

CBSE Class 10th Mathematics
Standard Sample Paper- 09

Maximum Marks:

Time Allowed: 3 hours

General Instructions:

- i. All the questions are compulsory.
 - ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
 - iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
 - iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
 - v. Use of calculators is not permitted.
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Section A

1. For every natural number 'n', 6^n always ends with the digit
 - a. 4
 - b. 8
 - c. 6
 - d. 0
2. HCF of two numbers is 113, their LCM is 56952. If one number is 904, the second number is

a. 7791

b. 7911

c. 7719

d. 7119

3. The mean of 'n' observations is \bar{x} . If the first item is increased by 1, second by 2 and so on, then the new mean is

a. $\bar{x} - \frac{n-1}{2}$

b. $\bar{x} - \frac{n+1}{2}$

c. $\bar{x} + \frac{n+1}{2}$

d. \bar{x}

4. If one root of the equation $ax^2 + bx + c = 0$ is three times the other, then $b^2 : ac =$

a. 16 : 3

b. 16 : 1

c. 3 : 16

d. 3 : 1

5. $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

a. $\sin 45^\circ$

b. 0

c. $\cos 45^\circ$

d. $\tan 45^\circ$

6. If $\sin A + 2 \cos A = 1$, then the value of $2 \sin A - \cos A$ is

a. 2

-
- b. 0
- c. -2
- d. 1
7. The _____ is the line drawn from the eye of an observer to the point in the object viewed by the observer.
- a. Horizontal line
- b. line of sight
- c. None of these
- d. Vertical line
8. The ratio in which the point $(1, 3)$ divides the line segment joining the points $(-1, 7)$ and $(4, -3)$ is
- a. $2 : 3$
- b. $7 : 2$
- c. $3 : 2$
- d. $2 : 7$
9. The triangle whose vertices are $(-3, 0)$, $(1, -3)$ and $(4, 1)$ is _____ triangle.
- a. Obtuse triangle
- b. equilateral
- c. right angled isosceles
- d. scalene
10. The king, queen and jack of clubs are removed from a deck of 52 cards and the remaining cards are shuffled. A card is drawn from the remaining cards. The probability of getting a king is

a. $\frac{4}{52}$

b. $\frac{3}{52}$

c. $\frac{3}{49}$

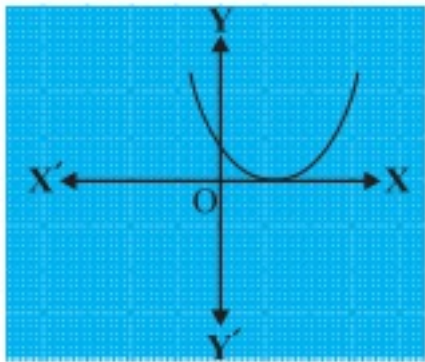
d. $\frac{4}{49}$

11. Fill in the blanks:

The base area of the cylinder is 80 sq.cm. If its height is 5cm, then its volume is _____.

12. Fill in the blanks:

The graph of $y = p(x)$ is given in the fig. below, for some polynomial $p(x)$. The number of zeroes of $p(x)$ is _____.



OR

Fill in the blanks:

If ' $x + a$ ' is a factor (zero) of the polynomial $2x^2 + 2ax + 5x + 10$, the value of ' a ' is _____.

13. Fill in the blanks:

The areas of two similar triangles are respectively 25cm^2 and 81cm^2 . Then the ratio of their corresponding sides is _____.

14. Fill in the blanks:

n^{th} term of an AP from the end can be found by the formula _____, where ' l ' is the last term.

15. Fill in the blanks:

Three points are said to be collinear, if area of triangle formed by these points is _____.

16. The HCF of two numbers is 16 and their product is 3072. Find their LCM.

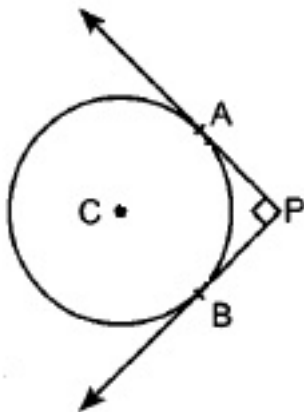
17. The sides of certain triangle are $(a - 1)\text{cm}$, $2\sqrt{a}\text{ cm}$, $(a+1)$ Determine whether the triangle is a right triangle.

