Sample Question Paper - 12
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

1. In an AP, the first term is 2, the last term is 29 and the sum of all the terms is 155. Find the common difference. **[2]**

2. Solve: $\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$ **[2]**

3. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC **[2]**

4. Two cubes each of 10 cm edge are joined end to end. find the surface area of the resulting cuboid. **[2]**

5. The arithmetic mean of the following data is 25, find the value of k. **[2]**

x_i	5	15	25	35	45
f_i	3	k	3	6	2

6. If the equation $(1 + m^2) x^2 + 2 mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2 (1 + m^2)$. **[2]**

OR

Solve the quadratic equation by factorization:

$x^2 + 2ab = (2a + b)x$

Section B

7. Compute the median for each of the following data: **[3]**

Marks	No. of students
Less than 10	0
Less than 30	10
Less than 50	25

Less than 70	43
Less than 90	65
Less than 110	87
Less than 130	96
Less than 150	100

8. Construct tangents to a circle of radius 3 cm from a point on concentric circle of radius 5 cm and measure its length. **[3]**

9. If the mean of the following frequency distribution is 18, find the missing frequency. **[3]**

Class interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

10. If the angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary, find the height of the tower. **[3]**

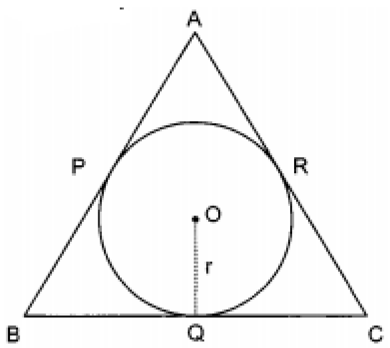
OR

Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the lighthouse are in the same straight line. The angles of depression of two ships are observed from the top of the lighthouse are 60° and 45° respectively. If the height of the lighthouse is 200 m, find the distance between the two ships. (Use $\sqrt{3}$ = 1.73)

Section C

11. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/ hr, in how much time will the tank be filled ? **[4]**

12. In figure the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P,Q and R respectively. **[4]**



Prove that:

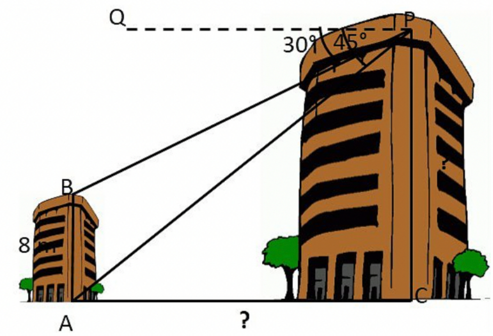
i. AB + CQ = AC + BQ
ii. $Area(\Delta ABC) = \frac{1}{2}(\text{Perimeter of } \Delta ABC) \times r$

OR

A is a point at a distance 13 cm from the centre 'O' of a circle of radius 5 cm. AP and AQ are the tangents to circle at P and Q. If a tangent BC is drawn at point R lying on minor arc PQ to intersect AP at B AQ at C. Find the perimeter of ΔABC.

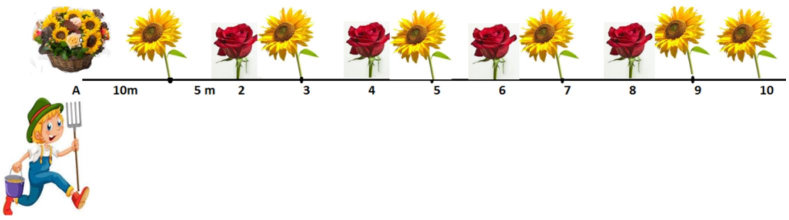
13. Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying **[4]**

to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30^{o} and 45^{o}, respectively.



i. Now help Vinod and Basant to find the height of the multi-storeyed building.
ii. Also, find he distance between two buildings.

