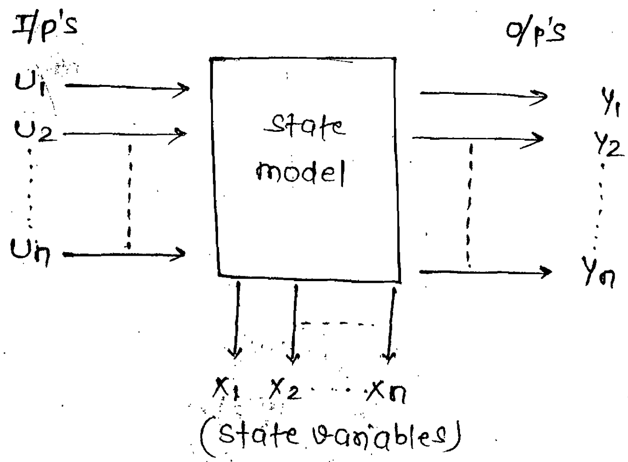
Chaper-(05)
State space variables



(1.) state eq^n →

$$\dot{X}(t) = A X(t) + B U(t)$$

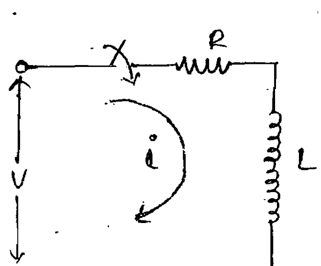
$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \vdots \\ \dot{X}_n \end{bmatrix} = \begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} + \begin{bmatrix} & & \\ & B & \\ & & \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}$$

(2.) O/p eq^n →

$$Y(t) = C X(t) + D U(t)$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} & & \\ & C & \\ & & \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} + \begin{bmatrix} & & \\ & D & \\ & & \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}$$

Ex:-



at $t = 0^-$

$i(0)^- = i_L(0)^- = 0$ Amps

At $t = 0^+$

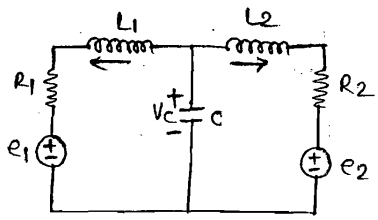
$$i_L(0)^+ = \frac{1}{L}\int_{t=0}^{0^+} V\,dt = 0 \text{ Amps}$$

$$V = iR + L\frac{di}{dt}$$

$$\boxed{\frac{di}{dt} = \frac{-R}{L}i + \frac{V}{L} \quad (A/s)}$$

state eqⁿ of N/ω.

$\boxed{\text{Conv(1)} \atop 80}$



Loop-(1)

$$L_1\frac{di_1}{dt} + i_1R_1 + e_1 - V_c = 0$$

$$\frac{di_1}{dt} = \frac{-R_1}{L_1}i_1 + \frac{V_c}{L_1} - \frac{e_1}{L_1} \text{ ------(i)}$$

Loop(2)

$$L_2\frac{di_2}{dt} + i_2R_2 + e_2 - V_c = 0$$

$$\frac{di_2}{dt} = \frac{-R_2}{L_2}i_2 + \frac{V_c}{L_2} - \frac{e_2}{L_2} \text{ -------(ii)}$$

KCL at Node V_c →

$$i_1 + i_2 + c\frac{dV_c}{dt} = 0$$

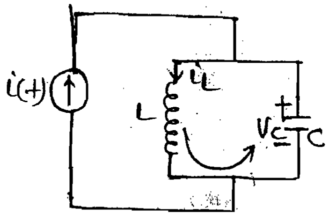
$$\frac{dV_c}{dt} = \frac{-i_1}{c} - \frac{i_2}{c} \text{ -----(iii)}$$

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R_1}{L_1} & 0 & \frac{1}{L_1} \\ 0 & \frac{-R_2}{L_2} & \frac{1}{L_2} \\ \frac{-1}{C} & \frac{-1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{-1}{L_1} & 0 \\ 0 & \frac{-1}{L_2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Let O/p $y = i_1$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} + 0$$

(b.)



