

Ex 25.1

Vector or Cross Product Ex 25.1 Q1

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix}$$

$$= \hat{i}(9 - 0) - \hat{j}(3 - 2) + \hat{k}(0 + 3)$$

$$\vec{a} \times \vec{b} = 9\hat{i} - \hat{j} + 3\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(9)^2 + (-1)^2 + (3)^2}$$

$$= \sqrt{81 + 1 + 9}$$

$$|\vec{a} \times \vec{b}| = \sqrt{91}$$

Vector or Cross Product Ex 25.1 Q2(i)

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(4 - 0) - \hat{j}(3 - 0) + \hat{k}(3 - 4)$$

$$= 4\hat{i} - 3\hat{j} - \hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(4)^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{16 + 9 + 1}$$

$$|\vec{a} \times \vec{b}| = \sqrt{26}$$

Vector or Cross Product Ex 25.1 Q2(ii)

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(0 - 1) - \hat{j}(2 - 1) + \hat{k}(2 - 0)$$

$$= -\hat{i} - \hat{j} + 2\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$

$$= \sqrt{1 + 1 + 4}$$

$$|\vec{a} \times \vec{b}| = \sqrt{6}$$

Magnitude of $\vec{a} \times \vec{b} = \sqrt{6}$.

Vector or Cross Product Ex 25.1 Q3(i)

A vector perpendicular to both $\vec{a}$ and $\vec{b} = \vec{a} \times \vec{b}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\vec{c} \text{ (say)} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix}$$

$$\vec{c} = \hat{i} (2 - 3) - \hat{j} (-8 + 6) + \hat{k} (4 - 2)$$

$$\vec{c} = -\hat{i} + 2\hat{j} + 2\hat{k}$$

$\vec{c}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|}$$

$$= \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{(-1)^2 + (2)^2 + (2)^2}}$$

$$= \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1 + 4 + 4}}$$

$$= \frac{1}{3}(-\hat{i} + 2\hat{j} + 2\hat{k})$$

So, unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \frac{1}{3}(-\hat{i} + 2\hat{j} + 2\hat{k})$.

Vector or Cross Product Ex 25.1 Q3(ii)

A vector perpendicular to the plane containing the vector $\vec{a}$ and $\vec{b}$ is given by

$$\vec{a} \times \vec{b} = \pm \vec{c} \text{ (Say)}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\vec{c} = \hat{i} (1 - 2) - \hat{j} (2 - 1) + \hat{k} (4 - 1)$$

$$\vec{c} = -\hat{i} - \hat{j} + 3\hat{k}$$

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|}$$

$$= \frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{(-1)^2 + (-1)^2 + (3)^2}}$$

$$= \frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{1 + 1 + 9}}$$

$$= \frac{1}{\sqrt{11}}(-\hat{i} - \hat{j} + 3\hat{k})$$

Unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b} = \pm \frac{1}{\sqrt{11}}(-\hat{i} - \hat{j} + 3\hat{k})$.

Vector or Cross Product Ex 25.1 Q4

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 3 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}(-4 - 3) - \hat{j}(0 - 3) + \hat{k}(0 - 4)$$

$$= -7\hat{i} + 3\hat{j} - 4\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(-7)^2 + (3)^2 + (-4)^2}$$

$$= \sqrt{49 + 9 + 16}$$

$$|\vec{a} \times \vec{b}| = \sqrt{74}$$

Vector or Cross Product Ex 25.1 Q5

$$\vec{b} = \hat{i} - 2\hat{k}$$

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|}$$

$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{(1)^2 + (-2)^2}}$$

$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{1+4}}$$

$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{5}}$$

$$2\hat{b} = \frac{2}{\sqrt{5}}\hat{i} - \frac{4}{\sqrt{5}}\hat{k}$$

$$\text{And, } \vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$$

$$\text{If } \vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k} \text{ and}$$

$$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k},$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$2\hat{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{2}{\sqrt{5}} & 0 & -\frac{4}{\sqrt{5}} \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}\left(0 + \frac{12}{\sqrt{5}}\right) - \hat{j}\left(\frac{2}{\sqrt{5}} + \frac{16}{\sqrt{5}}\right) + \hat{k}\left(\frac{6}{\sqrt{5}} - 0\right)$$

$$2\hat{b} \times \vec{a} = \frac{12}{\sqrt{5}}\hat{i} - \frac{18}{\sqrt{5}}\hat{j} + \frac{6}{\sqrt{5}}\hat{k}$$

$$|2\hat{b} \times \vec{a}| = \sqrt{\left(\frac{12}{\sqrt{5}}\right)^2 + \left(-\frac{18}{\sqrt{5}}\right)^2 + \left(\frac{6}{\sqrt{5}}\right)^2}$$

$$= \sqrt{\frac{144}{5} + \frac{324}{5} + \frac{36}{5}}$$

$$|2\hat{b} \times \vec{a}| = \sqrt{\frac{504}{5}}$$

Vector or Cross Product Ex 25.1 Q6

$$\begin{aligned}\vec{a} + 2\vec{b} &= (3\hat{i} - \hat{j} - 2\hat{k}) + 2(2\hat{i} + 3\hat{j} + \hat{k}) \\ &= 3\hat{i} - \hat{j} - 2\hat{k} + 4\hat{i} + 6\hat{j} + 2\hat{k}\end{aligned}$$

$$\vec{a} + 2\vec{b} = 7\hat{i} + 5\hat{j}$$

$$\begin{aligned}2\vec{a} - \vec{b} &= 2(3\hat{i} - \hat{j} - 2\hat{k}) - (2\hat{i} + 3\hat{j} + \hat{k}) \\ &= 6\hat{i} - 2\hat{j} - 4\hat{k} - 2\hat{i} - 3\hat{j} - \hat{k} \\ &= 4\hat{i} - 5\hat{j} - 5\hat{k}\end{aligned}$$

We know that if $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

Therefore,

$$\begin{aligned}(\vec{a} + 2\vec{b}) \times (2\vec{a} - \vec{b}) &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 5 & 0 \\ 4 & -5 & -5 \end{vmatrix} \\ &= \hat{i}(-25 - 0) - \hat{j}(-35 - 0) + \hat{k}(-35 - 20) \\ &= -25\hat{i} + 35\hat{j} - 55\hat{k}\end{aligned}$$

$$(\vec{a} + 2\vec{b}) \times (2\vec{a} - \vec{b}) = -25\hat{i} + 35\hat{j} - 55\hat{k}$$

Vector or Cross Product Ex 25.1 Q7(i)

