

Chapter 3

Transient Analysis (AC and DC)

CHAPTER HIGHLIGHTS

- ✎ DC Transients
- ✎ Source Free Circuits
- ✎ Singularity Functions
- ✎ Step, Impulse, and Ramp functions
- ✎ Step Response of an RC Circuit
- ✎ Transient and Steady-state Response
- ✎ Equivalent Circuits for R, L and C in S Domain
- ✎ Higher – Order Circuits
- ✎ Series and Parallel RLC Circuits
- ✎ A.C. Transients
- ✎ Transient Response with Sinusoidal Excitation
- ✎ Sinusoidal Steady – State Analysis
- ✎ Inductor
- ✎ Capacitor
- ✎ Sinusoidal Steady State Analysis of RLC Circuits
- ✎ RL Series Circuit
- ✎ Series RC Circuit
- ✎ Series RLC Circuit

INTRODUCTION

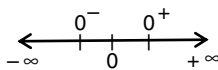
Whenever a circuit is switched from one condition to another, either by a change in the applied source or a change in the circuit elements, there is a transitional period during which the branch currents and element voltages change from their initial values to new ones. This period is called the transient period. After the transient period has passed, the circuit is said to be in steady state.

Transient in the system is because of the presence of energy storing elements (i.e., L and C).

Since the energy stored in a memory element cannot change instantaneously, that is, within zero time.

The network consists of only resistances. No transients in the system at the time of switching, since the resistor can accommodate any amount of voltage and currents.

The equivalent form of the elements in terms of the initial condition of the elements.



(i) $Z_L = sL\Omega$

(ii) $Z_C = \frac{1}{sC}\Omega$

At $t = 0^+ \Rightarrow f = \infty \Rightarrow Z_L = \infty \Rightarrow L \Rightarrow 0\text{ C}$

$Z_C = \frac{1}{2\pi fc} = 0\Omega \Rightarrow C$ is the short circuit.

S. No	Element (initial condition)	Equivalent circuit at $t = 0^+$
1.		
2.		
3.		
4.		
5.		

A long time after the switching action ($t \rightarrow \infty$) is the steady state (SS). In SS, the inductor behaves like a short circuit and capacitor behaves like an open circuit.

$t \rightarrow \infty \Rightarrow f = 0.$

$Z_L = sL\Omega \Rightarrow Z_L = 0\Omega \Rightarrow$ is the short circuit.

$Z_C = \frac{1}{sC}\Omega \Rightarrow Z_C = \infty$ is the open circuit.

* The equivalent form of the elements in terms of the final condition of the element.

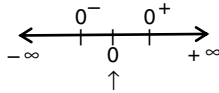
S. No	Element (and initial condition)	Equivalent circuit at $t = \infty.$ (steady state values)
1.		
2.		

(Continued)

S. No	Element (and initial condition)	Equivalent circuit at $t = \infty$, (steady state values)
3.		
4.		
5.		

NOTES

1. Inductor current at $t = 0^-$ and $t = 0^+$ instants



Current flowing through an inductor is given as

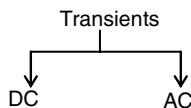
$$\begin{aligned}
 i_L(t) &= \frac{1}{L} \int_{-\infty}^t V_L(t) \cdot dt \\
 &= \frac{1}{L} \int_{-\infty}^{0^-} V_L(t) dt + \frac{1}{L} \int_{0^-}^{0^+} V_L(t) dt \\
 i_L(0^+) &= i_L(0^-) + 0 \\
 \therefore i_L(0^+) &= i_L(0^-)
 \end{aligned}$$

$\therefore$ Inductor current cannot change instantaneously.

2. Capacitor voltage at $t = 0^-$ and at $t = 0^+$ instants.

$$\begin{aligned}
 V_C(t) &= \frac{1}{C} \int_{-\infty}^t i_C \cdot dt \\
 &= V_C(0^-) + \frac{1}{C} \int_{0^-}^t i_C(t) \cdot dt \\
 \text{At } t = 0^+ \\
 V_C(0^+) &= V_C(0^-) + \frac{1}{C} \int_{0^-}^{0^+} i_C(t) \cdot dt \\
 V_C(0^+) &= V_C(0^-) \text{ Volts.}
 \end{aligned}$$

CLASSIFICATION OF TRANSIENTS



The transient effects are more for DC as compared to AC and the transient-free condition is possible to only for AC excitations.

DC Transients

Source-free Circuits

1. RL circuit $\rightarrow$ initial current through the inductor ($L \rightarrow I_0$)

2. RC Circuit $\rightarrow$ initial voltage (V_0) across the capacitor ($C \rightarrow V_0$)
3. RLC Circuit $\rightarrow$ initial current through inductor or voltage across capacitor.

Source-free RC circuits: A source-free RC circuit occurs when its DC source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors.

Consider a series combination of a resistor and an initially charged capacitor as shown in Figure 1.

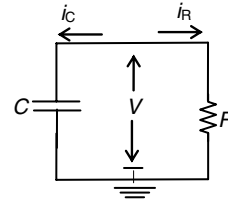


Figure 1 A source-free RC circuit.

The capacitor is initially charged; we can assume that at time $t = 0$, the initial voltage is

$$V(0) = V_0$$

By applying KCL at the node of circuit in Figure 1,

$$\begin{aligned}
 i_C + i_R &= 0 \\
 C \cdot \frac{dV}{dt} + \frac{V}{R} &= 0 \Rightarrow \frac{dV}{dt} + \frac{1}{RC} \cdot V = 0 \\
 \frac{dV}{V} &= -\frac{1}{RC} \cdot dt
 \end{aligned}$$

By integrating both sides, we get

$$\ln V = \frac{-t}{RC} + \ln A$$

where A is the integration constant. Thus,

$$\ln \frac{V}{A} = \frac{-t}{RC} \Rightarrow \frac{V}{A} = e^{-t/RC}$$

$$\therefore V(t) = A \cdot e^{-t/RC}$$

However, from initial conditions at $t = 0$

$$V(0) = A \cdot 1 \Rightarrow A = V(0) = V_0$$

$$\therefore V(t) = V_0 \cdot e^{-t/RC} \text{ Volts}$$

Therefore, it is called the natural response of the circuit. The natural response of a circuit refers to the behaviour of the circuit itself, with no external sources of excitation.

The natural response is illustrated graphically in Figure 2.

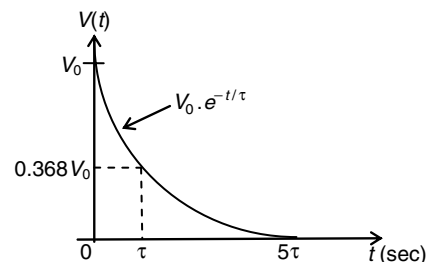


Figure 2 The voltage response of the RC circuit.

Time Constant

The time constant of a circuit is the time required for the response to decay to a factor of $1/e$ or 36.8% of its initial value.

At $t = \tau$

$$V_0 \cdot e^{-\tau/RC} = V_0 \cdot e^{-1} = 0.368V_0$$

$$\therefore \tau = RC$$

$$\therefore V(t) = V_0 \cdot e^{-t/RC} = V_0 \cdot e^{-t/\tau} \text{ Volts}$$

NOTE

In finding the time constant $\tau = RC$, R is obtained the Thevenin's equivalent resistance at the terminals of the capacitor.

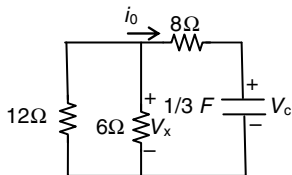
$$\therefore \tau = R_{eq} C = R_{th} C$$

$$i_R(t) = \frac{V(t)}{R} = \frac{V_0}{R} \cdot e^{-t/\tau} \text{ A}$$

Power dissipated in the resistor (R) is

$$P(t) = V \cdot i_R = \frac{V_0^2}{R} \cdot e^{-2t/\tau}$$

Let us consider the following example. The circuit shown in figure. Let $V_C(0) = 15 \text{ V}$



Solved Examples

Example 1

The value of $V_C(t)$, at $t = 2 \text{ s}$

- (A) 12 V (B) 13.62 V
(C) 2.04 V (D) 9.09 V

Solution

We know for source-free RC circuits

$$V_C(t) = V_0 \cdot e^{-t/RC}$$

$$\tau = R_{eq} C \Rightarrow R_{eq} = 8 + (6 \parallel 12) \Rightarrow 12 \Omega$$

$$\tau = 12 \times 1/3 = 4 \text{ s}$$

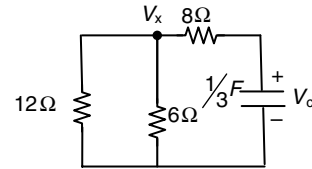
$$V_C(t) = 15e^{-t/4} \text{ V}$$

at $t = 2 \text{ s}$

$$V_C(2) = 15e^{-2/4} \Rightarrow 15e^{-0.5} = 9.09 \text{ V}$$

Example 2

The value of V_x for $t \geq 0$



- (A) $V_x = 5 \cdot e^{-4t} \text{ V}$ (B) $V_x = 12 \cdot e^{-0.25t} \text{ V}$
(C) $V_x = 5 \cdot e^{-0.25t} \text{ V}$ (D) $V_x = 15 \cdot e^{-0.71t} \text{ V}$

Solution

By applying KCL at node V_x .

$$\frac{V_x}{12} + \frac{V_x}{6} + \frac{V_x - V_C}{8} = 0$$

$$2V_x + 4V_x + 3(V_x - V_C) = 0$$

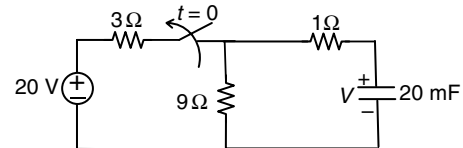
$$9V_x = 3V_C$$

$$V_x = 1/3 V_C \Rightarrow V_x = 1/3 \times 15 \cdot e^{-t/4} \text{ V}$$

$$V_x = 5 \cdot e^{-0.25t} \text{ V}$$

Example 3

The switch in the circuit in figure has been closed for a long time, and it is opened at $t = 0$.



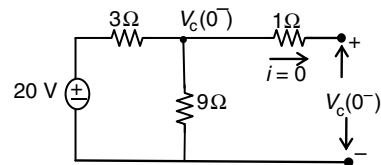
(i) The $V_C(t)$ for $t \geq 0$ is

- (A) $V_C(t) = 15e^{-t} \text{ V}$ (B) $V_C(t) = 12e^{-0.2t} \text{ V}$
(C) $V_C(t) = 15e^{-5t} \text{ V}$ (D) $V_C(t) = 12e^{-5t} \text{ V}$

Solution (i)

For $t < 0$:

The switch is closed, and the capacitor is open circuit to DC.



Circuit is in S and S.

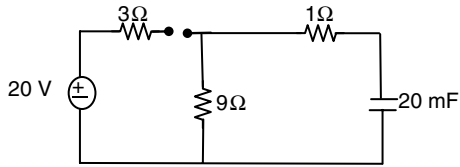
$$\therefore V_C(0^-) = 20 \times \frac{9}{9+3} = 15 \text{ V for } t < 0.$$

Since the voltage across capacitor does not change instantaneously,

$$V_C(0^-) = V_C(0^+) = 15 \text{ V}$$

For $t \geq 0$:

The switch is opened. The circuit is shown in the figure.



$$\therefore V_C(t) = V_0 e^{-t/\tau}$$

$$\tau = R_{eq} C.$$

$$R_{eq} \Rightarrow$$

$$R_{eq} = (9 + 1) = 10 \Omega$$

$$\tau = 10 \times 20 \times 10^{-3} \tau = 0.2 \text{ s}.$$

Thus, the voltage across the capacitor for $t \geq 0$ is

$$V(t) = V_0 \cdot e^{-t/\tau}$$

$$= 15e^{-t/0.2} \text{ V}$$

Therefore, $V(t) = 15e^{-5t} \text{ V}$

- (ii) The initial energy stored in the capacitor is
 (A) 4.5 J (B) 0 J (C) 2.25 J (D) 4 J

Solution (ii)

$$W_c(0) = 1/2 C V_c^2(0) = 1/2 C V_0^2$$

$$= 1/2 \times 20 \times 10^{-3} \times (15)^2 = 2.25 \text{ J}$$

Source-free RL circuits:

Consider the series connection of an RL circuit, as shown in the figure.

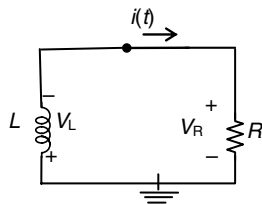


Figure 3 A source-free RL circuit.

At $t = 0$, we assume that the inductor has an initial current I_0 .
 $\therefore i(0) = I_0$.

By applying KVL around the loop in figure,

$$V_L + V_R = 0$$

$$L \cdot \frac{di}{dt} + iR = 0$$

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

Let $\frac{d}{dt} = D$

$$\left(D + \frac{R}{L}\right)i = 0$$

$$D = -R/L.$$

Therefore, $i(t) = A e^{-Dt} \Rightarrow i(t) = A e^{-Rt/L}$

$$\therefore i_L(t) = A \cdot e^{-t/(L/R)}$$

At $t = 0, i_L(0) = I_0$

$$i_L(0) = I_0 = A e^{-0} \Rightarrow A = I_0$$

$$i_L(t) = I_0 \cdot e^{-Rt/L} \text{ A}$$

$$\therefore \tau = \frac{L}{R} \text{ s}$$

$$\tau = \frac{L_{eq}}{R_{eq}} \text{ s}$$

$$\therefore i(t) = I_0 e^{-t/\tau} \text{ A}$$

This shows that the natural response of the RL circuit is an exponential decay of the initial current.

The current response shown in figure.

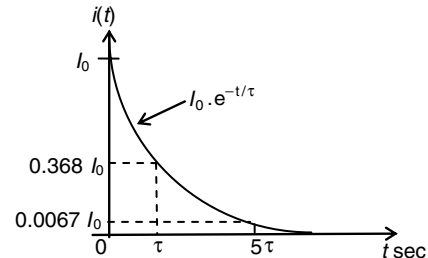


Figure 4 The current response of the RL circuit.

NOTE

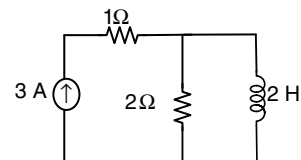
When

$$\tau = RC = \frac{L}{R}$$

$$R^2 = \frac{L}{C} \Rightarrow R = \sqrt{\frac{L}{C}} \Omega$$

Example 4

The time constant of the given circuit τ is

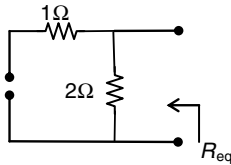


- (A) 1 s (B) 1.5 s
 (C) 2/3 s (D) 4 s

Solution

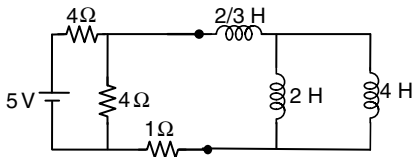
$$\tau = \frac{L_{eq}}{R_{eq}}$$

$$R_{eq} \Rightarrow R_{th}:$$



$$\Rightarrow R_{eq} = 2 \Omega$$

$$\tau = \frac{2}{2} = 1 \text{ sec.}$$

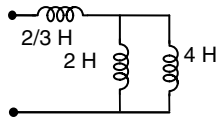
Example 5

The time constant τ is

- (A) 1 s (B) 1.5 s (C) $\frac{2}{3}$ s (D) 4 s

Solution

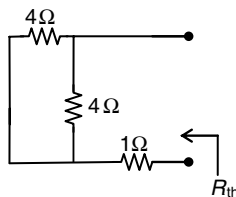
$$\tau = \frac{L_{eq}}{R_{eq}} \text{ s}$$



$$L_{eq} = \frac{2}{3} + (2 \parallel 4)$$

$$= \frac{2}{3} + \frac{4}{3} = 2 \text{ H.}$$

$$R_{eq} \Rightarrow R_{th}:$$



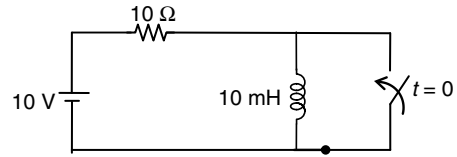
$$R_{th} = 1 + (4 \parallel 4)$$

$$= 3 \Omega$$

$$\tau = \frac{2}{3} \text{ s}$$

Example 6

The circuit shown in the figure is in a steady state. When the switch is closed at $t = 0$, the current through the inductor at $t = 0^+$ is



- (A) 0 A (B) 0.5 A (C) 1 A (D) 2 A

Solution

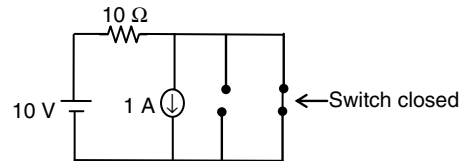
For $t < 0$:

at $t = 0^-$; the switch is opened.

In steady state, L behaves as a short circuit.

$$i_L(0^-) = \frac{10}{10} = 1 \text{ A.}$$

At $t = 0^+$, the circuit shown in figure.

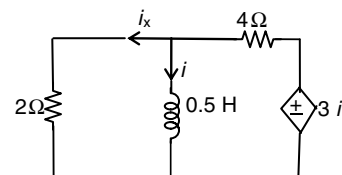


$\therefore$ Inductor does not allow sudden change in current.

$$\therefore i_L(0^+) = i_L(0^-) = 1 \text{ A}$$

Example 7

Assume that $i(0) = 10 \text{ A}$. Calculate the $i(t)$ and $i_x(t)$ in the circuit (for $t \geq 0$).