$i(t) = i_L + i_C$

$i(t) = i_L + \frac{C\,dV_C}{dt}$

$\frac{dV_C}{dt} = \frac{-i_L}{C} + \frac{i(t)}{C}$ ----(i)

$\frac{L\,di_L}{dt} - V_C = 0 \Rightarrow \frac{di_L}{dt} = \frac{V_C}{L}$ ---(ii)

* Type-(1.) Problem →

To obtain state model from differential eqn

$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 10y = 20u$$

$$[s^3 + 4s^2 + 6s + 10]\,Y(s) = 20\,U(s)$$

Let $y = x_1$

$\frac{dy}{dt} = \dot{x}_1 = x_2$; $\frac{d^2y}{dt^2} = \dot{x}_2 = x_3$; $\frac{d^3y}{dt^3} = \dot{x}_3$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = 20u - 4x_3 - 6x_2 - 10x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

$\boxed{\text{BUSH/ COMPANION FORM}}$

* Shortcut method for BUSH form →

(7/80)

$$\frac{Y(s)}{U(s)} = \frac{1}{s^4+5s^3+8s^2+6s+3}$$

← Reverse order with rev. sign

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & -6 & -8 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

constant numerator.

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Que.→ $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 9y = 2\frac{du}{dt} + u$

Soln→ $(s^2+7s+9)\,Y(s) = (2s+1)\,U(s)$

$$\frac{Y(s)}{U(s)} = \frac{(2s+1)}{s^2+7s+9}$$

Que.(8.) phase variable method →

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2+7s+9} \quad (2s+1)$$

← only rev order *

* Rev order with rev sign

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1^{*} & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Ex:-

$$\frac{Y(s)}{U(s)} = \frac{10(s^2+2s+4)}{s^3+6s^2+8s+12}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -8 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = \begin{bmatrix} 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

* Type (2) problem →

To obtain TF from state model.

$$\dot{X}(t) = AX(t) + BU(t) \text{ ---- (i)}$$

$$Y(t) = CX(t) + DU(t)$$

Applying LT on eqn (i)

$$sX(s) - X(0) = AX(s) + BU(s)$$

$$Y(s) = CX(s) + DU(s)$$

For TF $X(0) = 0$

$$sX(s) - AX(s) = BU(s)$$

$$(sI - A)X(s) = BU(s)$$

$$X(s) = (sI-A)^{-1}B \cdot U(s)$$

$$Y(s) = [C(sI-A)^{-1}B + D]\,U(s)$$

$$\frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D$$

(Transfer matrix)

Q.(9.) →

$$\dot{X} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u.$$

$$y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T \cdot x$$

Solⁿ →

$$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (S+1) & 0 \\ 0 & (S+2) \end{bmatrix}$$

$$adj(\cancel{SI-A}) = \begin{bmatrix} (S+2) & 0 \\ 0 & (S+1) \end{bmatrix}$$

$$|(SI-A)| = (S+1)(S+2)$$

$$(SI-A)^{-1} = \begin{bmatrix} \frac{1}{S+1} & 0 \\ 0 & \frac{1}{S+2} \end{bmatrix}$$

$$(SI-A)^{-1} \cdot B = \begin{bmatrix} \frac{1}{S+1} & 0 \\ 0 & \frac{1}{S+2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{S+1} \\ 0 \end{bmatrix}$$

$$C(SI-A)^{-1} \cdot B \Rightarrow \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{S+1} \\ 0 \end{bmatrix}$$

$$\boxed{\left[\frac{Y(S)}{U(S)}\right] = \frac{1}{S+1}}$$

Q.(2.)

$$\dot{X}(t) = -2X(t) + 2U(t)$$

$$Y(t) = 0.5X(t)$$

Solⁿ →

$$SX(S) = -2X(S) + 2U(S)$$

$$(S+2)X(S) = 2U(S)$$

$$X(S) = \frac{2}{S+2} U(S)$$

$$Y(s) = 0.5X(s)$$

$$Y(s) = 0.5 \times \frac{2}{s+2} U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{1}{s+2}}$$

*** Type(3.) Problem → Stability for sm →**

$$\frac{Y(s)}{U(s)} = C[SI-A]^{-1}B + D$$

$$= C \cdot \frac{Adj(SI-A) \cdot B}{|SI-A|} + D$$

$$\frac{Y(s)}{U(s)} = \frac{C\,Adj(SI-A) \cdot B + |SI-A| \cdot D}{|SI-A|}$$

Zeros → $C \cdot Adj(SI-A) \cdot B + |SI-A| \cdot D$

Poles → $(1+G(s) \cdot H(s) = 0) = |SI-A| = 0$

$$\boxed{\text{Eigen values of sys. matrix } [A] = \text{CL poles}}$$

$\boxed{\text{Conv}(2)\ 80}$

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$C = \begin{bmatrix} -17 & -5 \end{bmatrix} X + [1]\, r$$

