Chapter 3 Systems of Linear Equations and Inequalities

Ex 3.6

Answer 1e.

We know that if A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.
Therefore, the given statement can be completed as
"The product of matrices A and B is defined provided the number of columns in A is equal to the number of rows in B."

Answer 1gp.

The number of columns in A is equal to the number of rows in B.
Thus, the product AB is defined.

We know that when A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.
Since matrix A is 5×2 and matrix B is 2×2, the product AB is a 5×2 matrix.

Answer 2e.

If A and B are two matrices and the product AB is defined then we have to multiply each element in the first row of A by the corresponding element in the first column of B, then add the products.

This product is the required element in the product AB.

Answer 2gp.

The product of two matrices A and B is defined provided the number of column in A is equal to the number of rows in B.
If A is an $m\times n$ matrix and B is an $n\times p$ matrix, then the product AB is an $m\times p$ matrix.

For the given data, since A is a 3×2 matrix and B is a 3×2 matrix, the product of AB is not defined.

Answer 3e.

The number of columns in A is equal to the number of rows in B.
Thus, the product AB is defined.

We know that when A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.
Since A and B are 2×2 matrices, the product AB is a 2×2 matrix.

Answer 3gp.

Since the dimension of the first matrix is 2×2 and that of the second matrix is 2×2, the product of the matrices is defined and it is a 2×2 matrix.

STEP 1 Multiply the numbers in the first row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the first row, first column of the product.

$$\begin{bmatrix} -3 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} -3(1)+3(-3) & \\ & \end{bmatrix}$$

STEP 2 Multiply the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the first row, second column of the product.

$$\begin{bmatrix} -3 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} -3(1)+3(-3) & -3(5)+3(-2) \\ & \end{bmatrix}$$

STEP 3 Multiply the numbers in the second row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the second row, first column of the product.

$$\begin{bmatrix} -3 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} -3(1)+3(-3) & -3(5)+3(-2) \\ 1(1)+(-2)(-3) & \end{bmatrix}$$

STEP 4 Multiply the numbers in the second row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the second row, second column of the product.

$$\begin{bmatrix} -3 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} -3(1)+3(-3) & -3(5)+3(-2) \\ 1(1)+(-2)(-3) & 1(5)+(-2)(-2) \end{bmatrix}$$

STEP 5 Simplify the product matrix.

$$\begin{bmatrix} -3(1)+3(-3) & -3(5)+3(-2) \\ 1(1)+(-2)(-3) & 1(5)+(-2)(-2) \end{bmatrix} = \begin{bmatrix} -12 & -21 \\ 7 & 9 \end{bmatrix}$$

Therefore, $\begin{bmatrix} -3 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -21 \\ 7 & 9 \end{bmatrix}$.

Answer 4e.

The product of two matrices A and B is defined provided the number of column in A is equal to the number of rows in B.
If A is an $m\times n$ matrix and B is an $n\times p$ matrix, then the product AB is an $m\times p$ matrix.

For the given data, since A is a 3×4 matrix and B is a 4×2 matrix, the product of AB is defined and is a $\boxed{3\times 2}$ matrix.

Answer 4gp.

We have

$$A=\begin{pmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{pmatrix}, B=\begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix}, C=\begin{pmatrix} -4 & 5 \\ 1 & 0 \end{pmatrix}$$

Let us substitute this in the given expression,

$$\begin{aligned} B-C &= \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix} - \begin{pmatrix} -4 & 5 \\ 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 3-(-4) & 2-5 \\ -2-1 & -1-0 \end{pmatrix} \\ &= \begin{pmatrix} 7 & -3 \\ -3 & -1 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} A(B-C) &= \begin{pmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 7 & -3 \\ -3 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -7-6 & 3-2 \\ -21+0 & 9+0 \\ 28-3 & -12-1 \end{pmatrix} \\ &= \begin{pmatrix} -13 & 1 \\ -21 & 9 \\ 25 & -13 \end{pmatrix} \end{aligned}$$

Therefore, $\boxed{A(B-C)=\begin{pmatrix} -13 & 1 \\ -21 & 9 \\ 25 & -13 \end{pmatrix}}$.

Answer 5e.

The number of columns in A is not equal to the number of rows in B.
Thus, the product AB is not defined.

Answer 5gp.

Substitute the matrices for A and B in $AB - AC$.

$$AB - AC = \begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} -4 & 5 \\ 1 & 0 \end{bmatrix}$$

Multiply the matrices A and B.

$$\begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -1(3)+2(-2) & -1(2)+2(-1) \\ -3(3)+0(-2) & -3(2)+0(-1) \\ 4(3)+1(-2) & 4(2)+1(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -4 \\ -9 & -6 \\ 10 & 7 \end{bmatrix}$$

Now, multiply the matrices A and C.

$$\begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} -4 & 5 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1(-4)+2(1) & -1(5)+2(0) \\ -3(-4)+0(1) & -3(5)+0(0) \\ 4(-4)+1(1) & 4(5)+1(0) \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 \\ 12 & -15 \\ -15 & 20 \end{bmatrix}$$

Substitute the obtained products in $AB - AC$.

$$\begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} -4 & 5 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -7 & -4 \\ -9 & -6 \\ 10 & 7 \end{bmatrix} - \begin{bmatrix} 6 & -5 \\ 12 & -15 \\ -15 & 20 \end{bmatrix}$$

Subtract the matrices by subtracting the elements in the corresponding positions.

$$\begin{bmatrix} -7 & -4 \\ -9 & -6 \\ 10 & 7 \end{bmatrix} - \begin{bmatrix} 6 & -5 \\ 12 & -15 \\ -15 & 20 \end{bmatrix} = \begin{bmatrix} -7-6 & -4-(-5) \\ -9-12 & -6-(-15) \\ 10-(-15) & 7-20 \end{bmatrix}$$

$$= \begin{bmatrix} -13 & 1 \\ -21 & 9 \\ 25 & -13 \end{bmatrix}$$

Therefore, the value of $AB - AC$ is $\begin{bmatrix} -13 & 1 \\ -21 & 9 \\ 25 & -13 \end{bmatrix}$.

Answer 6e.

The product of two matrices A and B is defined provided the number of column in A is equal to the number of rows in B.
If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

For the given data, since A is a 1×2 matrix and B is a 2×3 matrix, the product of AB is defined and is a $\boxed{1\times 3}$ matrix.

Answer 6gp.

We have

$$A=\begin{pmatrix}-1 & 2\\ -3 & 0\\ 4 & 1\end{pmatrix},\ B=\begin{pmatrix}3 & 2\\ -2 & -1\end{pmatrix}$$

Let us substitute this in the given expression,

$$\begin{aligned}-\frac{1}{2}AB &= -\frac{1}{2}\times\begin{pmatrix}-1 & 2\\ -3 & 0\\ 4 & 1\end{pmatrix}\begin{pmatrix}3 & 2\\ -2 & -1\end{pmatrix}\\ &= -\frac{1}{2}\times\begin{pmatrix}-3-4 & -2-2\\ -9+0 & -6+0\\ 12-2 & 8-1\end{pmatrix}\\ &= -\frac{1}{2}\times\begin{pmatrix}-7 & -4\\ -9 & -6\\ 10 & 7\end{pmatrix}\\ &= \begin{pmatrix}7/2 & 2\\ 9/2 & 3\\ -5 & -7/2\end{pmatrix}\end{aligned}$$

Therefore, $\boxed{-\frac{1}{2}(AB)=\begin{pmatrix}7/2 & 2\\ 9/2 & 3\\ -5 & -7/2\end{pmatrix}}$.

Answer 7e.

The number of columns in A is not equal to the number of rows in B.
Thus, the product AB is not defined.

Answer 7gp.

Set up the matrices so that the columns of the equipment matrix match the rows of the cost matrix.

The equipment can be listed in rows of a matrix.

$$\begin{array}{r} \\ \text{Women's team} \\ \text{Men's team} \end{array} \begin{array}{c} \text{Sticks Pucks Uniforms} \\ \begin{bmatrix} 14 & 30 & 18 \\ 16 & 25 & 20 \end{bmatrix} \end{array}.$$

The cost can be listed in a column of a matrix.

$$\begin{array}{r} \text{Sticks} \\ \text{Pucks} \\ \text{Uniforms} \end{array} \begin{bmatrix} 75 \\ 1 \\ 45 \end{bmatrix}.$$

The total cost can be found by multiplying the equipment matrix by the cost matrix. The equipment matrix is 2×3 and the cost matrix is 3×1. Their product is a 2×1 matrix.

$$\begin{bmatrix} 14 & 30 & 18 \\ 16 & 25 & 20 \end{bmatrix} \begin{bmatrix} 75 \\ 1 \\ 45 \end{bmatrix} = \begin{bmatrix} 14(75) + 30(1) + 18(45) \\ 16(75) + 25(1) + 20(45) \end{bmatrix}$$

$$= \begin{bmatrix} 1890 \\ 2125 \end{bmatrix}$$

Therefore, the total cost of equipment for the women's team is \$1890, and that for the men's team is \$2125.

Answer 8e.

The product of two matrices A and B is defined provided the number of column in A is equal to the number of rows in B.

If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

For the given data, since A is a 2×1 matrix and B is a 1×5 matrix, the product of AB is defined and is a $\boxed{2 \times 5}$ matrix.

We know that when A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

It is given that A is a 2×3 matrix and B is a 3×2 matrix. Then, its product AB is a 2×2 matrix.

The correct answer is choice **A**.

Answer 10e.

Since A is a 1×2 matrix and B is a 2×1 matrix, the product of AB is defined.

On multiplying, to find the element in the ith row and jth column of the product matrix AB, multiply each element in the ith row of A by the corresponding element in the jth column of B, then add the products.

$$\begin{aligned} AB &= \begin{pmatrix} 3 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 7 \end{pmatrix} \\ &= \big(3(5)+(-1)(7)\big) \\ &= (15-7) \\ &= (8) \end{aligned}$$

Therefore, the product is $\boxed{(8)}$.

Answer 11e.

Since the dimension of the first matrix is 2×1, and that of the second matrix is 1×2, the product of the matrices is defined and it is a 2×2 matrix.

STEP 1 Multiply the numbers in the first row of the first matrix by the numbers in the first column of the second matrix. Put the result in the first row, first column of the product.

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \end{bmatrix} = \begin{bmatrix} 1(-2) & \\ & \end{bmatrix}$$

STEP 2 Multiply the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Put the result in the first row, second column of the product.

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \end{bmatrix} = \begin{bmatrix} 1(-2) & 1(1) \\ & \end{bmatrix}$$

STEP 3 Multiply the numbers in the second row of the first matrix by the numbers in the first column of the second matrix. Put the result in the second row, first column of the product.

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \end{bmatrix} = \begin{bmatrix} 1(-2) & 1(1) \\ 4(-2) & \end{bmatrix}$$

STEP 4 Multiply the numbers in the second row of the first matrix by the numbers in the second column of the second matrix. Put the result in the second row, second column of the product.

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \end{bmatrix} = \begin{bmatrix} 1(-2) & 1(1) \\ 4(-2) & 4(1) \end{bmatrix}$$

STEP 5 Simplify the product matrix.

$$\begin{bmatrix} 1(-2) & 1(1) \\ 4(-2) & 4(1) \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -8 & 4 \end{bmatrix}$$

Therefore, $\begin{bmatrix} 1 \\ 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -8 & 4 \end{bmatrix}$.

Answer 12e.

The product of an $m \times n$ matrix A and an $n \times p$ matrix B is an $m \times p$ matrix C where $c_{ij} = \sum a_{ik} b_{kj}$

From the given data, since A is a 2×2 matrix and B is a 1×2 matrix, the product of AB is not defined. The inner dimensions are not the same.

Answer 13e.

