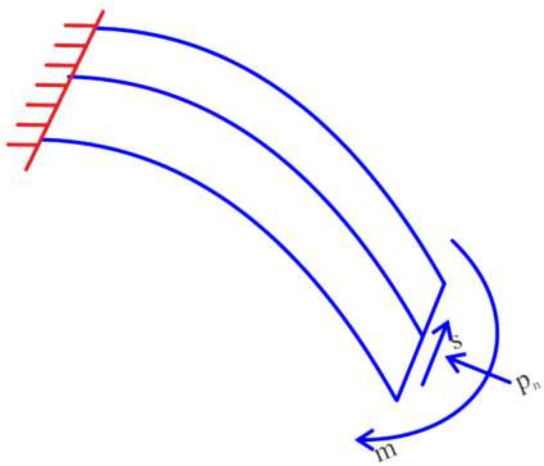
6 ARCHES & CABLES

6.1. Properties of Fluid

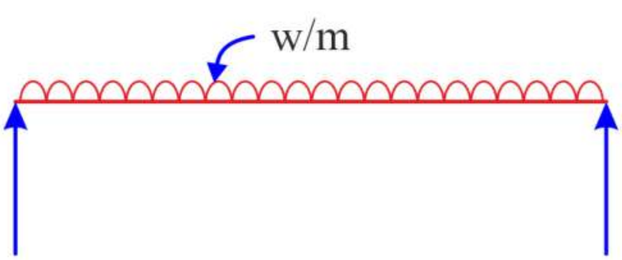
An Anch is a curved beam in vertical plane

- Design forces in an Arch:
- Rn : normal thrust or axial compression
- S : Radial shear force
- M : Bending moment

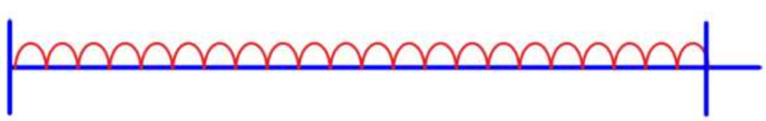


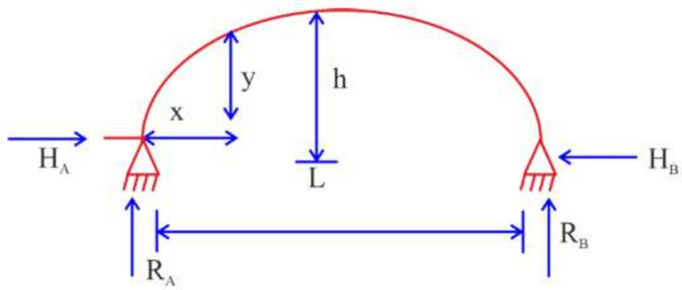
Advantages of Arches compared to SSB.

For a SSB:



For an Arch:





$$m_x = R_A x - wx\frac{x}{2}$$

$$m_{beam} = R_A x - \frac{wx^2}{2}$$

$$m_x = M_{arch} = \left(R_A.x - \frac{wx^2}{2}\right) - H_A y = M_{beam} - H_A y$$

$$M_{arch} = M_{beam} - H_{moment}$$

(a) An arch is economical for long spans compared to SSB

(b) The horizontal reaction developed of the support of each will reduce the net moment compared to that of SSB.

Note:

Arches are primarily suby to axial compression. Hence stone which strong in axial compression were used in olden days for contraction of arches.

