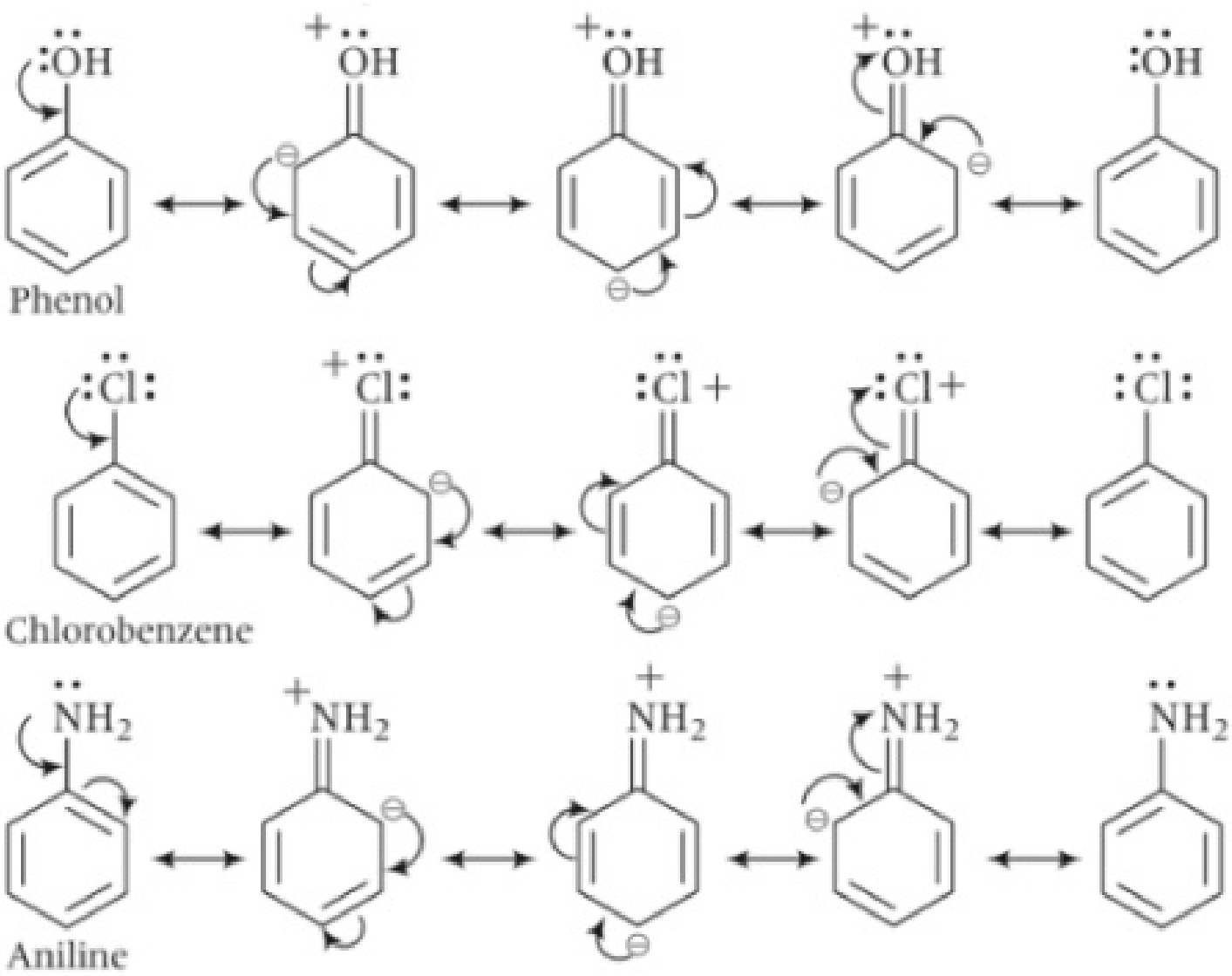
D. $C_6H_5NH_3$

Answer: D

Solution:

Solution:

$C_6H_5\overset{+}{N}H_3$ (protonated aniline) does not show resonance effect. The nitrogen of protonated aniline does not have any non-bonding electrons that can be delocalised. Whereas all other options have non-bonding electrons that can be delocalised.



Question 59

Petroleum refining involves

Options:

A. vacuum distillation

B. fractional distillation

C. steam distillation

D. simple distillation

Answer: B

Solution:

Solution:
Petroleum refining involves fractional distillation i.e. separating petroleum at different boiling ranges.
In this process, oil is washed with acidic or basic solution depending upon whether the impurities are acidic or basic in nature. The washed oil is then subjected to fractional distillation by heating it to about $725K$ and allowing the vapours to pass through a fractionating column.

Question 60

D-glucose units in cellulose are joint by

Options:

A. $\alpha - 1, 4$ glycosidic bond

B. $\beta - 1, 6$ glycosidic bond

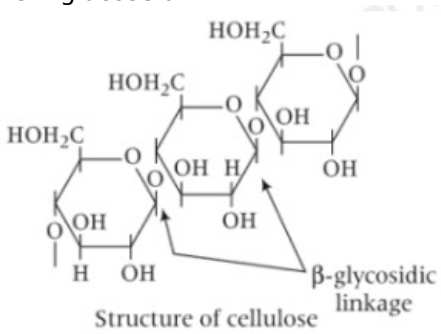
C. $\beta - 1, 4$ glycosidic bond

D. peptide bond

Answer: C

Solution:

Solution:
D-glucose units in cellulose are joint by $\beta - 1, 4$ glycosidic bond. Cellulose is a straight chain polysaccharide composed only of β -D glucose units that are linked together by glycosidic linkage between $C - 1$ of one glucose unit and C-4 of the next glucose unit.



Structure of cellulose

Question 61

Most reactive alocohol towards esterification is

Options:

A. primary alcohol

B. secondary alcohol

C. tertiary alcohol

D. All are same reactive

Answer: A

Solution:

Solution:
Most reactive alcohol towards esterification is primary alcohol. As the size of group R around - OH increases, the rate of esterification reaction decreases due to steric hindrance. Thus in esterification reaction, the order of reactivity of alcohols follows the order
$1° > 2° > 3°$

Question 62

The Markwnikoff's rule is best applicable to the reaction

Options:

A. $C_2H_4 + HCl$

B. $C_3H_6 + Br_2$

C. $C_3H_6 + HBr$

D. $C_3H_8 + Cl_2$

Answer: C

Solution:

Solution:
The Markownikoff's rule is best applicable to the reaction C_3H_6 + HBr i.e. unsymmetrical alkene with HBr. This rule is used as a guidance for electrophilic additions in those cases where an asymmetrical reagent is added to an asymmetrical alkene. Where the negative part of reagent attaches itself to that carbon atom that contains lesser number of hydrogen atoms.
In options (b) and (d), reagent used is Br_2 and Cl_2 that given addition reaction via free radial substitution. In option (a), alkene used is symmetrical. Hence, Markownikoff's rule is not applicable there.

Question 63

The addition of Br_2 to (E) -but -2 - ene gives

Options:

A. (R, R) $-$ 2 $-$ 3 - dibromobutane

B. (R, S) $-$ 2 $-$ 3 -dibromobutane

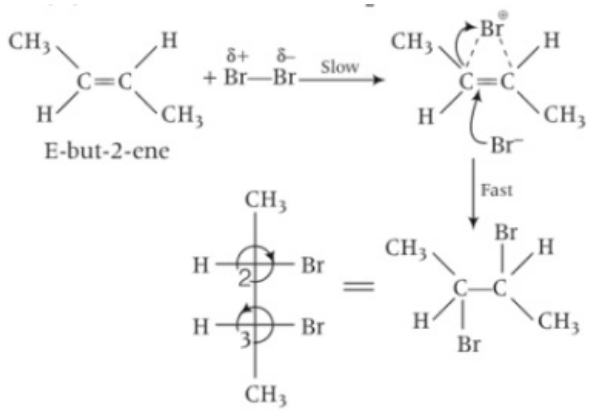
C. (R, S) $-$ 2 $-$ 3 -dibromobutane

D. a mixture of (R, R) and (S, S)2, 3 dibromobutane

Answer: C

Solution:

Solution:
The addition of bromine to E - but- 2 -ene gives (R, S) $-$ 2 $-$ 3 dibromobutane, it is always anti as bromine atoms added are on opposite sides.
In (E)-but-2-ene addition of Br_2 occurs as:



Question 64

Which among the following compounds will have meso form?

Options:

A. $CH_2OH - CHOH - CHOH - CHO$

B. $CH_2OH - CHOH - CHOH - COOH$

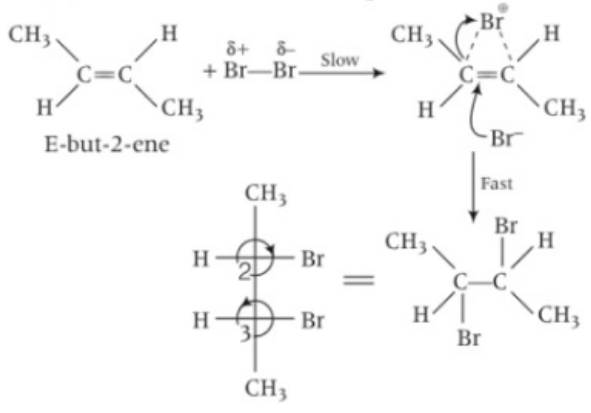
C. $CH_2OH - (CHOH)_2 - CH_2OH$

D. $C_6H_3 - CHCl - CHOH - CH_3$

Answer: C

Solution:

Solution:
$CH_2OH - CHOH - CHOH - COOH$
will have meso form.
The structures that have plane of symmetry, which can divide it into two equal and identical halves are called meso structures.
In the given compounds, only structure (b) has plane of symmetry, hence it is a meso compound.



Question 65

Which compound will be most reactive for electrophilic addition

reaction?

Options:

A. $CH_3 - CH = CH - CH_3$

B. $CH_3 - C \equiv C - CH_3$

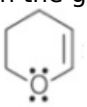
C. $CH_3 - CH = CHCOOCH_3$

D.

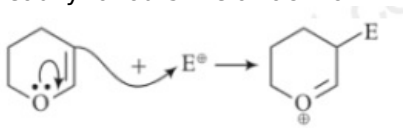
Answer: D

Solution:

Solution:
In the given options,



is most reactive for electrophilic addition reaction, due to electron donating effect of oxygen adjacent to double bond that readily favours the attack to $E^{\oplus}$.



Among other options such as alkenes and alkynes, alkenes gives fast electrophilic addition reaction.
This is because when a proton adds to an alkene, an alkyl cation is formed and when a proton adds to an alkyne, a vinylic cation is formed. A vinylic cation is less stable than alkyl cation. Hence alkenes are more reactive for electrophilic addition.

In $\overset{1}{C}H_3 - \overset{2}{C}H = \overset{3}{C}H - \overset{O}{\overset{\|}{\underset{4}{C}}} - O\overset{5}{C}H_3$ due to resonance $C_2 - C_3$ has less electron density, hence it won't undergo electrophilic addition easily.

$CH_3 - CN = CN - \overset{O}{\overset{\|}{C}} - OCH_3 \leftrightarrow CH_3 - \overset{\oplus}{C}H - CH = \overset{O^-}{\overset{|}{C}} - OCH_3$

Question 66

At 298K what will be the change in standard internal energy change for the given reaction
$OF_2(g) + H_2O(g) \rightarrow O_2(g) + 2HF(g)\ \Delta H = -310kJ$

Options:

A. $-312.5kJ$

B. $-125.03kJ$

C. $-310kJ$

D. $-156kJ$

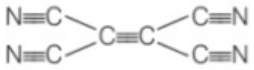
Answer: A

Solution:

Solution:
Relation between internal energy change ($\Delta E°$) and enthalpy change ($\Delta H°$) of a reaction is given by
$\Delta H° = \Delta E° + \Delta n_g RT$ .....(I)
Δn_g = moles of product − moles of reactant For the reaction given,
$OF_2(g) + H_2O(g) \rightarrow O_2(g) + 2HF(g)$
$\Delta n_g = (1 + 2) - (1 + 1)$
$\Delta n_g = 1\{\Delta H = -310kJ, = -310000J\}$
Putting the values in Eq (i)
$-310000J = \Delta E + 1(8.3141J\,K^{-1}mol^{-1})\,(298K)$
$\Delta E = -312,477.572J \approx -312.5kJ$

Question 67

How many σ -bond and π -bonds are present in?



Options:

A. 5σ + 8π

B. 9σ + 7π

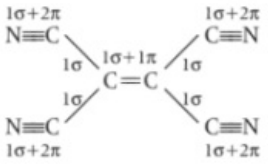
C. 9σ + 9π

D. 5σ + 9π

Answer: C

Solution:

Solution:
The given compound contains 9σ + 9πA double bond consists of 2π and 1σ bond, whereas a single bond consists of 1π and 1σ bond.
∴ In the given compound, 9σ and 9π bonds are present.



Question 68

Which of the following reactions is a redox reaction?

