SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	2(2)	1(2)*	1(3)	–	**4(7)**
2.	Inverse Trigonometric Functions	1(1)	–	–	–	**1(1)**
3.	Matrices	2(2)	–	–	–	**2(2)**
4.	Determinants	1(1)*	1(2)	–	1(5)*	**3(8)**
5.	Continuity and Differentiability	1(1)*	1(2)	2(6)	–	**4(9)**
6.	Application of Derivatives	1(1)	2(4)	1(3)	–	**4(8)**
7.	Integrals	2(2)#	1(2)*	1(3)*	–	**4(7)**
8.	Application of Integrals	–	1(2)	1(3)	–	**2(5)**
9.	Differential Equations	1(1)	1(2)*	1(3)*	–	**3(6)**
10.	Vector Algebra	1(1) + 1(4)	–	–	–	**2(5)**
11.	Three Dimensional Geometry	2(2)#	1(2)	–	1(5)*	**4(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	2(2)# + 1(4)	1(2)	–	–	**4(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours | ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Find the number of discontinuous functions $y(x)$ on [– 2, 2] satisfying $x^2 + y^2 = 4$.

OR

If $y = \sqrt{\sin x + y}$, then find $\dfrac{dy}{dx}$.

2. If matrix $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = pA$, then find the value of p.

3. Evaluate : $\displaystyle\int \frac{\cot x}{\sqrt[3]{\sin x}}\,dx$

OR

Evaluate : $\displaystyle\int_0^1 (x^2 + 2x + 5)\,dx$

4. $\vec{a}, \vec{b}, \vec{c}$ are three vectors, such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=1$, $|\vec{b}|=2$, $|\vec{c}|=3$, then find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

5. Given that, the events A and B are such that $P(A)=\dfrac{1}{2}$, $P(A\cup B)=\dfrac{3}{5}$ and $P(B)=p$. Then find the value of p, if A and B are mutually exclusive events.

OR

If E and F are event such that $0 < P(F) < 1$, then show that $P(E|F) + P(\overline{E}|F) = 1$.

6. Determine the degree and order of the differential equation $\dfrac{d^5y}{dx^5}+e^{dy/dx}+y^2=0$

7. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then find the value of x.

OR

If $A=\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$, then show that A^2 is a null matrix.

8. If x is real, then find the minimum value of $f(x) = x^2 - 8x + 17$.

9. Find the shortest distance between the lines $\dfrac{x}{m_1}=\dfrac{y}{1}=\dfrac{z-a}{0}$, $\dfrac{x}{m_2}=\dfrac{y}{1}=\dfrac{z+a}{0}$.

OR

If the lines $\dfrac{x-2}{1}=\dfrac{y-9}{2}=\dfrac{z-13}{3}$ and $\dfrac{x-a}{1}=\dfrac{y-1}{-2}=\dfrac{z+2}{3}$ are coplanar, then find the value of a.

10. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then find the values of x and y.

11. Using properties of definite integrals, prove that $\displaystyle\int_0^{\pi} \frac{x\tan x}{\sec x \operatorname{cosec} x}\,dx=\frac{\pi^2}{4}$.

12. If a line has the direction ratios 4, –12, 18, then find its direction cosines.

13. Find the value of $\tan^{-1}(1) + \tan^{-1}(0) - \tan^{-1}(\sqrt{3})$.

14. Let R be a relation on N defined by $x + 2y = 8$. Find the domain of R.

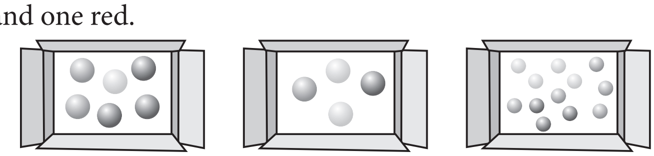
15. If $P(A)=\dfrac{3}{10}, P(B)=\dfrac{2}{5}$ and $P(A\cup B)=\dfrac{3}{5}$, then find the value of $P(B|A)$.