6.2. Classification of Arches

1. Based on Shape

(a) Parabolic
(b) Semi-circular
(c) Segmental

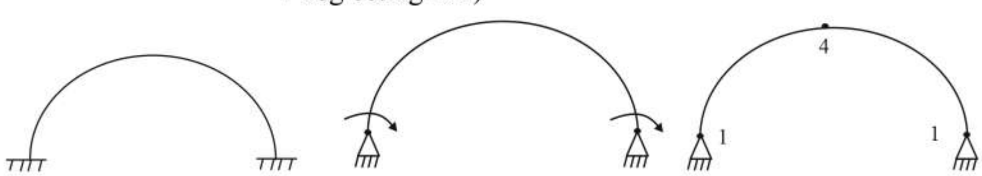
2. Based on number of hinges (or D_s) :

(a) Fixed arches ($D_s = 3$, $D_k = 0$)

(b) Two hinged arches ($D_s = 1$, $D_k = 2$)

(c) Three hinged arches ($D_s = D$, $D_k = 6$ considering AD

= 4 neglecting AD)



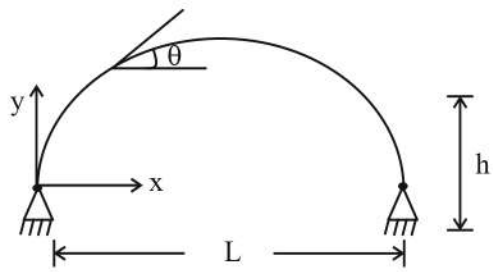
Parabolic Arches

$$y = \frac{4h}{l^2}x(1 - x)$$

(one of the support as origin)

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(1-2x)$$

$$\frac{x^2}{y} = \text{const. (Crown as origin)}$$

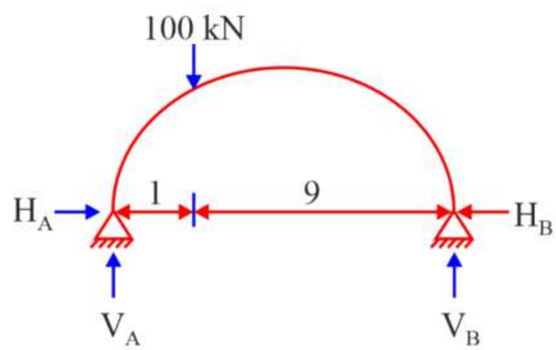


Calculation of Reactions at support of Arches

(a) Supports are at same level

To calculate vertical reactions, if the supports are at same level, analysis is similar to that of a SSB.

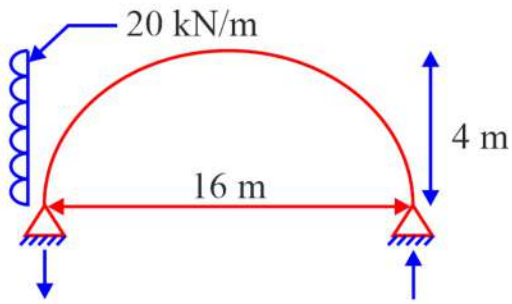
$$\Sigma M_A = 0$$
$$10\ V_B = 100 \times 1$$
$$V_B = 10\ KN\ \&\ V_A = 90\ KN$$



Horizontal reaction is not influencing the vertical reaction as their line of action through the support

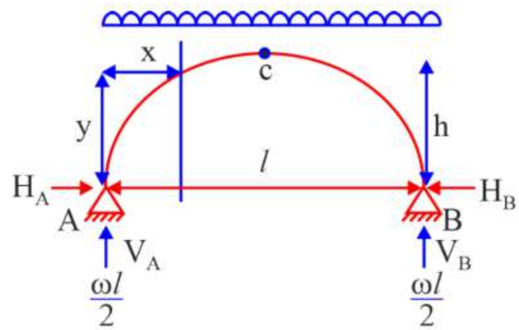
$$\Sigma M_A = 0$$

$\therefore$ $$16\ VB = 20 \times 4 \times 2$$
$$VB = 10\ KN\ (\uparrow)$$
$$VA = 10\ KN\ (\downarrow)$$



6.3. Calculation of Horizontal Reactions

(a) Three Hinged Arches. – parabolic arch suby. to wall throughout



Apply, $\Sigma M_C = 0$ (from right)