18. If they form an AP, find the common difference d and write three more terms. 0.2, 0.22, 0.222, 0.2222,

OR

Find 11^{th} term of the A.P. 10.0, 10.5, 11.0, 11.5,

19. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If $PA \perp PB$, find the length of each tangent.



20. Find the value of k for which the quadratic equation $x^2 - kx + 4 = 0$ has equal roots.

Section B

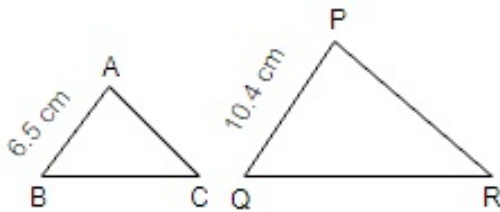
21. A bag contains 5 black, 7 red and 3 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is:

- i. red
- ii. black or white
- iii. not black

22. Find the values of k for which the following equation have real roots:

$$4x^2 + kx + 3 = 0$$

23. If $\triangle ABC \sim \triangle PQR$, $AB = 6.5\text{cm}$, $PQ = 10.4\text{cm}$ and perimeter of $\triangle ABC = 60\text{cm}$, find the perimeter of $\triangle PQR$.

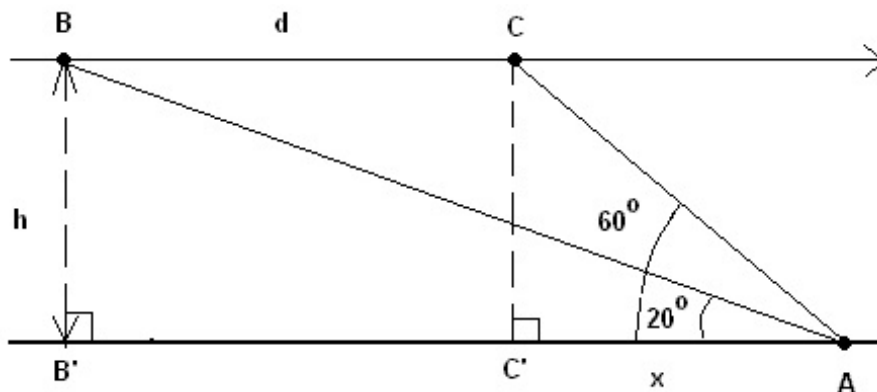


OR

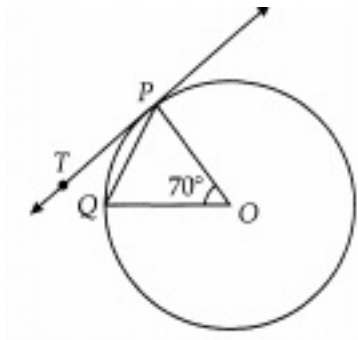
In a quadrilateral $ABCD$, $\angle B = 90^\circ$, $AD^2 = AB^2 + BC^2 + CD^2$, Prove that $\angle ACD = 90^\circ$

24. An airplane is approaching point A along a straight line and at a constant altitude h .

At 10:00 am, the angle of elevation of the airplane is 20° and at 10:01 am, it is 60° .

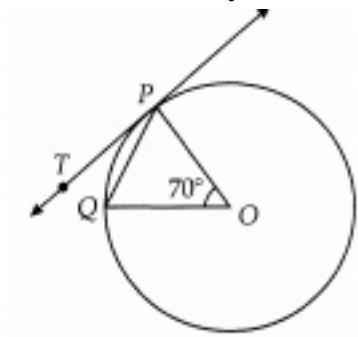


- i. What is the distance ' d ' is covered by airplane from 10:00 am to 10:01 am if the speed of the airplane is constant and equal to 600 miles/hour?
 - ii. What is the altitude ' h ' of the airplane? (round answer to 2 decimal places).
25. In fig., O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P . Find $\angle TPQ$?

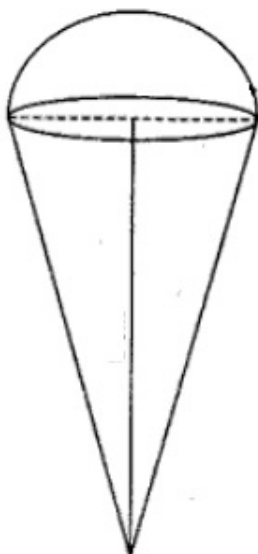


OR

In fig., O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P. Find $\angle TPQ$?