Options:

A. $CaCO_3 + 2HCl \rightarrow CaCl_2 + H_2O + CO_2$

B. $AgNO_3 + KI \rightarrow AgI + KNO_3$

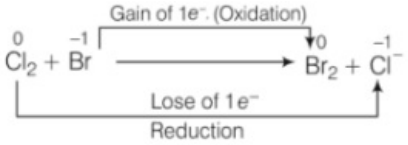
C. $Cl_2 + 2Br^- \rightarrow Br_2 + 2Cl^-$

D. $BaCl_2 + SO_4^{2-} \rightarrow BaSO_4 + 2Cl^-$

Answer: C

Solution:

Solution:
$Cl_2 + 2Br^- \rightarrow Br_2 + 2Cl^-$ is a redox reaction as it involves both oxidation and reduction reactions simultaneously.



In the other options (a), (b) and (d) there is no change in oxidation state of any atom, hence these are not redox reaction.

Question 69

The E°_{red} of Ag, Cu, Co and Zn are 0.799,0.337, −0.277 and −0.762V respectively,which of the following cells will have maximum cell emf?

Options:

A. $Zn \mid Zn^{2+}(1M) \parallel Cu^{2+}(1M) \mid Cu$

B. $Zn \mid Zn^{2+}(1M) \parallel Ag^{+}(1M) \mid Ag$

C. $Cu \mid Cu^{2+}(1M) \mid \mid Ag^{+}(1M) \mid Ag$

D. $Zn \mid Zn^{2+}(1M) \mid \mid Co^{2+}(1M) \mid Co$

Answer: B

Solution:

Solution:
$Zn \mid Zn^{2+}(1M) \mid Ag^{+}(IM) \mid Ag$
$Zn \mid Zn^{2+}(1M) \mid Ag^{+}(1M) \mid Ag$
given $E^\circ_{Zn^{2+}/Zn} = -0.762$
$E^\circ_{Ag^{+}/Ag} = 0.799$
From Nernst equation,
$E = E^\circ - \frac{0.0591}{n} \log\left[\frac{\text{Products}}{\text{Reactants}}\right]$
Cell reaction for the given cell is
$Zn + 2Ag^{+} \rightarrow Zn^{2+} + 2Ag$
$E = E^\circ_{Ag^{+}/Ag} - E^\circ_{Zn^{2+}/Zn} - \frac{0.0591}{2} \log\frac{Zn^{2+}}{(Ag^{+})^2}$
$E = 0.799 - (-0.762) - \frac{0.0591}{2} \log\left(\frac{1}{1^2}\right)$
$E = 1.569V$
cell B has maximum emf.

Question 70

What will be the value of ΔG and ΔG° for the reaction, A + B ⇌ C + D at 27°C for which$K = 10^2$?

Options:

A. $\Delta G = 0; \Delta G° = -11.48 kJ\ mol^{-1}$

B. $\Delta G = 0; \Delta G° = 11.48 kJ\ mol^{-1}$

C. $\Delta G = -11.48 kJ\ mol^{-1}; \Delta G° = 0$

D. $\Delta G = 11.48 kJ\ mol^{-1}; \Delta G° = 0$

Answer: A

Solution:

Solution:
ΔG° of a reaction is given by the expression.
$\Delta G° = -2.303RT \log K$ .....(i)
Given, $K = 10^2$
$T = 27°C$ or 300K
on putting the values in Eq (i)
$\Delta G° = -2303 \times 8.314 J\ K^{-1} mol^{-1} \times 300 \times \log 10^2$
$\Delta G° = -1148.28 J\ mol^{-1}$ or $-11.48 kJ\ mol^{-1}$
Now, as the reaction is at equilibrium, hence $\Delta G = 0$.

Question 71

In which of the following changes entropy decreases?

Options:

A. Crystallisation of sucrose from solution

B. Dissolving sucrose in water

C. Melting of ice

D. Vaporisation of camphor

Answer: A

Solution:

Solution:
Entropy decreases in crystallisation of sucrose from solution.
It is a measure of randomness or disorder of the system. Greater the randomness, higher is the entropy. Order of randomness or entropy for solid liquid and gas is : gas > liquid > solid.
In dissolving sucrose in water and melting of ice solid state is changing to liquid state, hence entropy increases.
In vaporisation of camphor, solid state is changing to gaseous state, hence entropy increases.
In crystallisation of sucrose from solution, liquid state is changing to solid state, hence entropy decreases.

Question 72

Why is the heat evolved in neutralisation of HF is highest?

Options:

A. Due to low hydration energy of F^- ion

B. Due to high hydration energy of F^- ion

C. HF is a strong acid

D. None of the above

Answer: B

Solution:

Solution:
Due to high hydration energy of F^- ion.
Although HF is a weak acid, its enthalpy change of neutralisation is very high, (more negative) $-57.3 kJ$.
This is due to high hydration energy of F^- ion. The size of F^- is very small and thus it can form strong bond with water molecule when it is hydrated.

Question 73

The reaction between p-methyl benzaldehyde and conc. $NaOH$ is an example of

Options:

A. Cannizzaro reaction

B. Aldol condensation

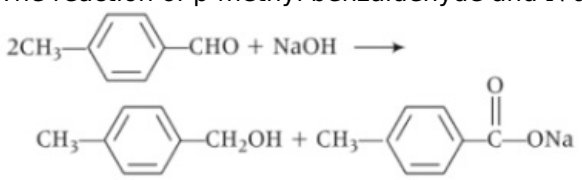
C. Hydrolysis

D. Haloform reaction

Answer: A

Solution:

Solution:
Cannizzaro reaction
Aldehydes that do not contain an α -hydrogen atom, when treated with conc. NaOH solution undergo disproportionation, i.e. self reduction oxidation. As a result, one molecule of the aldehyde is reduced to corresponding alcohol and the other is oxidised to corresponding carboxylic acid.This is known as Cannizzaro reaction.
The reaction of p-methyl benzaldehyde and $NaOH$ is an example of Cannizzaro reaction.



Question 74

How benzene is converted into n-propyl benzene?

Options:

A. Friedel-crafts alkylation

B. Friedel-crafts acylation

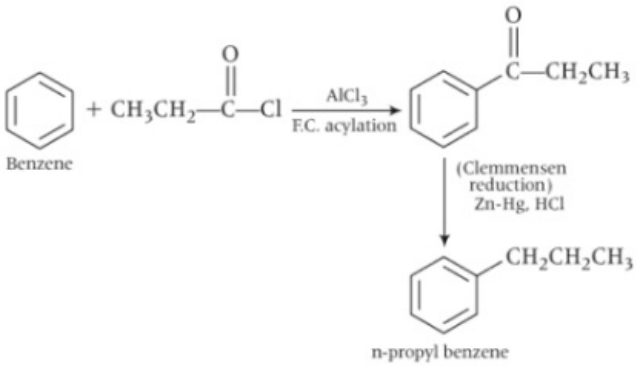
C. Friedel-crafts alkylation followed by clemmensen reduction

D. Friedel-crafts acylation followed by clemmensen reduction

Answer: D

Solution:

Solution:
Friedel crafts acylation followed by clemmensen reduction forms n -propyl benzene as follows:



Friedel craft alkylation method can also be used for conversion of benzene to n -propyl benzene. But the electrophile $CH_3 - CH_2 - \overset{\oplus}{\underset{\|}{C}}O$ formed in F. C acylation method is more electrophilic in nature than the electrophile $CH_3\left(CH_2\overset{\oplus}{C}H_2\right)$ formed in F. C alkylation method.
Hence, F.C acylation followed by clemmensen reduction is more preferrable.

Question 75

Which of the following does not depend on the attraction of the bonding pair towards the nucleus?

Options:

A. The number of protons in the nucleus

B. The repulsion by the electrons in the same valence shell

C. The amount of sheieding by inner shell electron

D. The distance from the nucleus

Answer: B

Solution:

Solution:
The repulsion by the electrons in the same valence shell does not depend on the attraction of the bonding pair towards the nucleus. This is because electrons are identical in charge.

The repulsion can be minimised, if two electrons move as far apart as possible by occupying different degenerate orbitals or if they have parallel spins.

Question 76

Which of the following oxide is of acidic nature?

Options:

A. B_2O_3

B. Al_2O_3

C. Ga_2O_3

D. In_2O_3

Answer: A

Solution:

Solution:
B_2O_3 is the acidic oxide. On moving down the group 13, the nature of oxide changes from acidic to basic.

$\underset{\text{Acidic}}{B_2O_3}$ $\{\underset{\text{Amphoteric}}{Al_2O_3 \text{ and } Ga_2O_3}\}$ $\{\underset{\text{Basic}}{In_2O_3 \text{ and } Tl_2O_3}\}$

Acidic character of boron oxide may be explained on the basis of small size of boron. Due to small size there is high positive charge density on atom.

Question 77

In activation energy for a smile chemical reaction A → B is E_a in forward direction. The activation energy for reverse reaction

Options:

A. can be less than or more than E_a

B. is always double of E_a

C. is negative of E_a

D. is always less than E_a

Answer: A

Solution:

Solution:
In activation energy for a simple chemical reaction A → B is E_a in forward direction. The activation energy for reverse reaction can be less than or more than E_a. The concept of activation energy gives an idea whether a given reaction is slow or fast at a given temperature. If the product is stable then activation energy for reverse reaction is more than E_a, but if the product is less stable then activation energy is less than E_a.

Question 78

The rate of first order reaction is 1.5×10^{-2} mol $L^{-1}min^{-1}$ at 0.5M concentration of the reactant. The half-life of the reaction is

Options:

A. 0.383min

B. 23.1min

C. 8.73min

D. 7.53min

Answer: B

Solution:

Solution:
Rate for first order reaction is defined as Rate $= k[A]$
$1.5 \times 10^{-2} \text{mol}\, L^{-1} \text{min}^{-1} = k[0.5M]$
$3 \times 10^{-2} \text{min}^{-1} = k$
For first order reaction, half life is
$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{3 \times 10^{-2}} \text{min}^{-1}$
$= 0.231 \times 10^{2} \times \text{min}^{-1}$
$t_{1/2} = 23.1 \text{min}^{-1}$

Question 79

0.5 molal aqueous solution of a weak acid (HX) is 20% ionised. If K_f for water is 1.86K kgmol^{-1}, the lowering in freezing point of the solution is

Options:

A. $-1.12K$

B. 0.56K

C. 1.12K

D. $-0.56K$

Answer: C

Solution:

Solution:
20% ionisation of weak acid

	HX	$\rightleftharpoons$	H^+	$+$	X^-
Initial moles	1		0		0
Final moles	$1-\alpha$		α		α

Total moles at equilibrium $= 1 - \alpha + \alpha + \alpha = 1 + \alpha$

$i = \frac{\text{Total moles}}{\text{Initial moles}} = \frac{1+\alpha}{1} = 1 + 0.2 = 1.2$

We know that, lowering in freezing point of the solutions $(\Delta T_b) = iK_f\ m$

$= 1.2 \times 1.86K\,kgmol^{-1} \times 0.5$

$= 1.116K \approx 1.12K$

Question 80

Which of the following is not an example of addition polymer?

Options:

A. Polythene

B. Polystyrene

C. Neoprene

D. Nylon-6.6

Answer: D

Solution:

Solution:
Nylon-6,6 is not an example of addition polymer. It is a condensation polymer. This polymer is prepared by the condensation copolymerisation of hexamethylene-diamine and adipic acid under high pressure and at high temperature.

$$\underset{\text{Hexamethylene diamine}}{nH_2N-(CH_2)_6-NH_2} + nHOOC-\underset{\text{Adipic acid}}{(CH_2)_4}-COOH \xrightarrow[\text{High pressure}]{553K} \left[-\overset{H}{\overset{|}{N}}-(CH_2)_6-\overset{H}{\overset{|}{N}}-\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{O}{\overset{\|}{C}}-\right]_N$$

All other options are examples of addition polymers i.e polythene, polystyrene and neoprene.

Question 81

The reactivity of the transition element usually decreases from Sc to Cu because of

Options:

A. lanthanide contraction

B. continuous increase in ionisation enthalpy

C. continuous decrease in ionisation enthalpy

D. increase in number of oxidation states

Answer: B

Solution:

Solution:
The reactivity of the transition element usually decreases from Sc to Cu because of continuous increase in ionisation enthalpy. In 3 d series, the ionisation energy increases form left to right but the increase is not pronounced as in the case of s-and p-block elements. This is due to the greater effective nuclear charge acting on outer valence electrons because

of the weak shielding of the nucleus by 4 f-electrons.

Question 82

SO_2 is considered as airpollutant because

Options:

A. its concentration increases with temperature increase of atmosphere

B. It is used as insecticide, which is airpollutant

C. It reacts with O_2 and H_2O to produce acid rain

D. It is a strong oxidant and oxidised other components of atmosphere

Answer: C

Solution:

Solution:
SO_2 is considered as air pollutant because it reacts with O_2 and H_2O to produce acid rain.
Oxides from nitrogen and sulphur, released into the atmosphere from thermal power plants, industries and automobiles are the main sources of acid rain. These oxides on oxidation followed by hydrolysis give sulphuric acid and nitric acid that along with HCl are responsible for the acidity of rain.