Since the dimension of the first matrix is 2×2 and that of the second matrix is 2×2, the product of the matrices is defined and it is a 2×2 matrix.

STEP 1 Multiply the numbers in the first row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the first row, first column of the product.

$$\begin{bmatrix} 9 & -3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 9(0) + (-3)(4) & \\ & \end{bmatrix}$$

STEP 2 Multiply the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the first row, second column of the product.

$$\begin{bmatrix} 9 & -3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 9(0) + (-3)(4) & 9(1) + (-3)(-2) \\ & \end{bmatrix}$$

STEP 3 Multiply the numbers in the second row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the second row, first column of the product.

$$\begin{bmatrix} 9 & -3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 9(0)+(-3)(4) & 9(1)+(-3)(-2) \\ 0(0)+2(4) & \end{bmatrix}$$

STEP 4 Multiply the numbers in the second row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the second row, second column of the product.

$$\begin{bmatrix} 9 & -3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 9(0)+(-3)(4) & 9(1)+(-3)(-2) \\ 0(0)+2(4) & 0(1)+2(-2) \end{bmatrix}$$

STEP 5 Simplify the product matrix.

$$\begin{bmatrix} 9(0)+(-3)(4) & 9(1)+(-3)(-2) \\ 0(0)+2(4) & 0(1)+2(-2) \end{bmatrix} = \begin{bmatrix} -12 & 15 \\ 8 & -4 \end{bmatrix}$$

Therefore, $\begin{bmatrix} 9 & -3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} -12 & 15 \\ 8 & -4 \end{bmatrix}$.

Answer 14e.

The product of an $m \times n$ matrix A and an $n \times p$ matrix B is an $m \times p$ matrix C where $c_{ij} = \sum a_{ik} b_{kj}$

Since A is a 2×2 matrix and B is a 2×2 matrix, the product of AB is defined.

Therefore,

$$\begin{aligned} \begin{pmatrix} 5 & 0 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} -3 & 2 \\ 6 & 2 \end{pmatrix} &= \begin{pmatrix} 5(-3)+0(6) & 5(2)+0(2) \\ -4(-3)+1(6) & -4(2)+1(2) \end{pmatrix} \\ &= \begin{pmatrix} -15+0 & 10+0 \\ 12+6 & -8+2 \end{pmatrix} \\ &= \begin{pmatrix} -15 & 10 \\ 18 & -6 \end{pmatrix} \end{aligned}$$

The product is $\boxed{\begin{pmatrix} -15 & 10 \\ 18 & -6 \end{pmatrix}}$.

Answer 15e.

Since the dimension of the first matrix is 3×2 and that of the second matrix is 2×2, the product of the matrices is defined and it is a 3×2 matrix.

STEP 1 Multiply the numbers in the first row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the first row, first column of the product.

$$\begin{bmatrix} 5 & 2 \\ 0 & -4 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 5(3)+2(-2) & \\ & \\ & \end{bmatrix}$$

STEP 2 Multiply the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the first row, second column of the product.

$$\begin{bmatrix} 5 & 2 \\ 0 & -4 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 5(3)+2(-2) & 5(7)+2(0) \\ & \\ & \end{bmatrix}$$

STEP 3 Multiply the numbers in the second row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the second row, first column of the product.

$$\begin{bmatrix} 5 & 2 \\ 0 & -4 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 5(3)+2(-2) & 5(7)+2(0) \\ 0(3)+(-4)(-2) & \\ & \end{bmatrix}$$

STEP 4 Multiply the numbers in the second row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the second row, second column of the product.

$$\begin{bmatrix} 5 & 2 \\ 0 & -4 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 5(3)+2(-2) & 5(7)+2(0) \\ 0(3)+(-4)(-2) & 0(7)+(-4)(0) \\ & \end{bmatrix}$$

STEP 5 Multiply the numbers in the third row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the third row, first column of the product.

$$\begin{bmatrix} 5 & 2 \\ 0 & -4 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 5(3)+2(-2) & 5(7)+2(0) \\ 0(3)+(-4)(-2) & 0(7)+(-4)(0) \\ 1(3)+6(-2) & \end{bmatrix}$$

STEP 6 Multiply the numbers in the third row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the third row, second column of the product.

$$\begin{bmatrix} 5 & 2 \\ 0 & -4 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 5(3)+2(-2) & 5(7)+2(0) \\ 0(3)+(-4)(-2) & 0(7)+(-4)(0) \\ 1(3)+6(-2) & 1(7)+6(0) \end{bmatrix}$$

STEP 7 Simplify the product matrix.

$$\begin{bmatrix} 5(3)+2(-2) & 5(7)+2(0) \\ 0(3)+(-4)(-2) & 0(7)+(-4)(0) \\ 1(3)+6(-2) & 1(7)+6(0) \end{bmatrix} = \begin{bmatrix} 11 & 35 \\ 8 & 0 \\ -9 & 7 \end{bmatrix}$$

Therefore, $\begin{bmatrix} 5 & 2 \\ 0 & -4 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 11 & 35 \\ 8 & 0 \\ -9 & 7 \end{bmatrix}$.

Answer 16e.

The product of an $m \times n$ matrix A and an $n \times p$ matrix B is an $m \times p$ matrix C where $c_{ij} = \sum a_{ik} b_{kj}$

From the given data, since A is a 3×2 matrix and B is a 3×2 matrix, the product of AB is not defined. The inner dimensions are not the same.

Answer 17e.

Since the dimension of the first matrix is 2×3 and that of the second matrix is 2×2, the product of the matrices is defined and it is a 2×2 matrix.

STEP 1 Multiply the numbers in the first row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the first row, first column of the product.

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 12 & -4 \end{bmatrix} \begin{bmatrix} 9 & 1 \\ 4 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 1(9)+3(4)+0(-2) & \\ & \end{bmatrix}$$

STEP 2 Multiply the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the first row, second column of the product.

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 12 & -4 \end{bmatrix}\begin{bmatrix} 9 & 1 \\ 4 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 1(9)+3(4)+0(-2) & 1(1)+3(-3)+0(4) \\ & \end{bmatrix}$$

STEP 3 Multiply the numbers in the second row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the second row, first column of the product.

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 12 & -4 \end{bmatrix}\begin{bmatrix} 9 & 1 \\ 4 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 1(9)+3(4)+0(-2) & 1(1)+3(-3)+0(4) \\ 2(9)+12(4)+(-4)(-2) & \end{bmatrix}$$

STEP 4 Multiply the numbers in the second row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the second row, second column of the product.

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 12 & -4 \end{bmatrix}\begin{bmatrix} 9 & 1 \\ 4 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 1(9)+3(4)+0(-2) & 1(1)+3(-3)+0(4) \\ 2(9)+12(4)+(-4)(-2) & 2(1)+12(-3)+(-4)(4) \end{bmatrix}$$

STEP 5 Simplify the product matrix.

$$\begin{bmatrix} 1(9)+3(4)+0(-2) & 1(1)+3(-3)+0(4) \\ 2(9)+12(4)+(-4)(-2) & 2(1)+12(-3)+(-4)(4) \end{bmatrix} = \begin{bmatrix} 21 & -8 \\ 74 & -50 \end{bmatrix}$$

Therefore, $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 12 & -4 \end{bmatrix}\begin{bmatrix} 9 & 1 \\ 4 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 21 & -8 \\ 74 & -50 \end{bmatrix}$.

Answer 18e.

The product of an $m\times n$ matrix A and an $n\times p$ matrix B is an $m\times p$ matrix C where $c_{ij}=\sum a_{ik}b_{kj}$

Since A is a 3×2 matrix and B is a 2×3 matrix, the product of AB is defined.

Therefore,

$$\begin{pmatrix} 2 & 5 \\ -1 & 4 \\ 3 & -7 \end{pmatrix}\begin{pmatrix} 0 & 1 & 5 \\ -3 & 10 & -4 \end{pmatrix}=\begin{pmatrix} 2(0)+5(-3) & 2(1)+5(10) & 2(5)+5(-4) \\ -1(0)+4(-3) & (-1)(1)+4(10) & (-1)(5)+4(-4) \\ 3(0)+(-7)(-3) & 3(1)+(-7)(10) & 3(5)+(-7)(-4) \end{pmatrix}$$

$$=\begin{pmatrix} 0-15 & 2+50 & 10-20 \\ 0-12 & -1+40 & -5-16 \\ 0+21 & 3-70 & 15+28 \end{pmatrix}$$

$$=\begin{pmatrix} -15 & 52 & -10 \\ -12 & 39 & -21 \\ 21 & -67 & 43 \end{pmatrix}$$

The product is $\boxed{\begin{pmatrix} -15 & 52 & -10 \\ -12 & 39 & -21 \\ 21 & -67 & 43 \end{pmatrix}}$.

Answer 19e.

The element in the ith row and jth column of a product matrix AB is the sum of the products of the element in the ith row of A by the corresponding element in the jth column of B.

The error is that the elements in the first row of the first matrix are multiplied by the corresponding elements in the first row of the second matrix.

Multiply the elements in the first row of the first matrix by the elements in the first column of the second matrix. Add the products and put the result in the first row, first column of the product.

$$\begin{bmatrix} 3 & -1 \\ 6 & 2 \end{bmatrix}\begin{bmatrix} 7 & 0 \\ 1 & -6 \end{bmatrix}=\begin{bmatrix} 3(7)+(-1)(1) & \\ & \end{bmatrix}$$

$$=\begin{bmatrix} 20 & \\ & \end{bmatrix}$$

Answer 20e.

On multiplying the matrix, to find the element in the ith row and jth column of the product matrix AB, multiply each element in the ith row of A by the corresponding element in the jth column of B, then add the products.

In the given example, this is not the case.

The given procedure of multiplying matrix is an error.

Answer 21e.

Since the dimension of the first matrix is 2×2 and that of the second matrix is 2×2, the product of the matrices is defined and it is a 2×2 matrix.

STEP 1 Multiply the numbers in the first row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the first row, first column of the product.

$$\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 1(4)+(-4)(0) & \\ & \end{bmatrix}$$

STEP 2 Multiply the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the first row, second column of the product.

$$\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 1(4)+(-4)(0) & 1(-1)+(-4)(-3) \\ & \end{bmatrix}$$

STEP 3 Multiply the numbers in the second row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the second row, first column of the product.

$$\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 1(4)+(-4)(0) & 1(-1)+(-4)(-3) \\ 3(4)+(-2)(0) & \end{bmatrix}$$

STEP 4 Multiply the numbers in the second row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the second row, second column of the product.

$$\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 1(4)+(-4)(0) & 1(-1)+(-4)(-3) \\ 3(4)+(-2)(0) & 3(-1)+(-2)(-3) \end{bmatrix}$$

STEP 5 Simplify the product matrix.

$$\begin{bmatrix} 1(4)+(-4)(0) & 1(-1)+(-4)(-3) \\ 3(4)+(-2)(0) & 3(-1)+(-2)(-3) \end{bmatrix} = \begin{bmatrix} 4 & 11 \\ 12 & 3 \end{bmatrix}$$

Therefore, $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 11 \\ 12 & 3 \end{bmatrix}$.

The correct answer is choice **B**.

Answer 22e.

We have

$$A=\begin{pmatrix}5 & -3\\ -2 & 4\end{pmatrix}, B=\begin{pmatrix}0 & 1\\ 4 & -2\end{pmatrix}$$

Let us substitute this in the given expression,

$$\begin{aligned}3AB &= 3\times\begin{pmatrix}5 & -3\\ -2 & 4\end{pmatrix}\begin{pmatrix}0 & 1\\ 4 & -2\end{pmatrix}\\ &= 3\times\begin{pmatrix}5(0)+(-3)(4) & 5(1)+(-3)(-2)\\ (-2)(0)+4(4) & (-2)(1)+(4)(-2)\end{pmatrix}\\ &= 3\times\begin{pmatrix}0-12 & 5+6\\ 0+16 & -2-8\end{pmatrix}\\ &= 3\times\begin{pmatrix}-12 & 11\\ 16 & -10\end{pmatrix}\\ &= \begin{pmatrix}-36 & 33\\ 48 & -30\end{pmatrix}\end{aligned}$$

Therefore, $\boxed{3AB=\begin{pmatrix}-36 & 33\\ 48 & -30\end{pmatrix}}$.