26. An 'ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day a child came to his shop and purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm.



By reading the above-given information, find the following:

- i. The volume of the ice-cream without hemispherical end.
- ii. The volume of the ice-cream with a hemispherical end.

Section C

27. What is the smallest number that, when divided by 35, 56 and 91 leaves remainders of 7 in each case?

OR

Prove $\frac{1}{2+\sqrt{3}}$ is an irrational number.

28. Split 207 into three parts such that these are in A.P. and the product of the two smaller parts is 4623.
29. Find the values of a and b for which $2x + 3y = 7$, $2ax + (a + b)y = 28$ has an infinite number of solutions.

OR

Find the values of a and b for which the following system of linear equations has infinite number of solutions:

$$2x - 3y = 7$$

$$(a + b)x - (a + b - 3)y = 4a + b$$

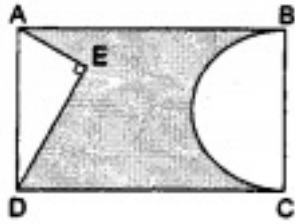
30. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$. if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.
31. Find the co-ordinates of the points of trisection of the line segment joining the points (3, - 2) and (- 3, - 4).
32. Prove the trigonometric identity:
if $\sin \theta + \cos \theta = x$, prove that $\sin^6 \theta + \cos^6 \theta = \frac{4-3(x^2-1)^2}{4}$

OR

If $\sec \theta = \frac{25}{7}$, find the values of $\tan \theta$ and $\operatorname{cosec} \theta$.

33. In the given figure, ABCD is a rectangle with AB = 80 cm and BC = 70 cm, $\angle AED = 90^\circ$

and $DE = 42$ cm. A semicircle is drawn, taking BC as diameter. Find the area of the shaded region.



34. Find the mean of the following frequency distribution:

Class-interval	0-10	10-20	20-30	30-40	40-50
No. of workers	7	10	15	8	10

Section D

35. Draw a circle of radius 4 cm. Take a point P outside the circle. Without using the centre of the circle, draw two tangents to the circle from point P .

OR

Construct a rhombus $ABCD$ in which $AB = 4$ cm and $\angle ABC = 60^\circ$. Divide it into two triangles ABC and ADC . Construct the triangle $AB'C'$ similar to $\triangle ABC$ with scale factor $\frac{2}{3}$. Draw a line segment CD' parallel to CD , where D' lies on AD . Is $AB'C'D'$ a rhombus? Give reasons.

36. Prove that the line segments joining the mid-points of the sides of a triangle form four triangles, each of which is similar to the original triangle.
37. Solve the following system of equations graphically:
 $2x - 3y + 13 = 0$ and $3x - 2y + 12 = 0$.

OR

The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and the denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

38. An iron pillar has some part in the form of a right circular cylinder and the remaining

in the form of a right circular cone. The radius of the base of each of the cone and the cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if 1 cubic centimetre of iron weighs 7.5 g.

OR

How many spherical lead shots each of diameter 4.2 cm can be obtained from a solid rectangular lead piece with dimensions 66 cm, 42 cm and 21 cm?

39. A tree is broken by the wind. The top struck the ground at an angle of 30° and at a distance of 30 metres from the root. Find the whole height of the tree.
40. The following table gives production yield per hectare of wheat of 100 farms of a village:

Production yield(kg/ha)	40 - 45	45 - 50	50 - 55	55 - 60	60 - 65	65 - 70
Number of farms	4	6	16	20	30	24

Change the distribution to a 'more than' type distribution and draw its Ogive.

CBSE Class 10th Mathematics Standard
Sample Paper - 05

Solution

Section A

1. (c) 6

Explanation:

Let us assume for some integer k we have $6^k = 10x + 6$

$$\therefore 6^{k+1} = 6 \cdot 6^k = 6(10x + 6)$$

$$\Rightarrow 6^{k+1} = 60x + 36$$

$$\Rightarrow 6^{k+1} = 60x + 30 + 6$$

$$\Rightarrow 6^{k+1} = 10(6x + 3) + 6$$

\therefore If 6^k ends with 6,

then 6^{k+1} ends with 6 for all natural numbers.