Question 83

Ferric chloride is used to stop bleeding in cuts because

Options:

A. Fe^{3+} coagulated blood which is positively charged sol

B. Fe^{3+} coagulated blood which is negatively charged sol

C. Cl^- coagulated blood which is positively charged sol

D. Cl^- coagulated blood which is negatively charged sol

Answer: B

Solution:

Solution:
Ferric chloride is used to stop bleeding in cuts because Fe^{3+} coagulates blood which is negatively charged sol. As we know blood is a colloidal sol.
On rubbing the injured part with Fe^{3+} then coagulation of blood takes place. Hence, main reason is coagulation, which stops bleeding.

Question 84

A method of removing excess solute from a colloidal solution is

Options:

A. distillation

B. crystallisation

C. dialysis

D. gaschromatography

Answer: C

Solution:

Solution:
A method of removing excess solute from a colloidal solution is dialysis. In this process, a sol containing dissolved ions or molecules is placed in a bag of permeable membrane dipping in water, the ions diffuse through the membrane. By using a continuous flow of water, the concentration of electrolyte outside the membrane is made zero.

Question 85

For a complex, MX_3Y_3 possessing trigonal prismatic geometry, the number of possible isomer is

Options:

A. 2

B. 4

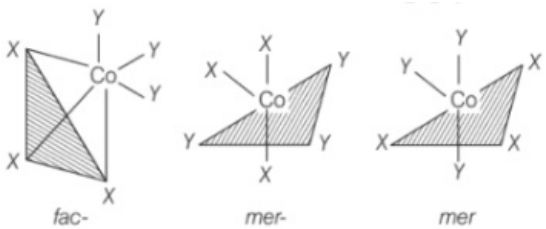
C. 3

D. 6

Answer: C

Solution:

Solution:
For a complex, MX_3Y_3 possessing trigonal prismatic geometry, the number of possible isomer is 3. These are as follows:



fac- and mer- isomerism occurs in octahedral coordination entities of the type $[Ma_3b_3]$. If three donor atoms of the same ligands occupy adjacent positions at the corners of an octahedral face, we have the facial (fac) isomer. When the positions are around the meridian of the octahedron, we get meridional (mer) isomer.

Question 86

On doping Ge metal with a little of In or Ga, we get

Options:

A. p - type semiconductor

B. insulator

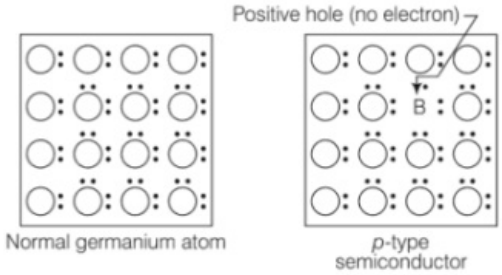
C. n - type semiconductor

D. rectifier

Answer: A

Solution:

Solution:
On doping Ge metal with a little of In or Ga, we get p- type semiconductor. Here, p specifies that conduction is through positive holes in them.



Question 87

Zn converts from its melted state to its solid state, it has hcp structure, then the number of nearest atoms will be

Options:

A. 6

B. 8

C. 12

D. 4

Answer: C

Solution:

Solution:
In hexagonal close packing, the coordination number is 12 . Since, Zn is in hcp structure, the number of nearest neighbours is 12, six are in the plane, 3 above the plane and 3 below the plane.

Question 88

If the pK_a of acetic acid and pK_b of NH_4OH are 4.76 and 4.75

respectively, what will be the pH of ammonium acetate solution?

Options:

A. 9.51

B. 7.005

C. 7.00

D. 6.9

Answer: B

Solution:

Solution:
When a weak acid (acetic acid) and weak base (NH_4OH) are mixed to form a solution, pH of the solution is then determined by following formula :

$pH = 7 + \frac{1}{2}[pK_a - pK_b]$

Given, $pK_a = 4.76$, $pK_b = 4.75$

$pH = 7 + \frac{1}{2}[4.76 - 4.75]$

$pH = 7 + 0.005$

$pH = 7.005$

Question 89

Which of the following has least oxidation state of Fe?

Options:

A. $K_2[Fe(OH)_6]$

B. $K_2[FeO_4]$

C. $FeSO_4(NH_4)_2SO_4 \cdot 6H_2O$

D. $Fe(CN)_6^{3-}$

Answer: C

Solution:

Solution:
Let oxidation state of Fe is 'x' in the given compounds.
(a) $K_2[FeO_4]$: $+2 + x + 4(-2) = 0$
$x = +6$
(b) $FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$
$x + 2(-2) + 2(1) + 6(0) = 0$
$x = +2$
(c) $[Fe(CN)_6]^{3-}$
$x + 6(-1) = -3$
$X = +3$
(d) $K_2[Fe(OH)_6]$
$2(1) + x + 6(-1) = 0$
$X = +4$

Hence, least oxidation state of Fe is in compound $FeSO_4 . (NH_4)_2SO_4 . 6H_2O$

Question 90

In electrochemical reaction of which set of reactants, the metal displacement will not take place?

Options:

A. $Mg + Cu^{2+}$

B. $Pb + Ag^{+}$

C. $Zn + Cu^{2+}$

D. $Cu + Mg^{2+}$

Answer: D

Solution:

Solution:
In electrochemical reaction of $Cu + Mg^{2+}$, the metal displacement will not take place.
Displacement reactions occurs when a metal from the electrochemical series is mixed with the ions of metal lower down in the series. In the electrochemical series Mg is placed above Cu.
Hence, metal displacement reaction won't occur in$Cu + Mg^{2+}$. $InPb + Ag^{+}$ and $Zn + Cu^{2+}$, lead and zinc are more reactive than silver and copper, hence they can displace silver and copper respectively from their salt solution.

Question 91

The IUPAC name of the compound $CH_3 - CH(CH_3) - CO - CH_3$ is

Options:

A. 3-methyl-2-butanone

B. 2-methyl-3-butanone

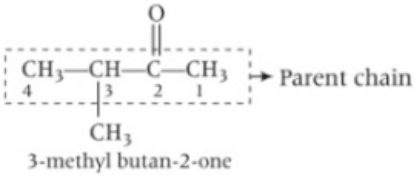
C. isopropyl methyl ketone

D. 2 ethyl-2 methyl pentane

Answer: A

Solution:

Solution:
(i) For the structure,



3-methyl butan-2-one

Functional group is ketone, hence its secondary suffix is 'one'. Now, the longest carbon chain containing the functional group has 4 carbon atom.
∴ The word root is 'but'.
(ii) Further, since the longest chain is saturated, therefore its primary suffix is 'ane'.

(iii) Number the carbon atoms right to left, it given lowest vacant 2 to functional group $\overset{\overset{O}{\|}}{C}$ and vacant 3 to substituent methyl group.
∴ The IUPAC name is 3 -methyl-2-butanone.

Question 92

CH_3MgI will give methane with

Options:

A. C_2H_5OH

B. $CH_3 - CH_2 - NH_2$

C. Both (a) and (b)

D. None of these

Answer: C

Solution:

Solution:
Methyl magnesium iodide, CH_3MgI will give methane with both C_2H_5OH and $CH_3CH_2NH_2$.

$$CH_3MgI + C_2H_5OH \rightarrow \underset{\text{Methane}}{CH_4} + Mg(OC_2H_5)I$$

$$CH_3MgI + CH_3CH_2NH_2 \rightarrow \underset{\text{Methane}}{CH_4} + Mg(NHCH_2CH_3)I$$

Question 93

On long standing, the bleaching powder undergoes auto-oxidation. The products formed are

Options:

A. calcium chloride only

B. calcium chlorate only

C. calcium chloride and calcium chlorate

D. calcium chloride and calcium chlorite

Answer: C

Solution:

Solution:
On long standing bleaching powder undergoes auto-oxidation to form calcium chloride, $CaCl_2$ and calcium chlorate

$Ca(ClO_3)_2$.
Following reaction takes place:

$$\underset{\text{Bleaching powder}}{6CaOCl_2} \rightarrow \underset{\text{Calcium chlorate}}{Ca(ClO_3)_2} + \underset{\text{Calcium chloride}}{5CaCl_2}$$

Question 94

Colour of the bead in borax bead test in mainly due to the formation of

Options:

A. metal oxides

B. boron oxides

C. metal metaborates

D. elemental boron

Answer: C

Solution:

Solution:
Colour of the bead in borax bead test is mainly due to the formation of metal metaborates. In this process, borax on strong heating loses its water of crystallisation and swells up to form puffy mass. On further heating it melts into clear liquid that solidifies to transparent glass like bead that consists of sodium metaborate ($NaBO_2$) and basic anhydride (B_2O_3).

$$Na_2B_4O_7 \xrightarrow{\text{Heat}} Na_2B_4O_7 + 10H_2O$$

$$Na_2B_4O_7 \xrightarrow{\text{Heat}} 2NaBO_2 + B_2O_3$$

Now, whenever a coloured salt containing cations like Ni^{2+}, Co^{2+}, Cr^{3+} etc. is heated with borax bead on platinum wire, the salt decomposes to form corresponding metal oxide that combines with B_2O_3 to form coloured metaborates. This test is called borax bead test.

e.g. $CoSO_4 \xrightarrow{\Delta} CoO + SO_3$

$$CoO + B_2O_3 \xrightarrow{\Delta} \underset{\text{Cobalt metaborate(blue)}}{Co(BO_2)_2}$$

Question 95

Product (P) of the given reaction is

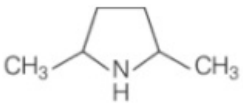
$$\mathbf{CH_3 - \overset{O}{\overset{\|}{C}} - CH_2 - COOC_2H_5 \xrightarrow[(II)I_2]{(I)NaH\ /\ THF} (A) \xrightarrow{H_3\overset{+}{O}\ /\ \Delta} (B) \longrightarrow (P) \xrightarrow[\Delta]{(NH_4)_2CO_3}}$$

Options:

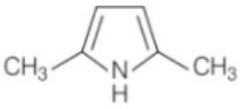
A. $CH_3 - \overset{O}{\overset{\|}{C}} - CH_2 - CH_2 - \overset{O}{\overset{\|}{C}} - CH_3$

B. $CH_3 - \overset{O}{\overset{\|}{C}} - \underset{COOH}{\underset{|}{CH}} - \underset{COOH}{\underset{|}{CH}} - \overset{O}{\overset{\|}{C}} - CH_3$

C.



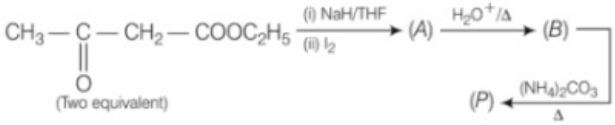
D.



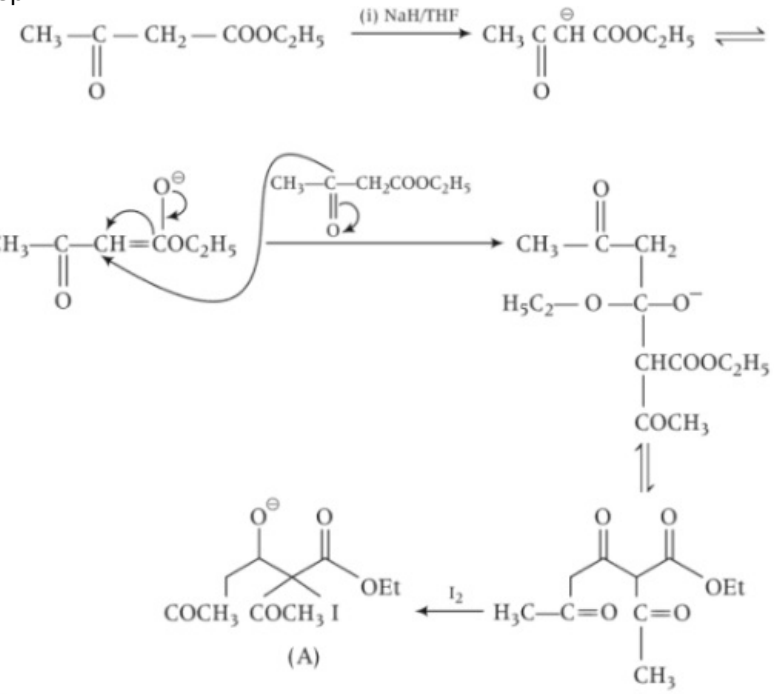
Answer: D

Solution:

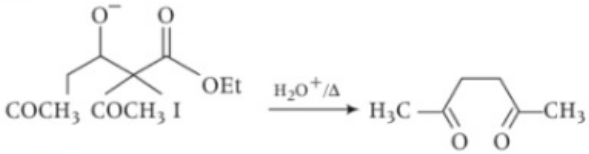
Solution:
Given.



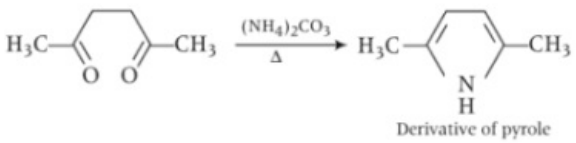
Ethyl acetoacetate gives self condensation reaction with other molecule of ethyl acetoacetate.
Step I



Step II



Step III



Question 96

Cordite is a mixture of

Options:

A. nitroglycerine, guncotton and vaseline

B. borax, glycerine

C. carborundum and charcoal

D. glycerol and $KMnO_4$

Answer: A

Solution:

Solution:
Cordite is a mixture of 30% nitroglycerine, 65% guncotton and vaseline.
The name cordite is most usually applied to the mixture of two explosives, nitrocellulose and nitroglycerine, made into cards and used as small arms (pistols, rifles).