Answer 23e.

Substitute the matrices for A and C in $-\frac{1}{2}AC$.

$$-\frac{1}{2}AC = -\frac{1}{2}\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}\begin{bmatrix}-6 & 3\\ 4 & 1\end{bmatrix}$$

Multiply the matrices.

$$\begin{aligned}\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}\begin{bmatrix}-6 & 3\\ 4 & 1\end{bmatrix} &= \begin{bmatrix}5(-6)+(-3)(4) & 5(3)+(-3)(1)\\ (-2)(-6)+4(4) & (-2)(3)+4(1)\end{bmatrix}\\ &= \begin{bmatrix}-42 & 12\\ 28 & -2\end{bmatrix}\end{aligned}$$

$$-\frac{1}{2}\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}\begin{bmatrix}-6 & 3\\ 4 & 1\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}-42 & 12\\ 28 & -2\end{bmatrix}$$

Multiply each element in the matrix by $-\frac{1}{2}$.

$$-\frac{1}{2}\begin{bmatrix} -42 & 12 \\ 28 & -2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}(-42) & -\frac{1}{2}(12) \\ -\frac{1}{2}(28) & -\frac{1}{2}(-2) \end{bmatrix}$$
$$= \begin{bmatrix} 21 & -6 \\ -14 & 1 \end{bmatrix}$$

Therefore, when $A = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} -6 & 3 \\ 4 & 1 \end{bmatrix}$, the value of $-\frac{1}{2}AC = \begin{bmatrix} 21 & -6 \\ -14 & 1 \end{bmatrix}$.

Answer 24e.

We have

$$A = \begin{pmatrix} 5 & -3 \\ -2 & 4 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ 4 & -2 \end{pmatrix}, C = \begin{pmatrix} -6 & 3 \\ 4 & 1 \end{pmatrix}$$

Let us substitute this in the given expression,

$$AB = \begin{pmatrix} 5 & -3 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 4 & -2 \end{pmatrix}$$
$$= \begin{pmatrix} 5(0)+(-3)(4) & 5(1)+(-3)(-2) \\ (-2)(0)+4(4) & (-2)(1)+4(-2) \end{pmatrix}$$
$$= \begin{pmatrix} 0-12 & 5+6 \\ 0+16 & -2-8 \end{pmatrix}$$
$$= \begin{pmatrix} -12 & 11 \\ 16 & -10 \end{pmatrix}$$

$$AC = \begin{pmatrix} 5 & -3 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} -6 & 3 \\ 4 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 5(-6)+(-3)(4) & 5(3)+(-3)(1) \\ (-2)(-6)+4(4) & (-2)(3)+4(1) \end{pmatrix}$$
$$= \begin{pmatrix} -30-12 & 15-3 \\ 12+16 & -6+4 \end{pmatrix}$$
$$= \begin{pmatrix} -42 & 12 \\ 28 & -2 \end{pmatrix}$$

$$AB+AC=\begin{pmatrix}-12 & 11\\ 16 & -10\end{pmatrix}+\begin{pmatrix}-42 & 12\\ 28 & -2\end{pmatrix}$$
$$=\begin{pmatrix}-12-42 & 11+12\\ 16+28 & -10-2\end{pmatrix}$$
$$=\begin{pmatrix}-54 & 23\\ 44 & -12\end{pmatrix}$$

Therefore, $\boxed{AB+AC=\begin{bmatrix}-54 & 23\\ 44 & -12\end{bmatrix}}$.

Answer 25e.

Substitute the matrices for A and B in $AB-BA$.

$$AB-BA=\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}\begin{bmatrix}0 & 1\\ 4 & -2\end{bmatrix}-\begin{bmatrix}0 & 1\\ 4 & -2\end{bmatrix}\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}$$

Multiply the matrices.

$$\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}\begin{bmatrix}0 & 1\\ 4 & -2\end{bmatrix}=\begin{bmatrix}(5)(0)+(-3)(4) & 5(1)+(-3)(-2)\\ (-2)(0)+(4)(4) & (-2)(1)+(4)(-2)\end{bmatrix}$$
$$=\begin{bmatrix}-12 & 11\\ 16 & -10\end{bmatrix}$$

$$\begin{bmatrix}0 & 1\\ 4 & -2\end{bmatrix}\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}=\begin{bmatrix}(0)(5)+(1)(-2) & (0)(-3)+(1)(4)\\ (4)(5)+(-2)(-2) & (4)(-3)+(-2)(4)\end{bmatrix}$$
$$=\begin{bmatrix}-2 & 4\\ 24 & -20\end{bmatrix}$$

$$\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}\begin{bmatrix}0 & 1\\ 4 & -2\end{bmatrix}-\begin{bmatrix}0 & 1\\ 4 & -2\end{bmatrix}\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}=\begin{bmatrix}-12 & 11\\ 16 & -10\end{bmatrix}-\begin{bmatrix}-2 & 4\\ 24 & -20\end{bmatrix}$$

Subtract the matrices by subtracting the elements in the corresponding positions.

$$\begin{bmatrix}-12 & 11\\ 16 & -10\end{bmatrix}-\begin{bmatrix}-2 & 4\\ 24 & -20\end{bmatrix}=\begin{bmatrix}-12-(-2) & 11-4\\ 16-24 & -10-(-20)\end{bmatrix}$$
$$=\begin{bmatrix}-10 & 7\\ -8 & 10\end{bmatrix}$$

Therefore, when $A=\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}$ and $B=\begin{bmatrix}0 & 1\\ 4 & -2\end{bmatrix}$, the value of $AB-BA=\begin{bmatrix}-10 & 7\\ -8 & 10\end{bmatrix}$.

Answer 26e.

We have

$$D=\begin{pmatrix}1 & 3 & 2\\ -3 & 1 & 4\\ 2 & 1 & -2\end{pmatrix},\ E=\begin{pmatrix}-3 & 1 & 4\\ 7 & 0 & -2\\ 3 & 4 & -1\end{pmatrix}$$

Let us substitute this in the given expression,

$$D+E=\begin{pmatrix}1 & 3 & 2\\ -3 & 1 & 4\\ 2 & 1 & -2\end{pmatrix}+\begin{pmatrix}-3 & 1 & 4\\ 7 & 0 & -2\\ 3 & 4 & -1\end{pmatrix}$$

$$=\begin{pmatrix}1-3 & 3+1 & 2+4\\ -3+7 & 1+0 & 4-2\\ 2+3 & 1+4 & -2-1\end{pmatrix}$$

$$=\begin{pmatrix}-2 & 4 & 6\\ 4 & 1 & 2\\ 5 & 5 & -3\end{pmatrix}$$

$$E(D+E)=\begin{pmatrix}-3 & 1 & 4\\ 7 & 0 & -2\\ 3 & 4 & -1\end{pmatrix}\begin{pmatrix}-2 & 4 & 6\\ 4 & 1 & 2\\ 5 & 5 & -3\end{pmatrix}$$

$$=\begin{pmatrix}6+4+20 & -12+1+20 & -18+2-12\\ -14+0-10 & 28+0-10 & 42+0+6\\ -6+16-5 & 12+4-5 & 18+8+3\end{pmatrix}$$

$$=\begin{pmatrix}30 & 9 & -28\\ -24 & 18 & 48\\ 5 & 11 & 29\end{pmatrix}$$

Therefore, $\boxed{E(D+E)=\begin{pmatrix}30 & 9 & -28\\ -24 & 18 & 48\\ 5 & 11 & 29\end{pmatrix}}$.

Answer 27e.

Substitute the matrices for A and B in $(D+E)D$.

$$(D+E)D=\left(\begin{bmatrix}1 & 3 & 2\\ -3 & 1 & 4\\ 2 & 1 & -2\end{bmatrix}+\begin{bmatrix}-3 & 1 & 4\\ 7 & 0 & -2\\ 3 & 4 & -1\end{bmatrix}\right)\begin{bmatrix}1 & 3 & 2\\ -3 & 1 & 4\\ 2 & 1 & -2\end{bmatrix}$$

Add the matrices by adding the elements in the corresponding positions.

$$\left(\begin{bmatrix}1&3&2\\-3&1&4\\2&1&-2\end{bmatrix}+\begin{bmatrix}-3&1&4\\7&0&-2\\3&4&-1\end{bmatrix}\right)\begin{bmatrix}1&3&2\\-3&1&4\\2&1&-2\end{bmatrix}=\begin{bmatrix}-2&4&6\\4&1&2\\5&5&-3\end{bmatrix}\begin{bmatrix}1&3&2\\-3&1&4\\2&1&-2\end{bmatrix}$$

Multiply the matrices.

$$\begin{bmatrix}-2&4&6\\4&1&2\\5&5&-3\end{bmatrix}\begin{bmatrix}1&3&2\\-3&1&4\\2&1&-2\end{bmatrix}=\begin{bmatrix}(-2)(1)+4(-3)+6(2)&(-2)(3)+4(1)+6(1)&(-2)(2)+4(4)+6(-2)\\4(1)+1(-3)+2(2)&4(3)+1(1)+2(1)&4(2)+1(4)+2(-2)\\5(1)+5(-3)+(-3)(2)&5(3)+5(1)+(-3)(1)&5(2)+5(4)+(-3)(-2)\end{bmatrix}$$

$$=\begin{bmatrix}-2&4&0\\5&15&8\\-16&17&36\end{bmatrix}$$

Therefore, the value of $(D+E)D=\begin{bmatrix}-2&4&0\\5&15&8\\-16&17&36\end{bmatrix}$.

Answer 28e.

We have

$$B=\begin{pmatrix}0&1\\4&-2\end{pmatrix}, C=\begin{pmatrix}-6&3\\4&1\end{pmatrix}$$

Let us substitute this in the given expression,

$$-2(BC)=-2\times\begin{pmatrix}0&1\\4&-2\end{pmatrix}\begin{pmatrix}-6&3\\4&1\end{pmatrix}$$

$$=-2\times\begin{pmatrix}0(-6)+1(4)&0(3)+1(1)\\4(-6)+(-2)(4)&4(3)+(-2)(1)\end{pmatrix}$$

$$=-2\times\begin{pmatrix}0+4&0+1\\-28-8&12-2\end{pmatrix}$$

$$=-2\times\begin{pmatrix}4&1\\-36&10\end{pmatrix}$$

$$=\begin{pmatrix}-8&-2\\72&-20\end{pmatrix}$$

Therefore, $\boxed{-2(BC)=\begin{pmatrix}-8&-2\\72&-20\end{pmatrix}}$.

Answer 29e.