- (A) $i(t) = 10 \cdot e^{-2t/3} \text{ A}; i_x(t) = -5/3 \cdot e^{-2t/3} \text{ A}$
 (B) $i(t) = 10 \cdot e^{-3t/8} \text{ A}; i_x(t) = 5/3 \cdot e^{-3t/8} \text{ A}$
 (C) $i(t) = 10 \cdot e^{-4t} \text{ A}; i_x(t) = -20 \cdot e^{-3t/8} \text{ A}$
 (D) None of the above

Solution

$$i(t) = I_0 \cdot e^{-t/\tau}.$$

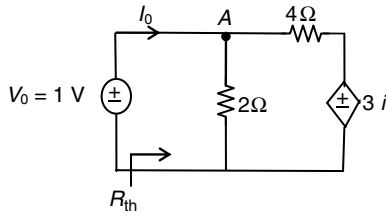
Given

$$I_0 = 10 \text{ A.}$$

$$\tau = \frac{L_{eq}}{R_{eq}}$$

$$R_{eq} = R_{th}:$$

The equivalent resistance is the same as the Thevenin's resistance at the inductor terminals.



Here, $I_0 = -i$

By applying KCL at node A,

$$\begin{aligned} \frac{V_0}{2} - I_0 + \frac{V_0 - 3i}{4} &= 0 \\ 2V_0 - 4I_0 + V_0 + 3I_0 &= 0 \\ 3V_0 &= I_0 \\ \frac{V_0}{I_0} &= R_{th} = \frac{1}{3} \Omega \\ \tau = \frac{L_{eq}}{R_{th}} &= \frac{0.5}{(1/3)} = 1.5 \text{ s} \end{aligned}$$

Thus, the current through the inductor is $i(t)$.

$$i(t) = 10 \cdot e^{-t/1.5} \text{ A}$$

$$i(t) = 10 \cdot e^{-2t/3} \text{ A for } t \geq 0.$$

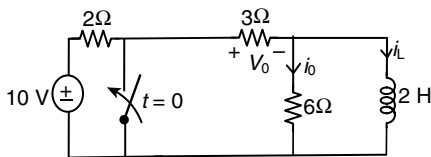
$$\begin{aligned} V_L(t) &= L \cdot \frac{di(t)}{dt} = 0.5 \times 10 \frac{d}{dt} e^{-2t/3} \\ &= 5 \times \left(-\frac{2}{3} \right) \cdot e^{-2t/3} \text{ V} = -\frac{10}{3} \cdot e^{-2t/3} \text{ V}. \end{aligned}$$

Voltage across inductor is equal to voltage across 2Ω resistors.

$$\therefore i_x(t) = \frac{V_L(t)}{2} = -\frac{5}{3} \cdot e^{-2t/3} \text{ A for } t > 0.$$

Example 8

In the circuit shown in figure assuming that the switch was open for a long time.



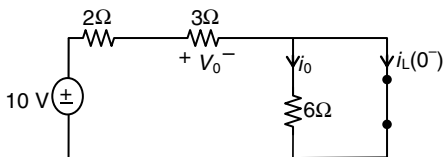
Find i_0 , V_0 , and i_L for all times $(-\infty \leq t \leq \infty)$.

Solution

$t < 0$:

For $t < 0$, the switch is open.

At $t = 0^-$, the inductor acts like a short circuit to DC.



The current through $6 \Omega \Rightarrow i_0 = 0 \text{ A}$.

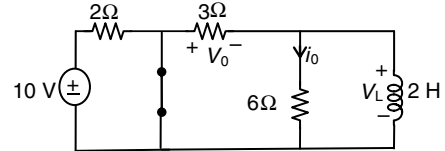
$$V_{6\Omega} = 0 \text{ V} \Rightarrow i_0 = 0 \text{ A}$$

$$i_L(0^-) = \frac{10}{5} = 2 \text{ A}.$$

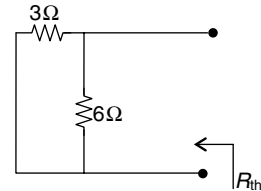
$$V_0 = 3 \times 2 = 6 \text{ V}.$$

(i) For $t > 0$, the switch is closed

$$i_L(t) = I_0 \cdot e^{-t/\tau} \text{ A}.$$



$\Rightarrow$ Find R_{th} :



$$\Rightarrow R_{th} = 6 \parallel 3 \Rightarrow \frac{6 \times 3}{9}$$

$$R_{th} = 2 \Omega.$$

$$\tau = \frac{L}{R_{th}} = \frac{2}{2} = 1 \text{ sec}.$$

Thus,

$$\begin{aligned} i_L(t) &= i(0^+) \cdot e^{-t/1} \\ &= 2 \cdot e^{-t} \text{ A for } t > 0. \end{aligned}$$

$$(ii) i_0(t) = \frac{V_L(t)}{6}$$

$$\begin{aligned} \Rightarrow V_L(t) &= L \cdot \frac{di(t)}{dt} = 2 \times \frac{d}{dt} (2 \times e^{-t}) \\ &= -4e^{-t} \text{ V}. \end{aligned}$$

$$i_0(t) = \frac{-2}{3} \cdot e^{-t} \text{ A for } t > 0.$$

$$(iii) V_0 = -V_L$$

$$\Rightarrow V_0 = 4e^{-t} \text{ V for } t > 0$$

Therefore, for all time,

$$i_0(t) = \begin{cases} 0 \text{ A}, & t < 0 \\ -\frac{2}{3} \cdot e^{-t} \text{ A}, & t \geq 0 \end{cases} \quad V_0(t) = \begin{cases} 6 \text{ V}, & t < 0 \\ 4 \cdot e^{-t} \text{ V}, & t > 0. \end{cases}$$

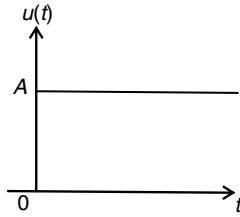
$$i(t) = \begin{cases} 2 \text{ A}, & t < 0 \\ 2 \cdot e^{-t} \text{ A}, & t \geq 0 \end{cases}$$

SINGULARITY FUNCTIONS

A basic understanding of singularity functions will help us make sense of the response of first-order circuits to a sudden application of an independent DC voltage or current source.

The most widely used singularity functions in circuit analysis are the step ramp and impulse functions.

Step Function



The mathematical terms:

$$u(t) = \begin{cases} 0 & t < 0 \\ A; & t > 0 \end{cases}$$

The unit step function is undefined at $t = 0$

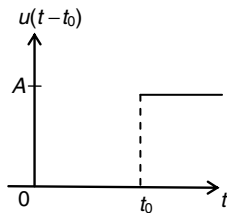


Figure 5 The step function delayed by t_0 units.

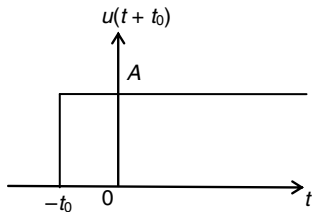
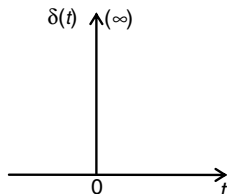


Figure 6 The step function advanced by t_0 .

Impulse Function $\delta(t)$

The derivative of the step function $u(t)$ is the impulse function $\delta(t)$.

$$\delta(t) = \frac{d}{dt} u(t) = \begin{cases} \text{undefined}; (\infty) & \text{for } t = 0 \\ 0; & \text{else where} \end{cases}$$



1. $\int_{0-}^{0+} \delta(t) dt = 1$
2. $\int_{t_1}^{t_2} f(t) \cdot \delta(t - t_0) dt = f(t_0); t_1 < t_0 < t_2$
 $= 0; \text{ else where}$

Ramp Function

Integrating the unit step function $u(t)$ results in the unit ramp function $r(t)$.

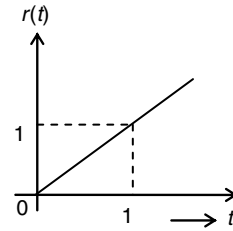


Figure 7 The unit ramp function.

$$r(t) = \int_{-\infty}^t u(t) \cdot dt = tu(t)$$

$$r(t) = \begin{cases} 0 & t < 0 \\ At & t \geq 0 \end{cases}$$

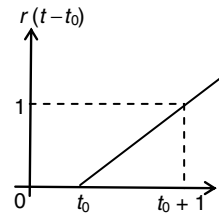


Figure 8 Unit ramp function delayed by t_0 .

$$r(t - t_0) = \begin{cases} 0 & t < t_0 \\ t - t_0 & t \geq t_0 \end{cases}$$

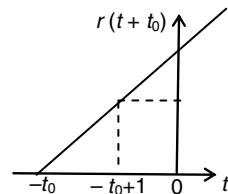


Figure 9 The unit ramp function advanced by t_0 .

$$r(t + t_0) = \begin{cases} 0 & t < -t_0 \\ t + t_0 & t \geq -t_0 \end{cases}$$

STEP RESPONSE OF AN RC CIRCUIT

When the DC source of an RC circuit is applied, the voltage or current source can be modelled as a step function, and the response is known as a step response.

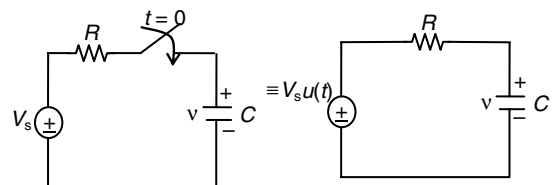


Figure 10 An RC circuit with voltage source.

We assume an initial voltage V_0 on the capacitor, since the voltage of the capacitor does not change instantaneously.

$$V(0^-) = V(0^+) = V_0$$

By applying KCL for $t \geq 0$,

$$\begin{aligned} C \cdot \frac{dV}{dt} + \frac{V - V_S}{R} &= 0. \\ \frac{dV}{dt} + \frac{V}{RC} &= \frac{V_S}{RC} \\ \frac{dV}{dt} &= -\frac{(V - V_S)}{RC} \\ \frac{dV}{V - V_S} &= -\frac{1}{RC} \cdot dt \end{aligned} \quad (1)$$

By integrating both sides and introducing the initial conditions,

$$\begin{aligned} \ln[V - V_S]_{V_0}^{V(t)} &= \left[-\frac{t}{RC} \right]_0^t \\ \ln[V(t) - V_S] - \ln[V_0 - V_S] &= \frac{-t}{RC} + 0 \\ \ln \left[\frac{V(t) - V_S}{V_0 - V_S} \right] &= \frac{-t}{RC} \\ \frac{V(t) - V_S}{V_0 - V_S} &= e^{-t/RC} \end{aligned} \quad (2)$$

We know $\tau = RC$

$$\begin{aligned} V(t) - V_S &= (V_0 - V_S) \cdot e^{-t/\tau} \\ V(t) &= V_S + (V_0 - V_S) \cdot e^{-t/\tau} \end{aligned}$$

for $t > 0$.

Thus,

$$V(t) = \begin{cases} V_0; & t \leq 0 \\ V_S + (V_0 - V_S) \cdot e^{-t/\tau}; & t > 0. \end{cases}$$

This is known as the complete response of the RC circuit.

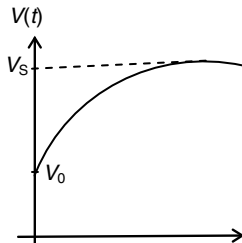


Figure 11 Response of an RC circuit with initially charged capacitor.

$$i(t) = C \cdot \frac{dV(t)}{dt}$$

$$i(t) = \frac{V_S}{R} \cdot e^{-t/\tau}; \quad t > 0.$$

with initial values zero.

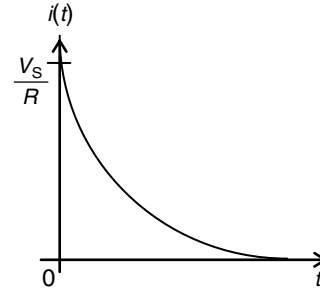


Figure 12 Current response.

Complete response = natural response + forced response.

$$\therefore V = V_n + V_f$$

where

$$V_n = V_0 \cdot e^{-t/\tau}.$$

$$V_f = V_S(1 - e^{-t/\tau}),$$

where V_n is the transient response and V_f is the steady state response.

Transient Response

The transient response is the circuit's temporary response that will die out with time.

Steady State Response

The steady state response is the behaviour of the circuit a long time after an external excitation is applied.

$\therefore$ complete response may be written as

$$\Rightarrow V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau} \text{ Volts}$$

where $V(0) = V_0$; the initial capacitor voltage

$V(\infty) \Rightarrow$ final capacitor voltage or SS value

τ is the time constant.

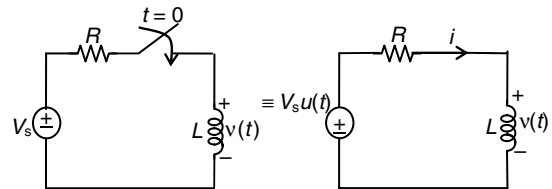
NOTE

If the switch changes position at time $t = t_0$ instead of $t = 0$, there is a time delay in response.

Therefore, the abovementioned equation becomes

$$V(t) = V(\infty) + [V(t_0) - V(\infty)] \cdot e^{-(t-t_0)/\tau} \text{ Volts}$$

STEP RESPONSE OF AN RL CIRCUIT



An RL circuit with a step input voltage. Let the response be the sum of the transient response and the steady state response.

$$i = i_{tr} + i_{ss} \quad (1)$$

We know that i_{tr} is always a decaying exponential.

$$\text{i.e.,} \quad i_{tr} = A \cdot e^{-t/\tau}, \quad \tau = \frac{L}{R}$$

The steady state is the value of current along time after the switch is closed. At that time, the inductor becomes a short circuit and the voltage across it is zero. The entire source voltage V_s appears across R .

Thus, the steady state response is

$$i_{ss} = \frac{V_s}{R} A.$$

$$i = A \cdot e^{-t/\tau} + \frac{V_s}{R}$$

We know $i(0^-) = i(0^+) = I_0$

Thus, at $t = 0$, Equation (2) becomes

$$i(0) = A + \frac{V_s}{R} = I_0$$

$$A = I_0 - \frac{V_s}{R}$$

Substituting A in Equation (2), we get

$$i(t) = \left(I_0 - \frac{V_s}{R} \right) \cdot e^{-t/\tau} + \frac{V_s}{R}$$

This is the compute response of the RL circuit.

$$i(t) = i(\infty) + [i(0) - i(\infty)] \cdot e^{-t/\tau} \text{ A.}$$

NOTE

If the switching takes place at time $t = t_0$ instead of $t = 0$,

$$i(t) = i(\infty) + [i(t_0) - i(\infty)] \cdot e^{-(t-t_0)/\tau}$$

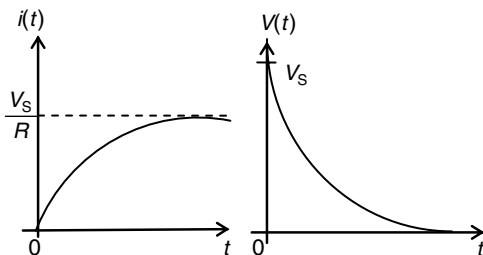


Figure 13 Step response of an RL circuit with no initial inductor current.

NOTE

With sources, elements behaviour at

$$t = 0^+ \text{ and } t \rightarrow \infty$$

(i) At $t = 0^+ \Rightarrow L$ is the open circuit and C is the short circuit.

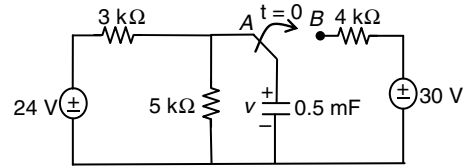
(ii) At $t \rightarrow \infty \Rightarrow f = 0$

$\Rightarrow X_L = j\omega L = 0 \Omega$ is the short circuit and

$X_C = \frac{1}{j\omega c} = \infty \Omega$ is the open circuit.

Example 9

The switch in figure has been in position A for a long time. At $t = 0$, the switch moves to B . Determine $V(t)$ for $t > 0$, and calculate its value at $t = 2$ s.



- (2) (A) $V_C(2) = 24.5$ V (B) $V_C(2) = 5.5$ V
(C) $V_C(2) = -14.5$ V (D) $V_C(2) = 18.4$ V

Solution

For $t < 0$, the switch is at position A .

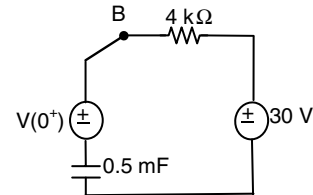
at $t = 0^-$, capacitor acts like open circuit.

$$v(0^-) = 5 \text{ k}\Omega \times \frac{24 \text{ V}}{8 \text{ k}\Omega} = 15 \text{ V.}$$

We know $v(0^+) = v(0^-) = 15$ V.

(capacitor voltages cannot change instantaneously.)

For $t > 0$, the switch is at position B



$$\tau = RC = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2 \text{ s.}$$

At $t \rightarrow \infty$,

capacitor acts like an open circuit to DC at steady state.

Thus, $V(\infty) = 30$ V

$$\begin{aligned} V_C(t) &= V(\infty) + [V(0^+) - V(\infty)] \cdot e^{-t/\tau} \\ &= 30 + (15 - 30) e^{-t/2} \text{ s.} \\ &= 30 - 15e^{-0.5t} \text{ s.} \end{aligned}$$

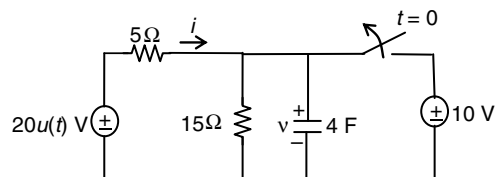
at

$$t = 2 \text{ s}$$

$$V_C(2) = 30 - 15e^{-1} = 30 - \frac{15}{e} = 24.5 \text{ V}$$

Example 10

The switch has been closed for a long time and is opened at $t = 0$



The value of $V_C(t)$ for $t > 0$ would be

- (A) $V_C(t) = 15 - 5 \cdot e^{-t/15}$ V.
(B) $V_C(t) = 5 - 15 \cdot e^{-t/15}$ V

- (C) $V_C(t) = 15 + 5 \cdot e^{-0.25t} \text{ V}$
 (D) $V_C(t) = 15 + 10 \cdot e^{-t/15} \text{ V}$

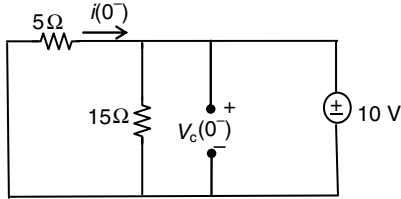
Solution

For $t < 0$:

At $t = 0^-$, switch is closed. Further, C is the open circuit.

$$u(t) = \begin{cases} 0 & ; \quad t < 0 \\ 1 & ; \quad t > 0. \end{cases}$$

At $t < 0$:



Circuit is in steady state.