Question 97

Which out of the following is the strongest acid?

Options:

A. $CH_3-\overset{O}{\overset{\|}{C}}-OH$

B. $CH_3-\overset{O}{\overset{\|}{C}}-COOH$

C. $CH_3-\underset{CH_3}{\overset{|}{C}}H-CH_3-\overset{O}{\overset{\|}{C}}-OH$

D. $CH_3-CH_2-\overset{O}{\overset{\|}{C}}-OH$

Answer: B

Solution:

Solution:
$CH_3-\overset{O}{\overset{\|}{C}}-COOH$ is strongest acid among all even stronger than $CH_3-\overset{O}{\overset{\|}{C}}-COOH$ due to electrophilic carbonyl group in it.

Among all carboxylic acids given, $CH_3-\overset{O}{\overset{\|}{C}}-COOH$ is strongest acid because it has least $+I$ effect of alkyl group present.
Alkyl group present in carboxylic acid has electron donating inductive effect, that increases electron density in the $O-H$ bond. As a result, release of H^+ ions form the carboxylic acid will be more difficult.

Question 98

Which element is not in first transition series?

Options:

A. F e

B. V

C. Ag

D. Cu

Answer: C

Solution:

Solution:
Ag (silver) element is not in first transition series.
First transition series includes metals: SC. Ti, V, Cr, Mn, Fe, Co, Ni, Cu, Zn.

Question 99

Which reagent is capable of giving test both of aldehyde and ketones?

Options:

A. Tollen reagent

B. Fehling solution

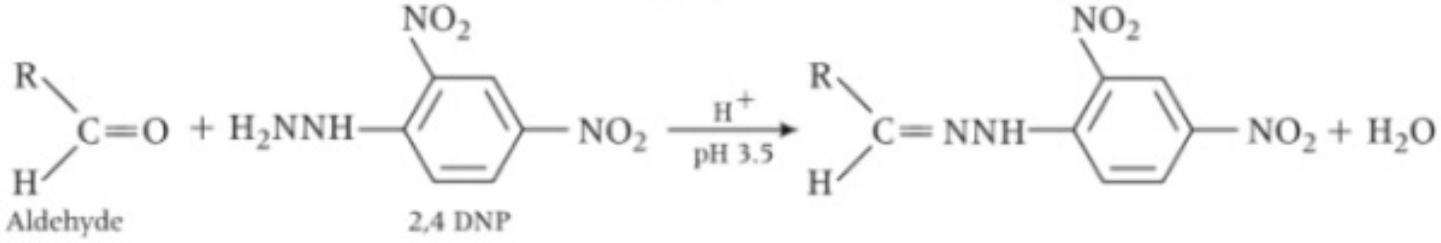
C. 2, 4-dinitrophenylhydrazine

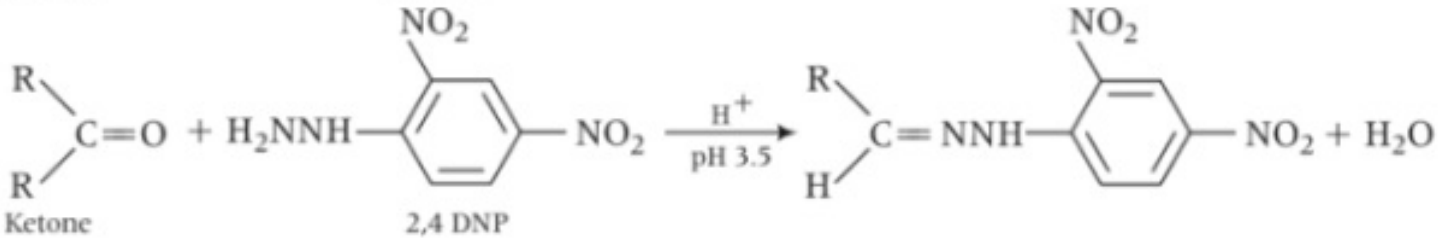
D. Shift reagent

Answer: C

Solution:

Solution:
Both aldehydes and ketone reacts with 2,4 DNP to form yellow, orange or red ppt. of 2,4-dinitrophenylhydrazones.





with Fehling solution, Shiff's reagent and Tollen reagent only aldehyde reacts. These are distinguishing test for aldehyde and ketones.

Question 100

Which reaction will not occur out of the following?

Options:

A. $Fe + H_2SO_4 \rightarrow H_2 + FeSO_4$

B. $Cu + 2AgNO_3 \rightarrow Cu(NO_3)_2 + 2Ag$

C. $2KBr + I_2 \rightarrow 2KI + Br_2$

D. $CuO + H_2 \rightarrow Cu + H_2O$

Answer: C

Solution:

Solution:
$2KBr + I_2 \rightarrow 2KI + Br_2$
reaction is not possible. Reaction of potassium bromide with iodine gives no product, as the activity of halogens decreases down the group. Hence, iodine is less reactive than bromine and thus, it cannot displace bromine from its compounds.
$KBr + I_2 \rightarrow$ No reaction
Reaction of copper with silver nitrate is possible, as copper is more reactive than silver, so it can displace silver from its salt solution.
$Cu + 2AgNO_3 \rightarrow Cu(NO_3)_2 + 2Ag$
Reaction of iron with sulphuric acid is also possible as iron is placed above hydrogen is activity series and thus can displace hydrogen from dilute acid like H_2SO_4. HCl.
$Fe + H_2SO_4 \rightarrow H_2 + FeSO_4$
$CuO + H_2 \rightarrow Cu + H_2O$; is a redox reaction, that proceeds via electrolysis.

Question 101

Mathematics
The value of $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$ is

Options:

A. $\frac{\sqrt{3}}{4}$

B. $\frac{3}{4}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{3}{2}$

Answer: D

Solution:

Solution:

We have, $\frac{7\pi}{8} = \pi - \frac{\pi}{8}$

and $\frac{5\pi}{8} = \pi - \frac{3\pi}{8}$

$\Rightarrow \sin\frac{7\pi}{8} = \sin\left(\pi - \frac{\pi}{8}\right)$

and $\sin\frac{5\pi}{8} = \sin\left(\pi - \frac{3\pi}{8}\right)$

$\Rightarrow \sin\frac{7\pi}{8} = \sin\frac{\pi}{8}$ and $\sin\frac{5\pi}{8} = \sin\frac{3\pi}{8}$

$\Rightarrow \sin^4\left(\frac{7\pi}{8}\right) = \sin^4\left(\frac{\pi}{8}\right)$ and $\sin^4\left(\frac{5\pi}{8}\right) = \sin^4\left(\frac{3\pi}{8}\right)$

Now, $\sin^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\frac{5\pi}{8}\right) + \sin^4\left(\frac{7\pi}{8}\right)$

$= \sin^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\frac{\pi}{8}\right)$

$= 2\sin^4\left(\frac{\pi}{8}\right) + 2\sin^4\left(\frac{3\pi}{8}\right)$

$= 2\left[\left(\sin^2\frac{\pi}{8}\right)^2 + \left(\sin^2\left(\frac{3\pi}{8}\right)\right)^2\right]$

$= 2\left[\left(\frac{1-\cos 2\left(\frac{\pi}{8}\right)}{2}\right)^2 + \left(\frac{1-\cos 2\left(\frac{3\pi}{8}\right)}{2}\right)^2\right] = 2\left[\frac{\left(1-\cos\frac{\pi}{4}\right)^2}{4} + \left(1-\cos\frac{3\pi}{4}\right)^2 4\right]$

$= \frac{1}{2}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2 + \left(1+\frac{1}{\sqrt{2}}\right)^2\right]$ $\left[\because \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \text{ and } \cos\frac{3\pi}{4} = \frac{-1}{\sqrt{2}}\right]$

$= \frac{1}{2}\left[1 + \frac{1}{2} - \frac{2}{\sqrt{2}} + 1 + \frac{1}{2} + \frac{2}{\sqrt{2}}\right]$

$= \frac{1}{2}\left[2 + \frac{2}{2}\right] = \frac{1}{2}[2+1] = \frac{3}{2}$

Question 102

For positive integer n, if f (n) = $\sin^n\theta + \cos^n\theta$. Then $\frac{f(3)-f(5)}{f(5)-f(7)}$ is

Options:

A. $\frac{f(1)}{f(3)}$

B. $\frac{f(3)}{f(1)}$

C. $\frac{f(3)}{f(5)}$

D. $\frac{f(5)}{f(7)}$

Answer: A

Solution:

Solution:

Given, $f(n) = \sin^n\theta + \cos^n\theta \; \forall n \in I^+$

Now, $\frac{f(3)-f(5)}{f(5)-f(7)} = \frac{\sin^3\theta + \cos^3\theta - \sin^5\theta - \cos^5\theta}{\sin^5\theta + \cos^5\theta - \sin^7\theta - \cos^7\theta}$

$= \frac{\sin^3\theta - \sin^5\theta + \cos^3\theta - \cos^5\theta}{\sin^5\theta - \sin^7\theta + \cos^5\theta - \cos^7\theta}$

$= \frac{\sin^3\theta(1-\sin^2\theta) + \cos^3\theta(1-\cos^2\theta)}{\sin^5\theta(1-\sin^2\theta) + \cos^5\theta(1-\cos^2\theta)}$

$= \frac{\sin^3\theta\cos^2\theta + \cos^3\theta\sin^2\theta}{\sin^5\theta\cos^2\theta + \cos^5\theta\sin^2\theta}$

$= \frac{\sin^2\theta\cos^2\theta(\sin\theta + \cos\theta)}{\sin^2\theta\cos^2\theta(\sin^3\theta + \cos^3\theta)}$

$= \frac{\sin^{(1)}\theta + \cos^{(1)}\theta}{\sin^{(3)}\theta + \cos^{(3)}\theta} = \frac{f(1)}{f(3)}$

Question 103

If $f_n(x) = \frac{1}{n}(\cos^n x + \sin^n x)$, for n = 1, 2, 3....., then $f_4(x) - f_6(x)$ is equal to

Options:

A. 10

B. $\frac{1}{12}$

C. $\frac{1}{10}$

D. 12

Answer: B

Solution:

Solution:

$f_n(x) = \frac{1}{n}(\cos^n x + \sin^n x)$,

where $f_n(x) = \frac{1}{n}(\cos^n x + \sin^n x)$,

Now, $f_4(x) - f_6(x) = \frac{1}{4}(\cos^4 x + \sin^4 x) - \frac{1}{6}(\cos^6 x + \sin^6 x)$

$= \frac{1}{4}[(\cos^2 x + \sin^2 x)^2 - 2\cos^2 x\sin^2 x] - \frac{1}{6}[(\cos^2 x)^3 + (\sin^2 x)^3]$

$= \frac{1}{4}[1 - 2\sin^2\cos^2 x] - \frac{1}{6}[(\cos^2 x + \sin^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)]$

$= \frac{1}{4}[1 - 2\sin^2 x\cos^2 x] - \frac{1}{6}[1 - 3\sin^2 x\cos^2 x]$

$= \frac{1}{4} - \frac{1}{2}\sin^2 x\cos^2 x - \frac{1}{6} + \frac{1}{2}\sin^2 x\cos^2 x$

$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

Question 104

If $(1 + \tan 1°)(1 + \tan 2°)..... . (1 + \tan 45°) = 2^n$, then n is

Options:

A. 22

B. 24

C. 23

D. 12

Answer: C

Solution:

Solution:
We have,
$(1+\tan 1°)(1+\tan 2°)\ldots\ldots.(1+\tan 45°) = 2^n$
Now, $(1+\tan\theta)(1+\tan(45°-\theta))$
$= (1+\tan\theta)\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right)$
$= (1+\tan\theta)\frac{(1+\tan\theta+1-\tan\theta)}{1+\tan\theta} = 2$
$\therefore (1+\tan 1°)(1+\tan 44°)(1+\tan 2°)\ (1+\tan 43°)\ldots\ldots.(1+\tan 22°)(1+\tan 23°)(1+\tan 45°)$
$= 2\times 2\times 2\ldots..22 \text{ times} \times(1+1)$
$= 2^{22}\times 2 = 2^{23}$
$\therefore n = 23$

Question 105

If $\cos^{-1}\frac{3}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}k$, then the value of k is

Options:

A. $\frac{16}{65}$

B. $\frac{12}{65}$

C. $\frac{11}{65}$

D. $\frac{19}{65}$

Answer: A

Solution:

Solution:
We have,
$\cos^{-1}\frac{3}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}k$
$\Rightarrow \cos^{-1}\left\{\frac{3}{5}\cdot\frac{12}{13} - \sqrt{1-\left(\frac{3}{5}\right)^2}\sqrt{1-\left(\frac{12}{13}\right)^2}\right\} = \cos^{-1}k$
$[\because \cos^{-1}x + \cos^{-1}y = \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}$, if $-1 \le x, y \le 1$ and $x + y \ge 0$]
$\Rightarrow \cos^{-1}\left\{\frac{36}{65} - \frac{4}{5}\times\frac{5}{13}\right\} = \cos^{-1}k$
$\Rightarrow \cos^{-1}\left\{\frac{36}{65} - \frac{20}{65}\right\} = \cos^{-1}k$
$\Rightarrow \cos^{-1}\frac{16}{65} = \cos^{-1}k$
$\Rightarrow k = \frac{16}{65}$