2. (d) 7119

Explanation:

$\text{LCM} \times \text{HCF} = \text{Product of two numbers}$

$$56952 \times 113 = 904 \times \text{second number}$$

$$\frac{56952 \times 113}{904} = \text{second number}$$

Therefore, second number = 7119

3. (c) $\overline{x} + \frac{n+1}{2}$

Explanation:

Let terms be $x_1, x_2, x_3, \dots, x_n$.

\therefore Mean (\bar{x})

$$= \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

$$\Rightarrow x_1 + x_2 + x_3 + \dots + x_n = n \cdot \bar{x}$$

New observations, are $x_1 + 1, x_2 + 2, x_3 + 3, \dots, x_n + n$.

$$\therefore \text{New Mean} = \frac{x_1 + 1 + x_2 + 2 + x_3 + 3 + \dots + x_n + n}{n}$$

$$= \frac{n \cdot \bar{x} + \frac{n(n+1)}{2}}{n}$$

$$= \bar{x} + \frac{n+1}{2}$$

4. (a) 16 : 3

Explanation:

Let one root be α then other root will be 3α

\therefore Sum of the roots

$$\Rightarrow \alpha + 3\alpha = \frac{-b}{a}$$

$$\Rightarrow 4\alpha = \frac{-b}{a}$$

$$\Rightarrow 16\alpha^2 = \frac{b^2}{a^2} \dots\dots(i)$$

$$\text{And Product of the roots} \Rightarrow \alpha(3\alpha) = \frac{c}{a} \Rightarrow 3\alpha^2 = \frac{c}{a} \dots\dots(ii)$$

Equating the coefficients of α^2 and subtracting eq. (ii) from eq. (i), we get

$$\frac{3b^2}{a^2} - \frac{16c}{a} = 0$$

$$\Rightarrow \frac{3b^2 - 16ac}{a^2}$$

$$\Rightarrow 3b^2 = 16ac$$

$$\Rightarrow \frac{b^2}{ac} = \frac{16}{3}$$

5. (b) 0

Explanation:

$$\text{Given: } \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$$

$$= \frac{1 - (1)^2}{1 + (1)^2} = \frac{1 - 1}{1 + 1} = 0/2$$

$$= 0$$

6. (a) 2

Explanation:

Given: $\sin A + 2 \cos A = 1$

Squaring both sides, we get

$$\Rightarrow \sin^2 A + 4\cos^2 A + 4\sin A \cos A = 1$$

$$\Rightarrow 1 - \cos^2 A + 4(1 - \sin^2 A) + 4\sin A \cos A = 1$$

$$\Rightarrow 1 - \cos^2 A + 4 - 4\sin^2 A + 4\sin A \cos A = 1$$

$$\Rightarrow \cos^2 A + 4\sin^2 A - 4\sin A \cos A = 4$$

$$\Rightarrow (2\sin A - \cos A)^2 = 4$$

\Rightarrow taking square root of both sides

$$\Rightarrow 2\sin A - \cos A = 2$$

7. (b) line of sight

Explanation:

The line of sight is the imaginary line drawn from the eye of an observer to the point in the object viewed by the observer. The angle between the line of sight and the ground is called angle of elevation

8. (a) 2 : 3

Explanation:

Given: $(x, y) = (1, 3), (x_1, y_1) = (-6, 10), (x_2, y_2) = (3, -8)$

Let $m_1 : m_2 = k : 1$

$$\therefore x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \Rightarrow$$

$$1 = \frac{k \times 4 + 1 \times (-6)}{k + 1} \Rightarrow$$

$$k + 1 = 4k - 6 \Rightarrow k = \frac{7}{3}$$

Therefore, the required ratio is 2 : 3.