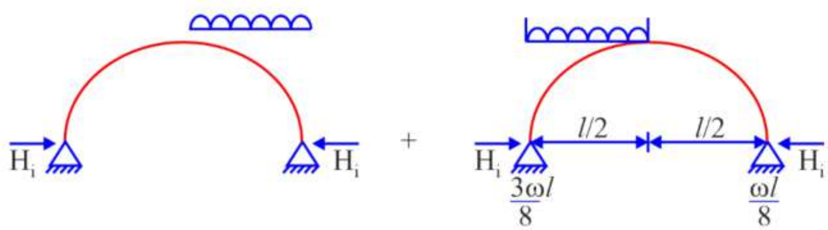
$$\Rightarrow \quad H_B \times h + \frac{wl}{2} \times \frac{l}{4} = \frac{wl}{2} \times \frac{1}{2}$$

$$H_B = \frac{wl^2}{8j}$$

$$H_A = H_B = H \text{ (no other horizontal forces)}$$

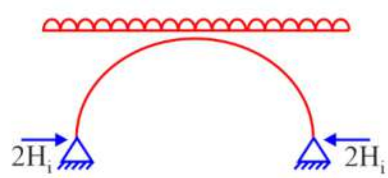
$$M_x = \frac{wl}{2} \times x - \frac{wl^2}{8h} \times y - \frac{wx^2}{2} = \frac{wlx}{2} - \frac{wl^2}{8h}\left(\frac{4h}{l^2} x(1-x)\right) - \frac{wx^2}{2} = 0$$

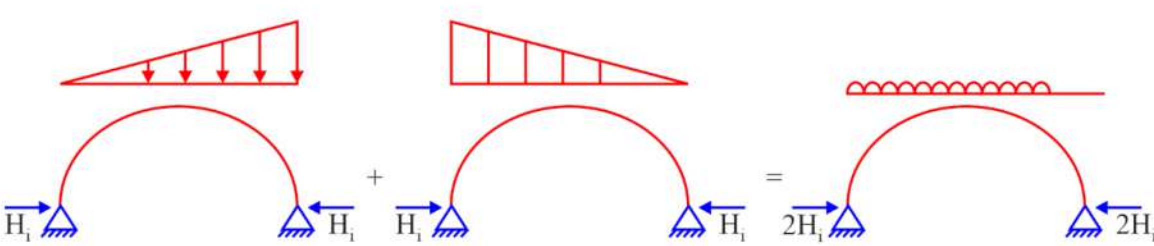
$$M_x = 0$$



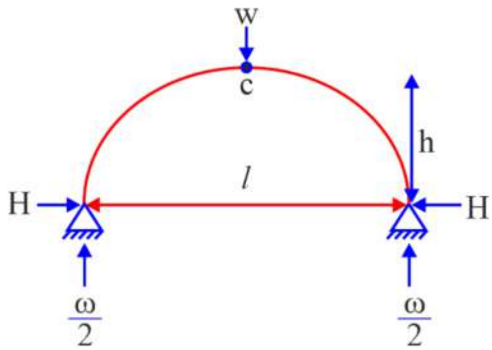
$$2H_i = \frac{wl^2}{8h}$$

$$H_i = \frac{wl^2}{16h}$$





$$2H_i = \frac{wl^2}{8h} \qquad H_i = \frac{wl^2}{16h}$$

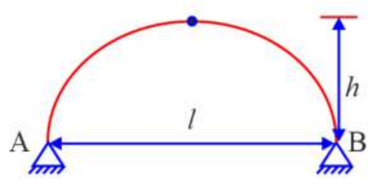


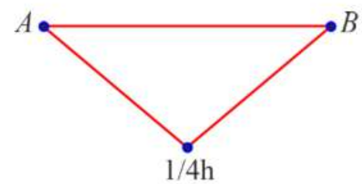
$$\Sigma m_c = 0 \;\Rightarrow\; \frac{W}{2} \times \frac{l}{2} = H \times h$$