16. Find the number of reflexive relations on a set having 6 elements.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. Box A contain 1 white, 2 black and 3 red balls. Box B contain 2 white, 1 black and 1 red ball. Box C contain 4 white, 5 black and 3 red balls. One box is chosen at random and two balls are drawn with replacement. These happen to be one white and one red.



If E_1, E_2, E_3 be the events that the balls drawn from box A, box B and box C respectively and E be the event that ball drawn are one white and one red.

Based on the above information answer the following questions.

(i) Probability that the ball drawn are one white and one red, given that the balls are from box A, is

(a) $\frac{1}{6}$ (b) $\frac{2}{6}$ (c) $\frac{3}{5}$ (d) $\frac{1}{7}$

(ii) Probability that the ball drawn are one white and one red, given that the balls are from box B, is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{3}{4}$

(iii) Probability that the ball drawn are one white and one red, given that balls are from box C, is

(a) $\frac{1}{12}$ (b) $\frac{3}{11}$ (c) $\frac{1}{6}$ (d) $\frac{4}{11}$

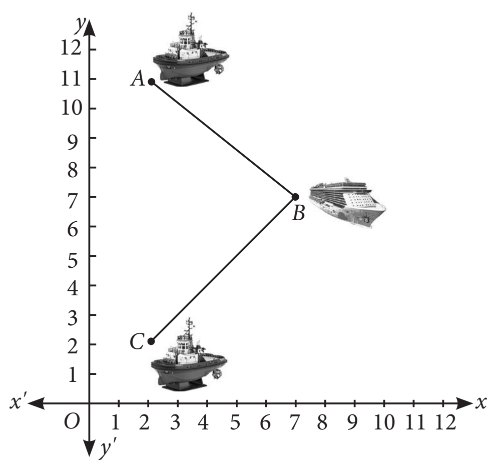
(iv) The value of $\sum_{i=1}^{3} P(E \mid E_i)$ is equal to

(a) $\frac{5}{7}$ (b) $\frac{2}{7}$ (c) $\frac{1}{12}$ (d) $\frac{7}{12}$

(v) The probability that the ball drawn are from box B, it is being given that the balls drawn are one white and one red, is

(a) $\frac{3}{7}$ (b) $\frac{4}{7}$ (c) $\frac{5}{7}$ (d) $\frac{1}{7}$

18. A ship is pulled into harbour by two tug boats as shown in the figure.



Based on the above information, answer the following questions :

(i) Position vector of A is

(a) $11\hat{i}+2\hat{j}$ (b) $2\hat{i}+11\hat{j}$ (c) $2\hat{i}-11\hat{j}$ (d) $11\hat{i}-2\hat{j}$

(ii) Position vector of B is

(a) $4\hat{i}+4\hat{j}$ (b) $6\hat{i}+6\hat{j}$ (c) $7\hat{i}+7\hat{j}$ (d) $3\hat{i}+3\hat{j}$

(iii) Find the vector $\overrightarrow{AC}$ in terms of $\hat{i}, \hat{j}$.

(a) $9\hat{j}$ (b) $-9\hat{j}$ (c) $9\hat{i}$ (d) None of these

(iv) If $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$, then its unit vector is

(a) $\frac{1\hat{i}}{\sqrt{14}}+\frac{2\hat{j}}{\sqrt{14}}+\frac{3\hat{k}}{\sqrt{14}}$ (b) $\frac{3\hat{i}}{\sqrt{14}}+\frac{2\hat{j}}{\sqrt{14}}+\frac{\hat{k}}{\sqrt{14}}$ (c) $\frac{2\hat{i}}{\sqrt{14}}+\frac{3\hat{j}}{\sqrt{14}}+\frac{\hat{k}}{\sqrt{14}}$ (d) None of these

(v) If $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=3\hat{i}+2\hat{j}$, then $|\vec{A}|=|\vec{B}|=$ _______ .

(a) $\sqrt{5}$ (b) $\sqrt{7}$ (c) $\sqrt{11}$ (d) $\sqrt{13}$

PART - B

Section - III

19. If $y = 3\cos(\log x) + 4\sin(\log x)$, then show that $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$.

20. Solve the differential equation $\dfrac{dy}{dx} = x\log x$.

OR

Find the solution of the differential equation $x\sin\left(\dfrac{y}{x}\right)\dfrac{dy}{dx} = y\sin\left(\dfrac{y}{x}\right) - x$.