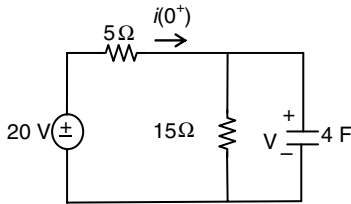
From the abovementioned circuit,

$$i(0^-) = \frac{10}{5} = 2 \text{ A.}$$

$$V_C(0^-) = 10 \text{ V} = V_C(0^+)$$

for $t > 0$: switch opened.

At $t = 0^+$



$$\begin{cases} \tau = R_{eq} \cdot C \\ R_{eq} = \frac{5 \times 15}{5 + 15} = \frac{5 \times 15}{20} = \frac{15}{4} \Omega \\ \tau = \frac{15}{4} \times 4 = 5 \text{ s} \end{cases}$$

C is the short circuit.

$$i(0^+) = \frac{20}{5} = 4 \text{ A.}$$

at $t \rightarrow \infty$; C is the open circuit.

$$V_C(\infty) = V_{15\Omega} = 20 \times \frac{15}{15 + 5} = 15 \text{ V}$$

We know that the total response $V_C(t)$ is

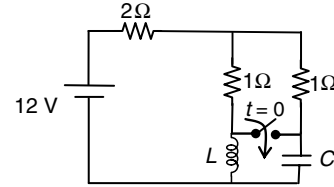
$$V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)] \cdot e^{-t/\tau}$$

$$= 15 + [10 - 15] \cdot e^{-t/15}$$

$$V_C(t) = 15 - 5 e^{-t/15} \text{ V}$$

Example 11

In the following circuit, the switch is closed at $t = 0$. What is the initial value of the current through the capacitor?



- (A) 0.8 A (B) 1.6 A (C) 2.4 A (D) 3.2 A

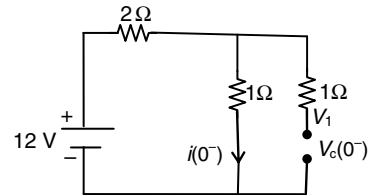
Solution

For $t < 0$:

At $t = 0^-$, the switch is opened.

L is short circuit and C is the open circuit.

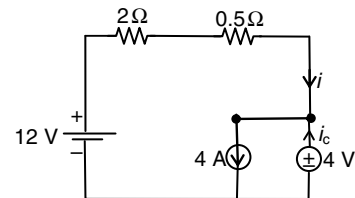
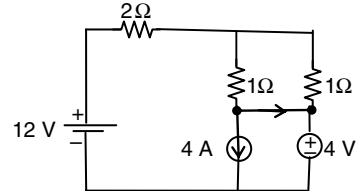
The following is the equivalent circuit.



$$i(0^-) = \frac{12}{3} = 4 \text{ A} = i_L(0^+)$$

$$V_C(0^-) = 1 \times 4 = 4 \text{ V} = V_C(0^+)$$

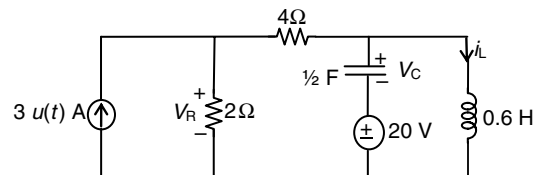
At $t = 0^+$, the equivalent circuit is shown in the following figure:



$$i = \frac{12 - 4}{2.5} = 3.2 \text{ A}$$

$$i + i_C = 4 \Rightarrow i_C = 0.8 \text{ A}$$

Direction for questions 12, 13, and 14:



Example 12

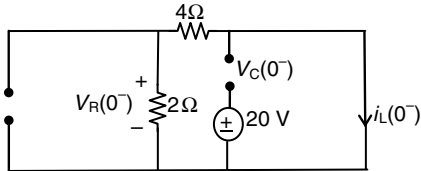
The values of $i_L(0^+)$, $V_C(0^+)$, and $V_R(0^+)$ would be.

- (A) $i_L(0^+) = 0$ A; $V_C(0^+) = 20$ V; and $V_R(0^+) = 4$ V
 (B) $i_L(0^+) = 5$ A; $V_C(0^+) = -15$ V; and $V_R(0^+) = 4$ V
 (C) $i_L(0^+) = 3$ A; $V_C(0^+) = -20$ V; and $V_R(0^+) = 4$ V
 (D) $i_L(0^+) = 0$ A; $V_C(0^+) = -20$ V; and $V_R(0^+) = +4$ V

Solution

For $t < 0$, $3.u(t) = 0$; at $t = 0^-$

When the circuit is in steady state, L is the short circuit and C is the open circuit.



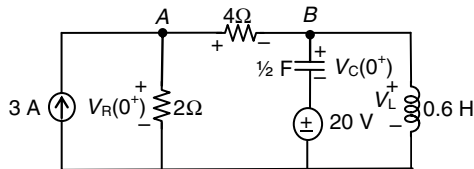
From this figure:

$$i_L(0^-) = 0; V_R(0^-) = 0; \text{ and } V_C(0^-) = -20 \text{ V}$$

For $t > 0$:

The circuit shown is at $t = 0^+$.

Therefore, L is the open circuit and C is the short circuit.



The inductor current and capacitor voltages cannot change instantaneously.

$$i_L(0^+) = 0; i_L(0^-) = 0; \text{ and } V_C(0^+) = V_C(0^-) = -20 \text{ V}$$

By applying KCL at node A,

$$3 = \frac{V_R(0^+)}{2} + \frac{V_0(0^+)}{4}$$

By applying KVL to the middle loop,

$$V_R(0^+) - V_0(0^+) - V_C(0^+) - 20 = 0$$

$$V_C(0^+) = -20 \text{ V}$$

$$\therefore V_R(0^+) = V_0(0^+).$$

$$3 V_R(0^+) = 12$$

$$\Rightarrow V_R(0^+) = 4 \text{ V}$$

$$\therefore V_R(0^+) = 4 \text{ V}, V_C(0^+) = -20 \text{ V}, i_L(0^+) = 0 \text{ A}$$

Example 13

The steady state values of the I_L , V_C , and V_R would be

- (A) $V_R(\infty) = 4$ V, $V_C(\infty) = -20$ V, $I_L(\infty) = 0$ A.
 (B) $V_R(\infty) = 4$ V, $V_C(\infty) = -20$ V, $I_L(\infty) = 1$ A.
 (C) $V_R(\infty) = -4$ V, $V_C(\infty) = 20$ V, $I_L(\infty) = 4$ A.
 (D) None of the above

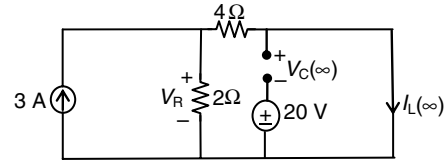
Solution

As $t \rightarrow \infty$, the circuit reaches steady state.

In steady state,

$\therefore L$ is the short circuit and C is the open circuit.

The circuit becomes.



$$i_L(\infty) = 2 \times \frac{3}{6} = 1 \text{ A.}$$

$$V_C(\infty) = -20 \text{ V.}$$

$$V_R(\infty) = 2 \times 2 = 4 \text{ V.}$$

$$\therefore V_R(\infty) = 4 \text{ V}, V_C(\infty) = -20 \text{ V and } i_L(\infty) = 1 \text{ A.}$$

Example 14

The values of $\frac{di_L(0^+)}{dt}$ and $\frac{dV_C(0^+)}{dt}$ would be

- (A) 0 A/s, 2 V/s (B) 0 A/s, 0.5 V/s
 (C) 2 A/s, 2 V/s (D) -2 A/s, -2 V/s

Solution

For $t > 0$:

At $t = 0^+$,

$$V_L(0^+) = L \cdot \frac{di_L(0^+)}{dt} \Rightarrow \frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

$$\text{However, } V_L(0^+) = 0 \text{ V} \Rightarrow \frac{di_L(0^+)}{dt} = 0 \text{ A/s}$$

$$i_C = C \cdot \frac{dV_C}{dt} \Rightarrow \frac{dV_C(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

$$\frac{dV_C(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

By applying KCL at node B,

$$\frac{V_0(0^+)}{4} = i_C(0^+) + i_L(0^+)$$

$$i_C(0^+) = \frac{4}{4} = 1 \text{ A.}$$

$$\frac{dV_C(0^+)}{dt} = \frac{1}{1/2} = 2 \text{ V/sec.}$$

EQUIVALENT CIRCUITS FOR R , L , AND C IN S DOMAIN

Laplace transform

$$i(t) \leftrightarrow I(s)$$

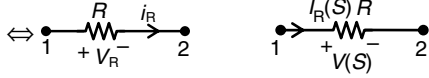
$$V(t) \leftrightarrow V(s)$$

$$R \leftrightarrow R$$

$$L \leftrightarrow sL \Omega$$

$$C \leftrightarrow \frac{1}{sC} \Omega$$

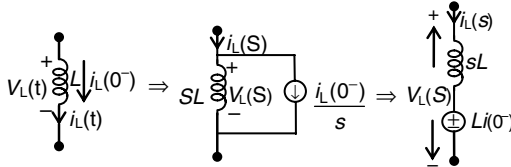
1. Resistor (R):



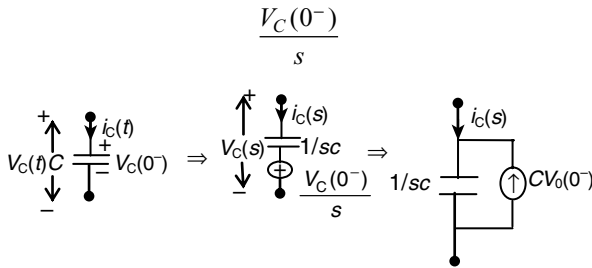
1. Time domain

2. Frequency domain.

2. Inductor (L):



3. Capacitor (C):

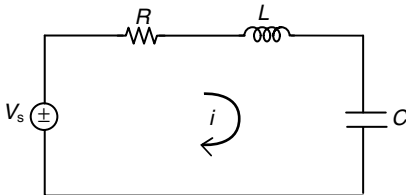


1. Time domain
2. s domain voltage source
3. s domain current source.

HIGH ORDER CIRCUITS

When two or more energy storage elements are present, the network equations will result in second-order differential equations.

Series RLC Circuits



By applying KVL

$$Ri + L \cdot \frac{di}{dt} + \frac{1}{C} \int i \cdot dt = V_s. \quad (1)$$

To eliminate the integral, differentiate with respect to t .

$$R \cdot \frac{di}{dt} + L \cdot \frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L} \cdot \frac{di}{dt} + \frac{i}{LC} = 0. \quad (2)$$

By applying Laplace transform to Equation (2)

$$\Rightarrow s^2 + \frac{R}{L} \cdot s + \frac{1}{LC} = 0.$$

$\Rightarrow$ Characteristic equation of the differential equation.

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad (3)$$

Second-order characteristic equation:

From Equation (3)

$$\omega_n = \frac{1}{\sqrt{LC}} \text{ and } 2\xi\omega_n = \frac{R}{L}$$

$$\xi\omega_n = \frac{R}{2L}$$

$$\tau = \frac{1}{\xi\omega_n} = 2 \frac{L}{R}$$

$$\xi = \frac{R}{2} \cdot \sqrt{\frac{C}{L}}$$

The roots of the characteristic equation are

$$s = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

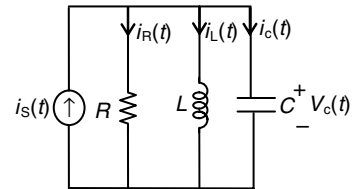
Let

$$\alpha = \frac{R}{2L} = \frac{1}{\tau}$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_n^2}$$

1. If $\alpha > \omega_n$ or $\xi > 1$, we have over damped oscillations.
2. $\xi = 1$ or $\alpha = \omega_n$, we have critically damped oscillator.
3. $0 < \xi < 1$ or $\alpha < \omega_n$, we have under damped oscillators.

Parallel RLC Circuit



$$i_s(t) = i_R(t) + i_L(t) + i_C(t)$$

Let us assume that the voltage across the capacitor is $V_C(t)$.

$$\therefore \frac{V_C(t)}{R} + \frac{1}{L} \int V_C(t) dt + C \cdot \frac{dV_C(t)}{dt} = i_s(t)$$

By simplifying the abovementioned equation, it becomes

$$\frac{d^2V_C(t)}{dt^2} + \frac{1}{RC} \cdot \frac{dV_C(t)}{dt} + \frac{1}{LC} \cdot V_C(t) = 0$$

In S domain,

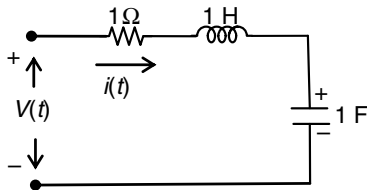
$$s^2 V_C(s) + \frac{1}{RC} \cdot s V_C(0) + \frac{1}{LC} \cdot V_C(s) = 0$$

$$s^2 + \frac{1}{RC} \cdot s + \frac{1}{LC} = 0.$$

$$\boxed{\xi = \frac{1}{2R} \sqrt{\frac{L}{C}}; \quad \omega_n = \frac{1}{\sqrt{LC}}}$$

Example 15

The circuit shown in figure has initial current $i_L(0^-) = 1$ A. Through the inductor and an initial voltage $V_C(0^-) = -1$ V across the capacitor for input $V(t) = u(t)$, the Laplace transform of the current $i(t)$ for $t \geq 0$ is



(A) $\frac{s}{s^2 + s + 1}$

(B) $\frac{s + 2}{s^2 + s + 1}$

(C) $\frac{s - 1}{s^2 + s + 1}$

(D) $\frac{s - 1}{s^2 + s + 1}$

Solution

By applying KVL, the loop equation is

$$V(t) = Ri(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int i(t) dt.$$

However, $V(t) = u(t)$, $i_L(0^-) = 1$ A and $V_C(0^-) = -1$ V. Take LPF both sides,

$$L[sI(s) - I(0^-)] + RI(s) + \frac{1}{sC} \cdot I(s) + \frac{V_C(0^-)}{s} = \frac{1}{s}$$

Substituting $R = 1 \Omega$, and $L = 1$ H
 $C = 1$ F

$$sI(s) - 1 + I(s) + \frac{1}{s} I(s) - \frac{1}{s} = \frac{1}{s}$$

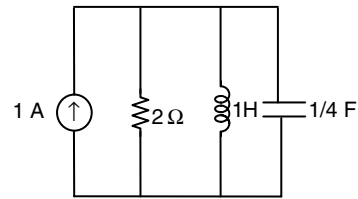
$$I(s) \left[s + \frac{1}{s} + 1 \right] = 1 + \frac{2}{s}$$

$$I(s) [s^2 + s + 1] = s + 2$$

$$I(s) = \frac{s + 2}{s^2 + s + 1}$$

Example 16

The circuit is



(A) critically damped

(B) undamped

(C) under damped

(D) over damped

Solution

For RLC parallel circuit

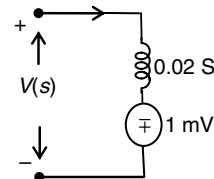
$$\xi = \frac{1}{2R} \sqrt{\frac{L}{C}}$$

$$= \frac{1}{2 \times 2} \times \sqrt{\frac{4}{1}}; \xi = \frac{1}{2}.$$

$\therefore \xi < 1$ is under damped system

Example 17

The value of Initial current is



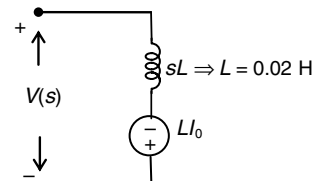
(A) 0.5 A

(B) 0.05 A

(C) 0.01 A

(D) 0.2 μ A.

Solution



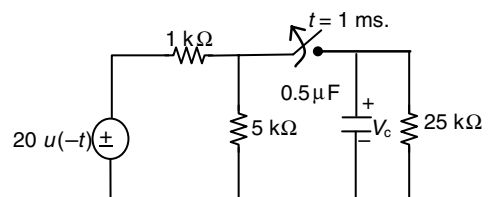
$$LI_0 = 1 \times 10^{-3}$$

$$I_0 = \frac{1 \times 10^{-3}}{0.02} = 0.05 \text{ A}$$

$$I_0 = 50 \text{ mA}$$

Example 18

In the following circuit, 20 V source has been applied for a long time. The switch is opened at $t = 1$ ms.



At $t = 3$ ms, the value of $V_C(t)$ is

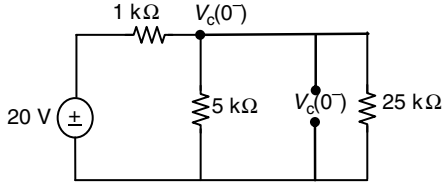
- (A) 1.314 V (B) 1.128 V
(C) 16.13 V (D) None of the above.

Solution

For $t < 0$:

At $t = 0^-$, switch was closed.