$$\boxed{H = \frac{Wl}{4h}}$$

6.4. ILD for 3-hinged Arches

(a) ILD for Horizontal Thrust



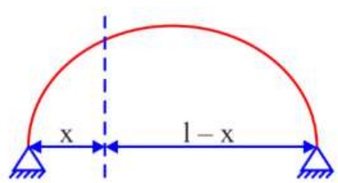


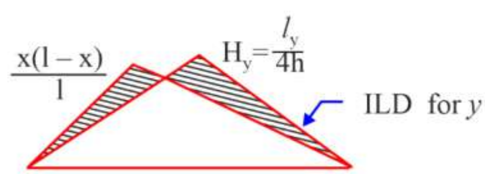
When unit to At support horizontal thrust = 0

(a) ILD for M_x

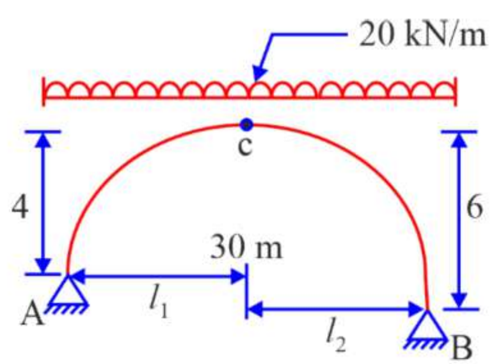
$$M_{arch} = M_{beam} - Hy$$

x & y are the coordinated of the choosen section where ILD is to be drawn for BM.





(b) Supports at different level



Calculate reaction the supports.

(1) There hinged parabolic Unsymmetric Arches

Step 1: Calculate horizontal distance of Ac & BC

$$\text{We known } \frac{x^2}{y} = \text{const. (For parabolic arch wrt crown as origin)}$$

$$\frac{x}{\sqrt{y}} = \text{const}$$

$$\frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} = \text{const.} = \frac{l_1 + l_2}{\sqrt{h_1} + \sqrt{h_2}} = \frac{l}{\sqrt{h_1} + \sqrt{h_2}}$$

$$\frac{l_1}{\sqrt{L_1}} = \frac{30}{\sqrt{L_1} + \sqrt{b}}$$

$\Rightarrow$ $$l_1 = \frac{60}{2+\sqrt{6}} = 13.48 \text{ m}$$

$$l_2 = 16.51 \text{ m}$$

As supports are not at same level, we cannot calculate vertical reactions by treating like a SSB initially

$$\text{Apply } \Sigma M_c = 0 \text{ (from left)}$$

$$R_A \times 13.48 = 20 \times \frac{13.48^2}{2} + 4H$$

$$R_A = 0.3\ H + 134.9$$

Apply $\Sigma M_c = 0$ (from right)