9. (c) right angled isosceles

Explanation:

Let A $(-3, 0)$, B $(1, -3)$ and C $(4, 1)$ are the vertices of a triangle ABC.

$$\therefore AB = \sqrt{(1 + 3)^2 + (-3 - 0)^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ units}$$

$$BC = \sqrt{(4 - 1)^2 + (1 + 3)^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units}$$

$$CA = \sqrt{(-3 - 4)^2 + (0 - 1)^2} = \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2} \text{ units}$$

Now, check if $AC^2 = AB^2 + BC^2$

$$\Rightarrow (5\sqrt{2})^2 = (5)^2 + (5)^2$$

$$\Rightarrow 50 = 50$$

Therefore, $\triangle ABC$ is a right-angled triangle. and also $AB = BC = 5$ units

Therefore triangle ABC is a right-angled isosceles triangle

10. (c) $\frac{3}{49}$

Explanation:

K, Q, J of clubs i.e 3 cards are removed, therefore remaining cards = $52 - 3 = 49$

3 kings are left in the pack

Number of possible outcomes = 3

Number of total outcomes = $52 - 3 = 49$

$$\therefore \text{Required Probability} = \frac{3}{49}$$

11. 400 cu.cm

12. 1 OR $a = 2$

13. 5:9

14. $l - (n - 1)d$

15. zero

16. Let the numbers are a and b

We are given here $ab=3072$, $HCF=16$, $LCM=?$

We know that,:

$$LCM \times HCF = ab$$

$$LCM \times 16 = 3072$$

$$\text{So LCM} = \frac{3072}{16}$$

$$= 192$$

17. Since, we know that by the converse of the Pythagoras theorem, if the square of the length of the longest side of the triangle is equal to the sum of the square of other two sides, then the triangle is a right triangle.

Let ABC be a triangle and let $AB = (a-1)$ cm, $BC = 2\sqrt{a}$ cm and $CA = (a+1)$ cm

Then,

$$AB^2 = (a-1)^2 = a^2 + 1 - 2a$$

$$BC^2 = (2\sqrt{a})^2 = 4a$$

$$CA^2 = (a+1)^2 = a^2 + 1 + 2a$$

$$\text{Thus, } AB^2 + BC^2 = a^2 + 1 - 2a + 4a = a^2 + 1 + 2a = (a+1)^2 = CA^2$$

Hence,

$$AB^2 + BC^2 = CA^2$$

Thus, the given triangle ABC is right angled triangle and right angles at B.

18. 0.2, 0.22, 0.222, 0.2222, 0.00

$$a_2 - a_1 = 0.22 - 0.2 = 0.02$$

$$a_3 - a_2 = 0.222 - 0.22 = 0.002$$

As $a_2 - a_1 \neq a_3 - a_2$, the given list of numbers does not form an AP.

OR

A.P = 10.0, 10.5, 11.0, 11.5.....

To find any term we need to have first term and common difference.

Now, here first term(a) = 10.0

Common difference (d) = 10.5 - 10.0 = 0.5

We have,

$$a_n = a + (n-1)d$$

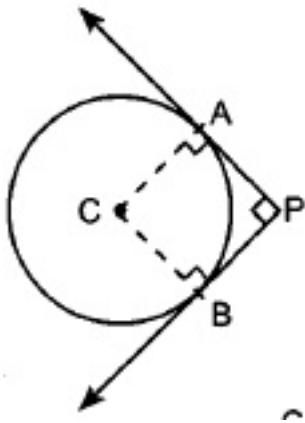
$$\Rightarrow a_{11} = 10 + (11-1) \times 0.5$$

$$= 10 + 10 \times 0.5$$

$$= 10 + 5$$

$$= 15$$

19. PA and PB are two tangents drawn from an external point P to a circle.



$$CA \perp AP$$

$$CB \perp BP$$

$$PA \perp PB$$

\therefore BPAC is a square.

$$\Rightarrow AP = PB = BC = 4cm$$

20. Given quadratic equation $x^2 - kx + 4 = 0$

$$\text{So, } a = 1, b = -k, c = 4.$$

If quadratic equation has equal root.

$$\text{So } D = 0$$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow (-k)^2 - 4(1)(4) = 0$$

$$\Rightarrow k^2 - 16 = 0$$

$$\Rightarrow k^2 = 16$$

$$\Rightarrow k = 4 \text{ (or) } -4.$$

Section B

21. Number of Black balls = 5

$$\text{Red balls} = 7$$

$$\text{White balls} = 3$$

$$\text{Total balls} = 5 + 7 + 3 = 15$$

$$\text{i. } P(\text{drawing a red ball}) = \frac{7}{15}$$

$$\text{ii. } P(\text{drawing black or white ball}) = \frac{5+3}{15} = \frac{8}{15}$$

$$\text{iii. } P(\text{drawing a ball which is not black}) = 1 - P(\text{drawing a black ball})$$

$$= 1 - \frac{5}{15}$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

22. $4x^2 + kx + 3 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 4, b = k, c = 3$$