Let, $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{b} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$$

$$= \hat{i}(6 + 36) - \hat{j}(4 - 18) + \hat{k}(-12 - 9)$$

$$= 42\hat{i} + 14\hat{j} - 21\hat{k}$$

$$= 7(6\hat{i} + 2\hat{j} - 3\hat{k})$$

$$\begin{aligned}|\vec{a} \times \vec{b}| &= 7\sqrt{(6)^2 + (2)^2 + (-3)^2} \\ &= 7\sqrt{36 + 4 + 9}\end{aligned}$$

$$|\vec{a} \times \vec{b}| = 7\sqrt{49}$$

$$|\vec{a} \times \vec{b}| = 7 \times 7$$

$$|\vec{a} \times \vec{b}| = 49$$

Vector or Cross Product Ex 25.1 Q7(ii)

Vector perpendicular to $\vec{a}$ and $\vec{b}$

$$\begin{aligned}\text{with magnitude } 1 &= \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} \\ &= \frac{1}{49} \left(7 \left(6\hat{i} + 2\hat{j} - 3\hat{k} \right) \right) \\ &= \frac{1}{7} \left(6\hat{i} + 2\hat{j} - 3\hat{k} \right)\end{aligned}$$

vector of magnitude 49, which is perpendicular to $\vec{a}$ and $\vec{b}$

$$\begin{aligned}&= 49 \left(\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} \right) \\ &= 49 \left[\frac{1}{7} \left(6\hat{i} + 2\hat{j} - 3\hat{k} \right) \right] \\ &= 42\hat{i} + 14\hat{j} - 21\hat{k}\end{aligned}$$

The required vector = $42\hat{i} + 14\hat{j} - 21\hat{k}$

Vector or Cross Product Ex 25.1 Q7(iii)

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then,

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -4 \\ 6 & 5 & -2 \end{vmatrix} \\ &= \hat{i} \left(-2 + 20 \right) - \hat{j} \left(-6 + 24 \right) + \hat{k} \left(15 - 6 \right) \\ &= 18\hat{i} - 18\hat{j} + 9\hat{k} \\ &= 9 \left(2\hat{i} - 2\hat{j} + \hat{k} \right)\end{aligned}$$

$$\begin{aligned}|\vec{a} \times \vec{b}| &= 9\sqrt{2^2 + (-2)^2 + (1)^2} \\ &= 9\sqrt{4 + 4 + 1}\end{aligned}$$

$$|\vec{a} \times \vec{b}| = 9\sqrt{9}$$

$$|\vec{a} \times \vec{b}| = 9 \times 3$$

$$|\vec{a} \times \vec{b}| = 27$$

Vector or Cross Product Ex 25.1 Q7(iv)

Unit vector perpendicular to the vector

$$\begin{aligned}\vec{a} \text{ and } \vec{b} &= \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} \\ &= \frac{1}{27} (9(2\hat{i} - 2\hat{j} + \hat{k})) \\ &= \frac{1}{3} (2\hat{i} - 2\hat{j} + \hat{k})\end{aligned}$$

vector with length 3 and which is perpendicular to both $\vec{a}$ and $\vec{b}$

$$\begin{aligned}&= 3 \left(\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} \right) \\ &= 3 \left[\frac{1}{3} (2\hat{i} - 2\hat{j} + \hat{k}) \right] \\ &= 2\hat{i} - 2\hat{j} + \hat{k}\end{aligned}$$

Required vector = $2\hat{i} - 2\hat{j} + \hat{k}$ **Vector or Cross Product Ex 25.1 Q8(i)**Here, $\vec{a} = 2\hat{i} + 0\hat{j} + 0\hat{k}$ $\vec{b} = 0\hat{i} + 3\hat{j} + 0\hat{k}$,

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{vmatrix} \\ &= \hat{i}(0 - 0) - \hat{j}(0 - 0) + \hat{k}(6 - 0) \\ &= 6\hat{k}\end{aligned}$$

$$\begin{aligned}\text{Area of parallelogram} &= |\vec{a} \times \vec{b}| \\ &= |0\hat{i} + 0\hat{j} + 6\hat{k}| \\ &= \sqrt{(0)^2 + (0)^2 + (6)^2}\end{aligned}$$

Area of parallelogram = 6 sq.unit

Vector or Cross Product Ex 25.1 Q8(ii)Let, $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ $\vec{b} = \hat{i} - \hat{j}$

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{vmatrix} \\ &= \hat{i}(0 + 3) - \hat{j}(0 - 3) + \hat{k}(-2 - 1) \\ &= 3\hat{i} + 3\hat{j} - 3\hat{k} \\ &= 3(\hat{i} + \hat{j} - \hat{k})\end{aligned}$$

$$\begin{aligned}\text{Area of parallelogram} &= |\vec{a} \times \vec{b}| \\ &= 3\sqrt{(1)^2 + (1)^2 + (-1)^2} \\ &= 3\sqrt{3}\end{aligned}$$

Area of parallelogram = $3\sqrt{3}$ sq.unit

Vector or Cross Product Ex 25.1 Q8(iii)

$$\text{Let, } \vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= \hat{i} (4 - 6) - \hat{j} (12 + 2) + \hat{k} (-9 - 1)$$

$$= -2\hat{i} - 14\hat{j} - 10\hat{k}$$

$$= -2(\hat{i} + 7\hat{j} + 5\hat{k})$$

$$\begin{aligned} \text{Area of parallelogram} &= |\vec{a} \times \vec{b}| \\ &= 2\sqrt{(1)^2 + (7)^2 + (5)^2} \\ &= 2\sqrt{1 + 49 + 25} \\ &= 2\sqrt{75} \\ &= 10\sqrt{3} \end{aligned}$$

$$\text{Area of parallelogram} = 10\sqrt{3} \text{ sq.unit}$$

Vector or Cross Product Ex 25.1 Q8(iv)

$$\text{Let, } \vec{a} = \hat{i} - 3\hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i} (-3 - 1) - \hat{j} (1 - 1) + \hat{k} (1 + 3)$$

$$= -4\hat{i} - 0\hat{j} + 4\hat{k}$$

$$\begin{aligned} \text{Area of parallelogram} &= |\vec{a} \times \vec{b}| \\ &= \sqrt{(-4)^2 + (0)^2 + (4)^2} \\ &= \sqrt{16 + 0 + 16} \\ &= \sqrt{32} \\ &= 4\sqrt{2} \end{aligned}$$

$$\text{Area of parallelogram} = 4\sqrt{2} \text{ sq.unit}$$

Vector or Cross Product Ex 25.1 Q9(i)