Capacitor behaves like an open circuit.



$$\frac{V_C(0^-) - 20}{1 \text{ k}\Omega} + \frac{V_C(0^-)}{5 \text{ k}\Omega} + \frac{V_C(0^-)}{25 \text{ k}\Omega} = 0$$

$$25(V_C(0^-) - 20) + 5V_C(0^-) + V_C(0^-) = 0$$

$$31V_C(0^-) = 25 \times 20$$

$$\Rightarrow V_C(0^-) = \frac{25 \times 20}{31} = 16.13 \text{ V}$$

$$R_{eq} = (1 \text{ k}\Omega \parallel 5 \text{ k}\Omega \parallel 25 \text{ k}\Omega)$$

$$\frac{1}{R_{eq}} = \frac{1}{1 \text{ k}\Omega} + \frac{1}{5 \text{ k}\Omega} + \frac{1}{25 \text{ k}\Omega}$$

$$= \frac{25 + 5 + 1}{25 \text{ k}\Omega}$$

$$R_{eq} = \frac{25}{31} \text{ k}\Omega = 806 \Omega$$

$$\tau = R_{eq} \times C = 806 \times 0.5 \times 10^{-6}$$

$$\tau = 0.4 \text{ ms.}$$

$$V_C(t) = V_0 e^{-t/\tau} = 16.13 \times e^{-2.500t} \text{ Volts}$$

$$V_C(1 \text{ ms}) = 16.13 \times e^{-2500 \times 1 \times 10^{-3}} \text{ V}$$

$$= 16.13 e^{-2.5} = 1.324 \text{ V for } 0 < t < 1 \text{ ms}$$

$$V_C(t) = 1.324 \times e^{-(t-1\text{ms})/25 \times 0.5 \times 10^{-3}} \text{ V}$$

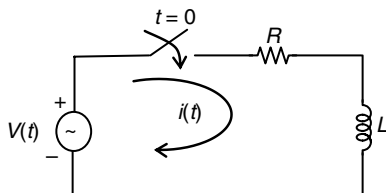
$$V_C(3 \text{ ms}) = 1.324 \times e^{-80 \times 2 \times 10^{-3}}$$

$$= 1.128 \text{ V}$$

AC TRANSIENTS

Transient Response with Sinusoidal Excitation

1. Series R - L Circuit:



Let $V(t) = V_m \cos(\omega t + \phi)$ volts

For $t > 0$: switch is closed.

By applying KVL,

$$Ri + L \frac{di}{dt} = V_m \cos(\omega t + \phi)$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V_m}{L} \cos(\omega t + \phi) \quad (1)$$

Let $\frac{d}{dt} = D.$

$$\left(D + \frac{R}{L}\right)i = \frac{V_m}{L} \cos(\omega t + \phi) \quad (2)$$

The complete solution = complement function + particular solution.

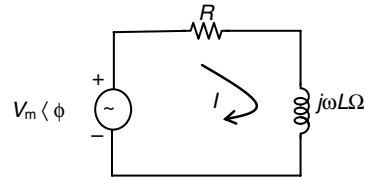
i.e., C.S. = C.F. + P.I.

The complementary function of Eq. (2) is

$$i_{tr}(t) = A e^{-\frac{R}{L}t} \text{ A.}$$

Next, we are to obtain the particular solution of current $i(t)$: $\Rightarrow i_{ss}(t)$

Transform the abovementioned network into phasor domain.



Network is phasor domain

By applying KVL to abovementioned circuit,

$$V_m \angle \phi - R.I - j\omega L I = 0.$$

$$I = \frac{V_m \angle \phi}{R + j\omega L}$$

$$\Rightarrow I = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cdot \angle \phi - \tan^{-1} \frac{\omega L}{R}$$

I in time domain $i_{ss}(t)$.

$$i_{ss}(t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cdot \cos\left(\omega t + \phi - \tan^{-1} \frac{\omega L}{R}\right) \text{ A.}$$

$$i(t) = i_{tr}(t) + i_{ss}(t)$$

= Transient + Steady state response

$$= A e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cos\left(\omega t + \phi - \tan^{-1} \frac{\omega L}{R}\right)$$

at $t = 0^- \Rightarrow i(0^-) = 0 = i(0^+).$

$$A + \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cos\left(\omega t + \phi - \tan^{-1} \frac{\omega L}{R}\right) = 0$$

$$A = \frac{-V_m}{\sqrt{R^2 + (\omega L)^2}} \cdot \cos\left(\omega t + \phi - \tan^{-1} \frac{\omega L}{R}\right) A.$$

⇒ As $A \ll 1$, the transient effects are less for AC at $t = 0$.

2. Transient-free condition:

$$A = 0.$$

$$\Rightarrow \cos\left(\phi - \tan^{-1} \frac{\omega L}{R}\right) = 0$$

$$\phi - \tan^{-1} \frac{\omega L}{R} = \frac{\pi}{2}$$

$$\phi = \frac{\pi}{2} + \tan^{-1} \frac{\pi L}{R}, \text{ at } t = 0.$$

Case 1:

If the switch is closed at $t = t_0$. Then, the condition for transient-free response is

$$\omega t_0 + \phi = \frac{\pi}{2} + \tan^{-1} \frac{\omega L}{R}$$

NOTE

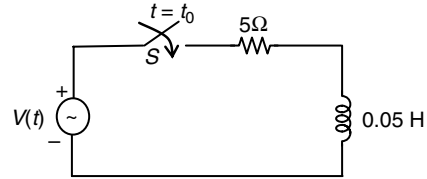
If the excitation is $V(t) = V_m \sin(\omega t + \phi)$, then the condition for transient-free response is $\omega t_0 + \phi = \tan^{-1} \omega \tau$ at $t = t_0$

$$\therefore i(t) = i_{ss}(t)$$

Table 1.1 Summary:

S. No.	Excitation	Circuit	Transient-free Condition (at $t = t_0$)
(i)	$V(t) = V_m \sin(\omega t + \phi)$	Series RL or	$\omega t_0 + \phi = \tan^{-1} \frac{\omega L}{R}$ $= \tan^{-1} \omega \tau.$
		Series RC	$\omega t_0 + \phi = \tan^{-1} \omega RC$ $= \tan^{-1} \omega \tau.$
(ii)	$V(t) = V_m \cos(\omega t + \phi)$	Series RL or Series RC	$\omega t_0 + \phi = \tan^{-1} \omega \tau$ $+ \frac{\pi}{2}$ where $\tau = \frac{L}{R}$ for RL $\tau = RC$ for RC.
(iii)	$i(t) = I_m \sin(\omega t + \phi)$	Parallel RL or Parallel RC	$\omega t_0 + \phi = \tan^{-1} \omega \tau$ where $\tau = RC$ for RC circuits. $\tau =$ $\frac{L}{R}$ for RL circuits.
(iv)	$i(t) = I_m \cos(\omega t + \phi)$	Parallel RL or Parallel RC	$\tau =$ $\frac{L}{R}$ for RL circuits, RC for RC circuits $\omega t_0 + \phi = \tan^{-1} \omega \tau$ $+ \frac{\pi}{2}$

Example 19



If $V(t) = 5 \cos(100\pi t + \pi/4)$ volts, the value of t_0 , which results in a transient-free response.

Solution

We know for RL series circuits transient-free condition. Given input cosinusoidal

$$\therefore \omega t_0 + \phi = \tan^{-1} \frac{\omega L}{R} + \frac{\pi}{2}$$

From the given data,

$$\omega = 100\pi \text{ and } \phi = \pi/4$$

$$100\pi t + \pi/4 = \tan^{-1} \left(\frac{100\pi \times 0.05}{5} \right) + \frac{\pi}{2}$$

$$100\pi t = 117.34$$

$$t = 0.3735 \text{ s}$$

SINUSOIDAL STEADY STATE ANALYSIS

A sinusoidal forcing function produces both a transient response and steady state response.

The transient response dies out with time so that only steady state response remains. When the transient response has become negligibly small when compared to steady state response, we say that the circuit is operating at sinusoidal steady state.

Sinusoids:

Let us consider a general expression for the sinusoidal.

$$V(t) = V_m \sin(\omega t + \phi)$$

where V_m is the amplitude; ω is the angular frequency; $\omega t + \phi$ is the argument of the sinusoidal; and ϕ is the phase.

A sinusoid can be expressed in either sine or cosine form.

This is achieved by using the following trigonometric identities:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B.$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

NOTE

$$\sin(\omega t \pm 180^\circ) = -\sin \omega t$$

$$\cos(\omega t \pm 180^\circ) = -\cos \omega t$$

$$\sin(\omega t \pm 90^\circ) = \pm \cos \omega t$$

$$\cos(\omega t \pm 90^\circ) = \pm \sin \omega t.$$

Example 20

Given the sinusoidal signal

$V(t) = 10 \sin(4\pi t + \pi/6)$. Calculate its power and period.

(A) 100 Wt, 2 s

(B) 50 Wt, 0.5 s

(C) 10 Wt, 0.5 s

(D) 0 Wt, 2 s

Solution

For sinusoidal signals,

$$\text{power} = \frac{V_m^2}{2R} \quad (\because V_m = \text{maximum voltage})$$

but R is not given.

Therefore, let $R = 1 \Omega$.

$$P = \frac{V_m^2}{2} = \frac{(10)^2}{2} = 50 \text{ Watts.}$$

$$\omega = 2\pi f = 4\pi$$

$$T = \frac{2\pi}{4\pi} = \frac{1}{2} \Rightarrow 0.5 \text{ s}$$

Example 21

Calculate the phase angle between $V_1 = -10 \cos(\omega t + 40^\circ)$ and $V_2 = 8 \sin(\omega t - 20^\circ)$.

(A) 30° (B) 60° (C) -60° (D) 20°

Solution

In order to compare V_1 and V_2 , we must express them in same form.

$$\begin{aligned} V_1 &= -10 \cos(\omega t + 40^\circ) \\ &= +10 \sin(\omega t + 40^\circ - 90^\circ). \\ &= 10 \sin(\omega t - 50^\circ). \end{aligned}$$

$$V_2 = 12 \sin(\omega t - 20^\circ)$$

$$V_2 \text{ leads } V_1 \text{ by } 30^\circ.$$

Phasor: A phasor is a complex number that represents the amplitude and phase of a sinusoid.

$$Z = x + jy \Rightarrow \text{rectangular form}$$

$$Z = r \angle \phi \Rightarrow \text{polar form}$$

$$Z = r \cdot e^{j\phi} \Rightarrow \text{exponential form}$$

Euler's identity:

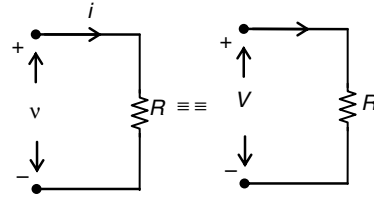
$$e^{\pm j\phi} = \cos \phi \pm j \sin \phi$$

Table 1.2 Sinusoidal phasor transformation:

S. No	Time domain	Phasor domain
(i)	$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
(ii)	$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
(iii)	$I_m \cos(\omega t + \phi)$	$I_m \angle \phi$
(iv)	$I_m \sin(\omega t + \phi)$	$I_m \angle \phi - 90^\circ$
(v)	$\frac{dv}{dt} \Leftrightarrow$	$j\omega v$
(vi)	$\int v dt \Leftrightarrow$	$\frac{V}{j\omega}$

Phasor relationships for circuit elements:

1. Resistor:



(i) time domain (ii) frequency domain

If the current through a resistor R is

$$i = I_m \cos(\omega t + \phi).$$

The voltage across it is given by

$$V = iR = RI_m \cos(\omega t + \phi)$$

The phasor form of this voltage is

$$V = RI_m \angle \phi$$

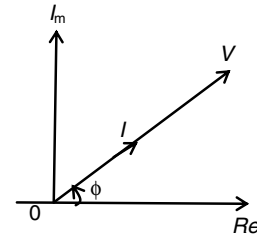
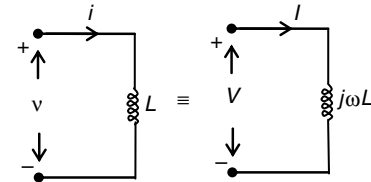


Figure 14 Phasor diagram for the resistor.

2. Inductor (L):



time domain

frequency domain

T - D

F - D

$$V = L \cdot \frac{di}{dt}$$

$$V = j\omega L \cdot I$$

The voltage across the inductor is

$$V = L \cdot \frac{di}{dt} = -\omega L \cdot I_m \sin(\omega t + \phi)$$

$$V = \omega L \cdot I_m \cos(\omega t + \phi + 90^\circ),$$

which transforms to the phasor.

$$V = \omega L \cdot I_m \cdot e^{j(\phi + 90^\circ)} = \omega L \cdot I_m \angle \phi + 90^\circ$$

However, $I_m \angle \phi = I$ and $e^{j90^\circ} = j$.

$$\therefore V = j\omega L \cdot I.$$

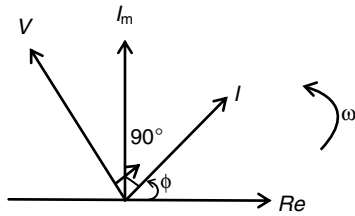
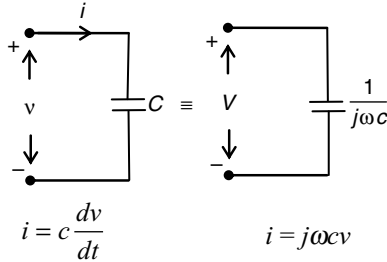


Figure 15 Phasor diagram for the inductor
 I lags V or V leads I .

3. Capacitor (C):



For the capacitor C , assume the voltage across it is $V = V_m \cos(\omega t + \phi)$.

$$i = C \frac{dV}{dt} \Leftrightarrow I = j\omega C V$$

$$T - D \quad F - D$$

$$\therefore V = \frac{I}{j\omega C}$$

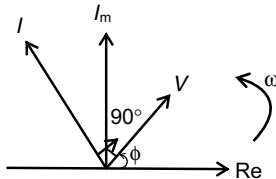


Figure 16 Phasor diagram for the capacitor (I leads V or V lags I)

NOTE

Summary of V - I relationships

Elements	$T - D$	$F - D$
R	$v = Ri$	$V = R.I$
L	$v = L.di/dt$	$V = j\omega L I$
C	$I = C dv/dt$	$I = I/j\omega C$

Example 22

Voltage $V = 5 \cos(40t + 60^\circ)$ is applied to a 0.5 H inductor. The steady state current through the inductor is

- (A) $i(t) = 4 \sin(40t + 60^\circ)$ A
- (B) $i(t) = 2 \cos(40t + 30^\circ)$ A
- (C) $i(t) = 0.25 \cos(40t - 30^\circ)$ A
- (D) $i(t) = 0.25 \cos(\omega t + 60^\circ)$ A

Solution

For the inductor, $V = j\omega L I$
from the given data.

$$\omega = 40 \text{ rad/s.}$$

$$L = 1/2 \text{ H and } V = 5 \angle 60^\circ \text{ V.}$$

$$I = \frac{V}{j\omega L} = \frac{5 \angle 60^\circ}{j \times 40 \times 0.5}$$

$$= \frac{1}{4} \angle 60^\circ - 90^\circ = 0.25 \angle -30^\circ.$$

$$i(t) = 0.25 \cos(40t - 30^\circ) \text{ A}$$

Example 23

If voltage $V = 4 \sin(50t + 30^\circ)$ is applied to a 100 μF capacitor. The steady state current through the capacitor is

- (A) $i(t) = 20 \sin(50t + 60^\circ)$ mA
- (B) $i(t) = 20 \cos(50t + 30^\circ)$ mA
- (C) $i(t) = 20 \cos(50t - 30^\circ)$ mA
- (D) $i(t) = 20 \cos(50t - 60^\circ)$ mA

Solution

Given $V = 4 \sin(50t + 30^\circ)$

$$i = C \frac{dV}{dt} \Leftrightarrow V = \frac{I}{j\omega C} ; V = 4 \angle -60^\circ$$

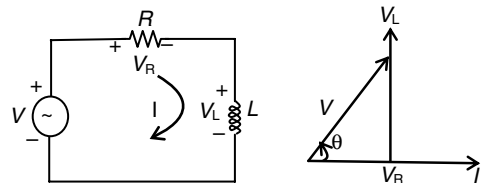
$$I = j\omega C V \Rightarrow j \times 50 \times 100 \times 10^{-6} \times 4 \angle -60^\circ.$$

$$I = 20 \cos(50t - 60^\circ + 90^\circ) \text{ mA}$$

$$i(t) = 20 \cos(50t + 30^\circ) \text{ mA}$$

SINUSOIDAL STEADY STATE ANALYSIS OF RLC CIRCUITS

1. RL Series Circuit:



$$V = V_R + V_L = I.R + j\omega L I$$

$$V = \sqrt{V_R^2 + V_L^2}$$

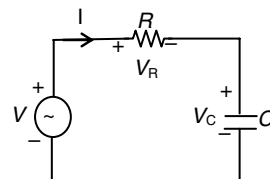
$$\theta = \tan^{-1} \left[\frac{V_L}{V_R} \right]$$

$$\text{where } V_R = IR$$

$$\text{Power factor} = \cos \theta = \frac{V_R}{V}$$

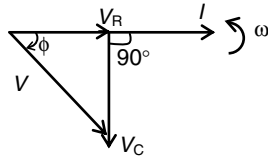
$$V_L = IX_L \angle 90^\circ \text{Lag.}$$

2. Series RC Circuit:



$$V = V_R + V_C$$

$$V_R = IR; V_C = IZ_C = IX_C \angle -90^\circ.$$

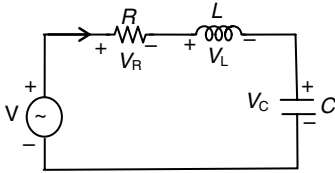


Phasor diagram

$$V = \sqrt{V_R^2 + V_C^2}$$

$$\text{p.f.} = \cos \phi = \frac{V_R}{V} \text{ Lead}$$

3. Series RLC Circuit:

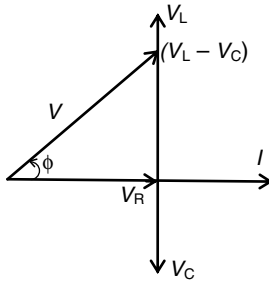


$$V = V_R + V_L + V_C$$

$$= IR + j\omega L \cdot I + \frac{1}{j\omega C} \cdot I$$

$$= IR + I \cdot X_L \angle 90^\circ + I \cdot X_C \angle -90^\circ.$$

(i) $V_L > V_C$:



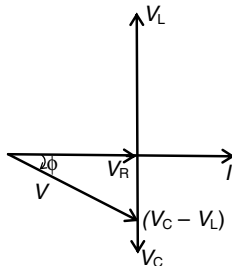
From the abovementioned phasor diagram

$$|V| = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\phi = \tan^{-1} \left[\frac{V_L - V_C}{V_R} \right] \Rightarrow \text{Impedance or admittance angle.}$$

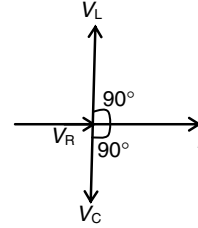
$$\text{Power factor} = \text{p.f.} = \cos \phi = \frac{V_R}{V}; \text{Lag}$$

(ii) If $V_L < V_C$:



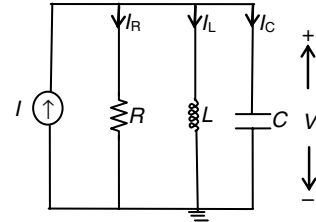
$$\cos \phi = \frac{V_R}{V}; \text{Lead}$$

(iii) If $V_L = V_C$
In this case,



$$\cos \phi = \frac{V_R}{V} = 1 \Rightarrow \text{unity power factor}$$

4. Parallel RLC Circuit:

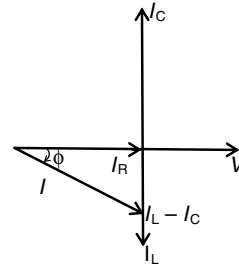


$$I = I_R + I_L + I_C$$

$$I = \frac{V}{R} + \frac{V}{j\omega L} + j\omega C \cdot V$$

$$\Rightarrow I = \frac{V}{R} + \frac{V}{X_L} \angle -90^\circ + \frac{V}{X_C} \angle 90^\circ.$$

(i) $I_L > I_C$

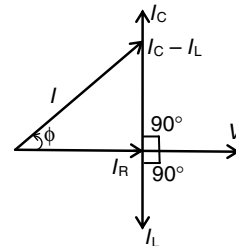


From the abovementioned phasor diagram

$$|I| = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$\text{p.f.} = \cos \phi = \frac{I_R}{I}; \text{Lag}$$

(ii) $I_L < I_C$



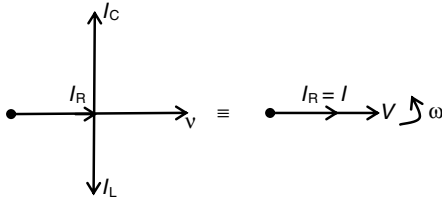
$$|I| = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\phi = \tan^{-1} \left[\frac{I_L - I_C}{I_R} \right] \Rightarrow \text{impedance (or) admittance angle}$$

$$\cos \phi = \text{p.f.} = \frac{I_R}{I} \text{ Lead}$$

$$(iii) I_L = I_C$$

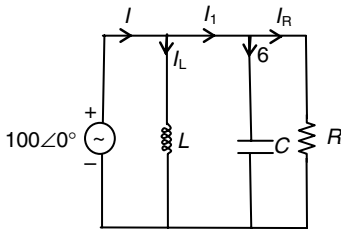
$$|I| = I_R$$



$$\cos \phi = \frac{I_R}{I} = 1 \Rightarrow \text{unity power factor.}$$

Example 24

Consider the following circuit.



If $|I_L| = 10$ A and $|I| = 12$ A, then the values of I_L and I_R would be

- (A) $I_L = -14.94$ A, $I_R = 8$ A (B) $I_L = 2$ A, $I_R = 4$ A
(C) $I_L = 14.94$ A, $I_R = 8$ A (D) None of the above

Solution

$$I = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$I_1 = \sqrt{I_R^2 + I_C^2}$$

$$100 = I_R^2 + 36$$

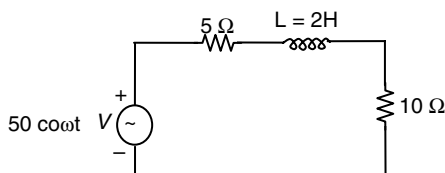
$$I_R^2 = 64 \Rightarrow I_R = 8 \text{ A}$$

$$12 = \sqrt{8^2 + (I_L - 6)^2}$$

$$(I_L - 6)^2 = 80$$

$$I_L = 6 + 8.944 = 14.94 \text{ A}$$

Example 25



If the power dissipated in the 5Ω resistor is 15 W, then the power factor of the circuit is

- (A) p.f. = 0.68 (B) p.f. = 0.854
(C) p.f. = 0.52 (D) None of the above.

Solution

For RL series circuit

$$\text{p.f.} = \cos \phi = \frac{V_R}{V}$$

$$P = VI = I^2 R$$

$$I^2 \times 5 = 15 \Rightarrow I^2 = 3$$

$$I = 1.732 \text{ A}$$

$\therefore$

$$V_R = 15 \times 1.732 \approx 26$$

$$|V| = 50 \angle 0^\circ.$$

$$\cos \phi = \frac{26}{50} = 0.52 \Rightarrow \text{power factor}$$

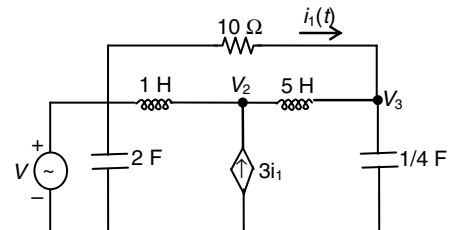
$$\text{Impedance angle } \phi = 58.66^\circ$$

Example 26

In the following circuit, it is known that

$$V_2(t) = 0.757 \cos(2t + 66.7^\circ) \text{ V,}$$

$$V_3(t) = 0.606 \cos(2t - 69.8^\circ) \text{ V, then } i_1(t) = ?$$



$$(A) 0.194 \cos(2t + 35.73^\circ) \text{ A.}$$

$$(B) 0.318 \cos(2t + 177^\circ) \text{ A.}$$

$$(C) 0.196 \cos(2t - 35.6^\circ) \text{ A.}$$

$$(D) 0.318 (2t - 177^\circ) \text{ A.}$$

Solution

By applying KCL at node V_3 .

$$I_1 = \frac{V_3}{1} (j/2) + \frac{V_3 - V_2}{j \times 2 \times 5}$$

$$I_1 = j 0.1 V_2 + j 0.4 \times V_3$$

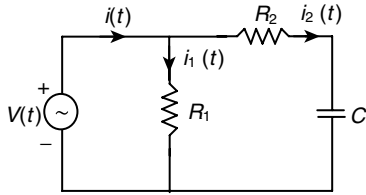
$$= 0.1 \times \angle 90^\circ \times 0.757 \angle 66.7^\circ + 0.4 \angle 90^\circ \times 0.606 \angle -69.8^\circ$$

$$= 0.0757 \angle 156.7^\circ + 0.2424 \angle 20.2^\circ$$

$$I_1 = 0.1945 \angle 35.736^\circ \text{ A}$$

LOCUS DIAGRAMS

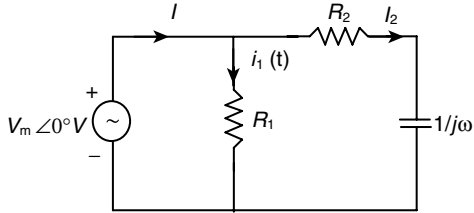
The locus diagram or circle diagram is the graphical representation of the electrical circuit. The frequency response of a circuit has been exhibited by drawing separately the angle and magnitude of a network function against variable parameter. For example, ω , L , R , and C .

Example 27


Consider $V(t) = V_m \cos \omega t$ volts, if frequency ' ω ' of the source is varying from 0 to ∞ . Draw the locus of the current phasor I_2 _____.

Solution

Transform the given network into phasor domain.



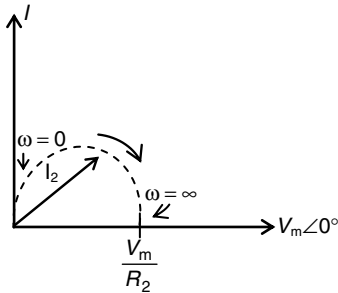
$$I = I_1 + I_2$$

$$I_1 = \frac{V_m \angle 0^\circ}{R_1}; I_2 = \frac{V_m \angle 0^\circ}{R_2 + 1/j\omega}$$

If $\omega = 0$, $I_2 = 0$ [$\because$ capacitor open circuit]

$$\omega = \infty, I_2 = \frac{V_m \angle 0^\circ}{R_2}$$

Locus of I_2 :



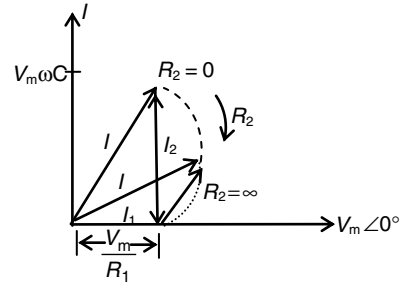
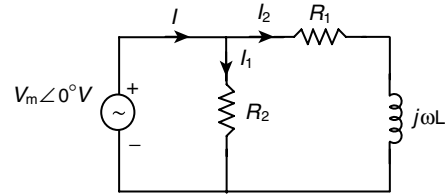
Case 2: In the abovementioned circuit, instead of ' ω '. If R_2 is varying from 0 to ∞ .

$$\text{If } R_2 = 0, I_2 = j\omega C V_m = \omega C V_m < 90^\circ$$

$$R_2 = \infty; I_2 = 0,$$

$$I_1 = \frac{V_m \angle 0^\circ}{R_1}$$

Locus diagram of I :


Example 28


If R_1 is varied from ' 0 ' to ' ∞ ', draw the locus diagram of I .

Solution

$$I = I_1 + I_2$$

$$I = \frac{V_m \angle 0^\circ}{R_2} + \frac{V_m}{R_1 + j\omega L}$$

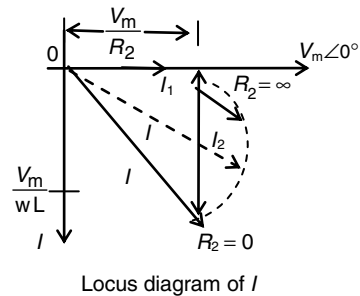
If $R_1 = 0$,

$$I = \frac{V_m}{R_2} + \frac{V_m \angle -90^\circ}{\omega L}$$

If $R_1 = \infty$,

$$I = \frac{V_m \angle 0^\circ}{R_2} + 0$$

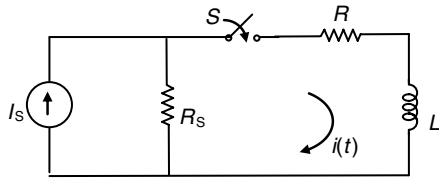
$$\therefore I = \frac{V_m \angle 0^\circ}{R_2}$$


EXERCISES
Practice Problems I

Direction for questions 1 to 32: Select the correct alternative from the given choices.

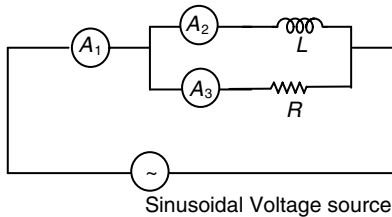
1. In the following circuit, the switch S is closed at $t = 0$.

The rate of change of current $\frac{di}{dt}(0+)$ is given by



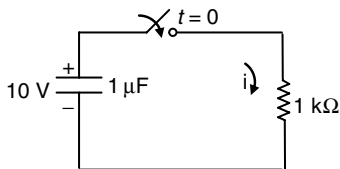
- (A) 0
(B) $\frac{R_S L_S}{L}$
(C) $\frac{(R + R_S) I_S}{L}$
(D) ∞

2. In the given figure, A_1 , A_2 , and A_3 are ideal ammeters. If A_2 and A_3 read 3 A and 4 A, respectively, then A_1 should read



- (A) 1 A
(B) 5 A
(C) 7 A
(D) None of these

3.



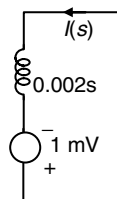
The current in the circuit when the switch is closed at $t = 0$

- (A) $10 e^{-100t}$
(B) $0.01 e^{-1,000t}$
(C) $0.1 e^{-1,000t}$
(D) $10 e^{-0.1t}$

4. An input voltage $V(t) = 10\sqrt{2} \cos(t + 10^\circ) + 10\sqrt{5} \cos(2t + 10^\circ)$ V is applied to a series combination of resistance $R = 1 \Omega$ and an inductance $L = 1$ H. The resulting steady state current $i(t)$ in ampere is

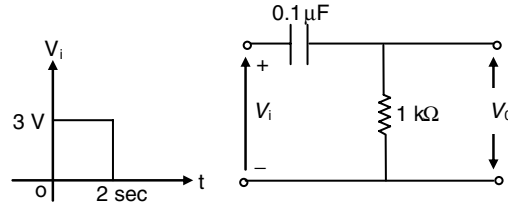
- (A) $10 \cos(t + 55^\circ) + 10 \cos(2t + 10^\circ + \tan^{-1} 2)$
(B) $10 \cos(t + 55^\circ) + 10 \sqrt{\frac{3}{2}} \cos(2t + 55^\circ)$
(C) $10 \cos(t - 35^\circ) + 10 \cos(2t + 10^\circ - \tan^{-1} 2)$
(D) $10 \cos(t - 35^\circ) + 10 \sqrt{\frac{3}{2}} \cos(2t - 35^\circ)$

5. A 2 mH inductor with some initial current can be represented as shown in the figure, where S is the Laplace transform variable. The value of initial current is



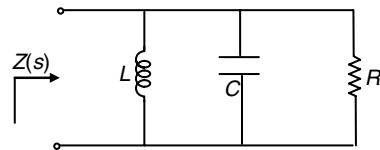
- (A) 0.5 A
(B) 2.0 A
(C) 1.0 A
(D) 0 A

6. A square pulse of 3 V amplitude is applied to C-R circuit shown in the figure. The capacitor is initially uncharged. The output voltage V_o at time $t = 2$ s is



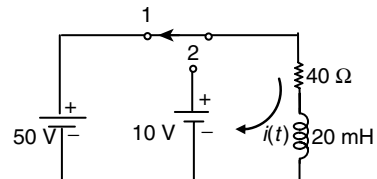
- (A) 3 V
(B) -3 V
(C) 0
(D) -4 V

7. The driving point impedance of the following network is given by $Z(s) = \frac{0.2s}{s^2 + 0.1s + 2}$. The component values are



- (A) $L = 5$ H, $R = 0.5 \Omega$, $C = 0.1$ F
(B) $L = 0.1$ H, $R = 0.5 \Omega$, $C = 5$ F
(C) $L = 5$ H, $R = 2 \Omega$, $C = 0.1$ F
(D) $L = 0.1$ H, $R = 2 \Omega$, $C = 5$ F

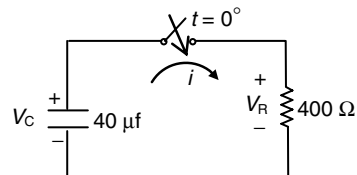
8.



The switch has been in position 1 for a long time, it is moved to position 2 at $t = 0$. The expression for $i(t)$ for $t > 0$ is

- (A) $0.25 e^{-2,000t}$
(B) $0.25 + e^{-2,000t}$
(C) $0.5 e^{-2,000t}$
(D) $0.5 + e^{-2,000t}$

9.



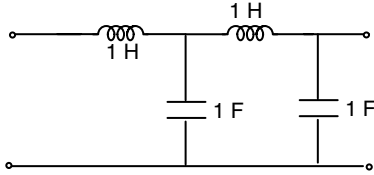
At $t = 0^-$, just before the switch is closed, $V_C = 100$ V. The current $i(t)$ for $t > 0$ is

- (A) $100 e^{-62.5t}$
(B) $50 e^{-160t}$
(C) $0.25 e^{-62.5t}$
(D) $50 e^{-62.5t}$

10. A series RL circuit with $R = 5 \Omega$ and $L = 2$ mH has an applied voltage $V = 150 \sin 5,000t$, the resulting current i is

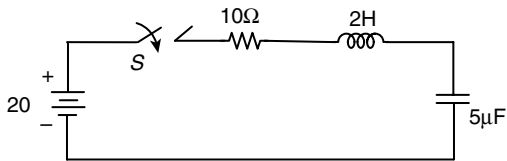
- (A) $13.4 \sin 5,000t$
 (B) $10 \cos(5,000t - 36.4^\circ)$
 (C) $13.4 \cos(5,000t - 63.4^\circ)$
 (D) $13.4 \sin(5,000t - 63.4^\circ)$

11. Driving point impedance of the network shown in the figure is



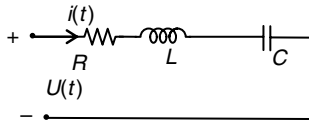
- (A) $\frac{s^4 + 3s^2 + 1}{s^3 + 2s}$ (B) $\frac{s^2 + 1}{s(s^2 + 2)}$
 (C) $\frac{s^4 + 3s^3 + 2s^2 + 1}{s^3 + 2s}$ (D) $\frac{s^2 + 1}{s + 1}$

12. Initially, the circuit shown in the given figure was relaxed. If the switch is closed at $t = 0$, the values of $i(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{d^2i(0^+)}{dt^2}$ will, respectively, be ____.