According to the question,

$$k^2 - 4 \times 4 \times 3 = 0$$

$$k^2 - 48 = 0$$

$$k^2 = 48$$

$$k = \pm\sqrt{48}$$

23. Given, $\triangle ABC \sim \triangle PQR$ and $AB = 6.5$ cm, $PQ = 10.4$ cm

Since, $\triangle ABC \sim \triangle PQR$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{6.5}{10.4}$ [\because corresponding sides of similar triangles are proportional]

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{65}{104}$$

$$\Rightarrow AB = \frac{65}{104}PQ, BC = \frac{65}{104}QR, AC = \frac{65}{104}PR$$

Also given, perimeter of $\triangle ABC = 60$ cm

$$\therefore AB + BC + AC = 60$$

$$\Rightarrow \frac{65}{104}PQ + \frac{65}{104}QR + \frac{65}{104}PR = 60$$

$$\Rightarrow \frac{65}{104}(PQ + QR + PR) = 60$$

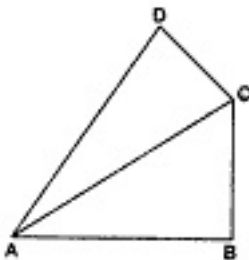
$$\Rightarrow PQ + QR + PR = \frac{60 \times 104}{65}$$

$$\Rightarrow PQ + QR + PR = 96$$
cm

Hence, the perimeter of $\triangle PQR$ is 96 cm.

OR

Given: In a quadrilateral ABCD, $\angle B = 90^\circ$ and $AD^2 = AB^2 + BC^2 + CD^2$



To prove: $\angle ACD = 90^\circ$

Proof: In Right triangle ABC

$$\therefore \angle B = 90^\circ$$

$$\therefore AC^2 = AB^2 + BC^2 \quad (1) \dots\dots [\text{By Pythagoras theorem}]$$

$$\text{But } AD = AB^2 + BC^2 + CD^2 \dots\dots \text{Given}$$

$$= AC^2 + CD^2 \dots\dots \text{From (1)}$$

$$\therefore \angle ACD = 90^\circ \text{ By Converse of Pythagoras theorem}$$

24. i. Time covered 10.00 am to 10.01 am = 1 minute = $\frac{1}{60}$ hour

Given: Speed = 600 miles/hour

$$\text{Thus, distance } d = 600 \times \frac{1}{60} = 10 \text{ miles}$$

ii. Now, $\tan 20^\circ = \frac{BB'}{B'A} = \frac{h}{10+x} \dots\dots \text{eq(1)}$

$$\text{And } \tan 60^\circ = \frac{CC'}{C'A} = \frac{BB'}{C'A} = \frac{h}{x}$$

$$x = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}}$$

Putting the value of x in eq(1), we get,

$$\tan 20^\circ = \frac{h}{10 + \frac{h}{\sqrt{3}}} = \frac{\sqrt{3}h}{10\sqrt{3} + h}$$

$$0.364(10\sqrt{3} + h) = \sqrt{3} h$$

$$6.3 + 0.364 h = 1.732 h$$

$$1.368 h = 6.3$$

$$h = 4.6$$

Thus, the altitude 'h' of the airplane is 4.6 miles.

25. In fig, O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P. We have to find $\angle TPQ$.

$$\angle OPQ = \angle OQP \quad [\text{angles in the same segment are equal}]$$

$$= \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

$$\therefore \angle TPQ = 90^\circ - 55^\circ$$

$$= 35^\circ$$

OR

In fig, O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P. We have to find $\angle TPQ$.