$$\text{Area of parallelogram} = \frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$$

$$\text{Here, } d_1 = 4\hat{i} - \hat{j} - 3\hat{k}$$

$$d_2 = -2\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & -3 \\ -2 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}(2+3) - \hat{j}(-8-6) + \hat{k}(4-2)$$

$$= 5\hat{i} + 14\hat{j} + 2\hat{k}$$

$$|\vec{d}_1 \times \vec{d}_2| = \sqrt{(5)^2 + (14)^2 + (2)^2}$$

$$= \sqrt{25 + 196 + 4}$$

$$= \sqrt{225}$$

$$= 15$$

$$\text{Area of parallelogram} = \frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$$

$$\text{Area of parallelogram} = \frac{15}{2} \text{ sq.unit}$$

Vector or Cross Product Ex 25.1 Q9(ii)

$$\text{Given, } d_1 = 2\hat{i} + \hat{k}$$

$$d_2 = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(2-1) + \hat{k}(2-0)$$

$$= -\hat{i} - \hat{j} + 2\hat{k}$$

$$|\vec{d}_1 \times \vec{d}_2| = \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$

$$= \sqrt{1+1+4}$$

$$= \sqrt{6}$$

$$\text{Area of parallelogram} = \frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$$

$$\text{Area of parallelogram} = \frac{1}{2} \sqrt{6} \text{ sq.unit}$$

Vector or Cross Product Ex 25.1 Q9(iii)

Given, $d_1 = 3\hat{i} + 4\hat{j}$

$$d_2 = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(4 - 0) - \hat{j}(3 - 0) + \hat{k}(3 - 4)$$

$$= 4\hat{i} - 3\hat{j} - \hat{k}$$

$$|\vec{d}_1 \times \vec{d}_2| = \sqrt{(4)^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{16 + 9 + 1}$$

$$= \sqrt{26}$$

$$\text{Area of parallelogram} = \frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$$

$$\text{Area of parallelogram} = \frac{\sqrt{26}}{2} \text{ sq.unit}$$

Vector or Cross Product Ex 25.1 Q9(iv)

Here, $d_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$$d_2 = 3\hat{i} - 6\hat{j} + 2\hat{k}$$

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$$

$$= \hat{i}(6 + 36) - \hat{j}(4 - 18) + \hat{k}(-12 - 9)$$

$$= 42\hat{i} + 14\hat{j} - 21\hat{k}$$

$$= 7(6\hat{i} + 2\hat{j} - 3\hat{k})$$

$$|\vec{d}_1 \times \vec{d}_2| = 7\sqrt{(6)^2 + (2)^2 + (-3)^2}$$

$$= 7\sqrt{36 + 4 + 9}$$

$$= 7\sqrt{49}$$

$$= 7 \times 7$$

$$= 49$$

$$\text{Area of parallelogram} = \frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$$

$$\text{Area of parallelogram} = \frac{49}{2} \text{ sq.unit}$$

Vector or Cross Product Ex 25.1 Q10

$$\text{Given, } \vec{a} = 2\hat{i} + 5\hat{j} - 7\hat{k},$$

$$\vec{b} = -3\hat{i} + 4\hat{j} + \hat{k},$$

$$\vec{c} = \hat{i} - 2\hat{j} - 3\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -7 \\ -3 & 4 & 1 \end{vmatrix}$$

$$= \hat{i}(5 + 28) - \hat{j}(2 - 21) + \hat{k}(8 + 15)$$

$$= 33\hat{i} + 19\hat{j} + 23\hat{k}$$

$$(\vec{a} \times \vec{b}) \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 33 & 19 & 23 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(-57 + 46) - \hat{j}(-99 - 23) + \hat{k}(-66 - 19)$$

$$(\vec{a} \times \vec{b}) \times \vec{c} = -11\hat{i} + 122\hat{j} - 85\hat{k} \quad \text{--- (i)}$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & 1 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(-12 + 2) - \hat{j}(9 - 1) + \hat{k}(6 - 4)$$

$$= -10\hat{i} - 8\hat{j} + 2\hat{k}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -7 \\ -10 & -8 & 2 \end{vmatrix}$$

$$= \hat{i}(10 + 56) - \hat{j}(4 - 70) + \hat{k}(-16 + 50)$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = 66\hat{i} + 66\hat{j} + 36\hat{k} \quad \text{--- (ii)}$$

From equation (i) and (ii)

$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$$

Vector or Cross Product Ex 25.1 Q11

We know that, if θ be the angle between $\vec{a}$ and $\vec{b}$, then,

$$\vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \sin \theta \cdot \hat{n}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \cdot |\sin \theta| \cdot |\hat{n}|$$

$$8 = 2.5 \cdot \sin \theta.1$$

[As $\hat{n}$ is a unit vector]

$$\sin \theta = \frac{8}{10}$$

$$\sin \theta = \frac{4}{5}$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$= 1 - \left(\frac{4}{5}\right)^2$$

$$= 1 - \frac{16}{25}$$

$$= \frac{25 - 16}{25}$$

$$= \frac{9}{25}$$

$$\cos \theta = \frac{3}{5}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos \theta$$

$$= 2.5 \cdot \frac{3}{5}$$

$$\vec{a} \cdot \vec{b} = 6$$

Vector or Cross Product Ex 25.1 Q12

$$\text{Given, } \vec{a} = \frac{1}{7} \{2\hat{i} + 3\hat{j} + 6\hat{k}\}$$

$$\vec{b} = \frac{1}{7} \{3\hat{i} - 6\hat{j} + 2\hat{k}\}$$

$$\vec{c} = \frac{1}{7} \{6\hat{i} + 2\hat{j} - 3\hat{k}\},$$

$$\begin{aligned} \vec{a} \times \vec{b} &= \frac{1}{7} \times \frac{1}{7} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix} \\ &= \frac{1}{49} [\hat{i}(6 + 36) - \hat{j}(4 - 18) + \hat{k}(-12 - 9)] \\ &= \frac{1}{49} [42\hat{i} + 14\hat{j} - 21\hat{k}] \\ &= \frac{7\{6\hat{i} + 2\hat{j} - 3\hat{k}\}}{49} \\ &= \frac{1}{7} \{6\hat{i} + 2\hat{j} - 3\hat{k}\} \end{aligned}$$

