Sample Question Paper - 35
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers.

OR

If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, then show that $c^2 = a^2(1 + m^2)$.

2. Draw a circle of radius 3.5 cm. Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents to the circle from that point.

3. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30°. Find the height of the tower.

OR

A kite is flying at a height of 30 m from the ground. The length of string from the kite to the ground is 60 m. Assuming that there is no slack in the string, find the angle of elevation of the kite at the ground.

4. If the median of the series exceeds the mean by 3, then by what number the mode exceeds its mean?

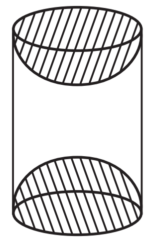
5. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. Find the distance of the car from the tower (in metres).

6. Find the mode of the following frequency distribution:

Class-interval	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	6

SECTION - B

7. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.

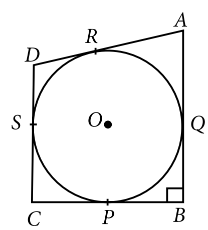


8. Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men. (Use $\sqrt{3} = 1.73$)

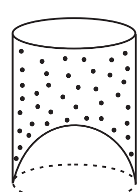
9. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

OR

In the given figure, a circle with centre O is inscribed in a quadrilateral $ABCD$ such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$ cm, $\angle B = 90°$ and $DS = 5$ cm, then find the radius of the circle (in cm).



10. A juice seller was serving his customers using glasses as shown in the given figure. The inner diameter of the cylindrical glass was 5 cm but bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of the glass was 10 cm, find the apparent and actual capacity of the glass. (Use $\pi = 3.14$)



SECTION - C

11. The following table gives weekly wages in rupees of workers in a certain commercial organization. The frequency of class 49-52 is missing. It is known that the mean of the frequency distribution is 47.2. Find the missing frequency.

Weekly wages (in ₹)	40-43	43-46	46-49	49-52	52-55
Number of workers	31	58	60	?	27

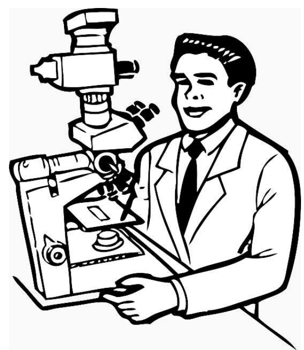
12. To fill a swimming pool two pipes are to be used. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. Find how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool.

OR

Some students planned a picnic. The total budget for food was ₹ 2,000. But 5 students failed to attend the picnic and thus the cost of food for each member increased by ₹ 20. How many students attended the picnic and how much did each student pay for the food?

Case Study - 1

13. In a pathology lab, a culture test has been conducted. In the test, the number of bacteria taken into consideration in various samples is all 3-digit numbers that are divisible by 7, taken in order.



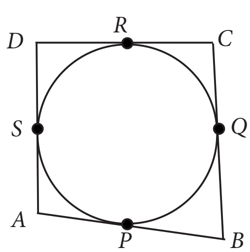
On the basis of above information, answer the following questions.

(i) How many bacteria are considered in the fifth sample?

(ii) Find the total number of bacteria in the first 10 samples.

Case Study - 2

14. In a park, four poles are standing at positions *A*, *B*, *C* and *D* around the fountain such that the cloth joining the poles *AB*, *BC*, *CD* and *DA* touches the fountain at *P*, *Q*, *R* and *S* respectively as shown in the figure.



Based on the above information, answer the following questions.

(i) If *O* is the centre of the circular fountain, then find $\angle OSA$.

(ii) If $DR = 7$ cm and $AD = 11$ cm, then find *AP*.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Let three consecutive natural numbers be $x, x+1, x+2$.