Substitute the matrices for A and B in $4AC + 3AB$.

$$4AC + 3AB = 4\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -6 & 3 \\ 4 & 1 \end{bmatrix} + 3\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}$$

Multiply the matrices.

$$\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -6 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 5(-6)+(-3)(4) & 5(3)+(-3)(1) \\ (-2)(-6)+4(4) & (-2)(3)+4(1) \end{bmatrix}$$

$$= \begin{bmatrix} -42 & 12 \\ 28 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 5(0)+(-3)(4) & 5(1)+(-3)(-2) \\ (-2)(0)+4(4) & (-2)(1)+4(-2) \end{bmatrix}$$

$$= \begin{bmatrix} -12 & 11 \\ 16 & -10 \end{bmatrix}$$

$$4\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -6 & 3 \\ 4 & 1 \end{bmatrix} + 3\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix} = 4\begin{bmatrix} -42 & 12 \\ 28 & -2 \end{bmatrix} + 3\begin{bmatrix} -12 & 11 \\ 16 & -10 \end{bmatrix}$$

Multiply each element of the matrices by the corresponding scalar.

$$4\begin{bmatrix} -42 & 12 \\ 28 & -2 \end{bmatrix} + 3\begin{bmatrix} -12 & 11 \\ 16 & -10 \end{bmatrix} = \begin{bmatrix} 4(-42) & 4(12) \\ 4(28) & 4(-2) \end{bmatrix} + \begin{bmatrix} 3(-12) & 3(11) \\ 3(16) & 3(-10) \end{bmatrix}$$

$$= \begin{bmatrix} -168 & 48 \\ 112 & -8 \end{bmatrix} + \begin{bmatrix} -36 & 33 \\ 48 & -30 \end{bmatrix}$$

Add the matrices by adding the elements in the corresponding positions.

$$\begin{bmatrix} -168 & 48 \\ 112 & -8 \end{bmatrix} + \begin{bmatrix} -36 & 33 \\ 48 & -30 \end{bmatrix} = \begin{bmatrix} -168+(-36) & 48+33 \\ 112+48 & -8+(-30) \end{bmatrix}$$

$$= \begin{bmatrix} -204 & 81 \\ 160 & -38 \end{bmatrix}$$

Therefore, when $A = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -6 & 3 \\ 4 & 1 \end{bmatrix}$, the value of

$$4AC + 3AB = \begin{bmatrix} -204 & 81 \\ 160 & -38 \end{bmatrix}.$$

Answer 30e.

Let us solve the matrix equation to find x and y,

$$\begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 4 \\ 0 & -2 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ x \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 19 \\ y \end{pmatrix}$$

Now the left side of the equation becomes,

$$\begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 4 \\ 0 & -2 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ x \\ 3 \end{pmatrix} = \begin{pmatrix} -2(1)+1(x)+2(3) \\ 3(1)+2(x)+4(3) \\ 0(1)+(-2)(x)+4(3) \end{pmatrix}$$

$$= \begin{pmatrix} -2+x+6 \\ 3+2x+12 \\ 0-2x+12 \end{pmatrix}$$

Equate the resultant matrix with that of the right matrix given.

$$\begin{pmatrix} -2+x+6 \\ 3+2x+12 \\ 0-2x+12 \end{pmatrix} = \begin{pmatrix} 6 \\ 19 \\ y \end{pmatrix}$$

Equate the rows of the both matrix.
The first row implies

$$-2+x+6=6$$
$$x=6-6+2$$
$$x=2$$

The third row implies

$$0-2x+12=y$$
$$y=-2x+12$$

Substituting the value of x

$$y=-2(2)+12$$
$$y=-4+12$$
$$y=8$$

Therefore, $\boxed{x=2, y=8}$.

Answer 31e.

Multiply the matrices.

$$\begin{bmatrix} 4 & 1 & 3 \\ -2 & x & 1 \end{bmatrix}\begin{bmatrix} 9 & -2 \\ 2 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4(9)+1(2)+3(-1) & 4(-2)+1(1)+3(1) \\ (-2)(9)+x(2)+1(-1) & (-2)(-2)+x(1)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 35 & -4 \\ -19+2x & 5+x \end{bmatrix}$$

$$\begin{bmatrix} 35 & -4 \\ -19+2x & 5+x \end{bmatrix} = \begin{bmatrix} y & -4 \\ -13 & 8 \end{bmatrix}$$

We know that if the dimensions of two matrices are the same and their elements in the corresponding positions are equal, then the matrices are equal.
To find the values of x and y, consider those elements and equate with their corresponding elements.

$y = 35$

and

$5 + x = 8$

Isolate x.

$$\begin{aligned} x &= 8 - 5 \\ &= 3 \end{aligned}$$

Therefore, $y = 35$ and $x = 3$.

Answer 32e.

To find the powers of the given matrix, we use matrix multiplication.

If $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$

We will find $A^2 = AA$

$$\begin{aligned} A^2 &= \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 1(1)+(-1)(0) & 1(-1)+(-1)2 \\ 0(1)+2(0) & 0(-1)+2(2) \end{pmatrix} \\ &= \begin{pmatrix} 1-0 & -1-2 \\ 0+0 & 0+4 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix} \end{aligned}$$

Now we will find $A^3 = AAA$.
Since we know the value of A^2, we can find A^3 by using A^2 result and A matrix.

$$A^3 = A^2 \times A$$
$$= \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 1-0 & -1-6 \\ 0+0 & 0+8 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & -7 \\ 0 & 8 \end{pmatrix}$$

Therefore, $\boxed{A^2 = \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix}, A^3 = \begin{pmatrix} 1 & -7 \\ 0 & 8 \end{pmatrix}}$.

Answer 33e.

Substitute the known matrix in AA.

$$AA = \begin{bmatrix} -4 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 2 & -1 \end{bmatrix}$$

Multiply the matrices.

$$\begin{bmatrix} -4 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} (-4)(-4)+1(2) & (-4)(1)+1(-1) \\ 2(-4)+(-1)(2) & 2(1)+(-1)(-1) \end{bmatrix}$$
$$= \begin{bmatrix} 18 & -5 \\ -10 & 3 \end{bmatrix}$$

$$A^2 = AA = \begin{bmatrix} 18 & -5 \\ -10 & 3 \end{bmatrix}$$

Substitute $\begin{bmatrix} 18 & -5 \\ -10 & 3 \end{bmatrix}$ for AA and $\begin{bmatrix} -4 & 1 \\ 2 & -1 \end{bmatrix}$ for A in AAA.

$$AAA = \begin{bmatrix} 18 & -5 \\ -10 & 3 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 2 & -1 \end{bmatrix}$$

Multiply.

$$\begin{bmatrix} 18 & -5 \\ -10 & 3 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -82 & 23 \\ 46 & -13 \end{bmatrix}$$

Therefore, $A^3 = AAA = \begin{bmatrix} -82 & 23 \\ 46 & -13 \end{bmatrix}$.

Answer 34e.

To find the powers of the given matrix, we use matrix multiplication.

If $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 3 & 2 \\ -2 & -1 & 0 \end{pmatrix}$

We will find $A^2 = AA$

$$A^2 = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 3 & 2 \\ -2 & -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 & -1 \\ 1 & 3 & 2 \\ -2 & -1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 4+0+2 & 0+0+1 & -2+0-0 \\ 2+3-4 & 0+9-2 & -1+6+0 \\ -4-1-0 & 0-3-0 & 2-2+0 \end{pmatrix}$$
$$= \begin{pmatrix} 6 & 1 & -2 \\ 1 & 7 & 5 \\ -5 & -3 & 0 \end{pmatrix}$$

Now we will find $A^3 = AAA$.
Since we know the value of A^2, we can find A^3 by using A^2 result and A matrix.

$$A^3 = A^2 \times A$$
$$= \begin{pmatrix} 6 & 1 & -2 \\ 1 & 7 & 5 \\ -5 & -3 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 & -1 \\ 1 & 3 & 2 \\ -2 & -1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 12+1+4 & 0+3+2 & -6+2-0 \\ 2+7-10 & 0+21-5 & -1+14+0 \\ -10-3-0 & 0-9-0 & 5-6+0 \end{pmatrix}$$
$$= \begin{pmatrix} 16 & 5 & -4 \\ -1 & 16 & 13 \\ -13 & -9 & -1 \end{pmatrix}$$

Therefore, $\boxed{A^2 = \begin{pmatrix} 6 & 1 & -2 \\ 1 & 7 & 5 \\ -5 & -3 & 0 \end{pmatrix}, A^3 = \begin{pmatrix} 16 & 5 & -4 \\ -1 & 16 & 13 \\ -13 & -9 & -1 \end{pmatrix}}$.

Answer 35e.

Since matrix multiplication is not commutative, any two different matrices A and B do not satisfy $AB = BA$.
The given condition will satisfy only when one of the matrices is an identity matrix.
Consider any matrix of order 2×2 and an identity matrix of order 2×2.

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Find AB by multiplying the matrices.

$$\begin{aligned} AB &= \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 5(1)+3(0) & 5(0)+3(1) \\ 2(1)+4(0) & 2(0)+4(0) \end{bmatrix} \\ &= \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} \end{aligned}$$

Find BA.

$$\begin{aligned} BA &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1(5)+0(2) & 1(3)+0(4) \\ 0(5)+1(2) & 0(3)+1(4) \end{bmatrix} \\ &= \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} \end{aligned}$$

Therefore. $AB = BA$.

Answer 36e.

To prove the associative property of scalar multiplication, we have to show
$k(AB) = (kA)B = A(kB)$

We have
$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$ and k, a scalar.

To find $k(AB)$

$$\begin{aligned} k(AB) &= k\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix}\right] \\ &= k\begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix} \\ &= \begin{pmatrix} kae+kbg & kaf+kbh \\ kce+kdg & kcf+kdh \end{pmatrix} \end{aligned}$$

To find $(kA)B$

$$(kA)B = \left[k\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]\begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$= \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$= \begin{pmatrix} kae+kbg & kaf+kbh \\ kce+kdg & kcf+kdh \end{pmatrix}$$

To find $A(kB)$

$$A(kB) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\left[k\begin{pmatrix} e & f \\ g & h \end{pmatrix}\right]$$

$$= \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} ke & kf \\ kg & kh \end{pmatrix}$$

$$= \begin{pmatrix} ake+bkg & akf+bkh \\ cke+dkg & ckf+dkh \end{pmatrix}$$

Hence, we observe that $k(AB) = (kA)B = A(kB)$ is true. Associative property of scalar multiplication is true.

Answer 37e.

Set up the matrices such that the columns of the inventory matrix match the rows of the cost matrix.

The things can be listed in a column of a matrix. The inventory matrix is

$$\begin{matrix} \text{Bats} \\ \text{Balls} \\ \text{Uniforms} \end{matrix} \begin{bmatrix} 12 \\ 45 \\ 15 \end{bmatrix}.$$

The cost of each item can be listed in a row of a matrix. The cost per item matrix is

$$\begin{matrix} & \text{Bat} & \text{Ball} & \text{Uniform} \\ \text{Cost} & [\,21 & 4 & 30\,] \end{matrix}.$$

The total cost can be found by multiplying the inventory matrix by the cost per item matrix. The inventory matrix is 1×3 and the cost matrix is 3×1.

$$\begin{bmatrix} 21 & 4 & 30 \end{bmatrix}\begin{bmatrix} 12 \\ 45 \\ 15 \end{bmatrix} = \begin{bmatrix} 21(12) + 4(45) + 30(15) \end{bmatrix}$$

$$= \begin{bmatrix} 882 \end{bmatrix}$$

Therefore, the total cost matrix is $\begin{matrix} & \text{Cost} \\ \text{Item} & [\,882\,] \end{matrix}$.

Answer 38e.

From the given data, we can write the inventory matrix as $\begin{pmatrix} 24 & 12 & 17 \\ 20 & 14 & 15 \end{pmatrix}$

Also cost per item matrix is $\begin{pmatrix} 3.35 \\ 1.75 \\ 4.50 \end{pmatrix}$

The total cost of products for each class can be found by multiplying the products matrix and the cost matrix.

