PRACTICE PAPER 23

MATHEMATICS

Time allowed: 45 minutes **Maximum Marks: 200**

General Instructions: *As given in Practice Paper – 1.*

Section–A

Choose the correct option:

1. If $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$ then PQ is equal to

   (*a*) $\begin{bmatrix} -2 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ (*b*) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$ (*c*) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$ (*d*) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. The value of determinant $\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix}$ is

   (*a*) $a^3 + b^3 + c^3$ (*b*) $3abc$ (*c*) $a^3 + b^3 + c^3 - 3abc$ (*d*) None of these

3. If A is a square matrix of order 3 such that $|A| = 2$, then the value of $|adj(adj\ A)|$ is

   (*a*) -16 (*b*) 16 (*c*) 0 (*d*) 2

4. Value of $\frac{d^2y}{dx^2}$, if $x = at^2$, $y = 2at$ is

   (*a*) $\frac{-1}{2at^3}$ (*b*) $\frac{1}{2at^2}$ (*c*) $\frac{-1}{2at^2}$ (*d*) 0

5. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point

   (*a*) (1, 2) (*b*) (2, 1) (*c*) (1, – 2) (*d*) (–1, 2)

6. The value of $\int \tan^2 x\, dx$ equals

   (*a*) $\tan x + x + C$ (*b*) $\tan x - x + C$ (*c*) $\cot x + x + C$ (*d*) $-\tan x + x + C$

7. The value of $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}\, dx$ equals

8. **The value of** $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x\, dx$ **is**

(a) $\log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$ (b) $\log\sqrt{2}$ (c) $\log\left(\frac{\sqrt{2}+1}{2+\sqrt{3}}\right)$ (d) $\log\left(\frac{\sqrt{2}-1}{2+\sqrt{3}}\right)$

9. **The value of** $\int_{0}^{\frac{\pi}{4}} \frac{dx}{1+\cos 2x}$ **is**

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) 0 (d) $\frac{1}{4}$

10. **The area of the region bounded by the curve** $x^2 = 4y$ **and the straight line** $x = 4y - 2$ **is**

(a) $\frac{3}{8}$ sq. unit (b) $\frac{5}{8}$ sq. unit (c) $\frac{7}{8}$ sq. unit (d) $\frac{9}{8}$ sq. units

11. **The order and degree of the differential equation** $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{1/5} = 0$ **respectively, are**

(a) 2 and 4 (b) 2 and 2 (c) 2 and 3 (d) 3 and 3

12. **The solution of differential equation** $\tan y \sec^2 x\, dx + \tan x \sec^2 y\, dy = 0$ **is**

(a) $\tan x + \tan y = k$ (b) $\tan x - \tan y = k$ (c) $\frac{\tan x}{\tan y} = k$ (d) $\tan x.\tan y = k$

13. **Consider a LPP given by**

Min $Z = 6x + 10y$

subject to $x \geq 6;\ y \geq 2;\ 2x + y \geq 10;\ x, y \geq 0$

Redundant constraints in this LPP are

(a) $x \geq 0, y \geq 0$ (b) $x \geq 6, 2x + y \geq 10$

(c) $2x + y \geq 10$ (d) none of these

14. **The probability distribution of a discrete random variable** X **is given below:**

X	2	3	4	5
$P(X)$	1/2	1/3	1/4	1/5

then the value of $E[X]$ **is**

(a) 1 (b) 2 (c) 3 (d) 4

15. **For binomial distribution** $B(n, p)$ **mean is**

(a) npq (b) nq (c) np (d) none of these

Section–B (B1)

16. **"Every relation is a function and every function is a relation" then which is correct for given statement ?**

(a) True (b) False

(c) Can't say anything (d) None of these

17. **Let * be a binary operation defined on Q – {1} by the rule** $a * b = a + b - ab$**. Then the value of 2 * 3 is**

(a) 1 (b) – 1 (c) 2 (d) – 2

18. **If** $f : R \to R$ **be given by** $f(x) = \tan x$**, then** $f^{-1}(1)$ **is**