- (A) 0, 10, and 100 (B) 0, 10, and 50
 (C) 0, 10, and -100 (D) 0, 10, and -50

13. Transient response of the following circuit ____.

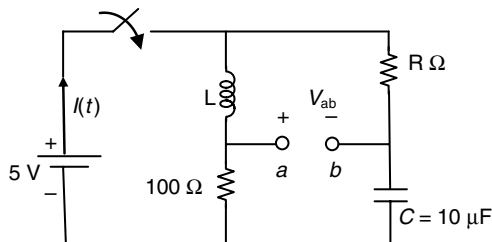


- (A) rises exponentially
 (B) decays exponentially
 (C) is oscillatory and the oscillations dies down with time
 (D) will have sustained oscillations

14. In the following circuit:

$$i_L(0^-) = 0, V_C(0^-) = 0.$$

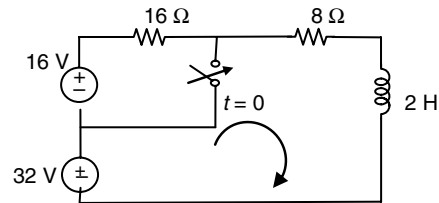
Switch S is closed at $t = 0$ $i(0^+) = 20$ mA and $V_{ab} = 0$ for $t \geq 0$



The value of R is ____.

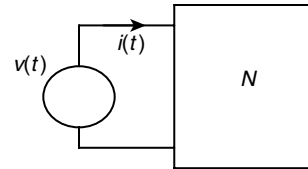
- (A) $\frac{1}{4}$ kΩ (B) 250 Ω
 (C) 350 Ω (D) 100 Ω

15. For the circuit shown in the following figure, if the switch is closed at $t = 0$, then $i(t)$ for $t \geq 0$ will be



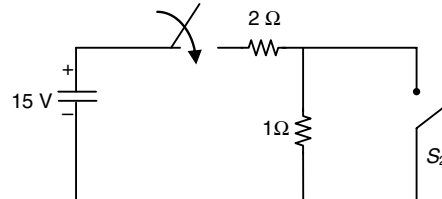
- (A) $4 + 2e^{-4t}$ (B) $4 - 2e^{-4t}$
 (C) $4 + 2e^{4t}$ (D) $4 - 2e^{4t}$

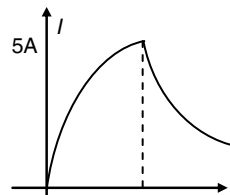
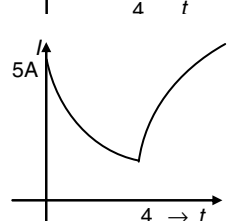
16. The network shown in the figure consists of only two elements. The response for unit step excitation is $i(t) = e^{-5t}$, then the elements are

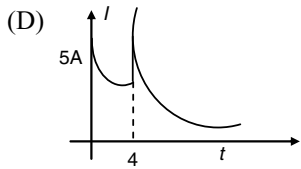
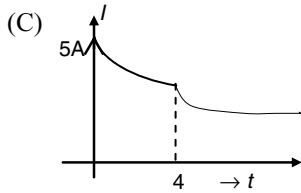


- (A) $R = 1 \Omega, L = 5$ H in series
 (B) $R = 1 \Omega, C = \frac{1}{5}$ F in series
 (C) $R = 1 \Omega, L = 5$ H in parallel
 (D) $R = 1 \Omega, C = \frac{1}{5}$ F in parallel

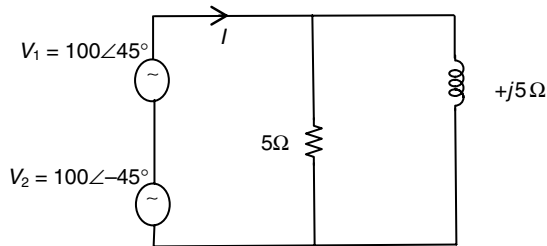
17. The capacitor in the circuit is initially charged to 15 V with S_1 and S_2 open. S_1 is closed at $t = 0$, while S_2 is closed at $t = 4$ s. The waveform of the capacitor current is



- (A) 
 (B) 

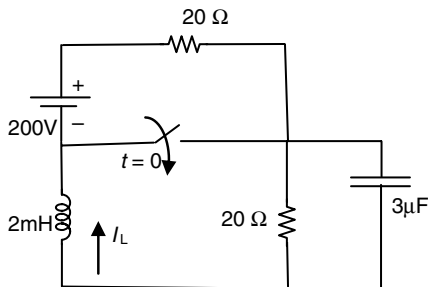


18. The phase angle of the current I with respect to the voltage V_2 in the circuit shown in the figure is



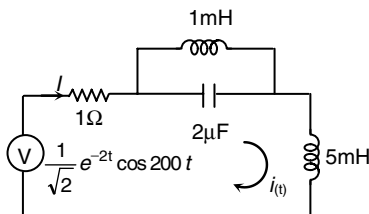
- (A) 0° (B) -45° (C) $+45^\circ$ (D) $+90^\circ$

19. In the circuit of figure, the switch S has been opened for a long time. It is closed at $t = 0$. The values of $V_L(0^+)$ and $I_L(0^+)$ are _____.



- (A) 200 V, -5 A (B) 0 V, 5 A
(C) 100 V, 5 A (D) 100 V, -5 A

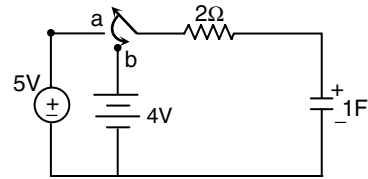
20. Obtain the value of current $i(t)$ in the given circuit at steady state.



- (A) $\frac{1}{\sqrt{2}} \cos(200t + 45^\circ)$ (B) $\frac{1}{\sqrt{2}}$
(C) 0 (D) ∞

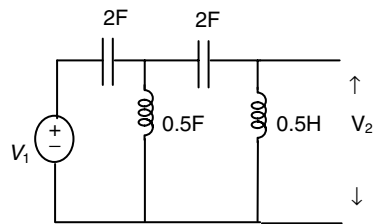
21. For the network shown, the switch is at position 'a' initially. At steady state, the switch is thrown to position

'b'. Now, $i(0^-) = 2$ A and $V_c(0^-) = 2$ V are the initial conditions. Find the circuit current.



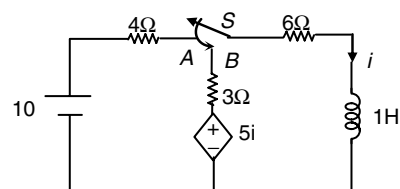
- (A) e^{-3t} (B) $-3e^{-3/2t}$
(C) $3e^{-3/2t}$ (D) $3e^{-2t}$

22. Obtain the driving point impedance of the network given in the diagram.



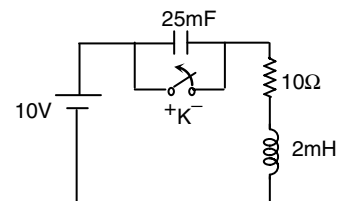
- (A) $\frac{s^3 + s^2 + 1}{(s+1)s}$ (B) $\frac{s^4 + s^2 + s}{1 + 2s^2}$
(C) $\frac{s^4 + s^3 + 2s^2 + 1}{(s+1)s}$ (D) $\frac{s^4 + s^2 + 1}{(1 + 2s^2)2s}$

23. For the given circuit switch S is at position 'A' when $t < 0$. At $t = 0$, the switch is thrown to position B. What will the value of current 'i' in the circuit at the instant $t = 4$ s?



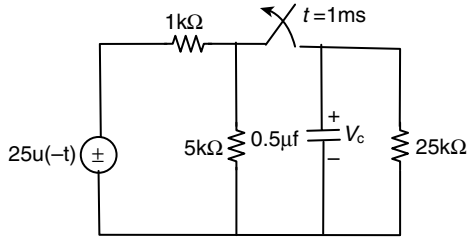
- (A) 0.1 μ A (B) 0.01 mA
(C) 10 μ A (D) 200 mA

24. In the given network, the switch K is closed for a long time and circuit is in steady state. Now, at $t = 0$, the switch is opened. Find $V_c(0^+)$ and $i(0^+)$ in the circuit.



- (A) 1 A, 0 V (B) 0 A, 5 V
(C) 0, α (D) 0, 0

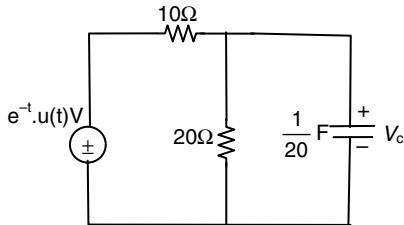
25. In the following circuit, the 25 V source has been applied for a long time and the switch is opened at $t = 1$ ms.



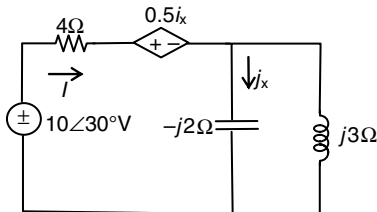
At $t = 5$ ms, the value of V_c is

- (A) 1.23 V (B) 20.16 V
(C) 1.69 V (D) -1.23 V
26. In the following circuit, capacitor is initially uncharged.

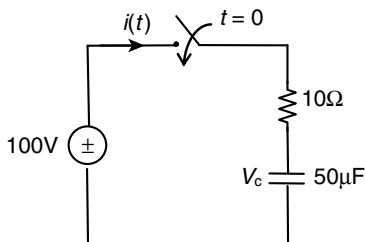
At $t = 0^+$, the value of $\frac{dV_c}{dt}$ and $\frac{d^2V_c}{dt^2}$



- (A) 0 V/s, 8 V/s² (B) -2 V/s, 8 V/s²
(C) 2 V/s, -8 V/s (D) None of these
27. In the following circuit, the current i_x is

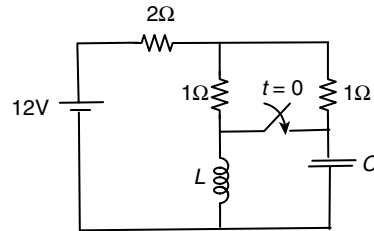


- (A) 3.94 ∠46.28° A (B) 4.62 ∠97.38° A
(C) 7.42 ∠92.49° A (D) 6.78 ∠49.27° A
28. In the following circuit, the initial charge on the capacitor is 2.5 mC, with the voltage polarity as indicated. The switch is closed at time $t = 0$. The current $i(t)$ at a time t after the switch is closed is



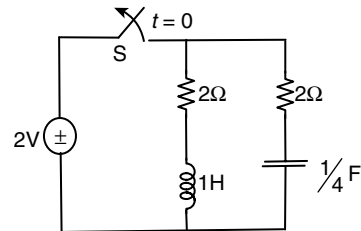
- (A) $i(t) = 15 \exp(-2 \times 10^3 t)$ A
(B) $i(t) = 5 \exp(-2 \times 10^3 t)$ A
(C) $i(t) = 10 \exp(-2 \times 10^3 t)$ A
(D) $i(t) = -5 \exp(-2 \times 10^3 t)$ A

29. In the following circuit, the switch is closed at $t = 0$. What is the initial value of the current through the capacitor?



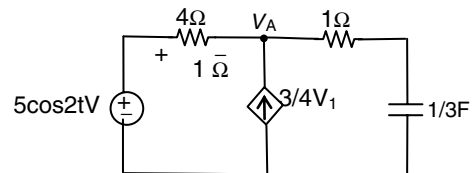
- (A) 0.8 A (B) 2.4 A (C) 1.6 A (D) 3.2 A

30.



The time constant of the circuit after the switch is opened would be

- (A) 2 s (B) 0.5 s
(C) 1 s (D) None of these
31. The power factor seen by the voltage source is

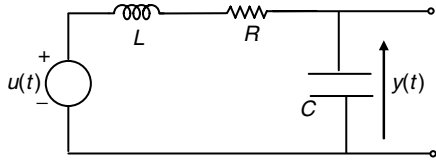


- (A) 0.8 (lagging) (B) 0.8 (leading)
(C) 36.9 (lagging) (D) -36.9 (leading)
32. An input voltage $V(t) = 10\sqrt{2} \cos(t + 10^\circ) + 10\sqrt{5} \cdot \cos(2t + 10^\circ)$ V is applied to a series combination of resistance $R = 1 \Omega$ and an inductance $L = 1$ H. The resulting steady state current $i(t)$ is
- (A) $10 \cos(t + 55^\circ) + 10 \cos(2t + 10^\circ + \tan^{-1}2)$ A
(B) $10 \cos(t + 55^\circ) + 10 \sqrt{\frac{3}{2}} \cdot \cos(2t + 55^\circ)$ A
(C) $10 \cos(t - 35^\circ) + 10 \cdot \cos(2t + 10^\circ - \tan^{-1}2)$ A
(D) $10 \cos(t - 35^\circ) + 10 \sqrt{\frac{3}{2}} \cos(2t - 35^\circ)$ A

Practice Problems 2

Direction for questions 1 to 24: Select the correct alternative from the given choices.

1. The condition on R , L , and C such that the step response $y(t)$ in the figure has no oscillations is



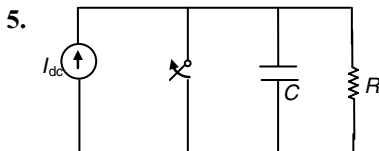
- (A) $R \geq \frac{1}{2} \sqrt{\frac{L}{C}}$ (B) $R \geq \sqrt{\frac{L}{C}}$
 (C) $R \geq 2 \sqrt{\frac{L}{C}}$ (D) $R = \frac{1}{\sqrt{LC}}$
2. A series circuit consists of two elements has the following current and applied voltage $i = 4 \cos(2,000t + 11.32^\circ)$ A
 $v = 200 \sin(2,000t + 50^\circ)$ V. The circuit elements are
 (A) resistance and capacitance
 (B) capacitance and inductance
 (C) inductance and resistance
 (D) both resistance

3. Transient current of an RLC circuit is oscillatory when

- (A) $R = 2\sqrt{L/C}$ (B) $R = 0$
 (C) $R > 2\sqrt{L/C}$ (D) $R < 2\sqrt{L/C}$

4. The transient response occurs

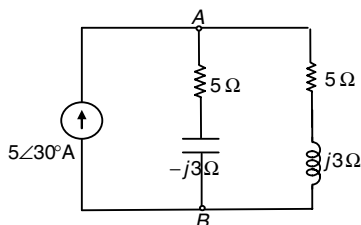
- (A) only in resistive circuits
 (B) only in inductive circuits
 (C) only in capacitive circuits
 (D) both in (B) and (C)



The initial voltage across the capacitor when the switch S is opened at $t = 0$

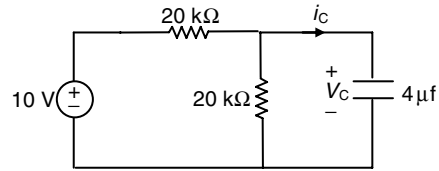
- (A) zero (B) $C \cdot \frac{I_{dc}}{s}$
 (C) $\frac{1}{Cs} I_{dc}$ (D) $Cs I_{dc}$

6. In the following AC network, the phasor voltage V_{AB} (in volt) is



- (A) 0 (B) $5 \angle 30^\circ$
 (C) $12.5 \angle 30^\circ$ (D) $17 \angle 30^\circ$

7. In the circuit shown, V_C is 0 volt at $t = 0$ V at $t = 0$ s. For $t > 0$, the capacitor current $i_C(t)$, where ' t ' is in seconds, is given by

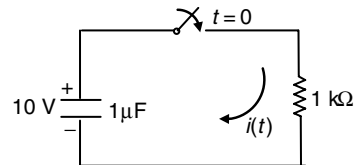


- (A) $0.50 \exp(-25t)$ mA
 (B) $0.25 \exp(-25t)$ mA
 (C) $0.50 \exp(-12.5t)$ mA
 (D) $0.25 \exp(-6.25t)$ mA

8. A series RL circuit, with $R = 10 \Omega$ and $L = 1$ H, has a 100 V source applied at $t = 0$. The current for $t > 0$ is

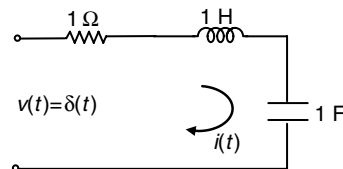
- (A) $10 e^{-10t}$ (B) $10 (1 - e^{-10t})$
 (C) $100 e^{-100t}$ (D) $100 (1 - e^{-100t})$

9. The current in the circuit when the switch is closed at $t = 0$ is



- (A) $10 e^{-100t}$ (B) $0.01 e^{-1,000t}$
 (C) $0.1 e^{-1,000t}$ (D) $10 e^{0.1t}$

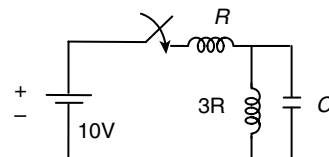
10.



The circuit shown in the figure is initially relaxed. The Laplace transform of the current $i(t)$ is

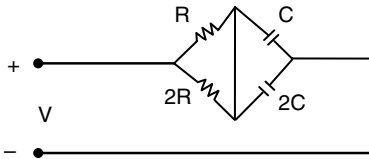
- (A) $\frac{s+1}{s^2+2s+1}$ (B) $\frac{s+1}{s^2+s+1}$
 (C) $\frac{s}{s^2+s+1}$ (D) $\frac{s}{s^2+2s+1}$

11. The time constant of the network shown in the figure is

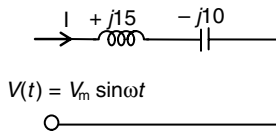


- (A) $4RC$ (B) $\frac{3}{4} RC$ (C) $3RC$ (D) RC

12. The time constant of the network shown in the figure is ____.

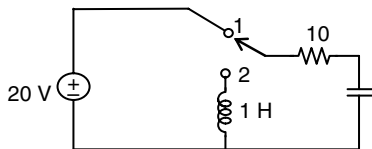


- (A) $\frac{2}{3} RC$ (B) $\frac{3}{2} RC$ (C) $2RC$ (D) RC
13. The network shown in the figure draws a current of ' I '



If the supply frequency is doubled, then the current drawn by the circuit is ____.