$$R_B \times 16.51 = 20 \times \frac{16.51^2}{2} + 6H$$

$$RB = 165.1 + 0.363\ H$$

Apply $\Sigma V = 0$ for the entire arch,

$$R_A + R_B = 20 \times 30$$

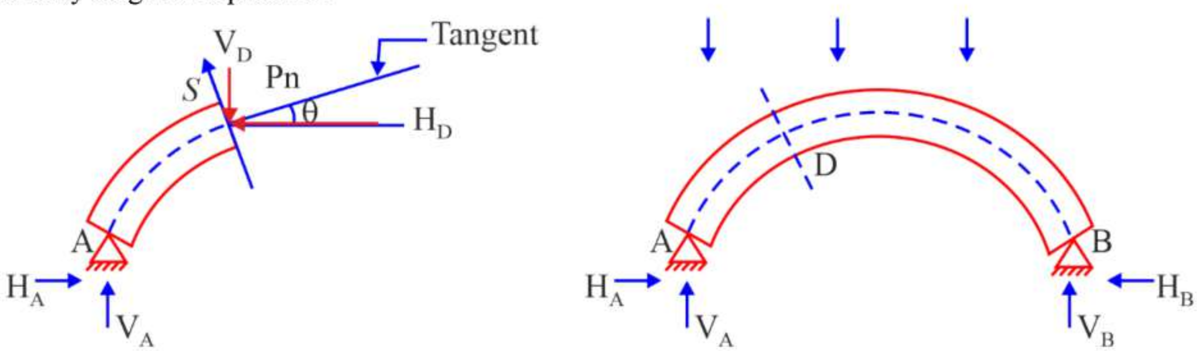
$H = 455.2$ KN

$R_A = 269.64$ KN

$R_B = 330.36$ KN

6.5. Radial shear & Normal Thrust

Consider free body diagram of part AD.



$V_D \rightarrow$ net vertical reaction at D

$H_D \rightarrow$ net horizontal reaction at D

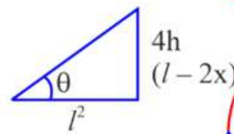
$\theta \rightarrow$ angle b/w the tangent at D and horizontal

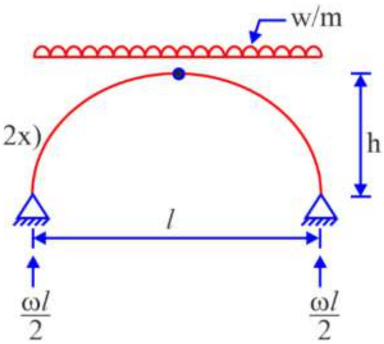
$p_n \rightarrow$ normal thrust or axial compression

$S \rightarrow$ radial SF.

- p_n is the resultant of H_D & V_D resolved in the direction of p_n
  $p_n = H_D \cos\theta + V_D \sin\theta$
- Radial shear, S is the resultant of H_D & V_D in the direction of s.
  $S = H_D \sin\theta - V_D \cos\theta$
- Prove that the shear force at any section of a 3-hinged parabolic arch subjected at well throughout is zero.

For parabolic arch at any section,





$$\frac{dy}{dx} = \frac{4h}{l^2}(1-2x) = \tan\theta$$

$$\sin\theta = \frac{4h(1-2x)}{\sqrt{l^4 + 16h^2(1-2x)^2}}$$

$$\cos\theta = \frac{l^2}{\sqrt{l^4 + lbh^2(1-2x)^2}}$$

$$S = H_D \sin\theta - V_D \cos\theta$$

$$= \frac{wl^2}{8h} \times \frac{4h(1-2x)}{\sqrt{l^4 + lbh^2(1-2x)^2}} - \frac{wl}{2}\frac{l^2}{\sqrt{l^4 + lbh^2(1-2x)^2}} + wx.\frac{l^2}{\sqrt{l^4 + 16h^2(1-2x)^2}}$$

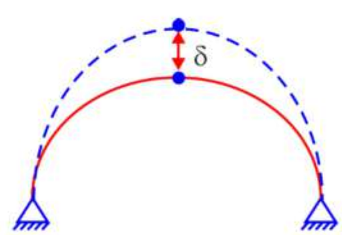
$$= 0$$

6.6. Effect of Temperature on 3 hinged Arches

As 3 hinged arch is statically determinate, no thermal stresses are developed. We know stresses depend upon BM. At a section. No stresses means no change in the moment of 3-hinged arch due to temperature change.

$$M = F_Z \Rightarrow F = \frac{M}{Z}$$

$$\partial = \left(\frac{l^2 + 4h^2}{4h}\right)\alpha T$$



As T ↑, y ↑, H ↓

$$M_{arch} = M_{beam} - Hy$$

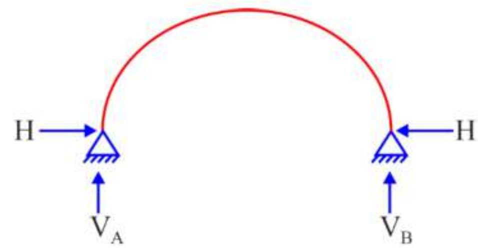
$$\frac{dH}{H} = \frac{-dh}{h}$$ –ve indicates that H and h vary in apposite directions.