Question 106

If $\cos 3x \cos 2x \cos x = \frac{1}{4}$ and $0 < x < \frac{\pi}{4}$, then the value of x is

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{5}$

C. $\frac{\pi}{8}$

D. $\frac{\pi}{7}$

Answer: C

Solution:

Solution:

Given, $\cos 3x \cos 2x \cos x = \frac{1}{4}$

$\Rightarrow 2(2\cos x \cos 3x)\cos 2x = 1$

$\Rightarrow 2(\cos 4x + \cos 2x)\cos 2x = 1$ [$\because 2\cos A \cos B = \cos(A + B) + \cos(A - B)$]

$\Rightarrow 2(2\cos^2 2x - 1 + \cos 2x)\cos 2x = 1$

$\Rightarrow 4\cos^3 2x - 2\cos 2x + 2\cos^2 2x - 1 = 0$

$\Rightarrow 2\cos 2x(2\cos^2 2x - 1) + 1(2\cos^2 x - 1) = 0$

$\Rightarrow (2\cos^2 2x - 1)(2\cos 2x + 1) = 0$

$\Rightarrow \cos 4x(2\cos 2x + 1) = 0$

$\Rightarrow \cos 4x = 0$ or $2\cos 2x + 1 = 0$

$\Rightarrow \cos 4x = \cos\frac{\pi}{2}$ or $\cos 2x = \frac{-1}{2}$

$\Rightarrow 4x = \frac{\pi}{2}$ or $\cos 2x = \cos\frac{2\pi}{3}$

$\Rightarrow x = \frac{\pi}{8}$ or $2x = \frac{2\pi}{3}$

$\Rightarrow x = \frac{\pi}{8}$ or $x = \frac{\pi}{3}$

Since, $x \in \left(0, \frac{\pi}{4}\right)$

$\therefore x = \frac{\pi}{8}$ is the required value.

Question 107

The number of solutions of the equation $\sin 2x + 2\sin x - \cos x - 1 = 0$ in the range$0 \le x \le 2\pi$ is

Options:

A. 3

B. 4

C. 2

D. None of these

Answer: A

Solution:

Solution:
We have, $\sin 2x + 2\sin x - \cos x - 1 = 0$
$\Rightarrow 2\sin x\cos x + 2\sin x - \cos x - 1 = 0$
$\Rightarrow 2\sin x(\cos x + 1) - 1(\cos x + 1) = 0$
$\Rightarrow (\cos x + 1)(2\sin x - 1) = 0$
$\Rightarrow \cos x + 1 = 0$ or $2\sin x - 1 = 0$
$\Rightarrow \cos x = -1$ or $\sin x = \frac{1}{2}$
$\Rightarrow \cos x = \cos \pi$
or $\sin x = \sin \frac{\pi}{6}, \sin \frac{5\pi}{6} [\because x \in [0, 2\pi]]$
$\Rightarrow x = \pi$ or $x = \frac{\pi}{6}, \frac{5\pi}{6}$
$\therefore x = \pi, \frac{\pi}{6}, \frac{5\pi}{6}$
Hence, number of solutions are 3 .

Question 108

The number of the distinct real roots of the equation

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0, \text{ in the interval } -\frac{\pi}{4} \leq x \leq \frac{\pi}{4} \text{ is}$$

Options:

A. 4

B. 3

C. 1

D. 2

Answer: C

Solution:

Solution:
We have,

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 2\cos x + \sin x & \cos x & \cos x \\ 2\cos x + \cos x & \sin x & \cos x \\ 2\cos x + \cos x & \cos x & \sin x \end{vmatrix} = 0$$

Taking $(2\cos x + \sin x)$ common from C_1, we get

$$(2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$(2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$$

Expanding along C_1, we get

$(2\cos x + \sin x)[1(\sin x - \cos x)^2] = 0$

$\Rightarrow (2\cos x + \sin x)(\sin x - \cos x)^2 = 0$

Now, if $2\cos x + \sin x = 0$, then $2\cos x = -\sin x$

$\Rightarrow \tan x = -2$

But here $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$, we have

$-1 \le \tan x \le 1$, so no solution possible.

or if $(\sin x - \cos x)^2 = 0$, then $\sin x = \cos x$

$\Rightarrow \tan x = 1 = \tan\frac{\pi}{4} \Rightarrow x = \frac{\pi}{4}$

So, only one distinct real root exist.

Question 109

In a triangle ABC, the sides are of length 17 , 25 and 28 units. Then, the length of the largest altitude is

Options:

A. 26.6

B. 27

C. 26

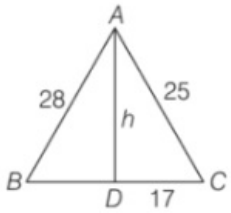
D. 420 / 17

Answer: D

Solution:

Solution:

Given, sides of ΔABC are 17,25 and 28 .



Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{17 + 25 + 28}{2} = 35$

$\Delta = \sqrt{35(35-17)(35-25)(35-28)}$

$\Delta = \sqrt{35 \times 18 \times 10 \times 7}$

$\Delta = 210$ .....(i)

Also, area of $\Delta = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 17 \times AD$ .....(ii)

From Eqs. (i) and (ii),

$210 = \frac{1}{2} \times 17 \times AD$

$\therefore AD = \frac{420}{17}$

Hence, length of largest altitude is $\frac{420}{17}$

Question 110

If α, β and γ are the roots of the equation $x^3 - 3x^2 + 3x + 7 = 0$, and w is cube root of unity, then the value of $\frac{\alpha-1}{\beta-1} + \frac{\beta-1}{\gamma-1} + \frac{\gamma-1}{\alpha-1}$ is equal to

Options:

A. $3w^2$

B. 3 / w

C. $2w^2$

D. None of these

Answer: A

Solution:

Solution:
We have,
$x^3 - 3x^2 + 3x + 7 = 0$
$(x + 1)(x^2 - 4x + 7) = 0$
$x + 1 = 0$ or $x^2 - 4x + 7 = 0$
$x = -1$ or $x = \frac{4 \pm \sqrt{16 - 28}}{2}$
$\Rightarrow x = -1$ or $x = \frac{4 \pm 2\sqrt{3}i}{2}$
$\Rightarrow x = -1$ or $x = 2 \pm \sqrt{3}i$
$\Rightarrow x = -1$ or $x = 1 - 2w, 1 - 2w^2$ [$2w = -1 + \sqrt{3}i$, $2w^2 = -1 - \sqrt{3}i$]
$\therefore \alpha = -1, \beta = 1 - 2w$ and $\gamma = 1 - 2w^2$
Now, $\frac{\alpha-1}{\beta-1} + \frac{\beta-1}{\gamma-1} + \frac{\gamma-1}{\alpha-1}$
$= \frac{-1-1}{1-2w-1} + \frac{1-2w-1}{1-2w^2-1} - \frac{1-2w^2-1}{-1-1}$
$= \frac{2}{2w} + \frac{2w}{2w^2} + \frac{2w^2}{2}$
$= \frac{1}{w} + \frac{1}{w} + w^2 = \frac{2}{w} + w^2$
$= 2w^2 + w^2 = 3w^2$

Question 111

The perimeter of the locus represented by $\arg\left(\frac{z+i}{z-i}\right) = \frac{\pi}{4}$ is equal to

Options:

A. 4π

B. $2\pi\sqrt{2}$

C. $2\pi\sqrt{3}$

D. $\frac{2\pi}{\sqrt{3}}$

Answer: B

Solution:

Solution:

We have, $\arg\left(\frac{z+i}{z-i}\right) = \frac{\pi}{4}$

Let $z = x + iy$

$\therefore \frac{z+i}{z-i} = \frac{x+iy+i}{x+iy-i}$

$= \frac{x+(y+1)i}{x+(y-1)i}$

$= \frac{(x+(y+1)i)(x-(y-1)i)}{(x+(y-1)i)(x-(y-1)i)}$

$= \frac{x^2+y^2-1+(-xy+x+xy+x)i}{x^2+(y-1)^2}$

$= \frac{x^2+y^2-1+2xi}{x^2+(y-1)^2}$

$\arg\left(\frac{x^2+y^2-1+2xi}{x^2+(y-1)^2}\right) = \tan^{-1}\left(\frac{2x}{x^2+y^2-1}\right)$

$\therefore \tan^{-1}\left(\frac{2x}{x^2+y^2-1}\right) = \frac{\pi}{4}$

$\Rightarrow 2x = x^2+y^2-1$

$\Rightarrow (x-1)^2+y^2 = (\sqrt{2})^2$

$\therefore$ Locus of $\arg\left(\frac{z+i}{z-i}\right) = \frac{\pi}{4}$ is circle,

whose centre (1, 0) and radius $= \sqrt{2}$

Perimetre of circle $= 2\pi r = 2\pi(\sqrt{2})$

$= 2\pi\sqrt{2}$

Question 112

In a triangle ABC if $\cos A + \cos B + \cos C = \frac{3}{2}$, then the triangle is

Options:

A. right angled

B. right angled isosceles

C. equilateral

D. None of the above

Answer: C

Solution:

Solution:

We have, in ΔABC

$\cos A + \cos B + \cos C = \frac{3}{2}$

We have, in ΔABC

$\cos A + \cos B + \cos C \leq \frac{3}{2}$

Since, in ΔABC

$\cos A + \cos B + \cos C = \frac{3}{2}$

$\therefore A = B = C = 60°$

Hence, ΔABC is an equilateral triangle.

Question 113

In a triangle ABC, the minimum value of the sum of the squares of sides is [Δ is the area of triangle ABC]

Options:

A. $3\sqrt{3}\Delta$

B. $4\sqrt{3}\Delta$

C. $2\sqrt{3}\Delta$

D. $5\sqrt{3}\Delta$

Answer: B

Solution:

Solution:

Let a, b and c are sides of ΔABC

Δ is area of ΔABC.

s = Semi-perimetre of ΔABC

$2s = a + b + c$

and $\Delta^2 = s(s-a)(s-b)(s-c)$

By apply property of AM and GM , we get

$\frac{s-a+s-b+s-c}{3} \geq ((s-a)(s-b)(s-c))^{1/3}$

$\frac{3s-2s}{3} \geq \left(\frac{\Delta^2}{s}\right)\frac{1}{3} \Rightarrow \frac{s}{3} \geq \left(\frac{\Delta^2}{s}\right)\frac{1}{3}$

$\Rightarrow \frac{\Delta^2}{s} \leq \frac{s^3}{27} \Rightarrow \Delta^2 \leq \frac{s^4}{27}$

$\Rightarrow s^2 \geq 3\sqrt{3}\Delta$

$\Rightarrow \left(\frac{a+b+c}{2}\right)^2 \geq 3\sqrt{3}\Delta$

$\Rightarrow (a + b + c^2 \geq 4 \times 3\sqrt{3}\Delta$

$\Rightarrow (a+b+c)^2 \geq 12\sqrt{3}\Delta$

We know that,

$a^2 + b^2 + c^2 \geq \frac{(a+b+c)^2}{3}$

$\therefore a^2 + b^2 + c^2 \geq \frac{12\sqrt{3}\Delta}{3}$

$\Rightarrow a^2 + b^2 + c^2 \geq 4\sqrt{3}\Delta$

Question 114

A tower subtends angles θ, 2θ and 3θ at three points A, B, C respectively

lying on a horizontal line through the foot of the tower. Then, the ratio $\frac{AB}{BC}$ equals

Options:

A. $\frac{\sin 3\theta}{\sin \theta}$

B. $\frac{\sin \theta}{\sin 3\theta}$

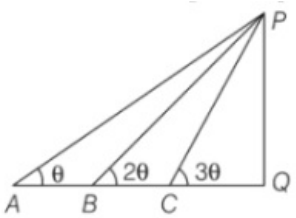
C. $\frac{\cos 3\theta}{\cos \theta}$

D. $\frac{\tan \theta}{\tan 3\theta}$

Answer: A

Solution:

Solution:
Let QP be tower subtends angle θ, 2θ and 3θ at three points A, B and C respectively.



