

THEORY

Differential equations with Variables Separable:

A first order, first deg. diff. eqn is of form.

$$\frac{dy}{dx} = F(x, y).$$

If $F(x, y)$ can be expressed as product of $g(x)h(y)$

where $g(x)$ is function of x and

$h(y)$ is function of y .

Then
$$\frac{dy}{dx} = g(x)h(y)$$

If $h(y) \neq 0$ then separating the variables, we get

$$\frac{1}{h(y)} dy = g(x) dx$$

$$\Rightarrow \int \frac{1}{h(y)} dy = \int g(x) dx.$$

Thus soln. of given diff. eqn will be of form

$$H(y) = G(x) + C$$

where $H(y) = \int \frac{1}{h(y)} dy$

$$G(x) = \int g(x) dx.$$

C : is arbitrary constant.

EXERCISE 9.4

For each of differential eqns from 1 to 10 find the general solution:

QNo.1

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

Sol.

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x} = \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = \tan^2 \frac{x}{2}$$

$$\frac{dy}{dx} = \sec^2 \frac{x}{2} - 1$$

$$\therefore dy = (\sec^2 \frac{x}{2} - 1) dx$$

$$\therefore \int dy = \int (\sec^2 \frac{x}{2} - 1) dx$$

$$\text{or } y = \frac{\tan \frac{x}{2}}{\frac{1}{2}} - x + C$$

$$\text{or } y = 2 \tan \frac{x}{2} - x + C$$

QNo.2

$$\frac{dy}{dx} = \sqrt{4 - y^2} \quad (-2 < y < 2)$$

Sol.

$$\frac{dy}{dx} = \sqrt{4 - y^2}$$

$$\text{or } \frac{1}{\sqrt{4 - y^2}} dy = dx$$

$$\text{or } \int \frac{1}{\sqrt{(2)^2 - (y)^2}} dy = \int dx$$

$$\text{or } \sin^{-1} \frac{y}{2} = x + C \quad \text{or } y = 2 \sin(x + C)$$

QNo.3

$$\frac{dy}{dx} + y = 1 \quad (y \neq 1)$$

Sol.

$$\frac{dy}{dx} = 1 - y \quad \text{or } \frac{dy}{dx} = -(y - 1)$$

$$\text{or } \frac{1}{y - 1} dy = -dx$$

$$\text{or } \int \frac{1}{y - 1} dy = - \int dx$$

$$\text{or } \log |y - 1| = -x + C$$

$$\text{or } |y - 1| = e^{(-x + C)} = e^{-x} \cdot e^C = C' e^{-x} \quad \text{where } C' = e^C$$

$$\therefore y = 1 + C'e^{-x} \quad \text{where } C' = \pm e^C$$

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QNo 4.

Sol.

$$\sec^2 x \tan y + \sec^2 y \tan x = 0$$

$$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$$

$$\Rightarrow \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0 \quad \left[\because \text{Dividing by } \tan x \tan y \right]$$

$$\Rightarrow \int \frac{\sec^2 x}{\tan x} \cdot dx + \int \frac{\sec^2 y}{\tan y} \cdot dy = 0$$

$$\Rightarrow \log |\tan x| + \log |\tan y| = C$$

$$\Rightarrow \log |\tan x \tan y| = C$$

$$\text{or } |\tan x \cdot \tan y| = e^C \quad \text{or } \tan x \cdot \tan y = \pm e^C = C' \text{ (say)}$$

QNo 5

Sol.

$$(e^x + e^{-x}) dy - (e^x - e^{-x}) dx = 0$$

$$(e^x + e^{-x}) dy - (e^x - e^{-x}) dx = 0$$

$$\text{or } (e^x + e^{-x}) dy = (e^x - e^{-x}) dx$$

$$\text{or } dy = \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$\text{or } \int dy = \int \frac{\frac{d}{dx}(e^x + e^{-x})}{e^x + e^{-x}} dx$$

$$\text{or } y = \log |e^x + e^{-x}| + C = \log [e^x + e^{-x}] + C$$

$$[\because e^x + e^{-x} \geq 2 \quad \forall x \in \mathbb{R}]$$

QNo. 6

Sol.