$$\vec{a} \times \vec{b} = \vec{c} \quad \text{--- (i)}$$

$$\begin{aligned} \vec{b} \times \vec{c} &= \frac{1}{7} \times \frac{1}{7} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 6 & 2 & -3 \end{vmatrix} \\ &= \frac{1}{49} [\hat{i}(18 - 4) - \hat{j}(-9 - 12) + \hat{k}(6 + 36)] \\ &= \frac{1}{49} [14\hat{i} + 21\hat{j} + 42\hat{k}] \\ &= \frac{7\{2\hat{i} + 3\hat{j} + 6\hat{k}\}}{49} \\ \vec{c} \times \vec{a} &= \frac{1}{7} \times \frac{1}{7} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 2 & -3 \\ 2 & 3 & 6 \end{vmatrix} \\ \vec{c} \times \vec{a} &= \frac{1}{49} [\hat{i}(12 + 9) - \hat{j}(36 + 6) + \hat{k}(18 - 4)] \\ &= \frac{1}{49} [21\hat{i} - 42\hat{j} + 14\hat{k}] \\ &= \frac{7\{3\hat{i} - 6\hat{j} + 2\hat{k}\}}{49} \\ &= \frac{1}{7} \{3\hat{i} - 6\hat{j} + 2\hat{k}\} \end{aligned}$$

$$\vec{c} \times \vec{a} = \vec{b} \quad \text{--- (ii)}$$

From (i), (ii), and (iii),

$$\left. \begin{aligned} \vec{a} \times \vec{b} &= \vec{c} \\ \vec{b} \times \vec{c} &= \vec{a} \\ \vec{c} \times \vec{a} &= \vec{b} \end{aligned} \right\} \text{--- (A)}$$

$$\begin{aligned} |\vec{a}| &= \frac{1}{7} \sqrt{(2)^2 + (3)^2 + (6)^2} \\ &= \frac{1}{7} \sqrt{4 + 9 + 36} \\ &= \frac{1}{7} \sqrt{49} \\ &= \frac{1}{7} \times 7 \end{aligned}$$

$$|\vec{a}| = 1 \quad \text{--- (iv)}$$

$$\begin{aligned} |\vec{b}| &= \frac{1}{7} \sqrt{(3)^2 + (-6)^2 + (2)^2} \\ &= \frac{1}{7} \sqrt{9 + 36 + 4} \\ &= \frac{1}{7} \sqrt{49} \\ &= \frac{7}{7} \end{aligned}$$

$$|\vec{b}| = 1 \quad \text{--- (v)}$$

$$\begin{aligned} |\vec{c}| &= \frac{1}{7} \sqrt{(6)^2 + (2)^2 + (-3)^2} \\ &= \frac{1}{7} \sqrt{36 + 4 + 9} \\ &= \frac{1}{7} \sqrt{49} \\ &= \frac{7}{7} \end{aligned}$$

$$|\vec{c}| = 1 \quad \text{--- (vi)}$$

From equation (iv), (v), (vi),

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1 \quad \text{--- (B)}$$

From (A) and (B), We can say that

$\vec{a}, \vec{b}, \vec{c}$ is a right handed orthogonal system of unit vectors

Vector or Cross Product Ex 25.1 Q13

We know that, if θ is angle between $\vec{a}$ and $\vec{b}$,

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$60 = 13.5 \cdot \cos \theta$$

$$\cos \theta = \frac{60}{65}$$

$$\cos \theta = \frac{12}{13}$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$= 1 - \left(\frac{12}{13}\right)^2$$

$$= 1 - \frac{144}{169}$$

$$= \frac{169 - 144}{169}$$

$$= \frac{25}{169}$$

$$\sin \theta = \pm \sqrt{\frac{25}{169}}$$

$$= \pm \frac{5}{13}$$

$$|\sin \theta| = \frac{5}{13}$$

We know that,

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| |\sin \theta| |\hat{n}|$$

$$= 13.5 \cdot \frac{5}{13} \cdot 1$$

[Since, $\hat{n}$ is a unit vector]

$$|\vec{a} \times \vec{b}| = 25$$

Vector or Cross Product Ex 25.1 Q14

We know that, if θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \quad \text{--- (i)}$$

$$\text{And, } \vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| |\sin \theta| |\hat{n}|$$

$$= |\vec{a}| |\vec{b}| |\sin \theta| \cdot 1$$

[Since, $\hat{n}$ is a unit vector]

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta \quad \text{--- (ii)}$$

$$\text{Given that, } |\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$$

$$|\vec{a}| |\vec{b}| \sin \theta = |\vec{a}| |\vec{b}| \cos \theta$$

$$\sin \theta = \cos \theta$$

$$\theta = \frac{\pi}{4}$$

$$\text{Angle between } \vec{a} \text{ and } \vec{b} = \frac{\pi}{4}$$

Vector or Cross Product Ex 25.1 Q15

We have,

$$\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$$

$$(\vec{a} \times \vec{b}) - (\vec{b} \times \vec{c}) = \vec{0}$$

$$(\vec{a} \times \vec{b}) + (\vec{c} \times \vec{b}) = \vec{0} \quad \left[\text{Since, } (\vec{b} \times \vec{c}) = -(\vec{c} \times \vec{b}) \right]$$

$$(\vec{a} + \vec{c}) \times \vec{b} = \vec{0} \quad [\text{Using distributive property}]$$

We know that, if $\vec{a} \times \vec{b} = \vec{0}$, then vector $\vec{a}$ is parallel to vector $\vec{b}$.

Thus, $(\vec{a} + \vec{c})$ is parallel to $\vec{b}$

$$(\vec{a} + \vec{c}) = m\vec{b}$$

Where m is any scalar

Vector or Cross Product Ex 25.1 Q16

We know that,

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$$

$$\begin{aligned} |\vec{a} \times \vec{b}| &= |\vec{a}| |\vec{b}| |\sin \theta| |\hat{n}| \\ &= |\vec{a}| |\vec{b}| |\sin \theta| \cdot 1 \quad [\text{as } \hat{n} \text{ is a unit vector}] \end{aligned}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| |\sin \theta|$$

$$\sqrt{(3)^2 + (2)^2 + (6)^2} = 2.7 |\sin \theta|$$

$$\sqrt{9 + 4 + 36} = 14 |\sin \theta|$$

$$\sqrt{49} = 14 |\sin \theta|$$

$$\sin \theta = \frac{7}{14}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \sin^{-1} \left(\frac{1}{2} \right)$$

$$\theta = \frac{\pi}{6}$$

Angle between $\vec{a}$ and $\vec{b} = \frac{\pi}{6}$

Vector or Cross Product Ex 25.1 Q17

Given that $\vec{a} \times \vec{b} = \vec{0}$

This gives us four conclusions about $\vec{a}$ and $\vec{b}$

$$(i) \vec{a} = \vec{0} \quad \text{or}$$

$$(ii) \vec{b} = \vec{0} \quad \text{or}$$

$$(iii) \vec{a} = \vec{b} = \vec{0} \quad \text{or}$$

$$(iv) \vec{a} \text{ is parallel to } \vec{b}.$$

Also, it is given that $\vec{a} \cdot \vec{b} = 0$

This also gives us four conclusions about $\vec{a}$ and $\vec{b}$.