In ΔPQA,

$\tan\theta = \frac{PQ}{AQ}$

$\Rightarrow AQ = \frac{PQ}{\tan\theta}$

In ΔPBQ,

$\Rightarrow \tan 2\theta = \frac{PQ}{BQ}$

$BQ = \frac{PQ}{\tan 2\theta}$

In ΔPQC

$\tan 3\theta = \frac{PQ}{QC} \Rightarrow QC = \frac{PQ}{\tan 3\theta}$

Now, $\frac{AB}{BC} = \frac{AQ - BQ}{BQ - CQ} = \frac{\frac{PQ}{\tan\theta} - \frac{PQ}{\tan 2\theta}}{\frac{PQ}{\tan 2\theta} - \frac{PQ}{\tan 3\theta}}$

$= \left(\frac{\tan 2\theta - \tan\theta}{\tan 3\theta - \tan 2\theta}\right)\left(\frac{\tan 3\theta}{\tan\theta}\right)$

$= \frac{\frac{\sin\theta}{\cos 2\theta \cos\theta}}{\frac{\sin\theta}{\cos 3\theta \cos 2\theta}} \times \frac{\tan 3\theta}{\tan\theta} \left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos B}\right]$

$= \frac{\cos 3\theta}{\cos\theta} \times \frac{\tan 3\theta}{\tan\theta} = \frac{\sin 3\theta}{\sin\theta}$

Question 115

From the top of a lighthouse, the angles of depression of two stations on the opposite sides of it at a distance d apart are α and β The height of the lighthouse is

Options:

A. $\frac{d \tan\alpha}{\tan\alpha + \tan\beta}$

B. $\frac{d}{\cot\alpha + \cot\beta}$

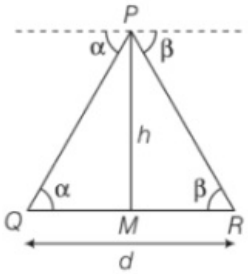
C. $\frac{d \tan\beta}{\tan\alpha + \tan\beta}$

D. $\frac{d \tan\beta}{\cot\alpha + \cot\beta}$

Answer: B

Solution:

Solution:
Let PM be lighthouse.



Given,
$\angle PQM = \alpha$, $\angle PRM = \beta$and $QR = d$
In ΔPQM,
$\tan\alpha = \frac{PM}{QM}$
$\Rightarrow QM = PM \cot\alpha$
In Δ PRM,
$\tan\beta = \frac{PM}{RM}$
$\Rightarrow RM = PM \cot\beta$
$\therefore QR = QM + MR$
$\Rightarrow d = PM \cot\alpha + PM \cot\beta$
$\Rightarrow d = PM(\cot\alpha + \cot\beta)$
$\Rightarrow PM = \frac{d}{\cot\alpha + \cot\beta}$
$\therefore$ Height of light house is $\frac{d}{\cot\alpha + \cot\beta}$.

Question 116

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar unit vectors then $[\vec{a}\,\vec{b}\,\vec{c}]$ is equal to

Options:

A. ± 2

B. ± 3

C. ± 1

D. 2

Answer: C

Solution:

Solution:
Here $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar unit vectors. So, we may consider $\vec{a} = \pm\hat{i}$, $\vec{b} = \pm\hat{j}$ and $\vec{c} = \pm\hat{k}$. Hence, $[\vec{a}\,\vec{b}\,\vec{c}] = \pm 1$

Question 117

If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar, then

Options:

A. $c^2 = ab$

B. $b^2 = ac$

C. $a + c = 2b$

D. $\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

Answer: A

Solution:

Solution:
We have,
$a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar.

$$\therefore \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$\Rightarrow a(0 - c) - a(b - c) + c(c - 0) = 0$
$\Rightarrow -ac - ab + ac + c^2 = 0$
$\Rightarrow c^2 = ab$

Question 118

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non coplanar vectors, then $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]$ is equal to

Options:

A. 0

B. $[\vec{a}\,\vec{b}\,\vec{c}]^2$

C. $[\vec{a}\,\vec{b}\,\vec{c}]$

D. $2[\vec{a}\,\vec{b}\,\vec{c}]$

Answer: B

Solution:

Solution:

$[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]$
$= (\vec{a} \times \vec{b}) . [(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})]$
$= (\vec{a} \times \vec{b}) . [\vec{\lambda} \times (\vec{c} \times \vec{a})] [\text{take } \vec{b} \times \vec{c} = \vec{\lambda}]$
$= (\vec{a} \times \vec{b}) . [(\vec{\lambda} . \vec{a})\vec{c} - (\vec{\lambda} . \vec{c})\vec{a}]$
$= (\vec{a} \times \vec{b}) . [(\vec{b} \times \vec{c}) . \vec{a})\vec{c} - (\vec{b} \times \vec{c} . \vec{c}]\vec{a}]$
$= (\vec{a} \times \vec{b}) . [[\vec{b}\vec{c}\vec{a}]\vec{c} - 0]$
$= (\vec{a} \times \vec{b}) . \vec{c}[\vec{b}\vec{c}\vec{a}] = [\vec{a}\vec{b}\vec{c}][\vec{a}\vec{b}\vec{c}]$
$= \vec{a}\vec{b}\vec{c}]^2$

Question 119

If $\vec{a}$ is a colinear vector with $\vec{b} = 3\hat{i} + 6\hat{j} + 6\hat{k}$ and $\vec{a} . \vec{b} = 27$. Then, $\vec{a}$ is

Options:

A. $3\left(\hat{i} + \hat{j} + \hat{k}\right)$

B. $2\left(\hat{i} + \hat{j} + \hat{k}\right)$

C. $\hat{i} + 2\hat{j} + 2\hat{k}$

D. $2\hat{i} + 3\hat{j} + 3\hat{k}$

Answer: C

Solution:

Solution:
Given,
$\vec{a}$ is a collinear vector with $\vec{b} = 3\hat{i} + 6\hat{j} + 6\hat{k} \therefore \vec{a} = \lambda\vec{b}$
Now $\vec{a} . \vec{b} = 27$
$\therefore \lambda\vec{b} . \vec{b} = 27$
$\Rightarrow \lambda\left|\vec{b}\right|^2 = 27$
$\Rightarrow \lambda = \frac{27}{9 + 36 + 36} = \frac{1}{3}$
Hence, $\vec{a} = 13\left(3\hat{i} + 6\hat{j} + 6\hat{k}\right)$
$\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$

Question 120

The value of $\lim_{x \to 0} \frac{(1 + x)^{1/2} - 1}{(1 + x)^{1/2} - 1}$ is equal to

Options:

A. 3 / 2

B. 2 / 3

C. 0

D. None of these

Answer: D

Solution:

Solution:
We have,
$\lim_{x \to 0} \frac{(1+x)^{1/2} - 1}{(1+x)^{1/2} - 1} = 1$

Question 121

The value of $\lim_{x \to \infty} \left[x\sqrt{x^2+4} - \sqrt{x^4+16} \right]$ is

Options:

A. 4

B. 8

C. 2

D. 16

Answer: C

Solution:

Solution:
We have,

$$\lim_{x \to \infty} \left[x\sqrt{x^2+4} - \sqrt{x^4+16} \right]$$

$$= \lim_{x \to \infty} \left[x\sqrt{x^2+4} - \sqrt{x^4+16} \right] \left(\frac{x\sqrt{x^2+4} + \sqrt{x^4+16}}{x\sqrt{x^2+4} + \sqrt{x^4+16}} \right)$$

$$= \lim_{x \to \infty} \frac{x^2(x^2+4) - (x^4+16)}{x\sqrt{x^2+4} + \sqrt{x^4+16}}$$

$$= \lim_{x \to \infty} \frac{4x^2 - 16}{x^2 \left(\sqrt{1 + \frac{4}{x^2}} + \sqrt{1 + \frac{16}{x^4}} \right)}$$

$$= \lim_{x \to \infty} \frac{4 - \frac{16}{x^2}}{\sqrt{1 + \frac{4}{x^2}} + \sqrt{1 + \frac{16}{x^4}}} = \frac{4}{2} = 2$$

Question 122

$\lim_{x \to 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}$ **is equal to (a > 0)**

Options:

A. $\log_e a$

B. 1

C. 0

D. e

Answer: A

Solution:

Solution:
We have,

$$\lim_{x \to 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x} = \lim_{x \to 0} a^{\sin x} \left(\frac{a^{\tan x - \sin x} - 1}{\tan x - \sin x} \right)$$

$$= \log_e a \left[\because \lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a \right]$$

Question 123

The area of triangle formed by the coordinate axes and tangent to the curve $y = \log_e x$ at (1, 0) is

Options:

A. 1

B. $\frac{1}{2}$

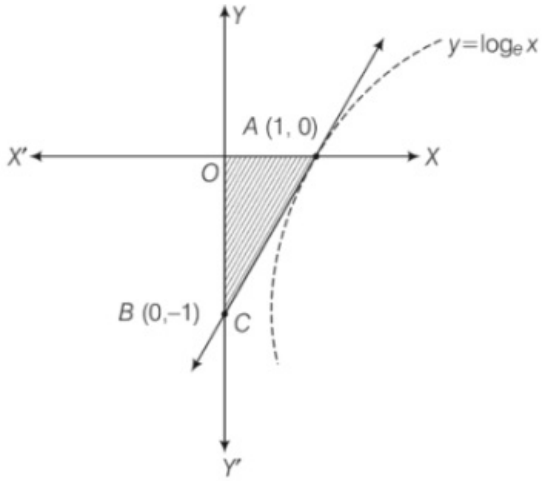
C. 2

D. $\frac{3}{2}$

Answer: B

Solution:

Solution:
Equation of tangent to the curve
$y = \log_e x$ at (1, 0)is
$y - 0 = 1(x - 1)$
We have,



$\Rightarrow y = x - 1$

Area of shaded region

$= \frac{1}{2}OB \times OA$

$= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Question 124

The slope of the tangent at $\left(\frac{\pi}{4}, 0\right)$ to the curve $1 + 16x^2y = \tan(x - 2y)$ is

Options:

A. $\frac{2}{\pi + 2}$

B. $\frac{1}{\pi^2 + 4}$

C. $\frac{1}{\pi + 4}$

D. $\frac{2}{\pi^2 + 4}$

Answer: D

Solution:

Solution:

We have,

$1 + 16x^2y = \tan(x - 2y)$

On differentiating w.r.t. x, we get

$32xy + 16x^2\frac{dy}{dx} = \sec^2(x - 2y)\left(1 - 2\frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx}(16x^2 + 2\sec^2(x - 2y) = \sec^2(x - 2y) - 32xy$

$\Rightarrow \frac{dy}{dx} = \frac{\sec^2(x - 2y) - 32xy}{16x^2 + 2\sec^2(x - 2y)}$

Slope of tangent at $\left(\frac{\pi}{4}, 0\right)$

i.e. $\left(\frac{dy}{dx}\right)_{\left(\frac{\pi}{4}, 0\right)} = \frac{\sec^2\left(\frac{\pi}{4} - 0\right) - 32\left(\frac{\pi}{4}\right)(0)}{16 \times \left(\frac{\pi}{4}\right)^2 + 2\sec^2\left(\frac{\pi}{4} - 0\right)}$

$\therefore$ Slope of tangent $= \frac{2}{\pi^2 + 4}$

Question 125

A man of height 2m walks at uniform speed of 5km / h away from a lamp post of 6m height. His shadow length increases at the rate of

Options:

A. 12.5km / h

B. 2.5km / h

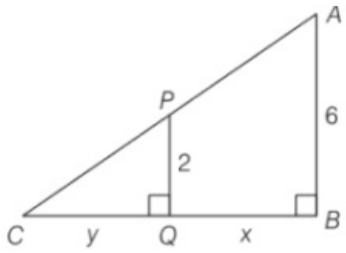
C. 3km / h

D. 3.5km / h

Answer: B

Solution:

Solution:
Let, AB = height of lamp post
PQ = height of man
and $\frac{dx}{dt} = 5\text{km / h}$



$\Delta ABC \sim \Delta PQC$
$\therefore \frac{AB}{PQ} = \frac{BC}{QC}$
$\Rightarrow \frac{6}{2} = \frac{x+y}{y}$
$\Rightarrow 3y = x + y \Rightarrow 2y = x$
On differentiating w.r.t. t, we get
$2\frac{dy}{dt} = \frac{dx}{dt}$
$\Rightarrow \frac{dy}{dt} = \frac{5}{2}\text{km / h}$
$\therefore$ Shadow length increases at the rate of 2.5km / h.

Question 126

If the function $f(x) = a\log|x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then the values of a and b.