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$

$$\text{or } \frac{1}{1+y^2} dy = (1+x^2) dx$$

$$\text{or } \int \frac{1}{1+y^2} dy = \int x^2 dx + \int dx$$

$$\Rightarrow \tan^{-1} y = \frac{x^3}{3} + x + c$$

QNo. 7.

$$y \log y \, dx - x \, dy = 0$$

Sol.

$$y \log y \, dx = x \, dy$$

$$\text{or. } \frac{dx}{x} = \frac{1}{y \log y} \, dy = \frac{1/y}{\log y} \, dy$$

$$\text{or } \int \frac{1}{x} \, dx = \int \frac{1/y}{\log y} \, dy$$

$$\text{or } \log |x| = \log |\log y| + c$$

$$\text{or } \log |x| - \log |\log y| = c$$

$$\text{or } \log \left| \frac{x}{\log y} \right| = c \quad \text{or } \left| \frac{x}{\log y} \right| = e^c$$

$$\text{or } \frac{x}{\log y} = \pm e^c = c' (\text{say})$$

$$\Rightarrow x = c' \log y$$

$$\text{or } \log y = \frac{1}{c'}, x = c'' x \quad ; \text{ where } c'' = 1/c'$$

$$\text{or } y = e^{c'' x} \quad \text{where } c'' \neq 0 \text{ is arbitrary const.}$$

QNo. 8.

$$x^5 \frac{dy}{dx} = -y^5$$

Sol.

$$\text{Given diff. eqn is } x^5 \frac{dy}{dx} = -y^5$$

$$\text{or } \int y^{-5} \, dy = - \int x^{-5} \, dx$$

$$\text{or } \frac{y^{-4}}{-4} = - \frac{x^{-5+1}}{-5+1} + c$$

$$\text{or } x^{-4} + y^{-4} = -4c = c' (\text{say})$$

$$\text{or } x^{-4} + y^{-4} = c'$$

which is required general soln. of given diff. eqn.

Q No 9

$$\frac{dy}{dx} = \sin^{-1} x.$$

Soln : Given diff. eqn is $\frac{dy}{dx} = \sin^{-1} x$

$$\text{or } dy = \sin^{-1} x \, dx$$

$$\text{or } \int dy = \int \sin^{-1} x \, dx.$$

$$\text{or } y = \sin^{-1} x \cdot x - \int \frac{1}{\sqrt{1-x^2}} x \, dx + C \quad \left[\begin{array}{l} \text{Integrating by parts taking} \\ \sin^{-1} x \text{ as first function} \end{array} \right]$$

$$= x \sin^{-1} x + \frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}} \, dx + C$$

$$= x \sin^{-1} x + \frac{1}{2} \frac{(1-x^2)^{-1/2+1}}{-1/2+1} + C = x \sin^{-1} x + \sqrt{1-x^2} + C$$

Q No 10

$$e^x \tan y \, dx + (1-e^x) \sec^2 y \, dy = 0$$

Sol. Given differential eqn is $e^x \tan y \, dy + (1-e^x) \sec^2 y \, dy = 0$

$$\text{or } \frac{e^x}{1-e^x} \, dx + \frac{\sec^2 y}{\tan y} \, dy = 0 \quad \left[\begin{array}{l} \text{Dividing throughout by} \\ (1-e^x) \tan y \end{array} \right]$$

$$\Rightarrow \int \frac{e^x}{1-e^x} \, dx + \int \frac{\sec^2 y}{\tan y} \, dy = C.$$

$$\text{or } - \int \frac{e^x}{1-e^x} \, dx + \int \frac{\frac{d}{dy}(\tan y)}{\tan y} \, dy = C$$

$$\text{or } - \log |1-e^x| + \log |\tan y| = C$$

$$\text{or } \log \left| \frac{\tan y}{1-e^x} \right| = C \quad \text{or } \left| \frac{\tan y}{1-e^x} \right| = e^C$$

$$\text{or } \frac{\tan y}{1-e^x} = \pm e^C = C' \text{ (say)}$$

Find

Q No 11

or $\tan y = C'(1-e^x)$
particular solution of differential eqns from 11 to 14.

$$(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x \quad ; \quad y = 1 \text{ when } x = 0.$$