A cantilever suby to temperature change uniformly or temperature gradient $\left(\frac{dT}{dy} = 0\right)$ zero, then the cantilever is free to elongate.

Here no resistance against deformation or no resistance against strain. No resistance means no strains

6.6.1. Two Hinged Arches

Assume supports of two hinged arch will not yield laterally. According to cartigliands theorem, if no deformation, assuming horizontal reaction as redundant,



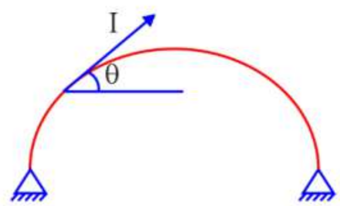
$$\frac{\partial U}{\partial H} = 0 \left(\frac{\partial U}{\partial R} = \partial\right)$$

$\Rightarrow$ $$H = \frac{\int M_y ds}{\int y^2 ds} \text{ ; M = beam moment}$$

$H = \frac{\int M_y ds}{\int y^2 ds}$ is useful for arches like 3–hinged arch with udl throughout. For unsymmetrical loads, numerator and denominator of above equation are not integrable.

In order to analyse it is assumed that, $I = I_o \sec\theta$ at any section I where IO is moment of inertia at the crown with this assumption,

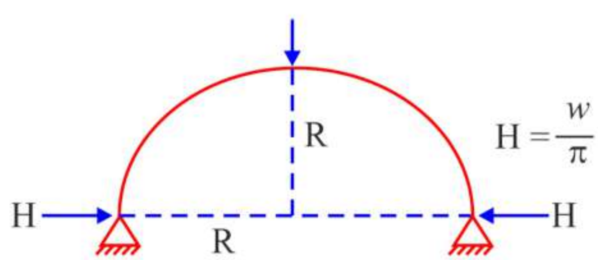
$$H = \frac{\int M \cdot y dx}{\int y^2 dx}$$



ILD for 2 hinged Arch:

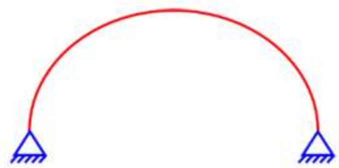
Two hinged semi circular Arches

(1) Point load at Crown.



(2) Temperature effect on 2 hinged Arches

M_{arch}	$= M_{beam}$	$- Hy$
(change)	(const)	(changes)



As there is no change at the crown, y won't change

But H changes

As T ↑, H↑

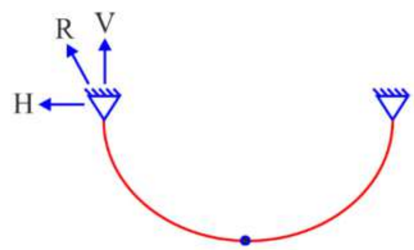
If temperature increases, H increases. If temperature increases, no change in the value of rise. Hence temperature will try to push the supports out. But they will not. In this process, H will increase. As H↑, Hy increases. M_{arch} decreases

Effects of Rib Shorbening in 2 hinged Arches

The effect of normal thrust in the arch is to shorten the rib of the arch and thus release part of horizontal thrust.

6.7. Cables

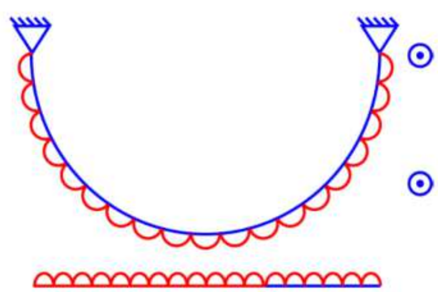
Assumptions:



(1) Cable is flexible

BM @ every point is zero.

(1) Self weight is neglected



- Load along the horizontal span-Shape of cable is parabola
- Udl is along the curve-shape of cable is catenary
- In chain surveying also, correction to sag is catenary