The inventory matrix is a 2×3 matrix and the cost matrix is a 3×1 matrix. So their product is a 2×1 matrix.

$$\begin{pmatrix} 24 & 12 & 17 \\ 20 & 14 & 15 \end{pmatrix}\begin{pmatrix} 3.35 \\ 1.75 \\ 4.50 \end{pmatrix} = \begin{pmatrix} 24(3.35)+12(1.75)+17(4.50) \\ 20(3.35)+14(1.75)+15(4.50) \end{pmatrix}$$

$$= \begin{pmatrix} 80.4+21+76.5 \\ 67+24.5+67.5 \end{pmatrix}$$

$$= \begin{pmatrix} 177.9 \\ 159 \end{pmatrix}$$

Therefore, the total cost of the inventories for the class 1 is $\boxed{\$177.9}$ and for the class 2 is $\boxed{\$159}$.

Answer 39e.

Set up the matrices such that the columns of the inventory matrix match the rows of the cost matrix.
The cost of the tickets can be listed in a column of a matrix. The cost per item matrix is

	Cost
Students	2
Adults	5
Senior citizens	4

The performance can be listed in the rows of a matrix. The inventory matrix is

	Students	Adults	Senior citizens
Friday's performance	120	150	40
Saturday's performance	192	215	54

The total cost can be found by multiplying the inventory matrix by the cost per item matrix. The inventory matrix is 2×3 and the cost matrix is 3×1.

$$\begin{bmatrix} 120 & 150 & 40 \\ 192 & 215 & 54 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 120(2) + 150(5) + 40(4) \\ 192(2) + 215(5) + 54(4) \end{bmatrix}$$

$$= \begin{bmatrix} 1150 \\ 1675 \end{bmatrix}$$

Therefore, the income from ticket sales for Friday night's performance is \$1150, and that for Saturday night's performance is \$1675.

Answer 40e.

From the given data, we can write the metals matrix as $\begin{pmatrix} 35 & 39 & 29 \\ 32 & 17 & 14 \\ 27 & 27 & 38 \end{pmatrix}$

Also points matrix is $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

The total number of points scored by each country can be found by multiplying the metals matrix and the points matrix.

The metals matrix is a 3×3 matrix and the points matrix is a 3×1 matrix. So their product is a 3×1 matrix.

$$\begin{pmatrix} 35 & 39 & 29 \\ 32 & 17 & 14 \\ 27 & 27 & 38 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 35(3)+39(2)+29(1) \\ 32(3)+17(2)+14(1) \\ 27(3)+27(2)+38(1) \end{pmatrix}$$

$$= \begin{pmatrix} 105+78+29 \\ 96+34+14 \\ 81+54+38 \end{pmatrix}$$

$$= \begin{pmatrix} 212 \\ 144 \\ 173 \end{pmatrix}$$

Therefore, the USA won $\boxed{\text{212 points}}$, China won $\boxed{\text{144 points}}$, and Russia won $\boxed{\text{173 points}}$.

Answer 41e.

Matrix S has 2 columns and matrix P has only 1 row. Since the number of columns in matrix S is not equal to the number of rows in matrix P, product SP is not defined.
Matrix P has 3 columns and matrix S has 3 rows. The number of columns in P is equal to the number of rows in S. Therefore, the product PS is defined.

Multiply matrix P by matrix S.

$$PS = \begin{bmatrix} 650 & 825 & 1050 \end{bmatrix} \begin{bmatrix} 21 & 16 \\ 40 & 33 \\ 15 & 19 \end{bmatrix}$$

$$= \begin{bmatrix} 650(21) + 825(40) + 1050(15) & 650(16) + 825(33) + 1050(19) \end{bmatrix}$$

$$= \begin{bmatrix} 62{,}400 & 57{,}575 \end{bmatrix}$$

The profit for dealer A is \$62,400, and that for dealer B is \$57,575.

Answer 42e.

From the given data, we can write the student's score matrix as $\begin{pmatrix} 82 & 88 & 86 \\ 92 & 88 & 90 \\ 82 & 73 & 81 \\ 74 & 75 & 78 \\ 88 & 92 & 90 \end{pmatrix}$

Also weights matrix is $\begin{pmatrix} 0.2 \\ 0.3 \\ 0.5 \end{pmatrix}$

The student's overall score can be found by multiplying the student's score matrix and the weights matrix.

The metals matrix is a 5×3 matrix and the points matrix is a 3×1 matrix. So their product is a 5×1 matrix.

$$\begin{pmatrix} 82 & 88 & 86 \\ 92 & 88 & 90 \\ 82 & 73 & 81 \\ 74 & 75 & 78 \\ 88 & 92 & 90 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.3 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 82(0.2)+88(0.3)+86(0.5) \\ 92(0.2)+88(0.3)+90(0.5) \\ 82(0.2)+73(0.3)+81(0.5) \\ 74(0.2)+75(0.3)+78(0.5) \\ 88(0.2)+92(0.3)+90(0.5) \end{pmatrix}$$

$$= \begin{pmatrix} 16.4+26.4+43 \\ 18.4+26.4+45 \\ 16.4+21.9+40.5 \\ 14.8+22.5+39 \\ 17.6+27.6+45 \end{pmatrix}$$

$$= \begin{pmatrix} 85.8 \\ 89.8 \\ 78.8 \\ 76.3 \\ 90.2 \end{pmatrix}$$

Therefore, the average score for Jean is $\boxed{85.8}$, Ted is $\boxed{89.8}$, Pat is $\boxed{78.8}$, Al is $\boxed{76.3}$, and Matt is $\boxed{90.2}$.

Answer 43e.

a. Write the percents as decimals.

$20\% = 0.2$

$5\% = 0.05$

Substitute 0.2 for p and 0.05 for q in the given transition matrix and simplify.

$$T = \begin{bmatrix} 1 - 0.20 & 0.05 \\ 0.20 & 1 - 0.05 \end{bmatrix}$$

$$= \begin{bmatrix} 0.80 & 0.05 \\ 0.20 & 0.95 \end{bmatrix}$$

b. From part (a), $T = \begin{bmatrix} 0.80 & 0.05 \\ 0.20 & 0.95 \end{bmatrix}$.

Substitute $\begin{bmatrix} 0.80 & 0.05 \\ 0.20 & 0.95 \end{bmatrix}$ for T and $\begin{bmatrix} 5000 \\ 8000 \end{bmatrix}$ for M_0 in $M_1 = TM_0$.

$$M_1 = \begin{bmatrix} 0.80 & 0.05 \\ 0.20 & 0.95 \end{bmatrix}\begin{bmatrix} 5000 \\ 8000 \end{bmatrix}$$

Multiply the matrices.

$$M_1 = \begin{bmatrix} 0.80(5000) + 0.05(8000) \\ 0.20(5000) + 0.95(8000) \end{bmatrix}$$

$$= \begin{bmatrix} 4400 \\ 8600 \end{bmatrix}$$

This matrix represents the number of commuters after one year.

c. Substitute $\begin{bmatrix} 0.80 & 0.05 \\ 0.20 & 0.95 \end{bmatrix}$ for T and $\begin{bmatrix} 4400 \\ 8600 \end{bmatrix}$ for M_1 in $M_2 = TM_1$.

$$M_2 = \begin{bmatrix} 0.80 & 0.05 \\ 0.20 & 0.95 \end{bmatrix}\begin{bmatrix} 4400 \\ 8600 \end{bmatrix}$$

Multiply.

$$M_2 = \begin{bmatrix} 0.80(4400) + 0.05(8600) \\ 0.20(4400) + 0.95(8600) \end{bmatrix}$$

$$= \begin{bmatrix} 3950 \\ 9050 \end{bmatrix}$$

This matrix represents the number of commuters after two years.

Similarly, find M_3 and M_4.

$$M_3 = \begin{bmatrix} 3612.5 \\ 9387.5 \end{bmatrix} \text{ and } M_4 = \begin{bmatrix} 3359.375 \\ 9640.625 \end{bmatrix}$$

M_3 represents the number of commuters after three years and M_4 represents the number of commuters after four years.

Answer 44e.

a. Let C denote a 4×1 matrix that gives the cost of making each style of scarf. Then from the given data,

$$C = \begin{pmatrix} 10 \\ 15 \\ 20 \\ 20 \end{pmatrix}$$

Let P denote a 4×1 matrix that gives the price of each style of scarf. Then from the given data,

$$P = \begin{pmatrix} 15 \\ 20 \\ 25 \\ 30 \end{pmatrix}$$

b. Let S denote a 3×4 matrix that gives the sales for the first three years. Then from the given data,

$$S = \begin{pmatrix} 0 & 20 & 100 & 0 \\ 10 & 100 & 50 & 30 \\ 20 & 300 & 100 & 50 \end{pmatrix}$$

c. To find SC

$$SC=\begin{pmatrix}0 & 20 & 100 & 0\\ 10 & 100 & 50 & 30\\ 20 & 300 & 100 & 50\end{pmatrix}\begin{pmatrix}10\\ 15\\ 20\\ 20\end{pmatrix}$$

$$=\begin{pmatrix}0+300+2000+0\\ 100+1500+1000+600\\ 200+4500+2000+1000\end{pmatrix}$$

$$=\begin{pmatrix}2300\\ 3200\\ 7900\end{pmatrix}$$

The matrix SC denotes the amount spent for making the scarves in the first three years.

To find SP

$$SP=\begin{pmatrix}0 & 20 & 100 & 0\\ 10 & 100 & 50 & 30\\ 20 & 300 & 100 & 50\end{pmatrix}\begin{pmatrix}15\\ 20\\ 25\\ 30\end{pmatrix}$$

$$=\begin{pmatrix}0+400+2500+0\\ 150+2000+1250+900\\ 300+6000+2500+1500\end{pmatrix}$$

$$=\begin{pmatrix}2900\\ 4300\\ 10300\end{pmatrix}$$

The matrix SP denotes the amount earned on selling the scarves in the first three years.

d. To find $SP-SC$

$$SP-SC=\begin{pmatrix}2900\\ 4300\\ 10300\end{pmatrix}-\begin{pmatrix}2300\\ 3200\\ 7900\end{pmatrix}$$

$$=\begin{pmatrix}600\\ 1100\\ 2400\end{pmatrix}$$

The above matrix denotes the amount gained on selling the scarves in the first three years.

Answer 45e.

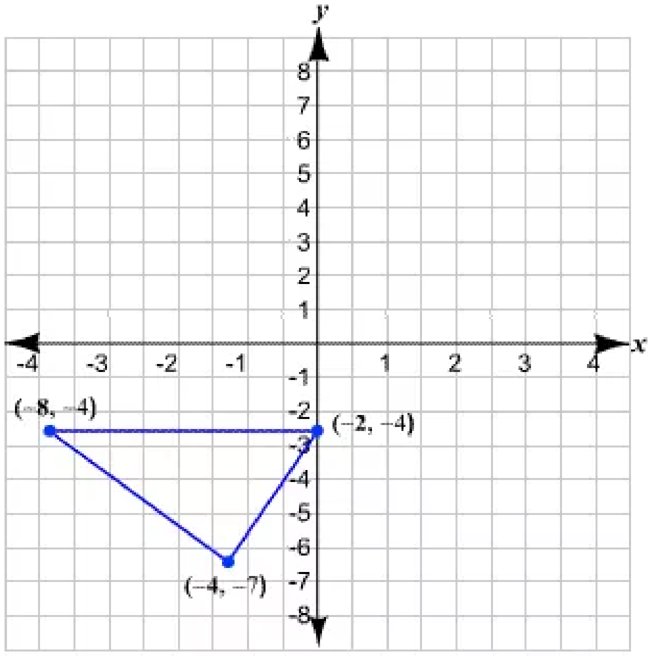
a. Multiply matrix A by matrix B.

$$AB = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -7 & -4 & -4 \\ 4 & 8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0(-7)+(-1)(4) & 0(-4)+(-1)(8) & 0(-4)+(-1)(2) \\ 1(-7)+0(4) & 1(-4)+0(8) & 1(-4)+0(2) \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -8 & -2 \\ -7 & -4 & -4 \end{bmatrix}$$

The first row of the product matrix represents the x-coordinates and the second row represents the corresponding y-coordinates.
The coordinates of the vertices of the triangle are $(-4, -7)$, $(-8, -4)$, and $(-2, -4)$.
Plot the vertices and join them to form a triangle.



b. The $180°$ rotational matrix is the product of $90°$ rotational matrix and AB. Find AAB.

$$AAB = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -4 & -8 & -2 \\ -7 & -4 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0(-4)+(-1)(-7) & 0(-8)+(-1)(-4) & 0(-2)+(-1)(-4) \\ 1(-4)+0(-7) & 1(-8)+0(-4) & 1(-2)+0(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 4 & 4 \\ -4 & -8 & -2 \end{bmatrix}$$

The $270°$ rotational matrix is the product of $90°$ rotational matrix and AAB. Find $AAAB$.

$$AAAB = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 7 & 4 & 4 \\ -4 & -8 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0(7)+(-1)(-4) & 0(4)+(-1)(-8) & 0(4)+(-1)(-2) \\ 1(7)+0(-4) & 1(4)+0(-8) & 1(4)+0(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 & 2 \\ 7 & 4 & 4 \end{bmatrix}$$

The coordinates of the vertices of the $180°$ rotated triangles are (7, −4), (4, −8), and (4, −2), and the coordinates of the vertices of the $270°$ rotated triangles are (4, 7), (8, 4), and (2, 4).