Sol. Given diff. eqn is $(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x$

$$\Rightarrow dy = \frac{2x^2 + x}{x^3 + x^2 + x + 1} \, dx.$$

$$\Rightarrow dy = \frac{2x^2+x}{(x^2+1)(x+1)} dx$$

Now. $\frac{2x^2+x}{(x^2+1)(x+1)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{x+1}$

$$\Rightarrow 2x^2+x = (Ax+B)(x+1) + C(x^2+1)$$

$$\Rightarrow 2x^2+x = (A+C)x^2 + (A+B)x + (B+C)$$

$$\Rightarrow A+C=2 \quad ; \quad A+B=1 \quad ; \quad B+C=0$$

Solving for A, B, C we get

$$A = \frac{3}{2} \quad ; \quad B = -\frac{1}{2} \quad ; \quad C = \frac{1}{2}$$

$$\therefore \int dy = \int \frac{2x^2+x}{(x^2+1)(x+1)} dx$$

$$\Rightarrow y = \int \frac{\frac{3}{2}x - \frac{1}{2}}{x^2+1} dx + \int \frac{\frac{1}{2}}{x+1} dx + C$$

$$= \frac{3}{2} \cdot \frac{1}{2} \int \frac{2x}{x^2+1} dx - \frac{1}{2} \int \frac{1}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x+1} dx + C$$

$$y = \frac{3}{4} \log|x^2+1| - \frac{1}{2} \tan^{-1}x + \frac{1}{2} \log|x+1| + C \quad \dots (1)$$

When $x=0$ $y=1$

$$\therefore 1 = \frac{3}{4} \log|1| - \frac{1}{2} \tan^{-1}(0) + \frac{1}{2} \log|1| + C$$

$$\Rightarrow 1 = 0 + C \quad \text{or} \quad C=1 \quad \dots (ii)$$

From (i) and (ii)

$$y = \frac{3}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1}x + \frac{1}{2} \log|x+1| + 1$$

Which is required particular soln. of given diff. eqn.

Q No 12 $x(x^2-1) \frac{dy}{dx} = 1$; $y=0$ when $x=2$.

Sol. The given diff. eqn is

$$x(x^2-1) \frac{dy}{dx} = 1 \quad \text{or} \quad dy = \frac{1}{x(x^2-1)} dx$$

Now Let $\frac{1}{x(x^2-1)} = \frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

$\therefore$ By resolving into partial fractions using short cut method.

$$\begin{aligned}\frac{1}{x(x-1)(x+1)} &= \frac{1}{x(-1)(+1)} + \frac{1}{(x-1)(1)(1+1)} + \frac{1}{(x+1)(-1)(-1-1)} \\ &= \frac{-1}{x} + \frac{1}{2(x-1)} + \frac{1}{2(x+1)}\end{aligned}$$

$$\therefore \int dy = \int \frac{1}{x(x^2-1)} dx$$

$$\begin{aligned}\Rightarrow y &= -\int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{1}{x+1} dx + C \\ &= -\log|x| + \frac{1}{2} \log|x-1| + \frac{1}{2} \log|x+1| + C\end{aligned}$$

$$y = \frac{1}{2} \log \left| \frac{(x-1)(x+1)}{x^2} \right| + C \quad \dots (1)$$

When $x=2$ $y=0$

$$\therefore 0 = \frac{1}{2} \log \left| \frac{(2-1)(2+1)}{(2)^2} \right| + C$$

$$0 = \frac{1}{2} \log \left(\frac{3}{4} \right) + C \quad \dots (2)$$

$\therefore$ from (1) and (2)

$$y = \frac{1}{2} \log \left| \frac{x^2-1}{x^2} \right| - \frac{1}{2} \log \left(\frac{3}{4} \right)$$

Which is required particular solution.