(a) $\pi/4$ (b) $\{n\pi + \pi/4 : n \in z\}$ (c) Does not exist (d) None of these

19. If $f(x) = x^2 - 1$ and $g(x) = 3x + 1$, then $g[f(x)]$ is

(a) $x^2 - 1$ (b) $2x^2 - 1$ (c) $3x^2 - 2$ (d) $2x^2 + 2$

20. If $f : R \to R$ is defined by $f(x) = 3x - 4$, then $f^{-1} : R \to R$ is

(a) $4 - 3x$ (b) $\frac{x+4}{3}$ (c) $\frac{1}{3x-4}$ (d) $\frac{3}{x+4}$

21. The value of $\sin [2 \tan^{-1} (0.75)]$ is

(a) 0.75 (b) 1.5 (c) 0.96 (d) 1.5

22. If $|x| \leq 1$, then $2 \tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

(a) $4 \tan^{-1} x$ (b) 0 (c) $\frac{\pi}{2}$ (d) π

23. If $\cos^{-1} x > \sin^{-1} x$, then

(a) $\frac{1}{\sqrt{2}} < x \leq 1$ (b) $0 \leq x < \frac{1}{\sqrt{2}}$ (c) $-1 \leq x < \frac{1}{\sqrt{2}}$ (d) $x > 0$

24. The value of $\cos^{-1} (2x^2 - 1)$, $0 \leq x \leq 1$ is equal to

(a) $2 \cos^{-1} x$ (b) $2 \sin^{-1} x$ (c) $\pi - 2 \cos^{-1} x$ (d) $\pi + 2 \cos^{-1} x$

25. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then $A^2 - 5A$ is

(a) I (b) 14 I (c) 0 (d) None of these

26. Which one of the following is correct?

(a) Skew - symmetric matrix of odd order is non-singular.

(b) Skew - symmetric matrix of odd order is singular.

(c) Skew - symmetric matrix of even order is always singular.

(d) None of these

27. The area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq. units. The value of k will be

(a) 9 (b) ± 3 (c) –9 (d) 6

28. The matrix A satisfying the equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 4 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) zero matrix

29. If the function $f(x)$ defined by

$$f(x) = \begin{cases} \dfrac{\log (1 + ax) - \log (1 - bx)}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$, then the value of k is

(a) a (b) b (c) $a + b$ (d) 0

30. Let $f(x) = |\sin x|$. Then,

(*a*) $f(x)$ is everywhere differentiable.

(*b*) $f(x)$ is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$.

(*c*) $f(x)$ is everywhere continuous but not differentiable at $x = (2n + 1)\frac{\pi}{2}$, $n \in Z$.

(*d*) None of these

31. If $x^y = y^x$, then $\frac{dy}{dx}$ is equal to

(*a*) $x \log x$ (*b*) $\frac{y}{x}\cdot\left(\frac{x\log y - y}{y\log x - x}\right)$ (*c*) 0 (*d*) None of these

32. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$, then $\frac{dy}{dx}$ is equal to

(*a*) $\frac{\cos x}{2y+1}$ (*b*) $\frac{\cos x}{2y-1}$ (*c*) 0 (*c*) None of these

33. The minimum value of x^x $(x > 0)$ is

(*a*) 1 (*b*) $e^{-1/e}$ (*c*) $\left(\frac{1}{e}\right)^e$ (*d*) none of these

34. The value of $\int \frac{x^2 + 5x + 1}{\sqrt{x}}dx$ equals

(*a*) $\frac{2}{5}x^{5/2} + \frac{2}{3} \times 5x^{3/2} - 2x^{1/2} + C$ (*b*) $\frac{2}{5}x^{5/2} - \frac{10}{3}x^{3/2} + 2x^{1/2} + C$

(*c*) $\frac{4}{5}x^{5/2} - \frac{10}{3}x^{3/2} + C$ (*d*) $\frac{2}{5}x^{5/2} + \frac{10}{3}x^{3/2} + 2x^{1/2} + C$