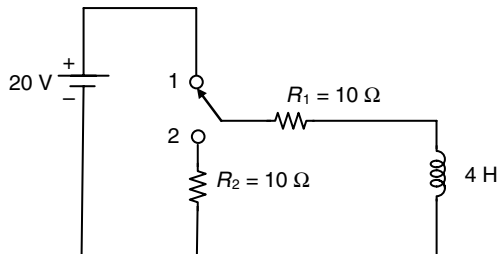
- (A) $\frac{I}{2}$ (B) $2I$ (C) $\frac{I}{5}$ (D) $\frac{2I}{5}$
14. In the circuit shown in the figure, the switch is thrown from position 1 to 2 at $t = 0$, after being at position 1 for a long time. The value of $\frac{d^2 i(0^+)}{dt^2}$ is ____.



- (A) 200 (B) -200 (C) -100 (D) 100

Direction for questions 15 and 16:

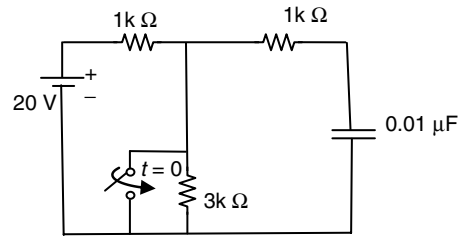
The circuit shown in the figure is initially under a steady state condition.



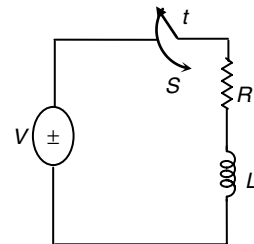
The switch is moved from position 1 to position 2 at $t = 0$.

15. The current through inductor immediately after switching is ____.
- (A) 2 A (B) $\frac{1}{2}$ A (C) 1 A (D) 5 A
16. The expression for current $i(t)$ is ____.
- (A) e^{-5t} (B) $2e^{-5t}$ (C) $\frac{2}{5}e^{-5t}$ (D) $5e^{-2t}$

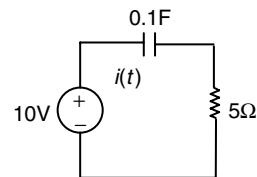
17. The switch in the circuit shown in the figure closes at $t = 0$. Find current i_c for all times.



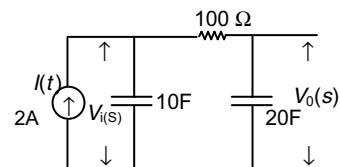
- (A) $5 \times 10^{-3} \times e^{-10^{5t}}$ A (B) $15 \times 10^{-2} \times e^{-10^{5t}}$ A
- (C) $15 \times 10^{-3} \times e^{-10^{5t}}$ A (D) $10 \times 10^{-3} \times e^{-10^{5t}}$ A
18. For the given circuit, the current passing through inductor ' L ' at the instant $t = 0^+$ is



- (A) $\frac{V}{R}$ (B) infinity
- (C) $\frac{V}{R + j\omega L}$ (D) 0
19. Determine the current i for $t \geq 0$, if $V_c(0) = 1$ V for the circuit shown

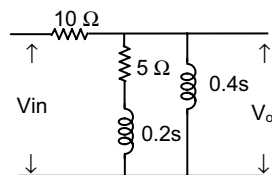


- (A) $\frac{0.9e^{-5t}}{2}$ (B) $1.8e^{-2t}$ (C) $5e^{-2t}$ (D) 3.6
20. Find the transfer function of the given system $\left(\frac{V_0(s)}{V_1(s)} \right)$



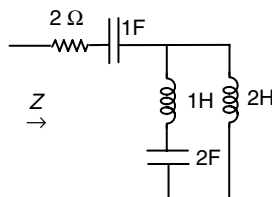
- (A) $\frac{1}{2000s + 1}$ (B) $\frac{10s}{2000s + 1}$
- (C) $\frac{100}{1000s + 1}$ (D) $\frac{1}{1000s + 1}$

21. Obtain the transfer function of the following system.



- (A) $\frac{0.4s}{0.2s+1s}$ (B) $\frac{s(2+0.4s)}{0.4s^2+8s+100}$
 (C) $\frac{s(2+.08s)}{0.08s^2+8s+50}$ (D) $\frac{2+.04s}{s^2+8s+50}$

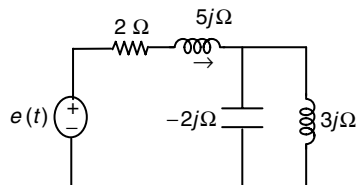
22. Find the driving point admittance of the following network.



- (A) $\frac{3s^3+2s}{2s^4+6s^3+7s^2+4s+2}$
 (B) $\frac{3s^2+2}{2s^4+5s^2+4s+2}$
 (C) $\frac{(3s^2+2)s}{2s^4+7s^3+6s^2+4s}$

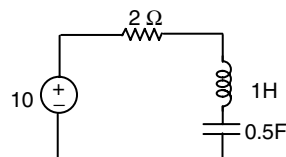
(D) $\frac{5s^3+2s}{s^4+7s^3+6s^2+4s+2}$

23. Find the voltage V_{ab} across the impedance of $(2+5j)\Omega$ in the network. The supply voltage $e(t) = 10 \sin(2\pi t + 45^\circ)$



- (A) $20 \angle 44^\circ$ (B) $24 \angle -40^\circ$
 (C) $4.48 \angle -108^\circ$ (D) $2.25 \angle 63.43^\circ$

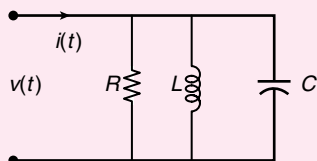
24. Find the current $i(t)$ through the circuit given



- (A) $10e^{-t} \cos t$ (B) $\frac{10}{\sqrt{2}} e^t \cos \sqrt{2}t$
 (C) $10e^{-t} \sin t$ (D) $\frac{10}{\sqrt{2}} e^t \sin \sqrt{2}t$

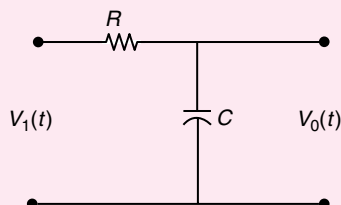
PREVIOUS YEARS' QUESTIONS

1. The circuit shown in figure, with $R = \frac{1}{3} \Omega$, $L = \frac{1}{4} \text{ H}$, and $C = 3 \text{ F}$ has input voltage $V(t) = \sin 2t$, the resulting current $i(t)$ is [2004]



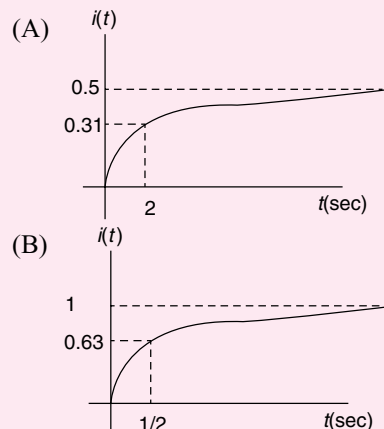
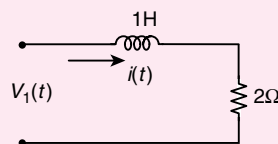
- (A) $5\sin(2t + 53.1^\circ)\text{A}$ (B) $5\sin(2t - 53.1^\circ)\text{A}$
 (C) $25\sin(2t + 53.1^\circ)\text{A}$ (D) $25\sin(2t - 53.1^\circ)\text{A}$

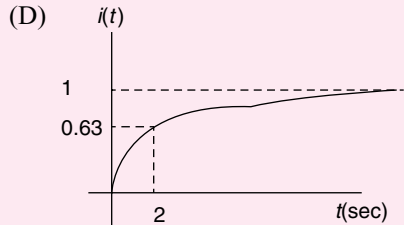
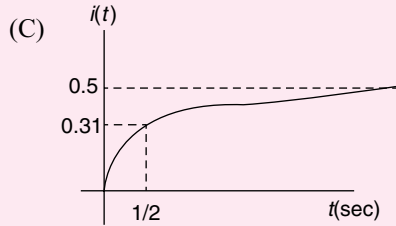
2. For the circuit shown in figure, the time constant $RC = 1 \text{ ms}$. The input voltage is $V_1(t) = \sqrt{2} \sin 10^3 t$. The output voltage $V_0(t)$ is equal to [2004]



- (A) $\sin(10^3 t - 45^\circ)\text{V}$ (B) $\sin(10^3 t + 45^\circ)\text{V}$
 (C) $\sin(10^3 t - 53^\circ)\text{V}$ (D) $\sin(10^3 t + 53^\circ)\text{V}$

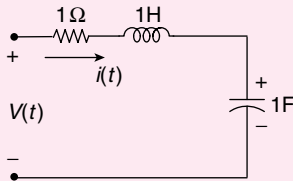
3. For the R - L circuit shown in figure, the input voltage $V_1(t) = u(t)$. The current $i(t)$ is [2004]





4. The circuit shown in figure, initial current $i_L(0^-) = 1$ A through the inductor and an initial voltage $V_C(0^-) = 1$ V across the capacitor. For input $V(t) = u(t)$, the Laplace transform of the current $i(t)$ for $t \geq 0$ is

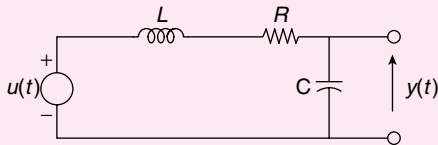
[2004]



- (A) $\frac{s}{s^2 + s + 1}$ (B) $\frac{s+2}{s^2 + s + 1}$
 (C) $\frac{s-2}{s^2 + s + 1}$ (D) $\frac{s-2}{s^2 + s - 1}$

5. The condition on R , L , and C such that the step response $y(t)$ in figure has no oscillations is

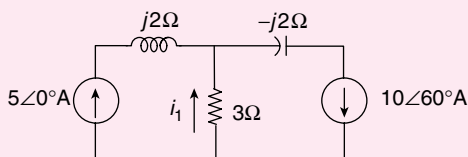
[2005]



- (A) $R \geq \frac{1}{2} \sqrt{\frac{L}{C}}$ (B) $R \geq \sqrt{\frac{L}{C}}$
 (C) $R \geq 2 \sqrt{\frac{L}{C}}$ (D) $R = \frac{1}{\sqrt{LC}}$

6. For the circuit in figure, the instantaneous current $i_1(t)$ is

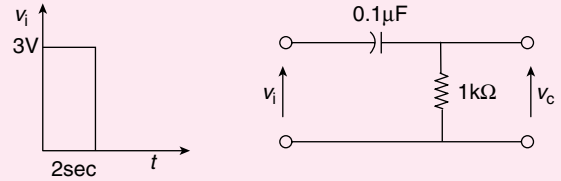
[2005]



- (A) $\frac{10\sqrt{3}}{2} \angle 90^\circ$ A (B) $\frac{10\sqrt{3}}{2} \angle -90^\circ$ A
 (C) $5 \angle 60^\circ$ A (D) $5 \angle -60^\circ$ A

7. A square pulse of 3 volts amplitude is applied to C – R circuit shown in figure. The capacitor is initially uncharged. The output voltage V_o at time $t = 2$ s is

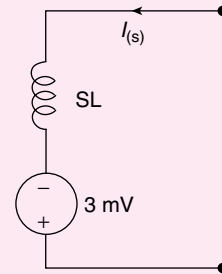
[2005]



- (A) 3 V (B) -3 V (C) 4 V (D) -4 V

8. A 2 mH inductor with some initial current can be represented as shown in the following figure, where s is the Laplace Transform variable. The value of initial current is

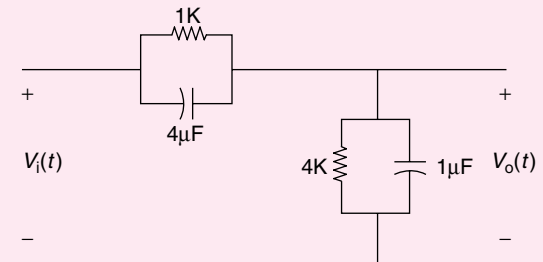
[2006]



- (A) 0.5 A (B) 2.0 A (C) 1.0 A (D) 0.0 A

9. In the following figure, assume that all the capacitors are initially uncharged. If $V_i(t) = 10 u(t)$ Volts, $V_o(t)$ is given by

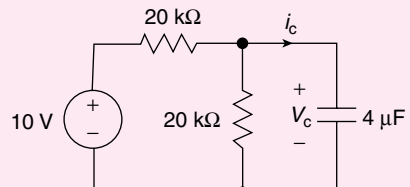
[2006]



- (A) $8e^{-t/0.004t}$ Volts (B) $8(1 - e^{-t/0.004t})$ Volts
 (C) $8u(t)$ Volts (D) 8 Volts

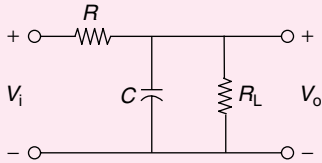
10. In the circuit shown, V_C is 0 Volts at $t = 0$ s. For $t > 0$, the capacitor current $i_C(t)$, where t in seconds is given by

[2007]



- (A) $0.50\exp(-25t)$ mA (B) $0.25\exp(-25t)$ mA
 (C) $0.50\exp(-12.5t)$ mA (D) $0.25\exp(-6.25t)$ mA

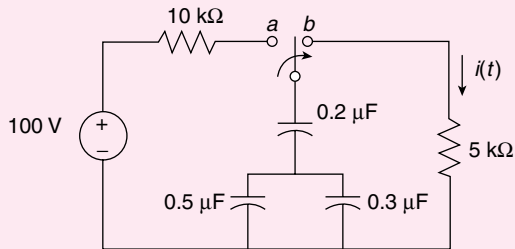
11. If the transfer function of the following network is $\frac{V_o(s)}{V_i(s)} = \frac{1}{2+sCR}$ [2009]



The value of the load resistance R_L is

- (A) $R/4$ (B) $R/2$ (C) R (D) $2R$

12. The switch in the circuit shown was on at position a for a long time, and is moved to position b at time $t = 0$. The current $i(t)$ for $t > 0$ is given by [2009]



- (A) $0.2e^{-125t} u(t)$ mA (B) $20e^{-1,250t} u(t)$ mA
 (C) $0.2e^{-1,250t} u(t)$ mA (D) $20e^{-1,000t} u(t)$ mA

13. The time domain behaviour of an RL circuit is represented by

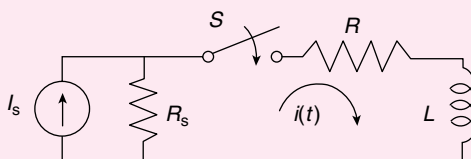
$$L \frac{di}{dt} + Ri = V_0 (1 + Be^{-Rt/L} \sin t) u(t).$$

For an initial current of $i(0) = \frac{V_0}{R}$, the steady state value of the current is given by [2009]

- (A) $i(t) \rightarrow \frac{V_0}{R}$ (B) $i(t) \rightarrow \frac{2V_0}{R}$
 (C) $i(t) \rightarrow \frac{V_0}{R}(1+B)$ (D) $i(t) \rightarrow \frac{2V_0}{R}(1+B)$

14. In the following circuit, the switch S is closed at $t = 0$.

The rate of change of current $\frac{di}{dt}(0^+)$ is given by [2008]



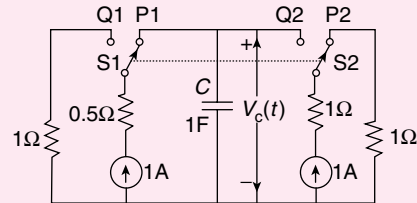
- (A) 0 (B) $\frac{R_s I_s}{L}$

- (C) $\frac{(R+R_s)I_s}{L}$ (D) ∞

15. The circuit shown in the figure is used to charge the capacitor C alternately from two current sources as indicated. The switches S_1 and S_2 are mechanically coupled and connected as follows:

For $2nT \leq t < (2n+1)T$ ($n = 0, 1, 2, \dots$) S_1 to P_1 and S_2 to P_2

For $(2n+1)T \leq t < (2n+2)T$ ($n = 0, 1, 2, \dots$) S_1 to Q_1 and S_2 to Q_2

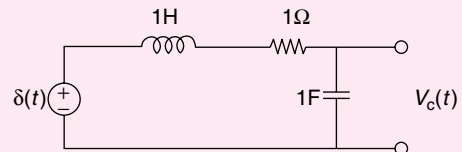


Assume that the capacitor has zero initial charge. Given that $u(t)$ is a unit step function, the voltage $V_c(t)$ across the capacitor is given by [2008]

- (A) $\sum_{n=0}^{\infty} (-1)^n tu(t-nT)$
 (B) $u(t) + 2 \sum_{n=1}^{\infty} (-1)^n u(t-nT)$
 (C) $tu(t) + 2 \sum_{n=1}^{\infty} (-1)^n (t-nT)u(t-nT)$
 (D) $\sum_{n=0}^{\infty} [0.5 - e^{-(t-2nT)} + 0.5e^{-(t-2nT-T)}]$

Direction for questions 16 and 17:

The following series RLC circuit with zero initial conditions is excited by a unit impulse function $\delta(t)$



16. For $t > 0$, the output voltage $V_c(t)$ is [2008]

- (A) $\frac{2}{\sqrt{3}} \left(e^{-\frac{1}{2}t} - e^{-\frac{\sqrt{3}}{2}t} \right)$ (B) $\frac{2}{\sqrt{3}} te^{-\frac{1}{2}t}$
 (C) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right)$ (D) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)$

17. For $t > 0$, the voltage across the resistor is [2008]

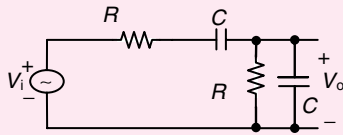
- (A) $\frac{1}{\sqrt{3}} \left(e^{-\frac{\sqrt{3}}{2}t} - e^{-\frac{1}{2}t} \right)$

(B) $e^{-\frac{1}{2}t} \left[\cos\left(\frac{\sqrt{3}t}{2}\right) - \frac{1}{\sqrt{3}} \sin\left(\frac{\sqrt{3}t}{2}\right) \right]$