Q.No. 13 $\cos\left(\frac{dy}{dx}\right) = a$ ($a \in \mathbb{R}$); $y=2$ when $x=0$

Soln: The given diff. eqn is $\cos \frac{dy}{dx} = a$

$$\text{or } \frac{dy}{dx} = \cos^{-1} a$$

$$\text{or } dy = \cos^{-1} a \, dx$$

$$\Rightarrow \int dy = \int \cos^{-1} a \, dx$$

$$\Rightarrow y = \cos^{-1} a \cdot x + C \quad \dots (1)$$

Now When $x=0$ $y=2$

$$\Rightarrow 2 = C \quad \dots (2)$$

$\therefore$ From (1) and (2) $y = x \cos^2 a + 2$

$$\text{or } \cos\left(\frac{y-2}{x}\right) = a.$$

QNo 14

$$\frac{dy}{dx} = y \tan x ; y=1 \text{ when } x=0$$

Sol.

$$\text{Given diff eqn is } \frac{dy}{dx} = y \tan x$$

$$\Rightarrow \frac{dy}{y} = \tan x \, dx$$

$$\Rightarrow \int \frac{1}{y} \, dy = \int \tan x \, dx$$

$$\Rightarrow \log|y| = -\log|\cos x| + C \Rightarrow \log|y \cos x| = C \Rightarrow y \cos x = \pm e^C = C'(y)$$

Now When $x=0$, $y=1$

$$1 \cdot \cos 0 = C' \Rightarrow C' = 1$$

$\therefore y \cos x = 1$ which is required particular soln.

QNo 15 find the equation of curve passing through the point $(0,0)$ and whose diff eqn $y' = e^x \sin x$.

Sol.

$$\text{Given differentl eqn is } \frac{dy}{dx} = e^x \sin x$$

$$\Rightarrow dy = e^x \sin x \, dx$$

$$\Rightarrow \int dy = \int e^x \sin x \, dx \Rightarrow y = \int e^x \sin x \, dx \quad \dots (1)$$

$$\Rightarrow y = e^x [-\cos x] - \int \left(\frac{d}{dx}(e^x)(-\cos x) \right) dx \quad \left[\begin{array}{l} \text{Integrating by} \\ \text{part taking} \\ e^x \text{ as first fn} \end{array} \right]$$
$$= -e^x \cos x + \left[e^x \sin x - \int e^x \sin x \, dx \right] \quad (\text{again "})$$

$$y = e^x [\sin x - \cos x] - y + C' \quad (\text{from (1)})$$

$$\Rightarrow 2y = e^x [\sin x - \cos x] + C'$$

$$\Rightarrow y = \frac{1}{2} e^x [\sin x - \cos x] + C \quad \left[\text{where } C = \frac{C'}{2} \right]$$

As $(0,0)$ lies on curve

$$\therefore 0 = \frac{1}{2} e^0 [\sin 0 - \cos 0] \Rightarrow C = \frac{1}{2}$$

$$\therefore y = \frac{1}{2} e^x [\sin x - \cos x] + \frac{1}{2} \text{ which is required soln.}$$

QNo. 16. For the differential eqn. $xy \frac{dy}{dx} = (x+2)(y+2)$, find the soln. Curve passing through point. $(1, -1)$

Sol. Given diff. eqn is $xy \frac{dy}{dx} = (x+2)(y+2) \dots (1)$

$$\Rightarrow \frac{y}{y+2} dy = \frac{x+2}{x} dx$$

$$\Rightarrow \frac{y+2-2}{y+2} dy = \left(1 + \frac{2}{x}\right) dx$$

$$\Rightarrow \left(1 - \frac{2}{y+2}\right) dy = \left(1 + 2 \cdot \frac{1}{x}\right) dx$$

$$\Rightarrow \int 1 \cdot dy - 2 \int \frac{1}{y+2} dy = \int 1 dx + 2 \int \frac{1}{x} dx$$

$$\Rightarrow y - 2 \log |y+2| = x + 2 \log |x| + C \dots (2)$$

But $(1, -1)$ lies on this curve.

$$\therefore -1 - 2 \log |-1+2| = 1 + 2 \log |1| + C$$

$$\Rightarrow C = -2 \quad [\because \log 1 = 0]$$

$\therefore$ from (2)

$$y - 2 \log |y+2| = x + 2 \log |x| - 2$$

$$\text{or } y - x = 2 \log |(x)(y+2)| - 2.$$

QNo 17. find the eqn of a curve passing through the point $(0, -2)$ given that at any point (x, y) on the curve the product of slope of its tangent and y-coordinate of the point is equal to the x-coordinate of point.

Soln.