According to question,

$(x+1)^2 - [(x+2)^2 - (x)^2] = 60$

$\Rightarrow (x+1)^2 - [x^2 + 4 + 4x - x^2] = 60$

$\Rightarrow x^2 + 1 + 2x - 4 - 4x = 60$

$\Rightarrow x^2 - 2x - 63 = 0 \Rightarrow x^2 - 9x + 7x - 63 = 0$

$\Rightarrow x(x-9) + 7(x-9) = 0 \Rightarrow (x+7)(x-9) = 0$

$\Rightarrow x = 9$ ($\because x \neq -7$ as x is a natural number)

$\therefore$ The numbers are 9, 10, 11.

OR

We have, $(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$

Since, equation has equal roots.

$\therefore$ Discriminant, $D = 0$

$\Rightarrow (2mc)^2 - 4(1+m^2)(c^2 - a^2) = 0$

$\Rightarrow 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4a^2m^2 = 0$

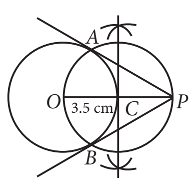
$\Rightarrow c^2 = a^2 + a^2m^2 = a^2(1+m^2)$

2. Steps of construction :

Step-I : Draw a circle of radius 3.5 cm, taking O as center and OC be its radius.

Step-II : Produce OC to P such that $OP = 7$ cm.

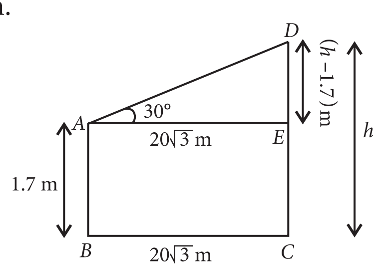
Step-III : Draw perpendicular bisector of OP that meets OP at C.



Step-IV : Taking C as centre and radius CP, draw a circle which intersect previous circle at points A and B.

Step-V : Join P to A and P to B. Now, PA and PB are the required tangents.

3. Let AB be the observer and CD be the tower of height h m.



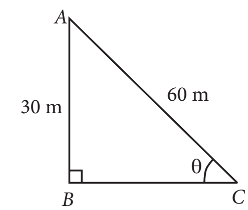
In ΔAED, $\tan 30° = \frac{DE}{AE} = \frac{h - 1.7}{20\sqrt{3}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-1.7}{20\sqrt{3}} \Rightarrow 20 = h - 1.7 \Rightarrow h = 21.7$

Hence, height of the tower is 21.7 m.

OR

Let A be the position of the kite, AC be the length of the string of the kite and θ be the angle of elevation of the kite at the ground.



$\therefore$ In right ΔABC,

$\sin\theta = \frac{AB}{AC} = \frac{30}{60}$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

4. Given, Median – Mean = 3 ...(i)

We know, Mode = 3 Median – 2 Mean

$\Rightarrow$ Mode – Mean = 3 Median – 3 Mean

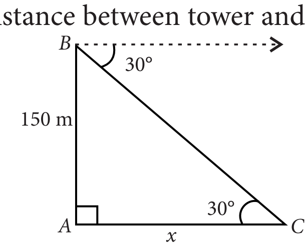
= 3(Median – Mean)

$\Rightarrow$ Mode – Mean = 3 × 3 [Using (i)]

$\Rightarrow$ Mode – Mean = 9

Hence, the mode exceeds the mean by 9.

5. Let, $AC = x$ m be the distance between tower and car and let $AB = 150$ m be height of tower.



In ΔABC,

$\tan 30° = \frac{150}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{150}{x} \Rightarrow x = 150\sqrt{3}$ m

Hence, distance between tower and car $= 150\sqrt{3}$ m .

6. From the given data, we observe that the highest frequency is 12, which lies in the interval 18-24.

$\therefore$ Modal class is 18-24.