Answer 46e.

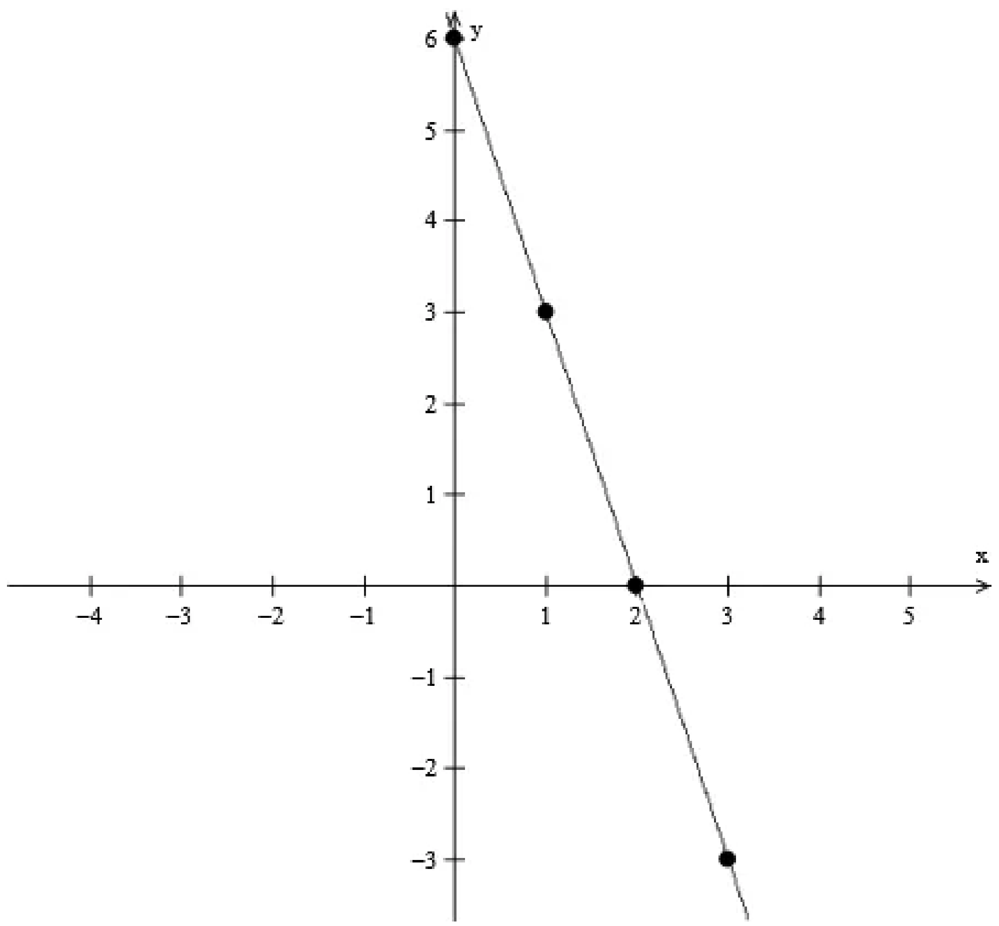
The intercept method is a convenient way to graph equations such as $3x+y=6$, because they are in standard $Ax+By=C$ form.

To graph this equation, we have to determine the coordinates of point which is to be plotted.
To find the y-intercept, let $x=0$ and solve for y.
To find the x-intercept, let $y=0$ and solve for x.
As a check, pick another x-value, such as 1 or 3.

$$3x+y=6$$

x	y	(x,y)
0	6	$(0,6)$
2	0	$(2,0)$
1	3	$(1,3)$
3	−3	$(3,-3)$

We shall now graph the equation



Answer 47e.

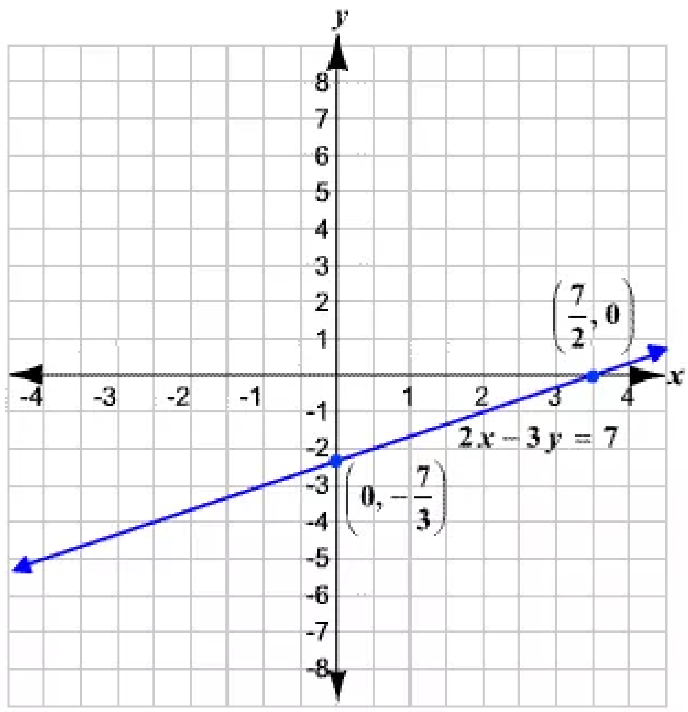
Find the intercept of the graph of the equation.
Substitute 0 for y in $2x - 3y = 7$ and solve for y.

$$2x - 3(0) = 7$$
$$2x = 7$$
$$x = \frac{7}{2}$$

Find the intercept of the graph of the equation.
Substitute 0 for y in $2x - 3y = 7$ and solve for y.

$$2(0) - 3y = 7$$
$$-3y = 7$$
$$y = -\frac{7}{3}$$

Plot $\left(\frac{7}{2}, 0\right)$ and $\left(0, -\frac{7}{3}\right)$ on a coordinate plane. Join the points with a straight line.



Answer 48e.

The intercept method is a convenient way to graph equations such as $x + 4y = 10$, because they are in standard $Ax + By = C$ form.

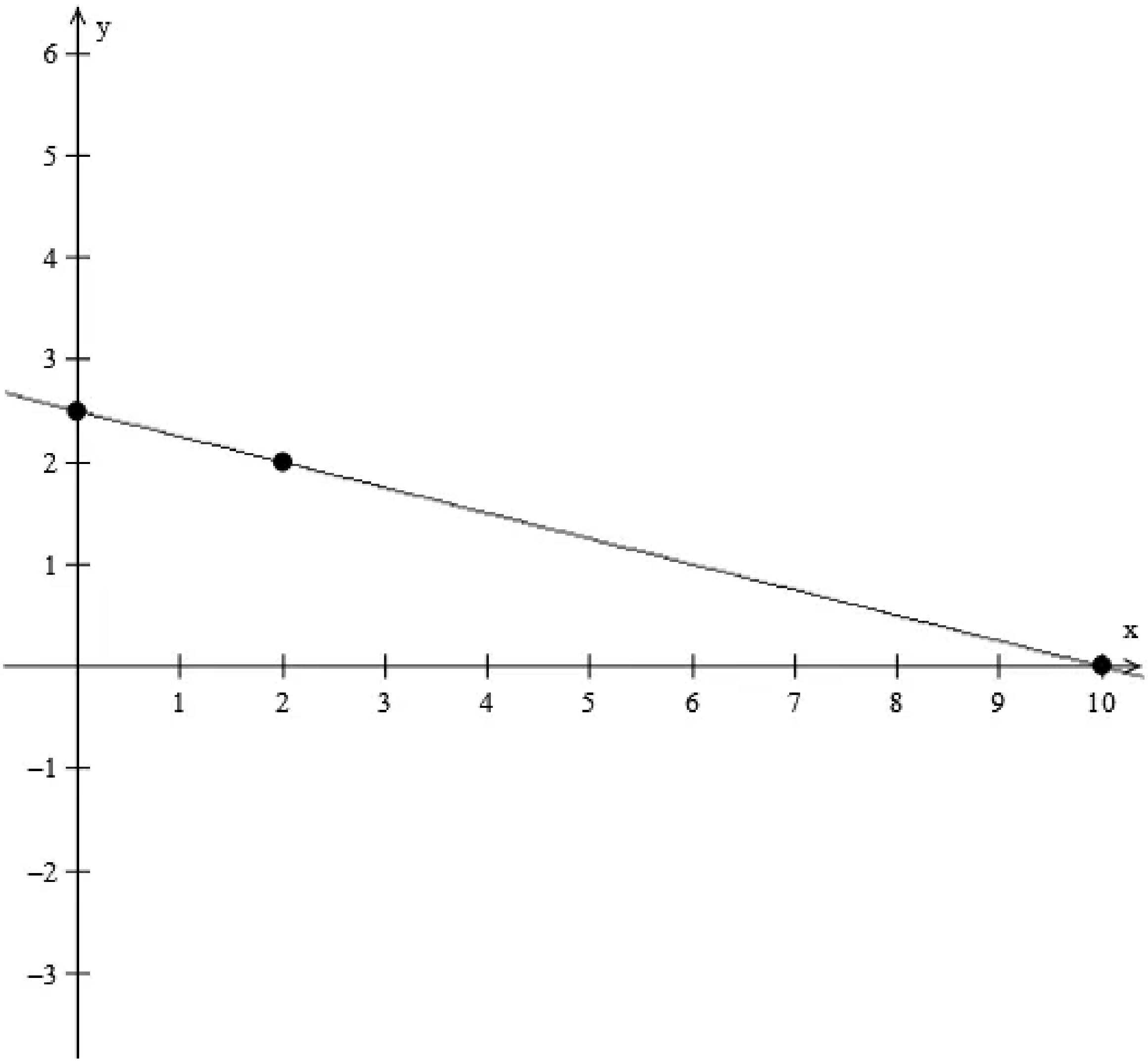
To graph this equation, we have to determine the coordinates of point which is to be

plotted.
To find the y-intercept, let $x=0$ and solve for y.
To find the x-intercept, let $y=0$ and solve for x.
As a check, pick another x-value, such as 2.

$$x+4y=10$$

x	y	(x,y)
0	$\frac{5}{2}$	$\left(0,\frac{5}{2}\right)$
10	0	$(10,0)$
2	2	$(2,2)$

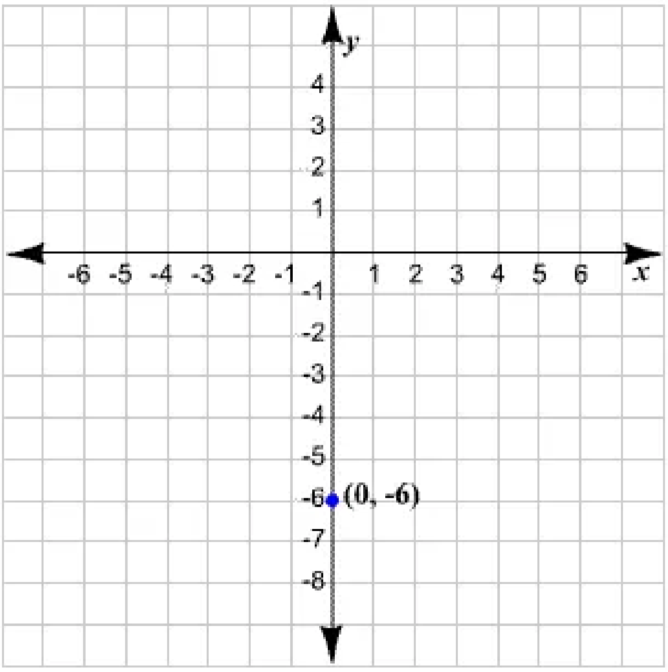
We shall now graph the equation



Answer 49e.