21. Find the maximum and the minimum values (if any) of $f(x) = x^2$.

22. Find the area bounded by the curve $y = x^2$ and the line $y = 3x$.

23. Prove that the inverse of an equivalence relation is also an equivalence relation.

OR

Using principal value, find the value of $\cos^{-1}\left(\cos\dfrac{13\pi}{6}\right)$.

24. The cartesian equation of a line is $3x + 1 = 6y - 2 = 1 - z$. Find the fixed point through which it passes, its direction ratios and also its vector equation.

25. Two balls are drawn one after another (without replacement) from a bag containing 2 white, 3 red and 5 blue balls. What is the probability that atleast one ball is red?

26. Find the interval in which $f(x) = x^3 - 3x^2 - 9x + 2$ is increasing.

27. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$, then find A^{-1}.

28. Evaluate : $\displaystyle\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)}\,dx$

OR

Evaluate : $\displaystyle\int_1^2 \frac{\sqrt{x}-\sqrt{3-x}}{1+\sqrt{x(3-x)}}\,dx$

Section - IV

29. For what value of λ, the function defined by $f(x) = \begin{cases} \lambda(x^2+2), & \text{if } x \le 0 \\ 4x+6, & \text{if } x > 0 \end{cases}$ is continuous at $x = 0$? Hence check the differentiability of $f(x)$ at $x = 0$.

30. Draw the graph of $y = |x + 1|$ and using integration, find the area below $y = |x + 1|$, above x - axis and between $x = -4$ to $x = 2$.

31. Evaluate : $\displaystyle\int \frac{\cos x}{(1-\sin x)(2-\sin x)}\,dx$

OR

Evaluate : $\displaystyle\int_0^{\pi/4} 2\tan^3 x\,dx$

32. The perimeter of a triangle is 10 cm. If one of the side is 4 cm, then what are the two sides of the triangle for its maximum area?

33. If $y = \cot^{-1}\left[\dfrac{x - \log x^{x^2}}{\log e^{x^2} + \log x^x}\right]$, then find $\dfrac{dy}{dx}$.

34. Find the solution of the differential equation $y - x\dfrac{dy}{dx} = 2\left(1 + x^2\dfrac{dy}{dx}\right)$ given that $y = 1$ when $x = 1$.

OR

Solve the following initial value problem : $(xe^{y/x} + y)dx = xdy, y(1) = 1$

35. Check whether the function f from the set of natural numbers to integers is defined by
$$f(n) = \begin{cases} \dfrac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\dfrac{n}{2}, & \text{when } n \text{ is even} \end{cases}$$ is one-one and onto or not.

Section - V

36. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$, then find the product AB and use this result to solve the following system of linear equations $2x - y + z = -1$, $-x + 2y - z = 4$ and $x - y + 2z = -3$.

OR

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, then find A^{-1}. Hence, solve the system of equations $x + y + z = 6$, $x + 2z = 7$, $3x + y + z = 12$.

37. Find the equation of plane passing through the points (3, 4, 1) and (0, 1, 0) and parallel to the line $\dfrac{x+3}{2} = \dfrac{y-3}{7} = \dfrac{z-2}{5}$.

OR

Find the image of the point (2, –1, 5) in the line $\dfrac{x-11}{10} = \dfrac{y+2}{-4} = \dfrac{z+8}{-11}$. Also, find the equation of the line joining the given point and its image. Find the length of that line segment also.

38. Determine graphically the minimum value of the objective function

$Z = 4x_1 + 5x_2$

Subject to the constraints :

$2x_1 + x_2 \geq 7;$

$2x_1 + 3x_2 \leq 15;$

$x_2 \leq 3;$

$x_1, x_2 \geq 0.$

OR

Solve the following LPP graphically.

Maximize $Z = x + 2y$

Subject to the constraints :

$x + 2y \geq 100$

$2x - y < 0$

$2x + y \leq 200$

$x, y \geq 0.$