So, $l = 18, f_0 = 10, f_1 = 12, f_2 = 6, h = 6$

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 18 + \left(\frac{12 - 10}{2 \times 12 - 10 - 6}\right) \times 6 = 18 + \frac{12}{8}$

$= 18 + 1.5 = 19.5$

7. Radius of the cylinder (r) = 3.5 cm

Height of the cylinder (h) = 10 cm

$\therefore$ Curved surface area of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{35}{10} \times 10\text{cm}^2 = 220\text{ cm}^2$

Curved surface area of a hemisphere $= 2\pi r^2$

$\therefore$ Curved surface area of both hemispheres

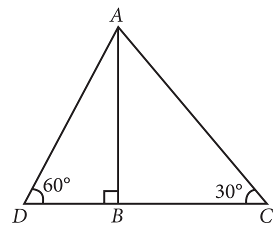
$$= 2 \times 2\pi r^2 = 4\pi r^2 = 4 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \text{cm}^2 = 154\text{cm}^2$$

Total surface area of the solid = (220 + 154) cm^2

= 374 cm^2.

8. Let AB = 75 m be the building and C, D be the positions of two men.

Now, in ΔABC,



$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BC}$$

$$\Rightarrow BC = 75\sqrt{3} \text{ m}$$

In ΔABD, $\tan 60° = \dfrac{AB}{BD}$

$$\Rightarrow \sqrt{3} = \frac{75}{BD} \Rightarrow BD = \frac{75}{\sqrt{3}} \text{ m} = 25\sqrt{3} \text{ m}$$

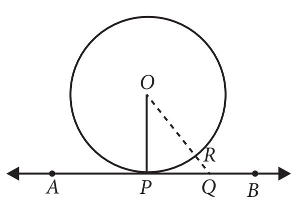
$\therefore$ Distance between the two men

$$= BC + BD = 75\sqrt{3} + 25\sqrt{3} = 100\sqrt{3} = 173 \text{ m}$$

9. Given : A circle $C(O, r)$ and a tangent AB at a point P.

To prove : $OP \perp AB$.

Construction : Take any point Q, other than P, on the tangent AB. Join OQ. Suppose OQ meets the circle at R.



Proof : We know that among all line segments joining the point O to a point on AB, the shortest one is perpendicular to AB. So, to prove that $OP \perp AB$, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.

Clearly, $OP = OR$ [radii of the same circle]

Now, $OQ = OR + RQ \Rightarrow OQ > OR$

$\Rightarrow OQ > OP$ [$\because OP = OR$]

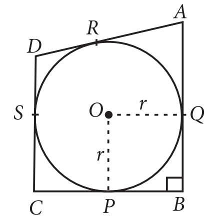
$\Rightarrow OP < OQ$

Thus, OP is shorter than any other segment joining O to any point of AB.

Hence, $OP \perp AB$.

OR

We know that, the lengths of the tangents drawn from an external point to a circle are equal.



$\therefore AQ = AR, DR = DS,$

$BQ = BP, CS = CP$

So, $DS = DR$ = 5 cm

$AR = AD - DR = 23 - 5 = 18$ cm

$\Rightarrow AQ = AR = 18$ cm

$QB = AB - AQ = 29 - 18 = 11$ cm

$\Rightarrow QB = BP = 11$ cm

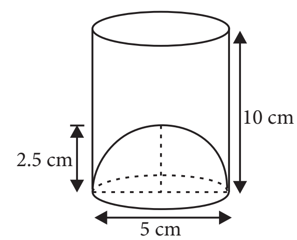
Since, $OQBP$ is a square

$\therefore BP = OP = OQ = QB = 11$cm

$\Rightarrow$ Radius of circle (OP) = 11 cm.

10. Base radius = $\dfrac{5}{2}$ = 2.5 cm

Apparent capacity of glass = Volume of cylindrical portion = $\pi r^2 h$



$= 3.14 \times (2.5)^2 \times 10$

$= 196.25 \text{ cm}^3$

Actual capacity of the glass = Volume of cylinder – Volume of hemisphere

$$= \pi r^2 h - \frac{2}{3}\pi r^3 = 196.25 - \frac{2}{3} \times 3.14 \times (2.5)^3$$

$= 196.25 - 32.71 = 163.54 \text{ cm}^3$

11. Let the missing frequency be f and h = 3.

Let us construct the following table for the given data.

