

## 8.6 Applications of Derivative

Functions:  $f, g, y$

Position of an object:  $s$

Velocity:  $v$

Acceleration:  $w$

Independent variable:  $x$

Time:  $t$

Natural number:  $n$

### 825. Velocity and Acceleration

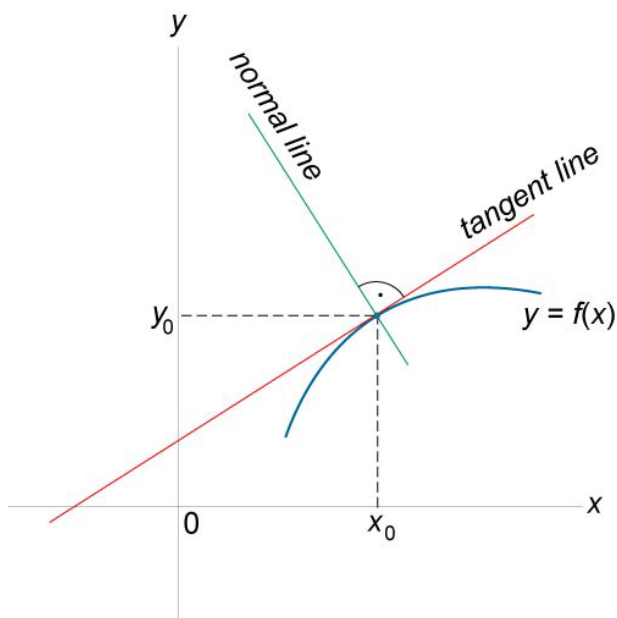
$s = f(t)$  is the position of an object relative to a fixed coordinate system at a time  $t$ ,

$v = s' = f'(t)$  is the instantaneous velocity of the object,

$w = v' = s'' = f''(t)$  is the instantaneous acceleration of the object.

### 826. Tangent Line

$$y - y_0 = f'(x_0)(x - x_0)$$



**Figure 176.**

**827. Normal Line**

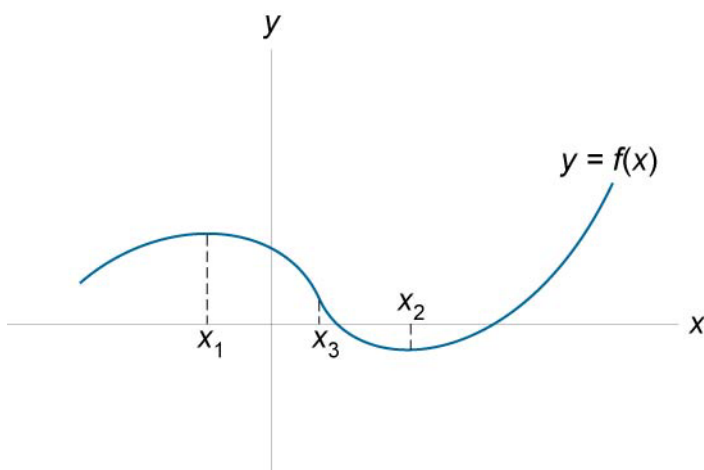
$$y - y_0 = -\frac{1}{f'(x_0)}(x - x_0) \quad (\text{Fig 176})$$

**828. Increasing and Decreasing Functions.**

If  $f'(x_0) > 0$ , then  $f(x)$  is increasing at  $x_0$ . (Fig 177,  $x < x_1$ ,  $x_2 < x$ ),

If  $f'(x_0) < 0$ , then  $f(x)$  is decreasing at  $x_0$ . (Fig 177,  $x_1 < x < x_2$ ),

If  $f'(x_0)$  does not exist or is zero, then the test fails.



**Figure 177.**

**829. Local extrema**

A function  $f(x)$  has a **local maximum** at  $x_1$  if and only if there exists some interval containing  $x_1$  such that  $f(x_1) \geq f(x)$  for all  $x$  in the interval (Fig.177).

A function  $f(x)$  has a **local minimum** at  $x_2$  if and only if there exists some interval containing  $x_2$  such that  $f(x_2) \leq f(x)$  for all  $x$  in the interval (Fig.177).

**830. Critical Points**

A critical point on  $f(x)$  occurs at  $x_0$  if and only if either  $f'(x_0)$  is zero or the derivative doesn't exist.

**831. First Derivative Test for Local Extrema.**

If  $f(x)$  is increasing ( $f'(x) > 0$ ) for all  $x$  in some interval  $(a, x_1]$  and  $f(x)$  is decreasing ( $f'(x) < 0$ ) for all  $x$  in some interval  $[x_1, b)$ , then  $f(x)$  has a local maximum at  $x_1$  (Fig.177).

**832.** If  $f(x)$  is decreasing ( $f'(x) < 0$ ) for all  $x$  in some interval  $(a, x_2]$  and  $f(x)$  is increasing ( $f'(x) > 0$ ) for all  $x$  in some interval  $[x_2, b)$ , then  $f(x)$  has a local minimum at  $x_2$ . (Fig.177).

**833.** Second Derivative Test for Local Extrema.

If  $f'(x_1) = 0$  and  $f''(x_1) < 0$ , then  $f(x)$  has a local maximum at  $x_1$ .

If  $f'(x_2) = 0$  and  $f''(x_2) > 0$ , then  $f(x)$  has a local minimum at  $x_2$ . (Fig.177)

**834.** Concavity.

$f(x)$  is concave upward at  $x_0$  if and only if  $f'(x)$  is increasing at  $x_0$  (Fig.177,  $x_3 < x$ ).

$f(x)$  is concave downward at  $x_0$  if and only if  $f'(x)$  is decreasing at  $x_0$ . (Fig.177,  $x < x_3$ ).

**835.** Second Derivative Test for Concavity.

If  $f''(x_0) > 0$ , then  $f(x)$  is concave upward at  $x_0$ .

If  $f''(x_0) < 0$ , then  $f(x)$  is concave downward at  $x_0$ .

If  $f''(x)$  does not exist or is zero, then the test fails.

**836.** Inflection Points

If  $f'(x_3)$  exists and  $f''(x)$  changes sign at  $x = x_3$ , then the point  $(x_3, f(x_3))$  is an **inflection point** of the graph of  $f(x)$ . If  $f''(x_3)$  exists at the inflection point, then  $f''(x_3) = 0$  (Fig.177).

**837.** L'Hopital's Rule

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)} \text{ if } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x) = \begin{cases} 0 \\ \infty \end{cases}.$$