(C) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}t}{2}\right)$

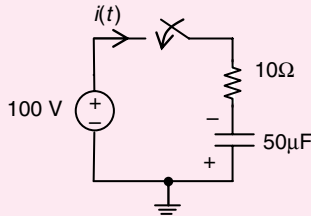
(D) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}t}{2}\right)$

18. The following circuit is driven by a sinusoidal input = $V_i = V_p \cos(t/RC)$. The steady state output V_o is [2011]



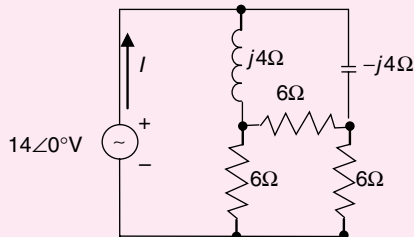
- (A) $(V_p/3) \cos(t/RC)$ (B) $(V_p/3) \sin(t/RC)$
 (C) $(V_p/2) \cos(t/RC)$ (D) $(V_p/2) \sin(t/RC)$

19. In the following circuit, the initial charge on the capacitor is 2.5 mC in which the voltage polarity as indicated. The switch is closed at time $t = 0$. The current $i(t)$ at time t after the switch is closed is [2011]



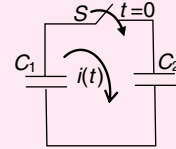
- (A) $i(t) = 15 \exp(-2 \times 10^3 t) \text{ A}$
 (B) $i(t) = 5 \exp(-2 \times 10^3 t) \text{ A}$
 (C) $i(t) = 10 \exp(-2 \times 10^3 t) \text{ A}$
 (D) $i(t) = -5 \exp(-2 \times 10^3 t) \text{ A}$

20. In the following circuit, the current I is equal to [2011]



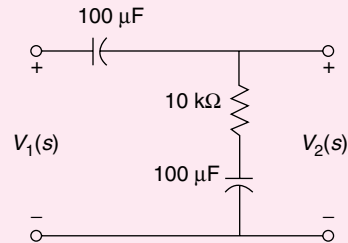
- (A) $1.4 \angle 0^\circ \text{ A}$ (B) $2.0 \angle 0^\circ \text{ A}$
 (C) $2.8 \angle 0^\circ \text{ A}$ (D) $3.2 \angle 0^\circ \text{ A}$

21. In the following figure, C_1 and C_2 are ideal capacitors. C_1 has been charged to 12 V before the ideal switch S is closed at $t = 0$. The current $i(t)$ for all t is [2012]



- (A) Zero
 (B) A step function
 (C) An exponentially decaying function
 (D) An impulse function

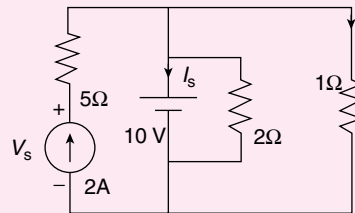
22. The transfer function $\frac{V_2(s)}{V_1(s)}$ of the following circuit is [2013]



- (A) $\frac{0.5s+1}{s+1}$ (B) $\frac{3s+6}{s+2}$
 (C) $\frac{s+2}{s+1}$ (D) $\frac{s+1}{s+2}$

Direction for questions 23 and 24:

Consider the following figure:

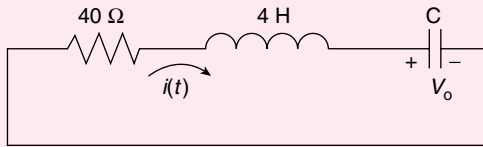


23. The current I_s in A in the voltage source and voltage V_s in Volts across the current source, respectively, are [2013]
 (A) 13, -20 (B) 8, -10
 (C) -8, 20 (D) -13, 20

24. The current in the 1Ω resistor in A is [2013]
 (A) 2 (B) 3.33 (C) 10 (D) 12

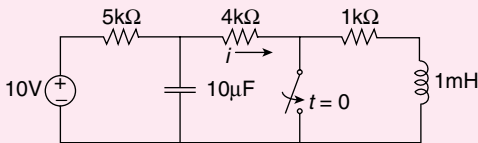
25. A 230 V RMS source supplies power to two loads connected in parallel. The first load draws 10 kW at 0.8 leading power factor and the second one draws 10 kVA at 0.8 lagging power factor. The complex power delivered by the source is [2014]
 (A) $(18 + j 1.5) \text{ kVA}$ (B) $(18 - j 1.5) \text{ kVA}$
 (C) $(20 + j 1.5) \text{ kVA}$ (D) $(20 - j 1.5) \text{ kVA}$

26. In the following circuit, the value of capacitor C (in mF) needed to have critically damped response $i(t)$ is [2014]



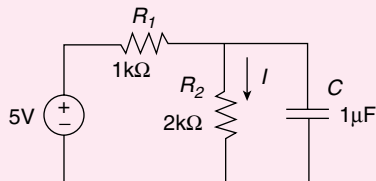
27. In the following figure, the ideal switch has been open for a long time. If it is closed at $t = 0$, then the magnitude of the current (in mA) through the $4 \text{ k}\Omega$ resistor at $t = 0^+$ is _____.

[2014]



28. In the figure shown, the capacitor is initially uncharged. Which one of the following expressions describes the current $I(t)$ (in mA) for $t > 0$?

[2014]



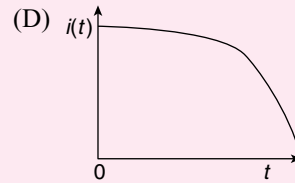
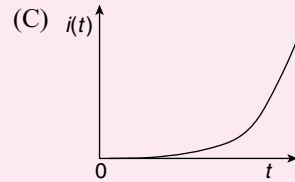
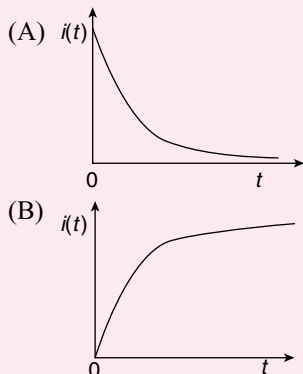
- (A) $I(t) = \frac{5}{3}(1 - e^{-t/\tau})$, $\tau = \frac{2}{3} \text{ msec}$
 (B) $I(t) = \frac{5}{2}(1 - e^{-t/\tau})$, $\tau = \frac{2}{3} \text{ msec}$
 (C) $I(t) = \frac{5}{3}(1 - e^{-t/\tau})$, $\tau = 3 \text{ msec}$
 (D) $I(t) = \frac{5}{2}(1 - e^{-t/\tau})$, $\tau = 3 \text{ msec}$

29. A series RC circuit is connected to a DC voltage source at time $t = 0$. The relationship between the source voltage V_s , resistance R , capacitance C , and current $i(t)$ is

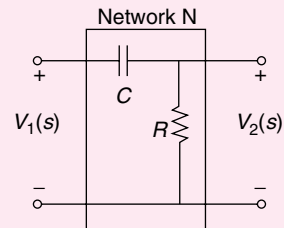
$$V_s = R i(t) + \frac{1}{C} \int_0^t i(u) du$$

Which one of the following represents the current $i(t)$?

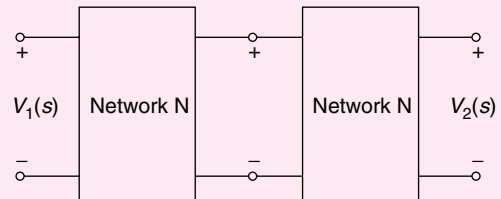
[2014]



30. Consider the building block called 'Network N ' shown in the figure. Let $C = 100 \text{ }\mu\text{F}$ and $R = 10 \text{ k}\Omega$.



Two such blocks are connected in cascade, as shown in the figure.



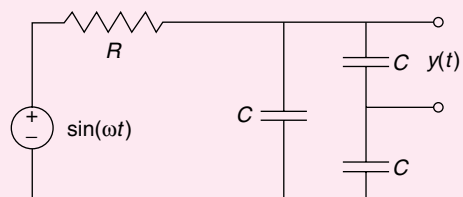
The transfer function $\frac{V_2(s)}{V_1(s)}$ of the cascaded network is

[2014]

- (A) $\frac{s}{1+s}$ (B) $\frac{s^2}{1+3s+s^2}$
 (C) $\left(\frac{s}{1+s}\right)^2$ (D) $\frac{s}{2+s}$

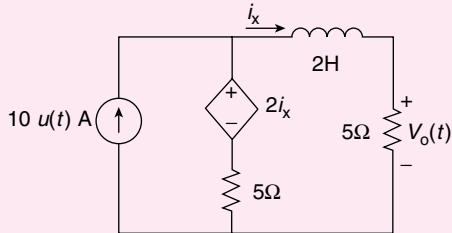
31. The steady state output of the circuit shown in the figure is given by $y(t) = A(\omega) \sin(\omega t + \Phi(\omega))$. If the amplitude $|A(\omega)| = 0.25$, then the frequency ω is

[2014]

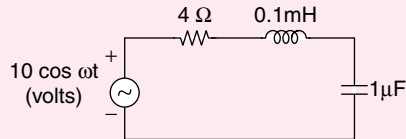


- (A) $\frac{1}{\sqrt{3}RC}$ (B) $\frac{2}{\sqrt{3}RC}$
 (C) $\frac{1}{RC}$ (D) $\frac{2}{RC}$

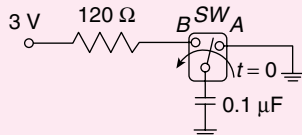
32. In the circuit shown in the figure, the value of $v_o(t)$ (in Volts) for $t \rightarrow \infty$ is _____. [2014]



33. In the circuit shown, at resonance, the amplitude of the sinusoidal voltage (in volts) across the capacitor is _____. [2015]



34. In the circuit shown, the switch SW is thrown from position A to position B at time $t = 0$. The energy (in μJ) taken from the 3 V source to charge the $0.1 \mu\text{F}$ capacitor from 0 V to 3 V is _____. [2015]

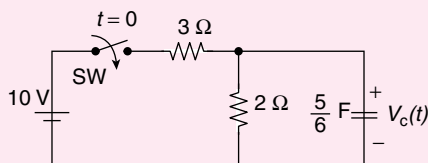


- (A) 0.3 (B) 0.45
 (C) 0.9 (D) 3

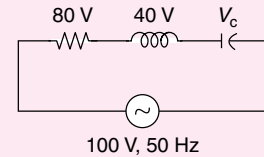
35. The damping ratio of a series RLC circuit can be expressed as [2015]

- (A) $\frac{R^2C}{2L}$ (B) $\frac{2L}{R^2C}$
 (C) $\frac{R}{2} \sqrt{\frac{C}{L}}$ (D) $\frac{2}{R} \sqrt{\frac{L}{C}}$

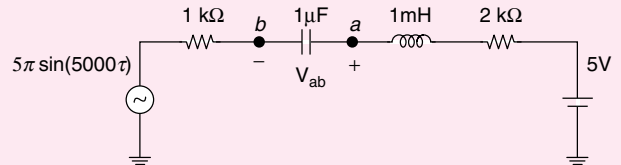
36. In the circuit shown, switch SW is closed at $t = 0$. Assuming zero initial conditions, the value of $V_C(t)$ (in volts) at $t = 1$ sec is _____. [2015]



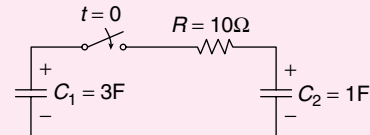
37. The voltage (V_C) across the capacitor (in volts) in the network shown is _____. [2015]



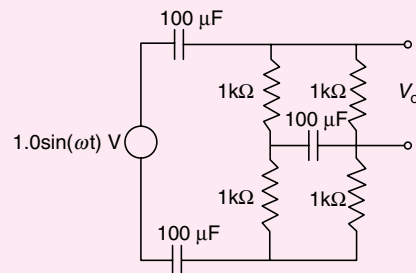
38. In the circuit shown, the average value of the voltage V_{ab} (in volts) in steady state condition is _____. [2015]



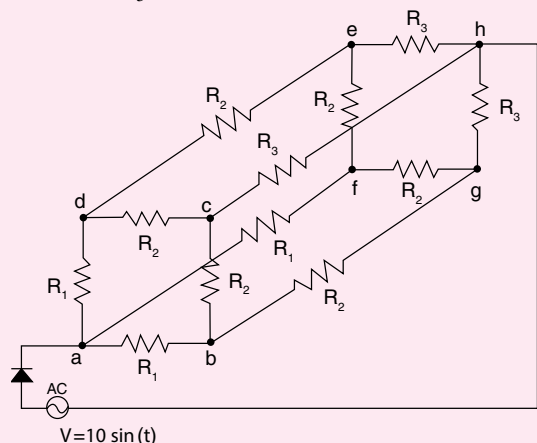
39. In the circuit shown, the initial voltages across the capacitors C_1 and C_2 are 1 V and 3 V, respectively. The switch is closed at time $t = 0$. The total energy dissipated (in Joules) in the resistor R until steady state is reached, is _____. [2015]



40. At very high frequencies, the peak output voltage V_o (in volts) is _____. [2015]

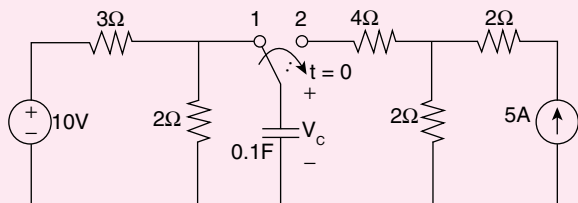


41. An AC voltage source $V = 10 \sin(t)$ volts is applied to the following network assume that $R_1 = 3 \text{ k}\Omega$, $R_2 = 6 \text{ k}\Omega$ and $R_3 = 9 \text{ k}\Omega$, and that the diode is ideal. [2016]



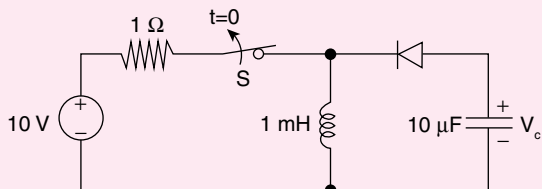
RMS current I_{rms} (in mA) through the diode is _____. [2016]

42. The switch has been in position 1 for a long time and abruptly changes to position 2 at $t = 0$.



If time t is in seconds, the capacitor voltage V_C (in volts) for $t > 0$ is given by [2016]

- (A) $4(1 - \exp(-t/0.5))$ (B) $10 - 6\exp(-t/0.5)$
 (C) $4(1 - \exp(-t/0.6))$ (D) $10 - 6\exp(-t/0.6)$
43. The switch S in the circuit shown has been closed for a long time. It is opened at time $t = 0$ and remains open after that. Assume that the diode has zero reverse current and zero forward voltage drop



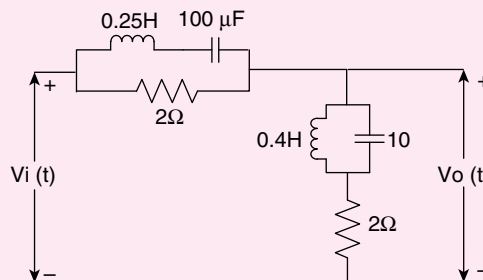
The steady state magnitude of the capacitor voltage V_c (in volts) is _____. [2016]

44. In the RLC circuit shown in the figure, the input voltage is given by

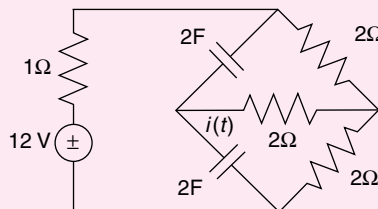
$$V_i(t) = 2\cos(200t) + 4\sin(500t).$$

The output voltage $v_o(t)$ is

[2016]



- (A) $\cos(200t) + 2\sin(500t)$
 (B) $2\cos(200t) + 4\sin(500t)$
 (C) $\sin(200t) + 2\cos(500t)$
 (D) $2\sin(200t) + 4\cos(500t)$
45. Assume that the circuit in the figure has reached the steady state before time $t = 0$ when the 3Ω resistor suddenly burns out, resulting in an open circuit. The current $i(t)$ (in ampere) at $t = 0$ is _____. [2016]



ANSWER KEYS

EXERCISES

Practice Problems 1

- | | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1. B | 2. B | 3. B | 4. C | 5. A | 6. C | 7. D | 8. B | 9. C | 10. D |
| 11. A | 12. D | 13. C | 14. B | 15. B | 16. B | 17. D | 18. A | 19. C | 20. C |
| 21. B | 22. D | 23. A | 24. A | 25. A | 26. C | 27. B | 28. A | 29. A | 30. B |
| 31. B | 32. C | | | | | | | | |

Practice Problems 2

- | | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1. C | 2. D | 3. D | 4. D | 5. A | 6. D | 7. A | 8. B | 9. B | 10. C |
| 11. B | 12. C | 13. C | 14. B | 15. A | 16. B | 17. C | 18. D | 19. B | 20. A |
| 21. C | 22. A | 23. B | 24. C | | | | | | |

Previous Years' Questions

- | | | | | | | | | | |
|--------------|----------------|-----------|----------------|------------------|-------------------|---------|------------------|-------|-------|
| 1. A | 2. A | 3. C | 4. B | 5. C | 6. A | 7. B | 8. A | 9. C | 10. A |
| 11. C | 12. B | 13. A | 14. B | 15. C | 16. D | 17. B | 18. A | 19. A | 20. B |
| 21. D | 22. D | 23. D | 24. C | 25. B | 26. 9.99 to 10.01 | | 27. 1.2 to 1.3 | | 28. A |
| 29. A | 30. B | 31. B | 32. 50 | 33. 24 to 26 | 34. C | 35. C | 36. 2.48 to 2.58 | | |
| 37. 100 | 38. 4.9 to 5.1 | | 39. 1.4 to 1.6 | 40. 0.49 to 0.51 | | 41. 0.7 | 42. D | | |
| 43. 100volts | 44. B | 45. 1 amp | | | | | | | |