Soln →

$$(SI-A) = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} S & -1 \\ 20 & S+9 \end{bmatrix}$$

$$Adj(SI-A) = \begin{bmatrix} S+9 & 1 \\ -20 & S \end{bmatrix}$$

$$\text{Adj}(SI-A)B \Rightarrow \begin{bmatrix} s+9 & 1 \\ -20 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$C \cdot \text{Adj}(SI-A) \cdot B \Rightarrow \begin{bmatrix} -17 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ s \end{bmatrix} = -17-5s$$

$$|SI-A| = s(s+9)+20$$

$$|SI-A|\,\cancel{D} = (s^2+9s+20)(1)$$

zeros $\Rightarrow -17-5s+s^2+9s+20=0$

$$s^2+4s+3=0$$

$$s=-1,-3$$

poles $\Rightarrow |SI-A|=0$

$$s^2+9s+20=0$$

$s=-4,-5$ (sys. is stable because of -ve eigen values)

$\boxed{\text{Que.(6)} \; 80}$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \qquad u = \begin{bmatrix} -0.5 & -3 & -5 \end{bmatrix} x + v$$

Sol$^n \Rightarrow$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times1} \begin{bmatrix} -0.5 & -3 & -5 \end{bmatrix}_{1\times3} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.5 & -3 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$\boxed{\text{The sum of principle diagonal elements} = \text{Sum of eigen values}}$

ans (9).

* Type (04) Diagonalization →

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

Let $X = PZ$ ⇒ transformation matrix

$$P\dot{Z} = APZ + BU$$
$$Y = CPZ + DU$$

$$\boxed{\begin{aligned} \dot{Z} &= [P^{-1}AP]Z + [P^{-1}B]u \\ Y &= CPZ + DU \end{aligned}}$$

$$P = \text{Vander mode matrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \cdots & \lambda_n^2 \end{bmatrix}$$

$P^{-1}AP$ ⇒ diagonal matrix with diagonal elements as eigen values.

Ex:→ Diagonalize

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} S & -1 \\ 6 & S+5 \end{bmatrix}$$

$$|SI - A| = S(S+5) + 6 = 0$$
$$S^2 + 5S + 6 = 0$$
$$S = -2, -3$$

$$P = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$$

$$P^{-1} = \frac{Adj(P)}{|P|} = \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$$

* TYPE (05) Controllability & observability →

Controllability → To control the state variables

observability → To measure the state variables.

KALMAN'S Test →

$$Q_c = [B \quad AB \quad A^2B \quad A^{n-1}B] = |Q_c| \neq 0$$

$$Q_o = [C^T \quad A^TC^T \quad (A^T)^2C^T \cdots (A^T)^{n-1}C^T] = |Q_o| \neq 0$$

Q.1 → $\dot{X} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = [1 \quad 1] X$

Sol^n → $Q_c = [B \quad AB]$

$AB = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$Q_c = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} = -1$

$Q_o = [C^T \quad A^TC^T]$

$A^TC^T = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$Q_o = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} = -4$

Q.(10) $\dot{X} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad y = [b \quad 0] X$

Sol^n → $A^TC^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 2b \end{bmatrix}$

$Q_o = \begin{bmatrix} b & b \\ 0 & 2b \end{bmatrix} = 2b^2 \neq 0$

ans.(c)

* Type(6) Solution of State eqn →

$$\dot{X}(t) = AX(t) + Bu(t)$$

(a) Free Responce → $U(t) = 0$

$$\dot{X}(t) = AX(t)$$

$$\dot{X}(t) - AX(t) = 0 \quad \text{-----(i)}$$

$$X(t) = ke^{At} \quad \text{-----(ii)}$$

Applying LT to eqn (i)

$$sX(s) - X(0) - AX(s) = 0$$

$$sX(s) - AX(s) = X(0)$$

$$(sI - A)X(s) = X(0)$$

$$X(s) = (sI - A)^{-1} \cdot X(0)$$

$$\phi(s) = (sI - A)^{-1} = \text{Resolvant matrix}$$

$$X(t) = \left[\mathcal{L}^{-1}(sI - A)^{-1}\right] X(0)$$

$$X(t) = e^{At} \qquad k$$

$$\boxed{\begin{array}{l}\phi(t) = e^{At} = \mathcal{L}^{-1}(sI - A)^{-1} \\ \text{State transition matrix}\end{array}}$$