$$(i) \vec{a} = \vec{0} \quad \text{or}$$

$$(ii) \vec{b} = \vec{0} \quad \text{or}$$

$$(iii) \vec{a} = \vec{b} = \vec{0} \quad \text{or}$$

$$(iv) \vec{a} \text{ is perpendicular to } \vec{b}.$$

Now,

$\vec{a}$ parallel $\vec{b}$ and $\vec{a}$ is perpendicular to $\vec{b}$ are not possible simultaneously.

So,

$$\vec{a} = \vec{0} \quad \text{or} \quad \vec{b} = \vec{0} \quad \text{or} \quad \vec{a} = \vec{b} = \vec{0}$$

Vector or Cross Product Ex 25.1 Q18

Given that $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that

$$\vec{a} \times \vec{b} = \vec{c}, \quad \vec{b} \times \vec{c} = \vec{a}, \quad \vec{c} \times \vec{a} = \vec{b},$$

$$\vec{a} \times \vec{b} = \vec{c}$$

$$\Rightarrow \quad \vec{c} \text{ is a vector perpendicular to both } \vec{a} \text{ and } \vec{b} \quad \text{--- (i)}$$

$$\vec{b} \times \vec{c} = \vec{a}$$

$$\Rightarrow \quad \vec{a} \text{ is a vector perpendicular to } \vec{b} \text{ and } \vec{c} \quad \text{--- (ii)}$$

$$\vec{c} \times \vec{a} = \vec{b}$$

$$\Rightarrow \quad \vec{b} \text{ is a vector perpendicular to } \vec{a} \text{ and } \vec{c} \quad \text{--- (iii)}$$

Using (i), (ii) and (iii), we can see that $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors.

$$\text{Since, } \vec{a} \times \vec{b} = \vec{c}$$

$$\vec{b} \times \vec{c} = \vec{a}$$

$$\vec{c} \times \vec{a} = \vec{b}$$

Therefore,

$\vec{a}, \vec{b}, \vec{c}$ form an orthonormal right handed triad of unit vectors.

Vector or Cross Product Ex 25.1 Q19

$$\text{Here, Position vector of } A = (3\hat{i} - \hat{j} + 2\hat{k})$$

$$\text{Position vector of } B = (\hat{i} - \hat{j} - 3\hat{k})$$

$$\text{Position vector of } C = (4\hat{i} - 3\hat{j} + \hat{k})$$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$

$$= (\hat{i} - \hat{j} - 3\hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k})$$

$$= \hat{i} - \hat{j} - 3\hat{k} - 3\hat{i} + \hat{j} - 2\hat{k}$$

$$= -2\hat{i} - 5\hat{k}$$

$$\overrightarrow{AC} = \vec{C} - \vec{A}$$

$$= (4\hat{i} - 3\hat{j} + \hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k})$$

$$= 4\hat{i} - 3\hat{j} + \hat{k} - 3\hat{i} + \hat{j} - 2\hat{k}$$

$$= \hat{i} - 2\hat{j} - \hat{k}$$

Vector perpendicular to the plane ABC

$$= \overrightarrow{AC} \times \overrightarrow{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -1 \\ -2 & 0 & -5 \end{vmatrix}$$

$$\overrightarrow{AC} \times \overrightarrow{AB} = \hat{i}(10 - 0) - \hat{j}(-5 - 2) + \hat{k}(0 - 4)$$

$$= 10\hat{i} + 7\hat{j} - 4\hat{k}$$

$$|\overrightarrow{AC} \times \overrightarrow{AB}| = \sqrt{(10)^2 + (7)^2 + (-4)^2}$$

$$= \sqrt{100 + 49 + 16}$$

$$= \sqrt{165}$$

$$\text{Therefore, unit vector perpendicular to the plane } ABC = \frac{\overrightarrow{AC} \times \overrightarrow{AB}}{|\overrightarrow{AC} \times \overrightarrow{AB}|}$$

$$= \frac{1}{\sqrt{165}} (10\hat{i} + 7\hat{j} - 4\hat{k})$$

$$\text{Unit vector perpendicular to the plane } ABC = \frac{1}{\sqrt{165}} (10\hat{i} + 7\hat{j} - 4\hat{k})$$

Vector or Cross Product Ex 25.1 Q20

Here, It is given that

In $\triangle ABC$

$$\begin{aligned}\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB} \\&= \overrightarrow{BA} + \overrightarrow{AB} \\&= \overrightarrow{BA} - \overrightarrow{BA} \quad \left[\text{Since, } \overrightarrow{BA} = -\overrightarrow{AB} \right]\end{aligned}$$

$$= \vec{0}$$

$$\begin{aligned}\text{Given that, } |\overrightarrow{BC}| &= a \\ |\overrightarrow{CA}| &= b \\ |\overrightarrow{AB}| &= c\end{aligned}$$

Let, $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$ and $\overrightarrow{AB} = \vec{c}$

We have,

$$\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB} = \vec{0}$$

$$\begin{aligned}\Rightarrow \quad \overrightarrow{BC} + \overrightarrow{CA} &= -\overrightarrow{AB} \\ \Rightarrow \quad \overrightarrow{BC} + \overrightarrow{CA} &= \overrightarrow{BA} \\ \Rightarrow \quad \vec{a} + \vec{b} &= -\vec{c} \\ \Rightarrow \quad \vec{a} + \vec{b} + \vec{c} &= \vec{0} \\ \Rightarrow \quad \vec{a} \times (\vec{a} + \vec{b} + \vec{c}) &= \vec{a} \times \vec{0} \\ \Rightarrow \quad \vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} &= \vec{0} \\ \Rightarrow \quad \vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} &= \vec{0} \quad \left[\text{Since, } \vec{a} \times \vec{a} = \vec{0} \right] \\ \Rightarrow \quad \vec{a} \times \vec{b} &= -(\vec{a} \times \vec{c}) \\ \Rightarrow \quad \vec{a} \times \vec{b} &= \vec{c} \times \vec{a} \quad \text{--- (i)}\end{aligned}$$