14. In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure. **[4]**



The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

Now answer the following questions:

i. Find the distance covered by Dinesh to plant the first 5 plants and return to basket. **(2)**
ii. Find the distance covered by Dinesh to plant all 10 plants and return to basket. **(1)**
iii. If the speed of Dinesh is 10 m/min and he takes15 minutes to plant a flower plant then find the total time taken by Dinesh to plant 10 plants. **(1)**

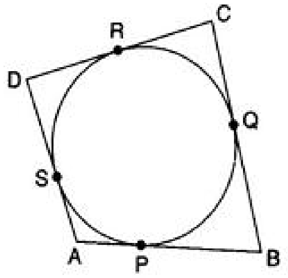
Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section A

1. In the given AP
   a=2, l =29 and S_n=155
   We know that
   Sum of n terms = $S_n=\frac{n}{2}$(a+l)=155
   $\therefore \frac{n}{2}$(2+29)=155
   $n = \frac{155\times2}{31} = 10$
   Now l=a+(n-1)d
   or 29=2+(10-1)d
   29=2+9d
   29-2=9d
   27 = 9d
   d = 3
2. We have, $\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$
   (x + 3)(2x - 3) = (x + 2)(3x - 7)
   $\Rightarrow x(2x-3)+3(2x-3) = x(3x-7)+2(3x-7)$
   $\Rightarrow 2x^2-3x+6x-9 = 3x^2-7x+6x-14$
   $\Rightarrow 2x^2+3x-9 = 3x^2-x-14$
   $\Rightarrow x^2 - 4x - 5 = 0$
   $\Rightarrow x^2 - 5x + x - 5 = 0$
   $\Rightarrow$ x(x - 5) + (x - 5) = 0
   $\Rightarrow$ (x - 5) (x + 1) = 0
   $\Rightarrow$ x - 5 = 0 or x + 1 = 0
   $\Rightarrow$ x = 5 or x = -1.
3. We know that the tangent segments from an external point to a circle are equal



   $\therefore$ AP = AS ....(1)
   BP = BQ ....(2)
   CR = CQ ....(3)
   DR = DS ....(4)
   Adding (1), (2), (3) and (4), we get
   (AP + BP) + (CR + DR) = (AS + BQ + CQ + DS)
   $\Rightarrow$ AB + CD = (AS + DS) + (BQ + CQ)
   $\Rightarrow$ AB + CD = AD + BC
4. If two cubes are joined end to end, we get a cuboid such that
   l = Length of the resulting cuboid = 10 cm +10 cm = 20 cm
   b = Breadth of the resulting cuboid = 10 cm
   h = Height of the resulting cuboid = 10 cm
   $\therefore$ Surface area of the cuboid = 2 (lb + bh + Ih)
   $\Rightarrow$ Surface area of the cuboid = $2(20\times10+10\times10+20\times10)\text{cm}^2$ = 1000 cm^2

5.

X	f	fx
5	3	15
15	k	15k
25	3	75
35	6	210
45	2	90
	N = k + 14	**Sum = 15k + 390**

Given mean = 25

$\frac{Sum}{N} = 25$

15k + 390 = 25k + 350

25k – 15k = 40

10k = 40

k = 4

6. The given quadratic equation is

$\left(1+m^2\right)x^2+2mcx+\left(c^2-a^2\right)=0$

Here, A = 1 + m^2, B = 2mc, C = c^2 - a^2

Therefore, discriminant = B^2- 4AC

$=(2\mathrm{mc})^2-4\left(1+\mathrm{m}^2\right)\left(\mathrm{c}^2-\mathrm{a}^2\right)$

$=4\mathrm{m}^2\mathrm{c}^2-4\left(\mathrm{c}^2-\mathrm{a}^2+\mathrm{m}^2\mathrm{c}^2-\mathrm{m}^2\mathrm{a}^2\right)$

$=4\mathrm{m}^2\mathrm{c}^2-4\mathrm{c}^2+4\mathrm{a}^2-4\mathrm{m}^2\mathrm{c}^2+4\mathrm{m}^2\mathrm{a}^2$