Class-interval	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
40-43	31	41.5	1286.5
43-46	58	44.5	2581
46-49	60	47.5	2850
49-52	f	50.5	50.5f
52-55	27	53.5	1444.5
Total	$\Sigma f_i = 176 + f$		$\Sigma f_i x_i = 8162 + 50.5f$

$$\therefore \text{Mean} = \left\{\frac{\Sigma f_i x_i}{\Sigma f_i}\right\}$$

$$\Rightarrow \frac{8162 + 50.5f}{176 + f} = 47.2$$

$\Rightarrow 8162 + 50.5f = 8307.2 + 47.2f$

$\Rightarrow 3.3f = 145.2$

$\Rightarrow f = 44$

Hence, the missing frequency is 44.

12. Let volume of the swimming pool = V

Time taken to fill the pool by the pipe of larger diameter = x hours

$\therefore$ Time taken to fill the pool by the pipe of smaller diameter = $(x + 10)$hours

Part of the pool filled by the pipe of larger diameter in 1 hour $= \frac{V}{x}$

Part of the pool filled by the pipe of smaller diameter in 1 hour $= \frac{V}{x+10}$

According to question,

$$\frac{4V}{x} + \frac{9V}{x+10} = \frac{1}{2}V$$

$$\Rightarrow \frac{4x+40+9x}{x(x+10)} = \frac{1}{2} \Rightarrow \frac{13x+40}{x^2+10x} = \frac{1}{2}$$

$\Rightarrow x^2 + 10x = 26x + 80$

$\Rightarrow x^2 + 10x - 26x - 80 = 0 \Rightarrow x^2 - 16x - 80 = 0$

$\Rightarrow (x + 4)(x - 20) = 0 \Rightarrow x = -4$ or $x = 20$

$\Rightarrow x = 20$ ($\because x$ cannot be negative)

Hence, the pipe with larger diameter fills the tank in 20 hours and the pipe with smaller diameter fills the tank in 30 hours.

OR

Let the original number of students be x.

$\therefore$ Share of each student = ₹ $\frac{2000}{x}$

If the number of students is decreased by 5.

Then, new share of each student = ₹ $\frac{2000}{x-5}$

According to question,

$$\frac{2000}{x-5} - \frac{2000}{x} = 20$$

$\Rightarrow 2000x - 2000x + 10000 = 20x(x - 5)$

$\Rightarrow 20x^2 - 100x = 10000$

$\Rightarrow x^2 - 5x - 500 = 0 \Rightarrow (x - 25)(x + 20) = 0$

$\Rightarrow x = 25$ or $x = -20$

$\Rightarrow x = 25$ ($\because x \neq -20$)

Thus, number of students attended the picnic = 25 – 5
= 20

and each student has to pay ₹ $\frac{2000}{20}$ = ₹ 100 for the food.

13. Here the smallest 3-digit number divisible by 7 is 105. So, the number of bacteria taken into consideration is 105, 112, 119, …., 994.

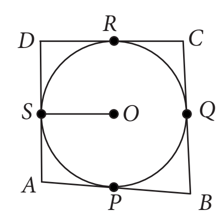
So, first term (a) = 105, d = 7 and last term = 994

(i) $t_5 = a + 4d = 105 + 28 = 133$

(ii) Total number of bacteria in first 10 samples

$$= S_{10} = \frac{10}{2}[2(105) + 9(7)] = 1365$$

14. (i)



Here, OS is the radius of the circle.

Since radius at the point of contact is perpendicular to tangent.

So, $\angle OSA = 90°$

(ii) Since, length of tangents drawn from an external point to a circle are equal.

$\therefore$ $AS = AP$, $BP = BQ$, $CQ = CR$ and $DR = DS$ ...(1)

Now, $AP = AS = AD - DS = AD - DR$ (Using (1)

= 11 – 7 = 4 cm