Options:

A. $a = 2b = \frac{1}{2}$

B. $a = \frac{1}{2}, b = 2$

C. $a = 2, b = -\frac{1}{2}$

D. $a = -2, b = -\frac{1}{2}$

Answer: C

Solution:

Solution:
We have,
$f(x) = a \log |x| + bx^2 + x$
$f'(x) = \frac{a}{x} + 2bx + 1$
Since, $f(x)$ has extremum value at $x = -1$ and $x = 2$
$\therefore f'(-1) = 0$ and $f'(2) = 0$
$\therefore f'(-1) = -a - 2b + 1 = 0$ .....(i)
$f'(2) = \frac{a}{2} + 4b + 1 = 0$ .....(i)
On solving Eqs. (i) and (ii), we get
$a = 2, b = -\frac{1}{2}$

Question 127

If α, β, γ are roots the equation $ax^3 + bx^2 + c = 0$, the value of determinant $\begin{vmatrix} \alpha\beta & \beta\gamma & \gamma\alpha \\ \beta\gamma & \gamma\alpha & \alpha\beta \\ \gamma\alpha & \alpha\beta & \beta\gamma \end{vmatrix}$ is

Options:

A. a

B. b

C. 0

D. c

Answer: C

Solution:

Solution:
We have, α, β, γ are roots of equation $ax^3 + bx^2 + c = 0$
$\therefore \alpha + \beta + \gamma = \frac{-b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = 0$
and $\alpha\beta\gamma = \frac{-c}{a}$
Now, $\begin{vmatrix} \alpha\beta & \beta\gamma & \gamma\alpha \\ \beta\gamma & \gamma\alpha & \alpha\beta \\ \gamma\alpha & \alpha\beta & \beta\gamma \end{vmatrix}$
Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} \alpha\beta+\beta\gamma+\gamma\alpha & \beta\gamma & \gamma\alpha \\ \alpha\beta+\beta\gamma+\gamma\alpha & \gamma\alpha & \alpha\beta \\ \alpha\beta+\beta\gamma+\gamma\alpha & \alpha\beta & \beta\gamma \end{vmatrix}$$

$$= (\alpha\beta+\beta\gamma+\gamma\alpha)\begin{vmatrix} 1 & \beta\gamma & \gamma\alpha \\ 1 & \gamma\alpha & \alpha\beta \\ 1 & \alpha\beta & \beta\gamma \end{vmatrix}$$

$= 0[\because \alpha\beta+\beta\gamma+\gamma\alpha = 0]$

Question 128

If a,b, c are pth, qth and rth terms respectively of a geometric progression, then the value of the determinant $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is equal to

Options:

A. 0

B. 1

C. -1

D. None of these

Answer: A

Solution:

Solution:
Given, a, b and c are pth, qth and rth terms of GP, respectively. Let a_1 and k be the first term and common ratio of a GP. Then,

$$\left.\begin{aligned} T_p &= a_1k^{p-1} \text{ i.e. } a = a_1k^{p-1} \\ T_q &= a_1k^{q-1} \text{ i.e. } b = a_1k^{q-1} \\ T_t &= a_1k^{r-1} \text{ i.e. } c = a_1k^{r-1} \end{aligned}\right\} \quad .....(i)$$

Now,

$$= \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = \begin{vmatrix} \log(a_1 . k^{p-1}) & p & 1 \\ \log(a_1 . k^{q-1}) & q & 1 \\ \log(a_1 . k^{r}-1) & r & 1 \end{vmatrix} \text{ [from Eq. (i)]}$$

$$= \begin{vmatrix} \log a_1+\log k^{p-1} & p & 1 \\ \log a_1+\log k^{q-1} & q & 1 \\ \log a_1+\log k^{r-1} & r & 1 \end{vmatrix} \quad [\because \log xy = \log x+\log y]$$

$$= \begin{vmatrix} \log a_1 & p & 1 \\ \log a_1 & q & 1 \\ \log a_1 & r & 1 \end{vmatrix} + \begin{vmatrix} p-1\log k & p & 1 \\ q-1\log k & q & 1 \\ r-1\log k & r & 1 \end{vmatrix} \text{ [by property of determinant]}$$

$$= \log a_1 \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log k \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$$

[$\because$ taking log a $_1$ common from C in I determinant and log k common from C_1 in II determinant]
On applying $C_1 \to C_1 + C_3$ in second determinant,
we get

$$= \log a_1(0) + \log k \begin{vmatrix} p & p & 1 \\ q & q & 1 \\ r & r & 1 \end{vmatrix}$$

[$\because C_1$ and C_3 are identical in first determinant]
$= 0 + \log k(0) = 0$
[$\because C_1$ and C_2 are identical]

Question 129

A matrix A is such that $A^2 = 2A - I$, where I is unity matrix, then for $n \geq 2$, A^n is equal to

Options:

A. $nA - (n-1)I$

B. $nA - I$

C. $2^{n-1}A - (n-1)I$

D. $2^nA - nI$

Answer: A

Solution:

Solution:
Given, $A^2 = 2A - I$ .....(i)
Now, $A^3 = A(A^2)$
$= A(2A - I)$ [from Eq. (i)]
$= 2A^2 - AI$
$= 2\sim A^2 - A$ [$\because AI = A$]
$= 2(2A - I) - A$ [from Eq. (i)]
$= 4A - 2I - A$
$\Rightarrow A^3 = 3A - 2I$ .....(ii)
and $A^4 = A(A^3) = A(3A - 2)$ [from Eq. (ii)]
$= 3A^2 - 2AI$
$= 3(2A - I) - 2A$ [from Eq. (ii)]
$= 6A - 3I - 2A$
$\Rightarrow A^4 = 4A - 3I$
following this, we can say
$A^n = nA - (n-1)I$

Question 130

The greatest coefficient in the expansion of $\left(x + \frac{1}{x}\right)^{2n}$ is

Options:

A. $\frac{1.3.5\ldots..(2n-1)}{n!}$

B. $\frac{2n!}{n!n!}$

C. $\frac{2n!}{\left(\frac{n}{2}!\right)^2}$

D. None of these

Answer:A, B

Solution:

Solution:
Since, middle term has greatest coefficient.
∴ Greatest coefficient = Coefficient of middle term... (i)
Since, 2n is an even.
∴ Middle term $= \left(\frac{2n}{2}+1\right)$ th term $= (n+1)$ th term
$= T_{n+1} = {}^{2n}C_n x^{n-n}\left(\frac{1}{x}\right)^n$
$= {}^{n}C_n\, x^n \frac{1}{x^n} = {}^{2n}C_n$
Now, coefficient of middle term
$= {}^{2n}C_n = \frac{2n!}{n!n!}$
∴ Greatest coefficient of
$\left(x+\frac{1}{x}\right)^{2n} = {}^{n}C_n$[from Eq. (i)]
$= \frac{2n!}{n!n!}$
$= \frac{2n(2n-1)(2n-2)(2n-3)\ldots\ldots..4.3.2.1}{n!n!}$
$= \frac{[1.3.5\ldots\ldots..(2n-1)]2^n[n(n-1)(n-2)\ldots..2.1]}{n!n!}$
$= \frac{[1.3.5\ldots\ldots..(2n-1)]2^n.n!}{n!n!}$
$= \frac{[1.3.5\ldots\ldots..(2n-1)2^n]}{n!}$

Question 131

The coefficient of x^5 in the expansion of $(1+x^2)^5(1+x)^4$ is

Options:

A. 40

B. 50

C. 60

D. -50

Answer: C

Solution:

Solution:

Coefficient of x^5 in the product $(1 + x^2)^5(1 + x)^4$

= The coefficient of x^5 in

$[{}^5C_0 + {}^5C_1(x^2) + {}^5C_2(x^2)^2 + {}^5C_3(x^2)^3 + {}^5C_4(x^2)^4 + {}^5C_5(x^2)^5] \times [{}^4C_0 + {}^4C_1x + {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4]$

= The coefficient of x^5 in

$[{}^5C_0 + {}^5C_1x^2 + {}^5C_2x^4 + {}^5C_3x^6 + {}^5C_4x^8 + {}^5C_5x^{10}] \times [{}^4C_0 + {}^4C_1x + {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4]$

$= {}^5C_1 \times {}^4C_3 + {}^5C_2 \times {}^4C_1$

$= 5 \times 4 + 10 \times 4$

$= 20 + 40 = 60$

Question 132

10% bulbs manufactured by a company are found to be defective. The probability that out of a sample of 5 bulbs none is defective is

Options:

A. $\left(\frac{1}{2}\right)^5$

B. $\left(\frac{1}{10}\right)^5$

C. $\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)$

D. $\left(\frac{9}{10}\right)^5$

Answer: D

Solution:

Solution:

Given, n = 5, r = 0, p = $\frac{1}{10}$

and q = $\frac{9}{10}$

∴ Required probability P(r = 0) = ${}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5 = \left(\frac{9}{15}\right)^5$

Question 133

Tho dice of different colours are thrown simultaneously. The probability that the sum of the faces appeared is either 7 or 11 is

Options:

A. $\frac{7}{36}$

B. $\frac{4}{9}$

C. $\frac{2}{9}$

D. $\frac{5}{9}$

Answer: C

Solution:

Solution:
We have two dice of different colours, are thrown simultaneous.
∴ Total number of sample space = 36
A be the event that sum of the faces appears is 7
i.e. A = { (1, 6)(2, 5)(3, 4)(4, 3) (5, 2)(6, 1)
∴P(A) = $\frac{6}{36}$
and B be the event that sum of the faces appears is 11
i.e. B = {(5, 6)(6, 5)}
P(B) = $\frac{2}{36}$
∴ Required probability = P(A) + P(B)
= $\frac{6}{36} + \frac{2}{36}$
= $\frac{8}{36} = \frac{2}{9}$

Question 134

Six students are to be selected for a quiz competition from 10 aspirants. The probability that two particular students are excluded is

Options:

A. $\frac{2}{15}$

B. $\frac{1}{3}$

C. $\frac{1}{5}$

D. $\frac{2}{3}$

Answer: B

Solution:

Solution:
Number of ways of selecting six students for a quiz competition from 10 aspirant = ${}^{10}C_6$
Number of ways when two particular students are excluded is ${}^{10-2}C_6 = {}^{8}C_6$
∴ Required probability = $\frac{{}^{8}C_6}{{}^{10}C_6} = \frac{2}{15}$

Question 135

If three distinct natural numbers are chosen randomly from the first natural numbers, then the probability that all three of them are

divisible by both 2 and 3 , is

Options:

A. $\frac{4}{25}$

B. $\frac{4}{35}$

C. $\frac{4}{1161}$

D. $\frac{4}{1155}$

Answer: D

Solution:

Solution:
Three distinct natural number can be chosen randomly from the first 100 natural number in ${}^{100}C_3$ ways.
Number of ways of selecting three natural number which is divisible by 2 and 3 from first 100 natural numbers is ${}^{16}C_3$.
$\therefore$ Required probability $= \frac{{}^{16}C_3}{{}^{100}C_3} = \frac{4}{1155}$

Question 136

If $y + \sqrt{1 + y^2} = e^x$, then the value of y is

Options:

A. $e^x - e^{-x}$

B. $e^x + e^{-x}$

C. $\frac{e^x + e^{-x}}{2}$

D. None of these

Answer: D

Solution:

Solution:
d) Given, $y + \sqrt{1 + y^2} = e^x$
$\Rightarrow \sqrt{1 + y^2} = e^x - y$
On squaring both sides, we get
$1 + y^2 = (e^x - y)^2$
$\Rightarrow 1 + y^2 = e^{2x} + y^2 - 2e^x y$ [$\because (a - b)^2 = a^2 + b^2 - 2ab$]
$\Rightarrow 1 = e^{2x} - 2e^x y$
$\Rightarrow 2e^x y = e^{2x} - 1$
$\Rightarrow y = \frac{e^{2x}}{2e^x} - \frac{1}{2e^x} = \frac{e^x - e^{-x}}{2}$

Question 137

The number of points having position vector $a\hat{i} + b\hat{j} + c\hat{k}$, where a, b, c ∈ {1, 2, 3, 4, 5}such that $2^a + 3^b + 5^c$ is divisible by 4, is

Options:

A. 140

B. 70

C. 100

D. 150

Answer: B

Solution:

Solution:
We have,
a, b, c ∈ {1, 2, 3, 4, 5)
$2^a + 3^b + 5^c$
$= 2^a + 4\lambda + (-1)^b + 4\mu + (1)^c$
Which is divisible by 4 if a ≥ 2 and b is odd number and c is any number.
$\therefore 4 \times 3 \times 5 = 60$
or if a = 1 and b is even number and c is any number.
$\therefore 1 \times 2 \times 5 = 10$
∴ Total number $= 60 + 10 = 70$

Question 138

There are 4 candidates for the post of a professor in Mathematics and one is to be selected by a opinions of 5 subject experts. The number of the ways in which the expert opinion can be expressed is

Options:

A. 1048

B. 1072

C. 1024

D. 1020

Answer: C

Solution:

Solution:
Total number of ways 5 subject experts opinion can be expressed for 4 candidates is
$4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$

Question 139

If $g(x) = (x^2 + 2x + 3)f(x)$, $f(0) = 5$ and $\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = 4$, then $g'(0)$ is equal to

Options:

A. 30

B. 18

C. 20

D. 22

Answer: D

Solution:

Solution:
We have,
$g(x) = (x^2 + 2x + 3)f(x)$
$g'(x) = (x^2 + 2x + 3)(f'(x)) + (2x + 2)f(x)$
$g'(0) = 3f'(0) + 2f(0)$
Now, $\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = 4$
$\Rightarrow f'(0) = 4$
$\therefore g'(0) = 3(4) + 2(5)[\because f(0) = 5]$
$= 12 + 10 = 22$

Question 140

The solution of the differential equation $\left(y - x\frac{dy}{dx}\right) = a\left(y^2 + \frac{dy}{dx}\right)$ is

Options:

A. $y = k(1 - ay)(x + a)$

B. $y = k(1 + ay)(x - a)$

C. $y = k(1 + ay)(x + a)$

D. $y = k(1 - ay)(x - a)$

Answer: A

Solution:

Solution:
Given, $\left(y - x\frac{dy}{dx}\right) = a\left(y^2 + \frac{dy}{dx}\right)$
$\Rightarrow y - x\frac{dy}{dx} = ay^2 + a\frac{dy}{dx}$
$\Rightarrow (x + a)\frac{dy}{dx} = y - ay^2$
$\Rightarrow (x + a)\frac{dy}{dx} = y(1 - ay)$