Step 1 The given function is of the form $y=|x-h|+k$, where (h, k) is the vertex of the function's graph.
Comparing the given equation with $y=|x-h|+k$, the value of h is 0 and that of k is -6. Thus, the vertex is $(0, -6)$.

Plot (0, −6) as the vertex.



Step 2 Use symmetry to find two more points.

Substitute any value, say, 0 for y in the given function.

$0 = |x| - 6$

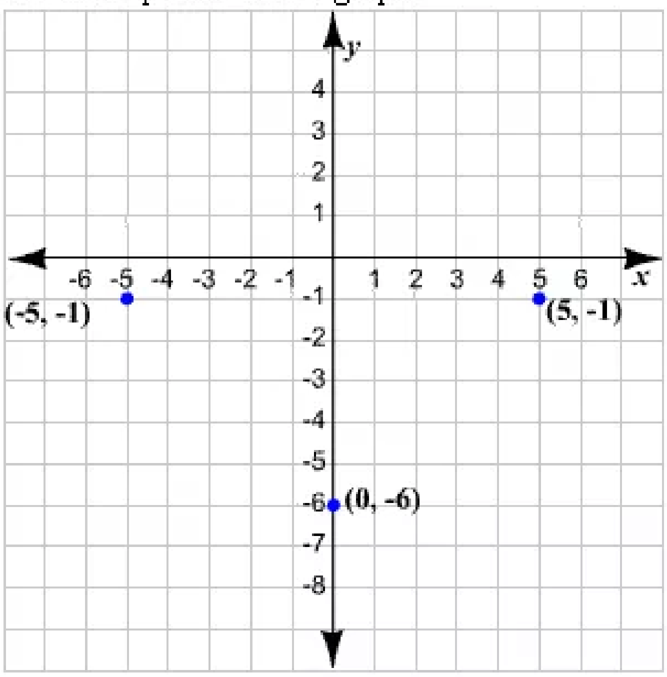
Add 7 to both the sides.

$$0 + 6 = |x| - 6 + 6$$

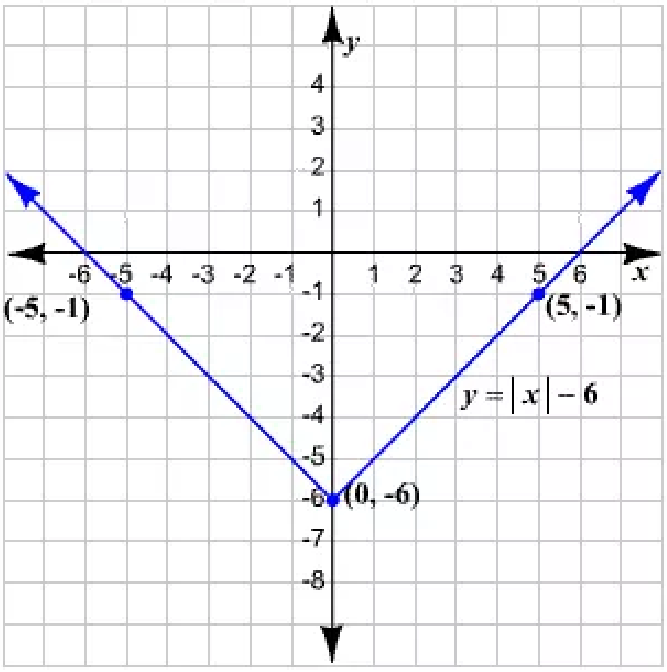
$$6 = |x|$$

We get the two values for x: 6 and −6. The two points are (6, 0) and (−6, 0).

Plot these points on the graph.



Step 3 Connect the points using straight lines to obtain a V-shaped graph.



Answer 50e.

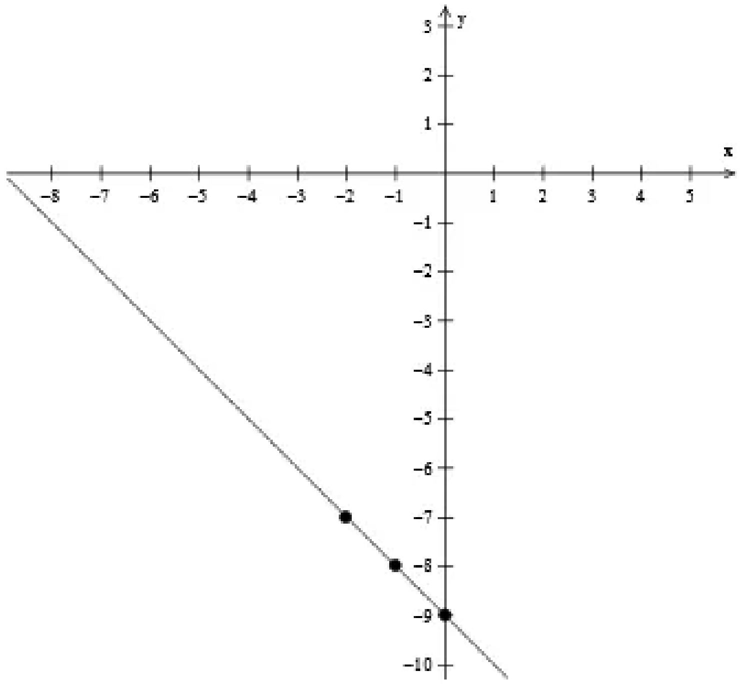
The intercept method is a convenient way to graph equations such as $y=-|x+4|-5$, because they are in standard $Ax+By=C$ form.

To graph this equation, we have to determine the coordinates of point which is to be plotted.
As a check, pick any x-value.

$$y=-|x+4|-5$$

x	y	(x,y)
0	-9	$(0,-9)$
-1	-8	$(-1,-8)$
-2	-7	$(-2,-7)$

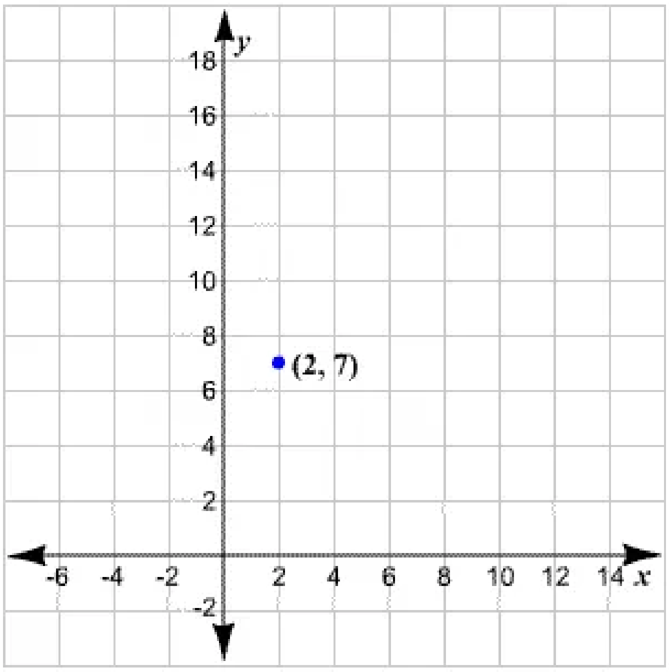
We shall now graph the equation



Answer 51e.

Step 1 The given function is of the form $y = |x - h| + k$, where (h, k) is the vertex of the function's graph.
Comparing the given equation with $y = |x - h| + k$, the value of h is 2, and that of k is 7. Thus, the vertex is (2, 7).

Plot the vertex.



Step 2 Use symmetry to find two more points.

Substitute any value, say, 0 for x in the given function.

$y = 2|0 - 2| + 7$

Simplify within the absolute value.

$y = 2|-2| + 7$

Evaluate the absolute value.

$y = 2(2) + 7$

Multiply.

$y = 4 + 7$

Add.

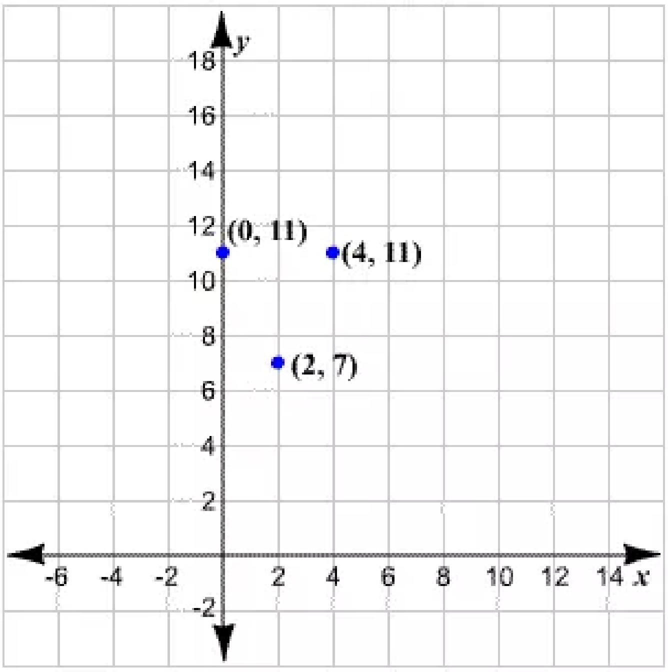
$y = 11$

A point on the graph is (0, 11).

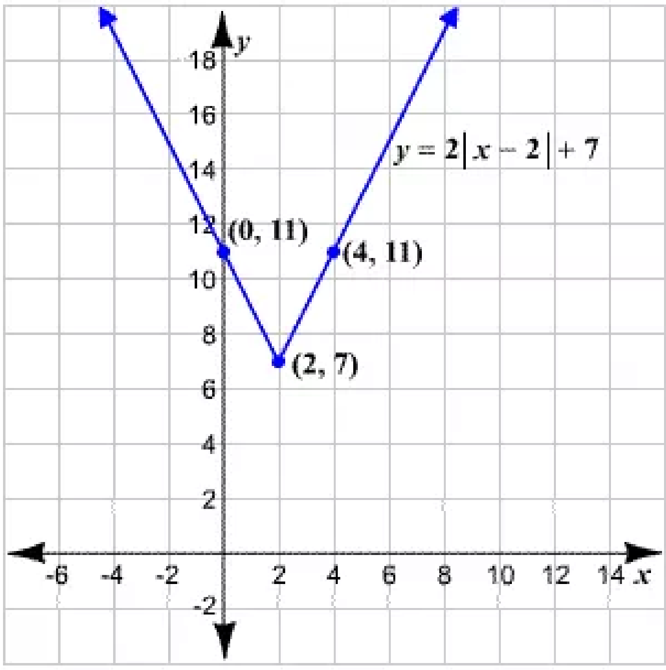
Since the given function is an absolute, any point on the graph will be symmetric about the line that passes through the vertex.