$=-4c^2+4a^2+4m^2a^2$

If the given quadratic equation has equal roots, then

B^2- 4AC = 0

$\Rightarrow -4c^2+4a^2+4m^2a^2=0$

$\Rightarrow 4c^2=4a^2+4m^2a^2$

$\Rightarrow 4c^2=4a^2\left(1+m^2\right)$

$\Rightarrow c^2=a^2\left(1+m^2\right)$............Dividing throughout by 4

OR

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

Here we have, $x^2+2ab=(2a+b)x$

$\Rightarrow$ x^2 – (2a + b)x + 2ab = 0

$\Rightarrow$ x^2 – 2ax – bx + 2ab = 0

$\Rightarrow$ x(x – 2a) – b(x – 2a) = 0

$\Rightarrow$ (x – b)(x – 2a) = 0

$\Rightarrow$ x = b, 2a

Section B

7.

Marks	No. of students	class interval	Frequency	Cumulative frequency
less than 10	0	0-10	0	0
less than 30	10	10-30	10	10
less than 50	25	30-50	15	25
less than 70	43	50-70	18	43(F)
less than 90	65	70-90	22(f)	65
less than 110	87	90-110	22	87
less than 130	96	110-130	9	96

less than 150	100	130-150	4	100
			N = 100	

We have

N = 100

$\therefore \frac{N}{2} = \frac{100}{2}$ = 50

The cumulative frequency just greater than $\frac{N}{2}$ is 65 then median class is 70 - 90 such that

l = 70, f = 22, F = 43, h = 90 - 70 =20

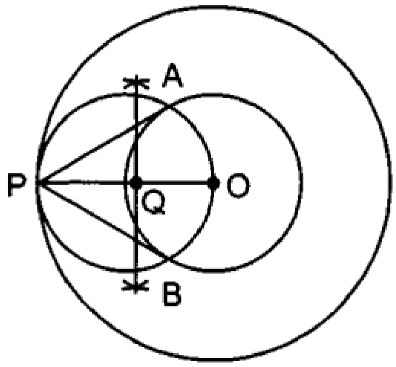
$\therefore$ Median = l + $\frac{\frac{N}{2}-F}{f} \times$ h

= 70 + $\frac{50-43}{22} \times 20$

= 70 + $\frac{7 \times 20}{22}$

= 70 + 6.36

= 76.36

8. 

Steps of Construction:

i. With O as centre a circle of radius 3 cm is drawn.
ii. With same centre O another circle of radius 5 cm is drawn.
iii. A point P is taken on outer circle and OP is joined.
iv. Perpendicular bisector of OP is drawn intersecting OP at Q.
v. With Q as centre and OQ as radius a circle is drawn intersecting the smaller circle at A and B.
vi. PA and PB is joined.
vii. PA and PB are the required tangents.
Length of tangent = 4 cm.

9.

Class interval	**Frequency f_i**	**Mid-value x_i**	**$f_i x_i$**
11-13	3	12	36
13-15	6	14	84
15-17	9	16	144
17-19	13	18	234
19-21	f	20	20f
21-23	5	22	110
23-25	4	24	96
	$\sum f_i$ = 40 + f		$\sum f_i x_i$ = 704 + 20 f

let the missing frequency is 'f '.

we know that , Mean = $\frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow$ 18 = $\frac{704+20f}{40+f}$

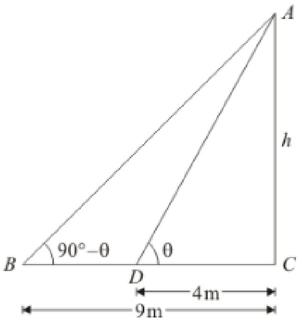
$\Rightarrow$ 18(40 + f) = 704 + 20f

$\Rightarrow$ 720 + 18f = 704 + 20f

$\Rightarrow$ 2f = 16

$\Rightarrow$ f = 8

10. Let AC be the height of tower is h meters.



Given that: angle of elevation are $\angle B = 90^0 - \theta$ and $\angle D = \theta$ and also CD = 4 m and BC = 9 m.