Again, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$$\begin{aligned}\Rightarrow \quad \vec{b} \times (\vec{a} + \vec{b} + \vec{c}) &= \vec{b} \times \vec{0} \\ \Rightarrow \quad \vec{b} \times \vec{a} + \vec{b} \times \vec{b} + \vec{b} \times \vec{c} &= \vec{0} \\ \Rightarrow \quad \vec{b} \times \vec{a} + \vec{0} + \vec{b} \times \vec{c} &= \vec{0} \quad \left[\text{Since, } \vec{b} \times \vec{b} = \vec{0} \right] \\ \Rightarrow \quad \vec{b} \times \vec{a} + \vec{b} \times \vec{c} &= \vec{0} \\ \Rightarrow \quad \vec{b} \times \vec{c} &= -(\vec{b} \times \vec{a}) \\ \Rightarrow \quad \vec{b} \times \vec{c} &= \vec{a} \times \vec{b} \quad \text{--- (ii)}\end{aligned}$$

From equation (i) and (ii), we get

$$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$$

$$\begin{aligned}\Rightarrow \quad |\vec{a} \times \vec{b}| &= |\vec{b} \times \vec{c}| = |\vec{c} \times \vec{a}| \\ \Rightarrow \quad |\vec{a}| |\vec{b}| \sin(\pi - C) &= |\vec{b}| |\vec{c}| \sin(\pi - A) = |\vec{c}| |\vec{a}| \sin(\pi - B) \\ \Rightarrow \quad ab \sin C &= bc \sin A = ca \sin B\end{aligned}$$

Dividing by abc

$$\begin{aligned}\Rightarrow \quad \frac{ab \sin C}{abc} &= \frac{bc \sin A}{abc} = \frac{ca \sin B}{abc} \\ \Rightarrow \quad \frac{\sin C}{c} &= \frac{\sin A}{a} = \frac{\sin B}{b} \\ \Rightarrow \quad \frac{c}{\sin C} &= \frac{a}{\sin A} = \frac{b}{\sin B}\end{aligned}$$

Vector or Cross Product Ex 25.1 Q21

Here, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -5 \end{vmatrix}$$

$$= \hat{i}(10 - 9) - \hat{j}(-5 - 6) + \hat{k}(3 + 4)$$

$$= \hat{i} + 11\hat{j} + 7\hat{k}$$

$$\text{Now, } \vec{a} \cdot (\vec{a} \times \vec{b}) = (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + 11\hat{j} + 7\hat{k})$$

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = (1)(1) + (-2)(11) + (3)(7)$$

$$= 1 - 22 + 21$$

$$= 22 - 22$$

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$$

Dot product of $\vec{a}$ and $\vec{a} \times \vec{b}$ is zero, then,

$\vec{a}$ is perpendicular to $(\vec{a} \times \vec{b})$

Vector or Cross Product Ex 25.1 Q22

Given $\vec{p}$ and $\vec{q}$ be unit vector with angle 30° between then

$$|\vec{p}| = |\vec{q}| = 1$$

$$\vec{p} \times \vec{q} = |\vec{p}| |\vec{q}| \sin 30^\circ \hat{n}$$

$$= 1 \cdot 1 \cdot \left(\frac{1}{2}\right) \hat{n}$$

$$|\vec{p} \times \vec{q}| = \left|\frac{\hat{n}}{2}\right|$$

$$|\vec{p} \times \vec{q}| = \frac{1}{2}$$

--- (i)

[Since, $\hat{n}$ is a unit vector]

$$\text{Area of parallelogram} = \frac{1}{2} |\vec{a} \times \vec{b}|$$

$$= \frac{1}{2} |(\vec{p} + 2\vec{q}) \times (2\vec{p} + \vec{q})|$$

$$= \frac{1}{2} |\vec{p} \times 2\vec{p} + \vec{p} \times \vec{q} + 2\vec{q} \times 2\vec{p} + 2\vec{q} \times \vec{q}|$$

$$= \frac{1}{2} |\vec{0} + \vec{p} \times \vec{q} + 2\vec{q} \times 2\vec{p} + \vec{0}| \quad [\text{Since, } \vec{p} \times 2\vec{p} = \vec{0} \text{ and } 2\vec{q} \times \vec{q} = \vec{0}]$$

$$= \frac{1}{2} |\vec{p} \times \vec{q} + 4(\vec{q} \times \vec{p})|$$

$$= \frac{1}{2} |(\vec{p} \times \vec{q}) - 4(\vec{p} \times \vec{q})| \quad [\text{Since, } \vec{q} \times \vec{p} = -\vec{p} \times \vec{q}]$$

$$= \frac{1}{2} |-3(\vec{p} \times \vec{q})|$$

$$= \frac{3}{2} |\vec{p} \times \vec{q}|$$

$$= \frac{3}{2} \times \frac{1}{2} \quad [\text{Using (i)}]$$

$$= \frac{3}{4}$$

$$\text{Area of parallelogram} = \frac{3}{4} \text{ sq. unit}$$

Vector or Cross Product Ex 25.1 Q23

We know that

$$\begin{aligned}\vec{a} \times \vec{b} &= |\vec{a}| |\vec{b}| \sin \theta \hat{n} \\ |\vec{a} \times \vec{b}| &= |\vec{a}| |\vec{b}| \sin \theta |\hat{n}| \\ &= |\vec{a}| |\vec{b}| \sin \theta \cdot 1 \quad \left[\text{Since, } \hat{n} \text{ is unit vector} \right]\end{aligned}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

Squaring both the sides,

$$\begin{aligned}|\vec{a} \times \vec{b}|^2 &= |\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta \\ &= |\vec{a}|^2 |\vec{b}|^2 (1 - \cos^2 \theta) \\ &= |\vec{a}|^2 |\vec{b}|^2 - |\vec{a}|^2 |\vec{b}|^2 \cos^2 \theta \\ &= |\vec{a}|^2 |\vec{b}|^2 - (|\vec{a}| |\vec{b}| \cos \theta)^2 \\ &= (\vec{a})^2 (\vec{b})^2 - (\vec{a} \cdot \vec{b})^2 \quad \left[\text{Since, } |\vec{a}| |\vec{b}| \cos \theta = \vec{a} \cdot \vec{b} \right] \\ |\vec{a} \times \vec{b}|^2 &= (\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{b}) - (\vec{a} \cdot \vec{b})(\vec{b} \cdot \vec{a}) \quad \left[\text{Since, } (\vec{a} \cdot \vec{b}) = (\vec{b} \cdot \vec{a}) \right]\end{aligned}$$

$$|\vec{a} \times \vec{b}|^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} \end{vmatrix}$$

Vector or Cross Product Ex 25.1 Q24

Define of $\vec{a} \times \vec{b}$:- Let $\vec{a}$, $\vec{b}$ be two non-zero, non-parallel vectors. Then $\vec{a} \times \vec{b}$, in that order, is defined as a vector whose magnitude is $|\vec{a}| |\vec{b}| \sin \theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$ and whose direction is perpendicular to the plane of $\vec{a}$ and $\vec{b}$ and this constitute a right handed system.