$\Rightarrow \frac{dy}{y(1-ay)} = \left(\frac{1}{x+a}\right)dx$
On integrating
$\int \frac{dy}{y(1-ay)} = \int \frac{dx}{x+a}$
$\Rightarrow \int \left(\frac{1}{y} + \frac{a}{1-ay}\right)dy = \int \frac{dx}{x+a}$
$\Rightarrow \log|y| - \log|1-ay| = \log|x+a| + \log k$

Question 141

The solution of the differential equation $\frac{dy}{dx} = \sin(10x + 6y)$ is

Options:

A. $5\tan(5x - 3y) = 4\tan(4x + k) + 3$

B. $5\tan(5x + 3y) = 4\tan(4x + k) - 3$

C. $5\tan(5x - 3y) = 4\tan(4y + k) - 3$

D. None of the above

Answer: B

Solution:

Solution:
We have,
$\frac{dy}{dx} = \sin(10x + 6y)$ .....(i)
Put $10x + 6y = v$
$\Rightarrow 10 + 6\frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{6}\left(\frac{dv}{dx} - 10\right)$
$\therefore$ Eq. (i) becomes reduces to
$\frac{1}{6}\left(\frac{dv}{dx} - 10\right) = \sin v \Rightarrow \frac{dv}{dx} = 6\sin v + 10$
$\Rightarrow \int \frac{dv}{2(3\sin v + 5)} = \int dx$
$\Rightarrow \frac{1}{2}\int \frac{dv}{3\left(\frac{2\tan\frac{v}{2}}{1+\tan^2\frac{v}{2}}\right) + 5} = \int dx$

$\left[\because \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]$

$\Rightarrow \frac{1}{2}\int \frac{\sec^2\frac{v}{2}dv}{5\tan^2\frac{v}{2} + 6\tan\frac{v}{2} + 5} = \int dx$
Put $\tan\frac{v}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{v}{2}dv = dt$
$\therefore \int \frac{dt}{5t^2 + 6t + 5} = \int dx$
$\Rightarrow \frac{1}{5}\int \frac{dt}{\left(t + \frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \int dx$
$\Rightarrow \frac{1}{4}\tan^{-1}\left(\frac{5t+3}{4}\right) = x + C$
$\Rightarrow 5t + 3 = 4\tan(4x + k)$

$\Rightarrow 5\tan\frac{v}{2} = 4\tan(4x + k) - 3$

where, $k = 4c$

$\Rightarrow 5\tan\left(\frac{10x + 6y}{2}\right) = 4\tan(4x + k) - 3$

$\Rightarrow 5\tan(5x + 3y) = 4\tan(4x + k) - 3$

Question 142

If $I = \int_0^{\pi/4} (\sqrt{\tan x} + \sqrt{\cot x})dx$, then value of Iis

Options:

A. $\pi 2$

B. $\frac{\pi}{2\sqrt{2}}$

C. $\frac{\pi}{\sqrt{3}}$

D. $\frac{\pi}{\sqrt{2}}$

Answer: B

Solution:

Solution:

We have,

$I = \int_0^{\pi/4} (\sqrt{\tan x} + \sqrt{\cot x})dx$

$\Rightarrow I = \int_0^{\pi/4} \left(\sqrt{\frac{\sin x}{\cos x}} + \sqrt{\frac{\cos x}{\sin x}}\right)dx$

$\Rightarrow I = \int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}dx$

Put $\sin x - \cos x = t$

$\Rightarrow \Rightarrow (\cos x + \sin x)dx = dt$

$x = 0, t = -1$

and $x = \frac{\pi}{4} \Rightarrow t = 0$

$\Rightarrow I = \int_{-1}^{0} \frac{\sqrt{2}dt}{\sqrt{1 - t^2}}$

$\Rightarrow I = \sqrt{2}[\sin^{-1}t]_{-1}{}^{0}$

$\Rightarrow I = \sqrt{2}[\sin^{-1}0 - \sin^{-1}(-1)]$

$\Rightarrow I = \sqrt{2}\left[0 + \frac{\pi}{2}\right]$

$\Rightarrow I = \frac{\pi}{\sqrt{2}}$

Question 143

The value of the integral $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx$ isequal to

Options:

A. $\pi\left(\frac{\pi}{2}-1\right)$

B. $\frac{\pi}{2}(\pi-1)$

C. $\pi(\pi-1)$

D. $\frac{\pi}{2}(\pi+1)$

Answer: A

Solution:

Solution:

Let $I=\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$

$I=\int_0^{\pi}\frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}dx\left[\because\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$I=\int_0^{\pi}\frac{(\pi-x)\tan x}{\sec x+\tan x}dx$

$\Rightarrow II=\pi\int_0^{\pi}\frac{\tan x}{\sec x+\tan x}dx$

$\Rightarrow 2I=\int_0^{\pi}\tan x(\sec x-\tan x)\,dx$

$\Rightarrow 2I=\int_0^{\pi}(\sec x\tan x-\tan^2 x)dx$

$\Rightarrow 2I=\int_0^{\pi}(\sec x\tan x-\sec^2 x+1)dx$

$\Rightarrow 2I=\pi[\sec x-\tan x+x]_0^{\pi}$

$\Rightarrow 2=\pi[(\sec\pi-\tan\pi+\pi)-(\sec 0-\tan 0+0)]$

$\Rightarrow I=\pi\left[\frac{\pi}{2}-1\right]$

Question 144

If the sum of the distances of a moving point from two perpendicular lines in a plane is always 1, then its locus is

Options:

A. Parabola

B. Ellipse

C. Hyperbola

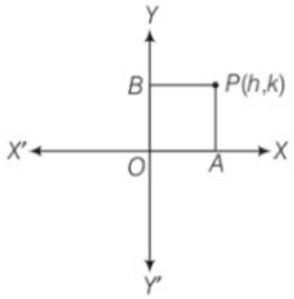
D. None of these

Answer: D

Solution:

Solution:

Let point P(h, k) is moving point.



Such that the sum of distance from two perpendicular line $x = 0$ and $y = 0$ in a plane is always 1 .
$\therefore AP + PB = 1$
$\Rightarrow k + h = 1 \Rightarrow h + k = 1$
$\therefore$ Locus of the curve is
$x + y = 1$
which represent the equation of line.

Question 145

The equation of an ellipse with focus at (1, −1), directrix x − y − 3 = 0 and eccentricity 1 / 2 is

Options:

A. $7x^2 + 2xy + 7y^2 + 7 = 0$

B. $7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$

C. $7(x^2 + y^2) + 2xy + 10x - 10y - 7 = 0$

D. $7(x^2 + y^2) + 2xy - 10x - 10y + 7 = 0$

Answer: B

Solution:

Solution:
We know that equation of an ellipse with focus (x_1, y_1) directrix $ax + by + c = 0$ and eccertricity e is

$$(x - x_1)^2 + (y - y_1)^2 = \frac{e^2(ax + by + c)^2}{a^2 + b^2}$$

Here, focus $= (1, -1)$
Directrix is $x - y - 3 = 0$
and $e = \frac{1}{2}$
$\therefore$ Equation of ellipse is

$$(x - 1)^2 + (y + 1)^2 = \left(\frac{1}{2}\right)^2 \frac{(x - y - 3)^2}{1 + 1}$$

$\Rightarrow x^2 - \&2x + 1 + y^2 + 2y + 1 = 18(x^2 + y^2 + 9 - 6x + 6y - 2xy)$
$\Rightarrow 8x^2 - \&16x + 8y^2 + 16y + 16 = x^2 + y^2 - 2xy - 6x + 6y + 9$
$\Rightarrow 7x^2 + 7y^2 + 2xy - 10x + 10y + 7 = 0$
$\Rightarrow 7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$

Question 146

The maximum and minimum of the resultant of two forces are F and G (the angle between these forces is 2α, Then, the resultant of F and G is

Options:

A. $(F^2\cos^2\alpha + G^2\sin^2\alpha)^{1/2}$

B. $(F^2\sin^2\alpha + G^2\cos^2\alpha)^{1/2}$

C. $(F^2\sin\alpha + G^2\cos^2\alpha)^{1/2}$

D. $(F^2\sin 2\alpha + G^2\cos 2\alpha)^{1/2}$

Answer: A

Solution:

Solution:
Let two forces be F_1 and F_2.
Given, F and G are maximum and minimum resultant of two forces F_1 and F_2 respectively and angle between two forces F_1 and F_2 is 2α.
$\therefore F = F_1 + F_2$ and $G = F_1 - F_2$
$\Rightarrow F_1 = F + G2$
and $F_2 = \frac{F - G}{2}$
Resultant $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 2\alpha}$
$= \sqrt{\left(\frac{F+G}{2}\right)^2 + \left(\frac{F-G}{2}\right)^2 + 2\left(\frac{F+G}{2}\right)\left(\frac{F-G}{2}\right)\cos 2\alpha}$
$= \sqrt{\frac{1}{4}[2(F^2 + G^2) + 2(F^2 - G^2)\cos 2\alpha]}$
$= \sqrt{\frac{1}{2}[F^2(1 + \cos 2\alpha) + G^2(1 - \cos 2\alpha)]}$
$= \sqrt{\frac{1}{2}[F^2(2\cos^2\alpha) + G^2(2\sin^2\alpha)]}$
$= \sqrt{F^2\cos^2\alpha + G^2\sin^2\alpha}$
$= (F^2\cos^2\alpha + G^2\sin^2\alpha)^{1/2}$

Question 147

The resultant of two forces 3P and 2P is R. If first-force is doubled then the resultant is also doubled. The angle between the forces is

Options:

A. 60°

B. 30°

C. 120°

D. 150°

Answer: C

Solution:

Solution:
We have two forces is 3P and 2P, let angle between two forces is θ.
$\therefore$ Resultant $R = \sqrt{(3P)^2 + (2P)^2 + 12P^2\cos\theta}$
$= \sqrt{P^2(13 + 12\cos\theta)}$ .....(i)
Now first force is doubled.

$\therefore$ First force $= 6P$
and resultant is also double
$\therefore R = 2R$
$\Rightarrow 2R = \sqrt{(6P)^2 + (2P)^2 + 24P^2\cos\theta}$
$\Rightarrow 2R = \sqrt{P^2(40 + 24\cos\theta)}$ .....(ii)
From Eqs. (i) and (ii),
$2P\sqrt{13 + 12\cos\theta} = P\sqrt{40 + 24\cos\theta}$
$\Rightarrow 4(13 + 12\cos\theta) = 40 + 24\cos\theta$
$\Rightarrow 52 + 48\cos\theta = 40 + 24\cos\theta$
$\Rightarrow 24\cos\theta = -12$
$\Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$

Question 148

A particle at rest starts moving with uniform acceleration covers a distances 114m in8th sec. The acceleration of the particle is

Options:

A. 20m / sec^2

B. 15m / sec^2

C. 14.5m / sec^2

D. 15.4m / sec^2

Answer: B

Solution:

Solution:
We know that,
Distance travelled in nth second
$S_n = u + \frac{a}{2}(2n - 1)$
Here, $S_8 = 114$, $u = 0$ and $n = 8$
$\therefore 114 = 0 + \frac{a}{2}(16 - 1)$
$\Rightarrow a = \frac{228}{15} = 15.2\text{m / sec}^2 \approx 15\text{m / s}^2$
$\therefore$ Acceleration of the particles is 15.2m / sec^2.

Question 149

A helicopter is flying at a height of 500m . If all of sudden its engines stops working, the helicopter will fall on the earth in

Options:

A. 10sec

B. 12sec

C. 15sec

D. 20sec

Answer: A

Solution:

Solution:
Here, h = 500m, u = 0 and g = 10m / s^2
$\therefore h = ut + \frac{1}{2}gt^2$
$\Rightarrow 500 = 0 + \frac{1}{2} \times 10 \times t^2$
$\Rightarrow t^2 = 100 \Rightarrow t = 10$
∴ The helicopter fall on earth in 10sec.

Question 150

If ax + by = 1 is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then the value of $a^2 - b^2$ is

Options:

A. b^2e^2

B. $\frac{1}{b^2e^2}$

C. a^2e^2

D. $\frac{1}{a^2e^2}$

Answer: D

Solution:

Solution:
Equation of any tangent to the hyperbola
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is
$y = mx \pm \sqrt{a^2m^2 - b^2}$ .....(i)
Now, ax + by = 1 is a tangent of given hyperbola
$\Rightarrow y = -\frac{a}{b}x + \frac{1}{b}$ .....(ii)
From Eqs. (i) and (ii),
$m = \frac{-a}{b}$
and $\sqrt{a^2m^2 - b^2} = \frac{1}{b}$
$\Rightarrow a^2m^2 - b^2 = \frac{1}{b^2}$
$\Rightarrow a^2\left(\frac{a^2}{b^2}\right) - b^2 = \frac{1}{b^2}\left[\because m = \frac{-a}{b}\right]$
$\Rightarrow a^4 - b^4 = 1$
$\Rightarrow a^2 - b^2 = \frac{1}{a^2 + b^2}$

$$\Rightarrow a^2 - b^2 = \frac{1}{a^2\left(1 + \frac{b^2}{a^2}\right)}$$

$$\Rightarrow a^2 - b^2 = \frac{1}{a^2e^2} \left[\because e^2 = 1 + \frac{b^2}{a^2}\right]$$