We are given $y \frac{dy}{dx} = x$. $\left[\because \text{slope of tangent at } (x, y) = \frac{dy}{dx} \right]$

$$\Rightarrow y \cdot dy = x dx$$

$$\Rightarrow \int y dy = \int x dx + C$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$$

Since $(0, -2)$ lies on curve.

$$\therefore \frac{(-2)^2}{2} = 0 + c \Rightarrow c = \frac{4}{2} = 2.$$

$$\therefore \frac{y^2}{2} = \frac{x^2}{2} + 2 \text{ or } x^2 - y^2 + 4 = 0$$

QNo. 18

At any point (x, y) of a curve, the slope of tangent is twice the slope of line segment joining the point of contact to the point $(-4, -3)$. Find the eqn of curve given that it passes through $(-2, 1)$

Sol.

Slope of segment of line joining (x, y) and $(-4, -3)$

$$= \frac{y - (-3)}{x - (-4)} = \frac{y+3}{x+4}.$$

$$\text{ATQ } \frac{dy}{dx} = 2 \left(\frac{y+3}{x+4} \right) \dots (1)$$

$$\Rightarrow \frac{dy}{y+3} = \frac{2}{x+4} dx.$$

$$\Rightarrow \int \frac{1}{y+3} dy = 2 \int \frac{1}{x+4} dx$$

$$\Rightarrow \log |y+3| = 2 \log |x+4| + c.$$

$$\Rightarrow \log \left| \frac{y+3}{(x+4)^2} \right| = c \Rightarrow \left| \frac{y+3}{(x+4)^2} \right| = e^c$$

$$\Rightarrow \frac{y+3}{(x+4)^2} = \pm e^c = c' \text{ (say)} \dots$$

$$\Rightarrow y+3 = c'(x+4)^2 \dots (2)$$

As $(-2, 1)$ lies on the curve.

$$\therefore 1+3 = c'(-2+4)^2$$

$$\Rightarrow 4 = 4c' \Rightarrow c' = 1$$

$$\therefore \text{From (2)} \quad y+3 = (x+4)^2.$$

QNo. 19

The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find radius of balloon after t seconds.

Sol. Let radius of the balloon at time t seconds be r .

$$\text{then } \frac{d}{dt} (\text{Volume}) = \text{Constant.}$$

$$\Rightarrow \frac{d}{dt} \left(\frac{4}{3} \pi r^3 \right) = k \text{ (say)}$$

$$\Rightarrow \frac{4}{3} \pi \cdot 3r^2 \frac{dr}{dt} = k$$

$$\Rightarrow 4\pi r^2 dr = k dt \quad \dots (1)$$

$$\Rightarrow 4\pi \int r^2 dr = k \int dt + C$$

$$\Rightarrow 4\pi \frac{r^3}{3} = kt + C \quad \dots (2)$$

When $t=0$, $r=3$

$$\therefore \text{from (2)} \cdot 4\pi \frac{3^3}{3} = k(0) + C$$

$\therefore$ from (2) and (3)

$$\frac{4\pi}{3} (r^3) = kt + 36\pi \quad \dots (3)$$

Also When $t=3$, $r=6$

$$\therefore \frac{4\pi}{3} (6)^3 = 3k + 36\pi \quad (\text{from (3)})$$

$$\Rightarrow 3k = 288\pi - 36\pi$$

$$\Rightarrow k = \frac{252\pi}{3} = 84\pi$$

$\therefore$ From (3) we get

$$\frac{4\pi}{3} r^3 = 84\pi t + 36\pi$$

$$\text{or } r^3 = \frac{3}{4\pi} (84\pi t + 36\pi) = 63t + 27$$

$$\therefore r = (63t + 27)^{1/3} \text{ or } [9(7t + 3)]^{1/3}$$

Q No 20

In a bank, Principal increases continuously at the rate of $r\%$ per year. Find the value of ' r ' if Rs 100 double itself in 10 years ($\log_e 2 = 0.6931$)

Sol.