The vertex of the graph is (2, 7) and so the graph is symmetric about the line $x = 2$. Another point is (4, 11).

Plot these two points on the graph.



Step 3 Connect the points using straight lines to obtain a V-shaped graph.



Answer 52e.

The slope-intercept form of a linear equation is $y = mx + c$,
where m is the slope of the line and c is its y-intercept.

It is given that the slope of the line is 2; substitute $m = 2$ in the above equation.

$$y = mx + c$$
$$y = 2x + c \quad \ldots\ldots(1)$$

Also the line passes through the point $(0, -4)$; therefore substituting $x = 0$ and $y = -4$ in the above equation.

$$y = 2x + c$$
$$-4 = 2(0) + c$$
$$-4 = c$$

Substitute the value of c in equation (1)

$$y = 2x - 4$$

The equation in slope-intercept form is $\boxed{y = 2x - 4}$

Answer 53e.

The equation $y - y_1 = m(x - x_1)$ is the point-slope form of the line with slope m that contains the point (x_1, y_1).

Substitute 2 for m, 5 for x_1, and 2 for y_1.

$y - 2 = 2(x - 5)$

Use the distributive property to open the parentheses.

$y - 2 = 2x - 10$

Add 2 to both the sides to rewrite the equation in slope-intercept form.

$$y - 2 + 2 = 2x - 10 + 2$$
$$y = 2x - 8$$

The equation of the line is $y = 2x - 8$.

Answer 54e.

The slope-intercept form of a linear equation is $y = mx + c$, where m is the slope of the line and c is its y-intercept.

It is given that the slope of the line is $-\frac{2}{3}$; substitute $m = -\frac{2}{3}$ in the above equation.

$$y = mx + c$$
$$y = \left(-\frac{2}{3}\right)x + c \qquad \ldots\ldots(1)$$

Also the line passes through the point $(0, -1)$; therefore substituting $x = 0$ and $y = -1$ in the above equation.

$$y = \left(-\frac{2}{3}\right)x + c$$
$$-1 = \left(-\frac{2}{3}\right)(0) + c$$
$$-1 = c$$

Substitute the value of c in equation (1)

$$y = \left(-\frac{2}{3}\right)x - 1$$

The equation in slope-intercept form is $\boxed{y = \left(-\frac{2}{3}\right)x - 1}$

Answer 55e.

The equation $y - y_1 = m(x - x_1)$ is the point-slope form of the line with slope m that contains the point (x_1, y_1).

Substitute $\frac{3}{4}$ for m, 0 for x_1, and 3 for y_1.

$$y - 3 = \frac{3}{4}(x - 0)$$

Simplify.

$$y - 3 = \frac{3}{4}x$$

Add 3 to both the sides to rewrite the equation in slope-intercept form.

$$y - 3 + 3 = \frac{3}{4}x + 3$$
$$y = \frac{3}{4}x + 3$$

The equation of the line is $y = \frac{3}{4}x + 3$.

Answer .

When two points (x_1, y_1) and (x_2, y_2) are given, the equation of a line is of the form:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

Substituting $(4, 8)$ as (x_1, y_1) and $(1, 2)$ as (x_2, y_2) in the above equation

$$\frac{x-(4)}{(1)-(4)} = \frac{y-(8)}{(2)-(8)}$$
$$\frac{x-4}{-3} = \frac{y-8}{-6}$$

Cross multiply the numerator and the denominator

$$-6(x-4) = -3(y-8)$$
$$-6x + 24 = -3y + 24$$
$$-6x + 3y = 0$$

Divide by -3 so that we can write in the standard form: $2x - y = 0$

The equation of the line is $\boxed{2x - y = 0}$.

Answer 57e.

First, find the slope of the line passing through the points (–8, 8) and (0, 1).
Ratio of the vertical change to the horizontal change will give us the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (–8, 8) for (x_1, y_1), and (0, 1) for (x_2, y_2) and evaluate.

$$\begin{aligned} m &= \frac{1-8}{0-(-8)} \\ &= \frac{-7}{8} \\ &= -\frac{7}{8} \end{aligned}$$

The slope of the line that passes through (–8, 8) and (0, 1) is $-\frac{7}{8}$.

An equation of a line in the point-slope form with slope m and passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$.

Substitute –8 for x_1, 8 for y_1, and $-\frac{7}{8}$ for m in the point-slope form.

$$\begin{aligned} y - 8 &= -\frac{7}{8}[x - (-8)] \\ y - 8 &= -\frac{7}{8}(x + 8) \\ y - 8 &= -\frac{7}{8}x - 7 \end{aligned}$$

Add 8 to both sides of the equation.

$$\begin{aligned} y - 8 + 8 &= -\frac{7}{8}x - 7 + 8 \\ y &= -\frac{7}{8}x + 1 \end{aligned}$$

Thus, the equation of the line that passes through (–8, 8) and (0, 1) is $y = -\frac{7}{8}x + 1$.

Answer 58e.

The given system of linear equations is

$$3x+2y=5 \quad \ldots\ldots(1)$$
$$-x+3y=13 \quad \ldots\ldots(2)$$

We can solve the above the linear equation using elimination method.

If we pick equations (1) and (2) multiply by 3, add the resulting equations, the variable x gets eliminated.

$$\begin{aligned} 3x+2y&=5 \\ -3x+9y&=39 \\ \hline 11y&=44 \\ y&=4 \end{aligned}$$

We can substitute y in any equation containing x and y (such as equation 2).

$$\begin{aligned} -x+3y&=13 \\ -x+3(4)&=13 \\ -x+12&=13 \\ -x&=13-12 \\ -x&=1 \\ x&=-1 \end{aligned}$$

The solution set is $\boxed{\{(-1,4)\}}$.

Answer 59e.

Number the equations.

$3x - 5y = 11$ Equation 1

$2x + 5y = 24$ Equation 2

Since the coefficients of x in the two equations are opposites, use the elimination method to solve the system.

STEP 1 Add the equations to eliminate x.

$$\begin{array}{r} 3x - 5y = 11 \\ \underline{2x + 5y = 24} \\ 5x \quad\quad = 35 \end{array}$$

Divide both the sides by 5.

$$\frac{5x}{5} = \frac{35}{5}$$

$$x = 7$$

STEP 2 Substitute 7 for x in either equations of the system, say, Equation 1 and simplify.

$$3(7) - 5y = 11$$

$$21 - 5y = 11$$

Subtract 21 from both the sides.

$$21 - 5y - 21 = 11 - 21$$

$$-5y = -10$$

Divide both the sides by -5.

$$\frac{-5y}{-5} = \frac{-10}{-5}$$

$$y = 2$$

CHECK Let us check the solution by substituting 2 for x, and 7 for y in the original equations.

$3x - 5y = 11$	$2x + 5y = 24$
$3(7) - 5(2) \stackrel{?}{=} 11$	$2(7) + 5(2) \stackrel{?}{=} 24$
$11 = 11$ ✓	$24 = 24$ ✓

The solution is (7, 2).

Answer 60e.

The given system of linear equations is

$$3x - y = 4 \quad \ldots\ldots(1)$$
$$-2x + 3y = -26 \quad \ldots\ldots(2)$$

We can solve the above the linear equation using elimination method.

If we pick equations (1) multiply by 3 and (2), add the resulting equation, the variable y gets eliminated.

$$\begin{aligned} 9x - 3y &= 12 \\ -2x + 3y &= -26 \\ \hline 7x &= -14 \\ x &= -2 \end{aligned}$$

We can substitute x in any equation containing x and y (such as equation 1).

$$\begin{aligned} 3(-2) - y &= 4 \\ -6 - y &= 4 \\ -y &= 4 + 6 \\ -y &= 10 \\ y &= -10 \end{aligned}$$

The solution set is $\boxed{\{(-2, -10)\}}$.

Answer 61e.

Number the equations.

$4x - 3y = 17$ Equation 1

$2x + 5y = 15$ Equation 2

STEP 1 We can eliminate one of the variables by obtaining coefficients that are opposites of each other.

In this case, multiply Equation 1 by 5 and Equation 2 by 3 as the first step in eliminating y.

$4x - 3y = 17 \xrightarrow{\times\ 5} 20x - 15y = 85$ Equation 3

$2x + 5y = 15 \xrightarrow{\times\ 3} 6x + 15y = 45$ Equation 4

STEP 2 Add Equation 3 and Equation 4 to eliminate y.

$$\begin{array}{rcl} 20x - 15y &=& 85 \\ 6x + 15y &=& 45 \\ \hline 26x &=& 130 \end{array}$$

Divide both the sides by 26.

$$\frac{26x}{26} = \frac{130}{16}$$

$$x = 5$$

STEP 3 Substitute 5 for x in either equations of the system, say, Equation 2 and simplify.

$$4(5) - 3y = 17$$

$$20 - 3y = 17$$

Subtract 20 from both the sides.

$$20 - 3y - 15 = 17 - 20$$

$$-3y = -3$$

Divide both the sides by -3.

$$\frac{-3y}{-3} = \frac{-3}{-3}$$

$$y = 1$$

CHECK Let us check the solution by substituting 5 for x, and 1 for y in the original equations.

$4x - 3y = 17$	$2x + 5y = 15$
$4(5) - 3(1) \stackrel{?}{=} 17$	$2(5) + 5(1) \stackrel{?}{=} 15$
$20 - 3 \stackrel{?}{=} 17$	$10 + 5 \stackrel{?}{=} 15$
$17 = 17$ ✓	$15 = 15$ ✓

The solution is (5, 1).

Answer 62e.

The given system of linear equations is

$$4x-2y=14$$
$$-2x+y=-7$$

We can solve the above the linear equation using elimination method.

If we pick equations (1) and (2) multiply by 2, add the resulting equation, the variable y gets eliminated.

$$\begin{array}{r} 4x-2y=14 \\ \underline{-4x+2y=-14} \\ 0=0 \end{array}$$

This statement implies that this system has no solution.

Answer 63e.

Number the equations.

$x+4y = 4$ Equation 1

$3x-2y = 19$ Equation 2

Since the coefficient of x in the first equation is 1, use the substitution method to solve the system.

STEP 1 Solve Equation 1 for x.

Subtract $4y$ from both the sides.

$$x+4y-4y = 4-4y$$
$$x = 4-4y \quad \text{Revised Equation 1}$$

STEP 2 Substitute $4-4y$ for x in Equation 2.

$$3(4-4y)-2y=19$$

Clear the parentheses using the distributive property.

$$3(4)+3(-4y)-2y = 19$$
$$12-12y-2y = 19$$
$$12-14y = 19$$

Solve for y. For this, subtract 12 from both the sides.

$$12 - 14y - 12 = 19 - 12$$
$$-14y = 7$$

Divide both the sides by -14.

$$\frac{-14y}{-14} = \frac{7}{-14}$$
$$y = -\frac{1}{2}$$

STEP 3 Substitute $-\frac{1}{2}$ for y in Revised Equation 1.

$$x = 4 - 4\left(-\frac{1}{2}\right)$$
$$= 4 + 2$$
$$= 6$$

CHECK Let us check the solution by substituting 6 for x, and $-\frac{1}{2}$ for y in the original equations.

$x + 4y = 4$	$3x - 2y = 19$
$6 + 4\left(-\frac{1}{2}\right) \stackrel{?}{=} 4$	$3(6) - 2\left(-\frac{1}{2}\right) \stackrel{?}{=} 19$
$4 = 4$ ✓	$19 = 19$ ✓

The solution is $\left(6, -\frac{1}{2}\right)$.