Here we have to find height of tower.

So we use trigonometric ratios.

In a triangle ADC,

$\tan\theta = \frac{h}{4}$

Again in a triangle ABC,

$\Rightarrow \tan(90^\circ - \theta) = \frac{AC}{BC}$

$\Rightarrow \cot\theta = \frac{h}{9}$

$\Rightarrow \frac{1}{\tan\theta} = \frac{h}{9}$

Put $\tan\theta = \frac{h}{4}$

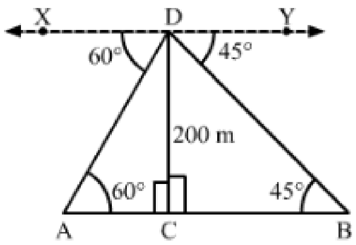
$\Rightarrow \frac{4}{h} = \frac{h}{9}$

$\Rightarrow h^2 = 36$

$\Rightarrow$ h = 6

Hence height of tower is 6 meters.

OR



Let CD be the lighthouse and A and B be the positions of the two ships.

Height of the lighthouse, CD = 200 m

Now,

$\angle CAD = \angle ADX = 60^o$ (Alternate angles)

$\angle CBD = \angle BDY = 45^o$ (Alternate angles)

In right $\triangle$ACD,

$\tan 60^\circ = \frac{\text{CD}}{\text{AC}}$

$\Rightarrow \sqrt{3} = \frac{200}{\text{AC}}$

$\Rightarrow \text{AC} = \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$ m

In right $\triangle$BCD,

$\tan 45^o = \frac{\text{CD}}{\text{BC}}$

$\Rightarrow 1 = \frac{200}{BC}$

$\Rightarrow$ BC = 200 m

$\therefore$ Distance between the two ships, AB = BC + AC

$= 200 + \frac{200\sqrt{3}}{3}$

$= 200 + \frac{200 \times 1.73}{3}$
= 200 + 115.33
= 315.33 m (approx)
Hence, the distance between the two ships is approximately 315.33 m.

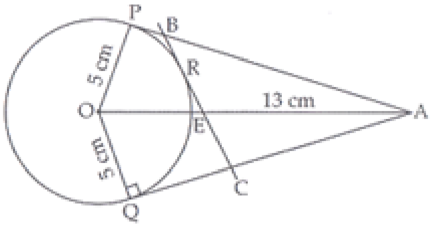
Section C

11. According to question,A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep.
Diameter of pipe = 20 cm.
$\therefore$ Radius of pipe $= \frac{20}{2} = 10\text{cm}$
$= 0.10m$
Diameter of tank = 10 m
$\therefore$ radius of the tank $= \frac{20}{2} = 5\text{m}$
Depth of tank = 2m
Volume of tank $= \pi r^2 h$
$= \pi \times 5 \times 5 \times 2$
$= 50\pi$
Speed of the water 3 km/ hr.
$= \frac{3000}{60} = 50\text{m/min}$
Volume of water supplied in one minute
$= \pi r^2 h$
$= \pi \times 0.10 \times 0.10 \times 50$
Let time taken by $t = \frac{50\pi}{\pi \times .10 \times 0.10 \times 50} = 100$
Hence time taken to fill the tank = 100 minutes.

12. As we know that the lengths of tangents to a circle are equal which are drawn from an external point.
Therefore,AP = AR, BP = BQ and CQ = CR
Proof of Part (i):
AB + CQ = AP + PB + CQ
= AR + BQ + CQ [$\because$AP = AR and PB = BQ]
= (AR + CR) + BQ [$\because$CQ = CR]
= AC + BQ [$\because$AR + CR = AC]
Hence proved.
Proof of Part (ii):
Area (ΔABC)= Area (ΔOBC)+ Area (ΔOAB)+ Area (ΔOAC)
$= \frac{1}{2}(BC \times OQ) + \frac{1}{2}(AB \times OP) + \frac{1}{2}(AC \times OR)$
$= \frac{1}{2}(BC \times r) + \frac{1}{2}(AB \times r) + \frac{1}{2}(AC \times r)$
$= \frac{1}{2}(BC + AB + AC) \times r$
$= \frac{1}{2}(\text{ Perimeter of } \Delta ABC) \times r$
Hence proved.