Given that principal increases at rate of $r\%$ per annum. Let P be the principal at the end of t years.

$$\text{Then } \frac{dP}{dt} = r\% \text{ of } P = \frac{rP}{100}$$

$$\Rightarrow \frac{dP}{P} = \frac{r}{100} dt$$

$$\Rightarrow \log P = \frac{r}{100} t + c$$

$$\Rightarrow \log P = \frac{rt}{100} + c \quad \dots\dots (1)$$

When $t=0$, Let $P = P_0$ then

$$\log P_0 = 0 + c \Rightarrow c = \log P_0$$

$$\therefore \log P = \frac{r}{100} t + \log P_0$$

$$\Rightarrow \log P - \log P_0 = \frac{r}{100} t \Rightarrow \log \frac{P}{P_0} = \frac{r}{100} t \quad \dots\dots (2)$$

When $t=10$, $P = 2P_0$

$$\Rightarrow \log 2 = \frac{r(10)}{100}$$

$$\Rightarrow r = 10 \log 2 = 10 \times 0.6931 \Rightarrow r = 6.931$$

Q.No. 21

In a bank, principal increases continuously at rate of 5% per year. An amount of Rs 1000 is deposited with the bank, How much will it worth after 10 years. ($e^{0.5} = 1.648$)

Sol.

ATQ (According to question) $\frac{dP}{dt} = \frac{5}{100} P$

where P is amount at end of t years.

$$\Rightarrow \frac{dP}{P} = \frac{1}{20} dt \Rightarrow \int \frac{dP}{P} = \int \frac{1}{20} dt + c$$

$$\Rightarrow \log P = \frac{1}{20} t + c \quad \dots\dots (1)$$

When $t=0$, $P = 1000$

$$\Rightarrow \log 1000 = \frac{1}{20} \times 0 + c \Rightarrow c = \log 1000$$

$\therefore$ (1) becomes $\log P = \frac{1}{20} t + \log 1000$

$$\Rightarrow \log \frac{P}{1000} = \frac{1}{20} t$$

When $t = 10$ then.

$$\log \frac{P}{1000} = \frac{1}{20} \times 10 = \frac{1}{2}$$

$$\Rightarrow \frac{P}{1000} = e^{1/2} \Rightarrow P = 1000 \times e^{1/2} = 1000 \times 1.648$$

$$\Rightarrow P = 1648$$

QNo 22. In a culture, the bacteria count is 100000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

Sol. Let P be the count of the bacteria at the end of t hours.

$$\text{Then } \frac{dP}{dt} \propto P \Rightarrow \frac{dP}{dt} = kP$$

where k is constant of proportionality.

$$\Rightarrow \frac{dP}{P} = k dt$$

$$\Rightarrow \int \frac{dP}{P} = k \int dt + C$$

$$\Rightarrow \log P = kt + C \quad \dots (1)$$

$$\text{When } t=0, P=100000$$

$$\text{and When } t=2$$

$$P = 100000 + \frac{10}{100}(100000) = 110000$$

$$\therefore \log 100000 = C \quad \dots (2)$$

$$\text{and } \log 110000 = 2k + C \quad \dots (3)$$

Subtracting (2) from (3), we get

$$\log \frac{110000}{100000} = 2k \Rightarrow k = \frac{1}{2} \log(1.1)$$

$\therefore$ from (1)

$$\log P = \frac{1}{2} \log(1.1) t + \log 100000$$

$$\text{When } P = 200000$$

$$\log 200000 = \frac{1}{2} \log(1.1) t + \log 100000$$

$$\Rightarrow 2 \log\left(\frac{200000}{100000}\right) = \log(1.1) t$$

$$\Rightarrow t = \frac{2 \log 2}{\log 1.1} = 2 \cdot \log_{1.1} 2 = \frac{2 \log_{10} 2}{2 \log_{10} (1.1)}$$

$$\Rightarrow t = \frac{2 \times 0.3010}{0.0414} \approx 14.545$$

QNo. 23 The general Soln. of differential equation

$$\frac{dy}{dx} = e^{x+y} \text{ is}$$

(A) $e^x + e^{-y} = c$ (B) $e^x + e^y = c$ (C) $e^{-x} + e^y = c$ (D) $e^x + e^{-y} = c$

Sol : Given $\frac{dy}{dx} = e^{x+y} = e^x \cdot e^y$

$$\Rightarrow \frac{dy}{e^y} = e^x dx.$$

$$\Rightarrow \int e^{-y} dy = \int e^x dx.$$

$$\Rightarrow \frac{e^{-y}}{-1} = e^x + c'$$

$$\Rightarrow e^x + e^{-y} = -c' = c \text{ (say)}$$

$\therefore$ option (A) is correct option.

##

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