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ such that $\vec{a}, \vec{b}, \hat{n}$ form a right handed system.

Now,

$$\begin{aligned}\vec{a} \times \vec{b} &= |\vec{a}| |\vec{b}| \sin \theta \hat{n} \\ |\vec{a} \times \vec{b}| &= |\vec{a}| |\vec{b}| \sin \theta |\hat{n}| \\ &= |\vec{a}| |\vec{b}| \sin \theta \cdot 1\end{aligned}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

$$= \frac{\vec{a} \cdot \vec{b}}{\cos \theta} \cdot \sin \theta \quad \left[\text{Since, } \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \right]$$

$$|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b} \cdot \tan \theta$$

Vector or Cross Product Ex 25.1 Q25

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \cdot \hat{n}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta |\hat{n}|$$

$$35 = \sqrt{26} \cdot 7 |\sin \theta| \cdot 1$$

$$\sin \theta = \frac{35}{\sqrt{26} \cdot 7}$$

$$\sin \theta = \frac{5}{\sqrt{26}}$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$= 1 - \left(\frac{5}{\sqrt{26}} \right)^2$$

$$= \frac{1}{1} - \frac{25}{26}$$

$$= \frac{26 - 25}{26}$$

$$= \frac{1}{26}$$

$$\cos \theta = \frac{1}{\sqrt{26}}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$= \sqrt{26} \cdot 7 \cdot \frac{1}{\sqrt{26}}$$

$$\vec{a} \cdot \vec{b} = 7$$

Vector or Cross Product Ex 25.1 Q26

$$\text{Area of triangle} = \frac{1}{2} |\vec{OA} \times \vec{OB}|$$

$$|\vec{OA} \times \vec{OB}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix}$$

$$= \hat{i} (2 + 6) - \hat{j} (1 + 9) + \hat{k} (-2 + 6)$$

$$= 8\hat{i} - 10\hat{j} + 4\hat{k}$$

$$= 2(4\hat{i} - 5\hat{j} + 2\hat{k})$$

$$\text{Area of triangle} = \frac{1}{2} |\vec{OA} \times \vec{OB}|$$

$$= \frac{1}{2} \left[2\sqrt{(4)^2 + (-5)^2 + (2)^2} \right]$$

$$= \frac{1}{2} \left[2\sqrt{16 + 25 + 4} \right]$$

$$= \sqrt{45}$$

$$= 3\sqrt{5}$$

$$\text{Area of triangle} = 3\sqrt{5} \text{ Sq.unit}$$

Vector or Cross Product Ex 25.1 Q27

$$\text{Let } \vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}.$$

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have:

$$\vec{d} \cdot \vec{a} = 0$$

$$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \quad \dots(i)$$

And,

$$\vec{d} \cdot \vec{b} = 0$$

$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \quad \dots(ii)$$

Also, it is given that:

$$\vec{c} \cdot \vec{d} = 15$$

$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \quad \dots(iii)$$

On solving (i), (ii), and (iii), we get:

$$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3} \text{ and } d_3 = -\frac{70}{3}$$

$$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$$

Hence, the required vector is $\frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$.

Vector or Cross Product Ex 25.1 Q28

$$\text{Given, } \vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\text{Let, } \vec{d} = \vec{a} + \vec{b}$$

$$= (3\hat{i} + 2\hat{j} + 2\hat{k}) + (\hat{i} + 2\hat{j} - 2\hat{k})$$

$$\vec{d} = 4\hat{i} + 4\hat{j} + 0\hat{k}$$

$$\text{And, } \vec{e} = \vec{a} - \vec{b}$$

$$= (3\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} - 2\hat{k})$$

$$\vec{e} = 2\hat{i} + 4\hat{k}$$

Let, $\vec{f}$ be any vector perpendicular to both $\vec{d}$ and $\vec{e}$

$$\vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$$= \hat{i}(16 - 0) - \hat{j}(16 - 0) + \hat{k}(0 - 8)$$

$$\vec{f} = 16\hat{i} - 16\hat{j} - 8\hat{k}$$

$$= 8(2\hat{i} - 2\hat{j} - \hat{k})$$

Let $\vec{g}$ be the required vector, then

$$\vec{g} = \lambda \vec{f} \quad \text{and} \quad |\vec{g}| = 1$$

$$\vec{g} = 8\lambda(2\hat{i} - 2\hat{j} - \hat{k}) \quad \dots(i)$$

$$|\vec{g}| = 1$$

$$8\lambda\sqrt{(2)^2 + (-2)^2 + (-1)^2} = 1$$

$$8\lambda\sqrt{4+4+1} = 1$$

$$8\lambda\sqrt{9} = 1$$

$$24\lambda = 1$$

$$\lambda = \frac{1}{24}$$

Put λ in (i)

$$\vec{g} = 8 \left(\frac{1}{24} \right) (2\hat{i} - 2\hat{j} - \hat{k})$$

$$\vec{g} = \frac{1}{3} (2\hat{i} - 2\hat{j} - \hat{k})$$

Thus,

Unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b}) = \frac{1}{3} (2\hat{i} - 2\hat{j} - \hat{k})$

