# 0293

Total No. of Questions - **30**
Total No. of Printed Pages - **4**

**Regd. No.**

## Part - III

# MATHEMATICS, Paper - IIB

### (English Version)

### MODEL QUESTION PAPER (FOR IPE 2020-21 ONLY)

***Time :*** **3 Hours** ***Max. Marks :*** **75**

**Note:** This question paper consists of three section A, B and C.

**Section - A**

**I. Very short answer type questions.**

**(i) Answer all questions.**

**(ii) Each question carries 2 marks.** **10×2=20**

1. If $ax^2 + bxy + 3y^2 - 5x + 2y - 3 = 0$ represents a circle, find the values of *a* and *b*. Also find its radius and centre.

2. State the necessary and sufficient condition for $lx + my + n = 0$ to be a normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

3. Find the angle between the circle $x^2 + y^2 - 12x - 6y + 41 = 0$ and $x^2 + y^2 + 4x + 6y - 59 = 0$.

4. Find the equation of the parabola whose focus is S(1, –7) and vertex is A(1, –2).

5. If the eccentricity of a hyperbola is $\frac{5}{4}$, then find the eccentricity of its conjugate hyperbola.

6. Evaluate: $\int \frac{1}{(x+3)\sqrt{x+2}}\,dx$

7. Evaluate: $\int \frac{\sin\left(\tan^{-1} x\right)}{1+x^2}\,dx$

Turn Over

8. Evaluate: $\int_0^1 \frac{x^2}{x^2+1}\,dx$

9. Evaluate: $\int_0^{\pi/2} \sec^4\theta\, d\theta$

10. Find the order and degree of the differential equation $\left(\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^3\right)^{6/5} = 6y$.

## Section - B

**II. Short answer type questions.** **5×4=20**

**(i) Answer any FIVE questions.**

**(ii) Each question carries four marks.**

11. Show that the tangent at (–1, 2) of the circle $x^2 + y^2 - 4x - 8y + 7 = 0$ touches the circle $x^2 + y^2 + 4x + 6y = 0$, also find its point of contact.

12. Find the angle between the tangents drawn from (3, 2) to the circle $x^2 + y^2 - 6x + 4y - 2 = 0$.

13. Find the equation of the circle passing through the points of intersection of the circles $x^2 + y^2 - 8x - 6y + 21 = 0$, $x^2 + y^2 - 2x - 15 = 0$ and (1, 2).

14. Find the lengths of major axis, minor axis, latus rectum, eccentricity of the ellipse $9x^2 + 16y^2 = 144$.

15. If the length of the latus rectum is equal to half of its minor axis of an ellipse in the standard form, then find the essentricity of the ellipse.

16. Find the centre, eccentricity, foci, length of latus rectum for the hyperbola $4x^2 - 9y^2 - 8x - 32 = 0$.

17. Evalute: $\int_0^{\pi/2} x^2 \sin x\, dx$

18. Evalute: $\int_0^{\pi/2} \frac{\cos^{5/2} x}{\sin^{5/2} x + \cos^{5/2} x}\,dx$

19. Solve : $\frac{dy}{dx} + 1 = e^{x+y}$

20. Solve $\tan y\, dx + \tan x\, dy = 0$

Turn Over

## Section - C

**III. Long Answer type questions.** **5×7=35**

**(i) Answer any FIVE questions.**

**(ii) Each question carries seven marks.**

21. Show that the points (1, 1), (–6, 0), (–2, 2) and (–2, –8) are concyclic.

22. Find the direct common tangents to the circles $x^2 + y^2 + 22x - 4y - 100 = 0$ ; $x^2 + y^2 - 22x + 4y + 100 = 0$ .

23. Show that the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ touch each other, also find the point of contact and common tangent at this point of contact.

24. Show that the common chord of the circles $x^2 + y^2 - 6x - 4y + 9 = 0$ and $x^2 + y^2 - 8x - 6y + 23 = 0$ is a diameter of the second circle and also find its length.

25. If $y_1, y_2, y_3$ are the y-coordinates of the vertices of the triangle inscribed in the parabola $y^2 = 4ax$, then show that the area of the triangle is $\frac{1}{8a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right|$ square units.

26. Evalute: $\int \frac{2x+5}{\sqrt{x^2 - 2x + 10}}\,dx$

27. Evalute: $\int \sqrt{3 + 8x - 3x^2}\,dx$

28. Find: $\int \frac{1}{5 - 2x^2 + 4x}\,dx$

29. Evalute: $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$

30. Solve: $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$