OR

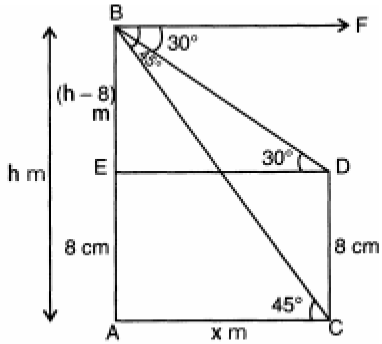
OA = 13 cm
OP = OQ = 5 cm
OP and PA are radius and tangent respectively at contact point P.
Therefore, $\angle$OPA = 90°



In right angled ΔOPA by Pythagoras theorem
$PA^2 = OA^2 - OP^2 = 13^2 - 5^2 = 169 - 25 = 144$
$\Rightarrow$ PA = 12 cm

Points A, B and C are exterior to the circle and tangents drawn from an external point to a circle are equal so
PA = QA
BP = BR
CR = CQ
Perimeter of $\triangle$ABC = AB + BC + AC
= AB + BR + RC + AC [From figure]
= AB + BP + CQ + AC = AP + AQ
= AP + AP = 2AP = 2 $\times$ 12 = 24 cm
So, the perimeter of $\triangle$ABC = 24 cm.

13. Let h is height of big building,here as per the diagram.
AE = CD = 8 m (Given)
BE = AB-AE = (h - 8) m
Let AC = DE = x
Also, $\angle FBD = \angle BDE = 30^{\circ}$
$\angle FBC = \angle BCA = 45^{\circ}$



In $\triangle$ACB, $\angle A = 90^{\circ}$
$\tan 45^{\circ} = \frac{AB}{AC}$
$\Rightarrow$ x = h, ...(i)
In $\triangle$BDE, $\angle E = 90^{\circ}$
$\tan 30^{\circ} = \frac{BE}{ED}$
$\Rightarrow \quad x = \sqrt{3}(h-8)$.(ii)
From (i) and (ii), we get
$h = \sqrt{3}h - 8\sqrt{3}$
h(√3 - 1) = 8√3
h = $\frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$
= $\frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84) = 18.92m$
Hence height of the multistory building is 18.92 m and the distance between two buildings is 18.92 m.

14. i. The distance covered by Dinesh to pick up the first flower plant and the second flower plant,
= 2 $\times$ 10 + 2 $\times$ (10 + 5) = 20 + 30
therefore, the distance covered for planting the first 5 plants
= 20 + 30 + 40 + ... 5 terms
This is in AP where the first term a = 20
and common difference d = 30 - 20 = 10
We know that: $\mathrm{S_n} = \frac{\mathrm{n}}{2}[2\mathrm{a} + (\mathrm{n}-1)\mathrm{d}]$
so, the sum of 5 terms
$\mathrm{S}_5 = \frac{5}{2}[2 \times 20 + 4 \times 10] = \frac{5}{2} \times 80 = 200 \mathrm{~m}$
hence, Dinesh will cover 200 m to plant the first 5 plants.
ii. As a = 20 ,d = 10 and here n = 10
so, $\mathrm{S}_{10} = \frac{10}{2}[2 \times 20 + 9 \times 10] = 5 \times 130 = 650 \mathrm{~m}$
hence Ramesh will cover 650 m to plant all 10 plants.
iii. Total distance covered by Ramesh = 650 m
$\mathrm{Time} = \frac{\mathrm{distance}}{\mathrm{speed}} = \frac{650}{10} = 65 \text{ minutes}$

Time taken to plant all 10 plants=15 $\times$ 10 = 150 minutes
Total time = 65 + 150 = 215 minutes = 3 hrs 35 minutes