35. The value of $\int \log x\, dx$ is

(*a*) $x \log x - x + C$ (*b*) $\log x - x + C$ (*c*) $x \log x + x + C$ (*d*) $\log x + C$

36. The value of $\int_2^8 \frac{\sqrt{10 - x}}{\sqrt{x} + \sqrt{10 - x}}dx$ is equal to

(*a*) 3 (*b*) −3 (*c*) 5 (*d*) 0

37. The area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x-axis is

(*a*) 8π sq. units (*b*) 20π sq. units (*c*) 16π sq. units (*d*) 256π sq. units

38. The differential equation for $y = A \cos \alpha x + B \sin \alpha x$, where A and B are arbitrary constants is

(*a*) $\frac{d^2y}{dx^2} - \alpha^2 y = 0$ (*b*) $\frac{d^2y}{dx^2} + \alpha^2 y = 0$ (*c*) $\frac{d^2y}{dx^2} + \alpha y = 0$ (*d*) $\frac{d^2y}{dx^2} - \alpha y = 0$

39. The integrating factor of differential equation $(1 - x^2)\frac{dy}{dx} - xy = 1$ is

(*a*) $-x$ (*b*) $\frac{x}{1 + x^2}$ (*c*) $\sqrt{1 - x^2}$ (*d*) $\frac{1}{2}\log(1 - x^2)$

40. In the triangle ABC, which of the following is not true?

(*a*) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ (*b*) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

(*c*) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$ (*d*) $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

41. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following is incorrect?

(*a*) $\vec{b} = \lambda\vec{a}$, for some scalar λ.

(*b*) $\vec{a} = \pm\vec{b}$

(*c*) the respective components of $\vec{a}$ and $\vec{b}$ are proportional.

(*d*) both the vectors $\vec{a}$ and $\vec{b}$ will always have same direction, but different magnitudes.

42. If $\vec{a}$ is a nonzero vector of magnitude a and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector if

(*a*) $\lambda = 1$ (*b*) $\lambda = -1$ (*c*) $a = |\lambda|$ (*d*) $a = 1/|\lambda|$

43. Let the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

(*a*) $\pi / 6$ (*b*) $\pi / 4$ (*c*) $\pi / 3$ (*d*) $\pi / 2$

44. If a line makes angles α, β, γ with the positive directions of coordinate axes, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is

(*a*) 2 (*b*) 1 (*c*) – 1 (*d*) none of these

45. If the line makes an angle of $\frac{\pi}{4}$ with each of y and z axes, then the angle which it makes with x-axis is

(*a*) 0 (*b*) π (*c*) $\frac{\pi}{2}$ (*d*) $\frac{\pi}{4}$

46. Distance of the point (α, β, γ) from y-axis is

(*a*) β (*b*) $|\beta|$ (*c*) $|\beta| + |\gamma|$ (*d*) $\sqrt{\alpha^2 + \gamma^2}$

47. If the direction cosines of a line are k, k, k then

(*a*) $k > 0$ (*b*) $0<k<1$ (*c*) $k=1$ (*d*) $k = \frac{1}{\sqrt{3}}$ or $\frac{-1}{\sqrt{3}}$

48. If A and B be two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, then $P(A/B).P(A'/B)$ is equal to

(*a*) $\frac{12}{5}$ (*b*) $\frac{3}{8}$ (*c*) $\frac{31}{20}$ (*d*) $\frac{6}{25}$

49. Two events E and F are independent . If $P(E) = 0.3$ and $P(E \cup F) = 0.5$, then $P(E/F) - P(F/E)$ equals to

(*a*) $\frac{12}{7}$ (*b*) $\frac{31}{35}$ (*c*) $\frac{1}{70}$ (*d*) $\frac{1}{7}$

50. Three persons A, B and C, fire at a target in turn, starting with A. Their probability of hitting the target are 0.4, 0.3 and 0.2, respectively. The probability of two hits is

(*a*) 0.025 (*b*) 0.188 (*c*) 0.339 (*d*) 0.475