(b) Forced Responce →

$$sX(s) - X(0) = AX(s) + BU(s)$$

$$sX(s) - AX(s) = X(0) + BU(s)$$

$$(sI - A)X(s) = X(0) + BU(s)$$

$$X(s) = (sI - A)^{-1}X(0) + (sI - A)^{-1}B \cdot U(s)$$

$$X(t) = \left[\mathcal{L}^{-1}(sI - A)^{-1}\right]X(0) + \mathcal{L}^{-1}\left[(sI - A)^{-1}BU(s)\right]$$

Que.(3) → $\dot{X} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Sol^n →

$\boxed{\begin{array}{l} \text{ZIR (zero impulse responce)} \\ \text{short cut method} \\ \text{At } t=0 \\ X(t) = X(0) \end{array}}$

$SI-A \Rightarrow \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (S-1) & 0 \\ -1 & (S-1) \end{bmatrix}$

$(SI-A)^{-1} = \begin{bmatrix} \frac{1}{(S-1)} & 0 \\ \frac{1}{(S-1)^2} & \frac{1}{(S-1)} \end{bmatrix}$

at $t=0$

$X(t) = X(0)$, $e^{A(t)} = I$

$e^{At} = \phi(t) = \mathcal{L}^{-1}(SI-A)^{-1}$

$S.T.M. \Rightarrow \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$

$\boxed{\begin{array}{l} \text{property of STM} \\ \text{At } t=0, \ e^{A0} = I \end{array}}$

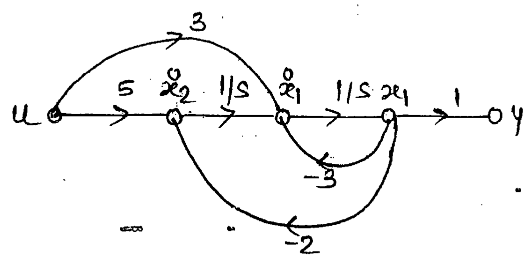
$X(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$

$\boxed{\text{Ans. (C)}}$

* TYPE(7) State diagrams →

(1). Physical variable Form →

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{s^2+3s+2} = \frac{\frac{3}{s}+\frac{5}{s^2}}{1-\left[\frac{-3}{s}-\frac{2}{s^2}\right]}$$



$$\dot{x}_1 = -3x_1 + x_2 + 3u$$

$$\dot{x}_2 = -2x_1 + 5u$$

$$y = x_1$$

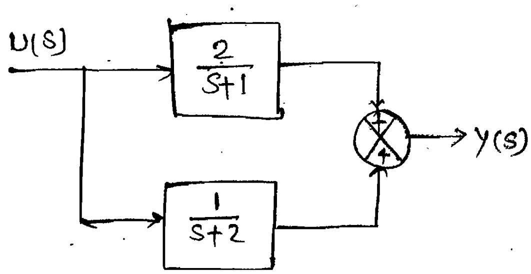
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}3\\ 5\end{bmatrix}u$$

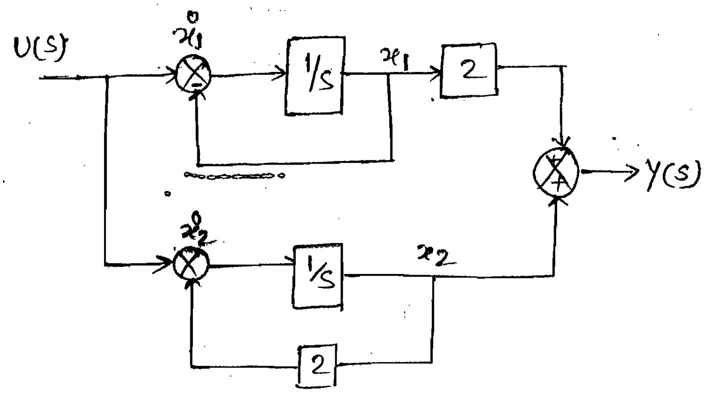
$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

(2.) Canonical variable Form →

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{(s+1)(s+2)} = \frac{2}{s+1} + \frac{1}{s+2}$$

$$Y(s) = \frac{2U(s)}{s+1} + \frac{U(s)}{s+2}$$





$$\dot{x}_1 = -x_1 + 4$$

$$\dot{x}_2 = -2x_2 + u$$

$$y = 2x_1 + x_2$$

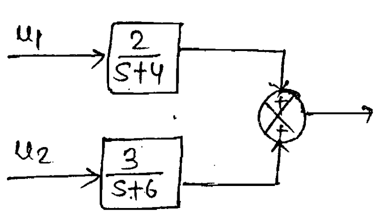
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

Unit matrix

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Residues of PF's

Ex:-



$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$