Vector or Cross Product Ex 25.1 Q29

Given, $A = (2, 3, 5)$

$$B = (3, 5, 8)$$

$$C = (2, 7, 8)$$

Position vector of $A = 2\hat{i} + 3\hat{j} + 5\hat{k}$

Position vector of $B = 3\hat{i} + 5\hat{j} + 8\hat{k}$

Position vector of $C = 2\hat{i} + 7\hat{j} + 8\hat{k}$

$\overrightarrow{AB}$ = Position vector of B - Position vector of A

$$= (3\hat{i} + 5\hat{j} + 8\hat{k}) - (2\hat{i} + 3\hat{j} + 5\hat{k})$$

$$= 3\hat{i} + 5\hat{j} + 8\hat{k} - 2\hat{i} - 3\hat{j} - 5\hat{k}$$

$$\overrightarrow{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$\overrightarrow{AC}$ = Position vector of C - Position vector of A

$$= (2\hat{i} + 7\hat{j} + 8\hat{k}) - (2\hat{i} + 3\hat{j} + 5\hat{k})$$

$$= 2\hat{i} + 7\hat{j} + 8\hat{k} - 2\hat{i} - 3\hat{j} - 5\hat{k}$$

$$\overrightarrow{AC} = 4\hat{j} + 3\hat{k}$$

$$\text{Area of triangle} = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$$= \hat{i} (6 - 12) - \hat{j} (3 - 0) + \hat{k} (4 - 0)$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-6)^2 + (-3)^2 + (4)^2}$$

$$= \sqrt{36 + 9 + 16}$$

$$= \sqrt{61}$$

$$\text{Area of triangle} = \frac{1}{2} \sqrt{61} \text{ Sq. unit}$$

Vector or Cross Product Ex 25.1 Q30

$$\text{Let } \vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}.$$

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have:

$$\begin{aligned}\vec{d} \cdot \vec{a} &= 0 \\ \Rightarrow d_1 + 4d_2 + 2d_3 &= 0 \quad \dots(i)\end{aligned}$$

And,

$$\begin{aligned}\vec{d} \cdot \vec{b} &= 0 \\ \Rightarrow 3d_1 - 2d_2 + 7d_3 &= 0 \quad \dots(ii)\end{aligned}$$

Also, it is given that:

$$\begin{aligned}\vec{c} \cdot \vec{d} &= 15 \\ \Rightarrow 2d_1 - d_2 + 4d_3 &= 15 \quad \dots(iii)\end{aligned}$$

On solving (i), (ii), and (iii), we get:

$$\begin{aligned}d_1 &= \frac{160}{3}, d_2 = -\frac{5}{3} \text{ and } d_3 = -\frac{70}{3} \\ \therefore \vec{d} &= \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})\end{aligned}$$

Hence, the required vector is $\frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$.

Vector or Cross Product Ex 25.1 Q31

$$\vec{a} \cdot \vec{b} = 0$$

Then,

(i) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \perp \vec{b}$ (in case $\vec{a}$ and $\vec{b}$ are non-zero)

$$\vec{a} \times \vec{b} = 0$$

(ii) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \parallel \vec{b}$ (in case $\vec{a}$ and $\vec{b}$ are non-zero)

But, $\vec{a}$ and $\vec{b}$ cannot be perpendicular and parallel simultaneously.

Hence, $|\vec{a}| = 0$ or $|\vec{b}| = 0$.

Vector or Cross Product Ex 25.1 Q32

Take any parallel non-zero vectors so that $\vec{a} \times \vec{b} = \vec{0}$.

$$\text{Let } \vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}.$$

Then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{i}(24 - 24) - \hat{j}(16 - 16) + \hat{k}(12 - 12) = 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

It can now be observed that:

$$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$\therefore \vec{a} \neq \vec{0}$$

$$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$$

$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Vector or Cross Product Ex 25.1 Q33

We have,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$(\vec{b} + \vec{c}) = (b_1 + c_1)\hat{i} + (b_2 + c_2)\hat{j} + (b_3 + c_3)\hat{k}$$

$$\text{Now, } \vec{a} \times (\vec{b} + \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$$

$$= \hat{i}[a_2(b_3 + c_3) - a_3(b_2 + c_2)] - \hat{j}[a_1(b_3 + c_3) - a_3(b_1 + c_1)] + \hat{k}[a_1(b_2 + c_2) - a_2(b_1 + c_1)]$$

$$= \hat{i}[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{j}[-a_1b_3 - a_1c_3 + a_3b_1 + a_3c_1] + \hat{k}[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1] \quad \dots(1)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \hat{i}[a_2b_3 - a_3b_2] + \hat{j}[a_1b_3 - a_3b_1] + \hat{k}[a_1b_2 - a_2b_1] \quad (2)$$

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \hat{i}[a_2c_3 - a_3c_2] + \hat{j}[a_1c_3 - a_3c_1] + \hat{k}[a_1c_2 - a_2c_1] \quad (3)$$

Vector or Cross Product Ex 25.1 Q34

Given that

$$A = (1, 1, 2)$$

$$B = (2, 3, 5)$$

$$C = (1, 5, 5)$$

Position vector of $A = \hat{i} + \hat{j} + 2\hat{k}$

Position vector of $B = 2\hat{i} + 3\hat{j} + 5\hat{k}$

Position vector of $C = \hat{i} + 5\hat{j} + 5\hat{k}$

$\vec{AB} = \text{Position vector of } B - \text{Position vector of } A$

$$= 2\hat{i} + 3\hat{j} + 5\hat{k} - (\hat{i} + \hat{j} + 2\hat{k})$$

$$= \hat{i} + 2\hat{j} + 3\hat{k}$$

$\vec{AC} = \text{Position vector of } C - \text{Position vector of } A$

$$= \hat{i} + 5\hat{j} + 5\hat{k} - (\hat{i} + \hat{j} + 2\hat{k})$$

$$= 4\hat{j} + 3\hat{k}$$

Area of triangle = $\frac{1}{2} |\vec{AB} \times \vec{AC}|$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$$= \hat{i}(6 - 12) - \hat{j}(3 - 0) + \hat{k}(4 - 0)$$

$$= -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{(-6)^2 + (-3)^2 + 4^2}$$

$$= \sqrt{36 + 9 + 16}$$

$$= \sqrt{61}$$

Area of the triangle = $\frac{1}{2} \sqrt{61}$ Sq. unit

Vector or Cross Product Ex 25.1 Q35

Let

$$\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$$

The unit vector parallel to one of its diagonals is $\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$.

Now

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix} \\ &= \hat{i}(12+10) - \hat{j}(-6-5) + \hat{k}(-4+4) \\ &= 22\hat{i} + 11\hat{j} \\ &= 11(2\hat{i} + \hat{j})\end{aligned}$$

$$\begin{aligned}|\vec{a} \times \vec{b}| &= 11\sqrt{2^2 + 1^2} \\ &= 11\sqrt{5}\end{aligned}$$

Therefore

$$\begin{aligned}\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} &= \frac{11(2\hat{i} + \hat{j})}{11\sqrt{5}} \\ &= \frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})\end{aligned}$$

The unit vector parallel to one of its diagonals is $\frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})$.

Again, the area of the parallelogram is $|\vec{a} \times \vec{b}| = 11\sqrt{5}$ Sq. unit