$$\angle OPQ = \angle OQP \text{ [angles in the same segment are equal]}$$

$$= \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

$$\therefore \angle TPQ = 90^\circ - 55^\circ$$

$$= 35^\circ$$

26. For cone, Radius of the base (r)

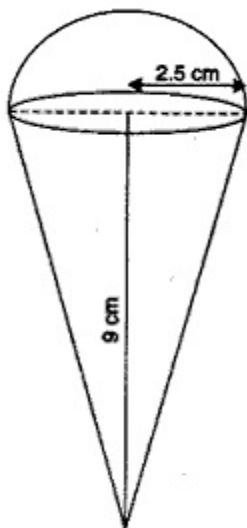
$$= 2.5\text{cm} = \frac{5}{2}\text{cm}$$

$$\text{Height (h)} = 9\text{ cm}$$

$$\therefore \text{Volume} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$$

$$= \frac{825}{14}\text{cm}^3$$



For hemisphere,

$$\text{Radius (r)} = 2.5\text{cm} = \frac{5}{2}\text{cm}$$

$$\therefore \text{Volume} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$$

i. The volume of the ice-cream without hemispherical end = Volume of the cone

$$= \frac{825}{14}\text{cm}^3$$

ii. Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere

$$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$$

$$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$$

Section C

27. $35 = 5 \times 7$

$$56 = 2^3 \times 7$$

$$91 = 13 \times 7$$

$$\text{L.C.M of } 35, 56 \text{ and } 91 = 2^3 \times 7 \times 5 \times 13 = 3640$$

The smallest number that when divided by 35, 56, 91 leaves a remainder 7 in each case = $3640 + 7 = 3647$.

Hence 3647 is the smallest number that, when divided by 35, 56 and 91 leaves a remainder of 7 in each case.

OR

Let $\frac{1}{2+\sqrt{3}}$ be a rational number.

A rational number can be written in the form of $\frac{p}{q}$ where p,q are integers.

$$\frac{1}{2+\sqrt{3}} = \frac{p}{q}$$
$$\Rightarrow \sqrt{3} = \frac{q-2p}{p}$$

p, q are integers then $\frac{q-2p}{p}$ is a rational number.

Then $\sqrt{3}$ is also a rational number.

But this contradicts the fact as $\sqrt{3}$ is an irrational number.

So, our supposition is false.

Therefore, $\frac{1}{2+\sqrt{3}}$ is an irrational number.

28. Let the four parts be (a - d), a and (a + d).

$$\therefore a - d + a + a + d = 207$$

$$\Rightarrow 3a = 207$$

$$\Rightarrow a = 69$$

According to given information,

$$\Rightarrow (a - d) \times a = 4623$$

$$\Rightarrow (69 - d) \times 69 = 4623$$

$$\Rightarrow 69 - d = 67$$

$$\Rightarrow d = 2$$

Thus, the three parts are a - d, a, a + d i.e., 67, 69, 71.

29. We know that,

if a system of linear equations

$$a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$$

has infinite number of solutions, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Given that,

$$2x + 3y = 7, 2ax + (a + b)y = 28$$

have an infinite number of solutions.

$$\Rightarrow 2x + 3y - 7 = 0, 2ax + (a + b)y - 28 = 0$$

Since, the pair of lines have an infinite number of solutions,

$$\text{so, } \frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$$

$$\Rightarrow \frac{1}{a} = \frac{3}{a+b} = \frac{7}{28}$$

$$a = 4$$

$$\text{and } a + b = 3a \Rightarrow 4 + b = 12 \Rightarrow b = 8$$

Hence, $a = 4$ and $b = 8$.

OR

The given system of equation may be written as,

$$2x - 3y - 7 = 0$$

$$(a + b)x - (a + b - 3)y - (4a + b) = 0$$

The given system of equation is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

$$\text{Where, } a_1 = 2, b_1 = -3, c_1 = -7$$

$$a_2 = (a + b), b_2 = -(a + b - 3), c_2 = -(4a + b)$$

The given system of equation will have infinitely many solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\frac{2}{(a+b)} = \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)}$$

$$\Rightarrow \frac{2}{(a+b)} = \frac{-3}{(a+b-3)}$$

$$\Rightarrow 2(a + b - 3) = 3(a + b)$$

$$\Rightarrow 2a + 2b - 6 = 3a + 3b$$

$$\Rightarrow a + b = -6 \text{ and } 5a - 4b = -21$$

$$\Rightarrow a + b = -6$$

$$\Rightarrow a = -6 - b$$

Also,

$$\Rightarrow \frac{3}{-(a+b-3)} = \frac{7}{(4a+b)}$$

$$\Rightarrow 3(4a + b) = 7(a + b - 3)$$

$$\Rightarrow 12a + 3b = 7a + 7b - 21$$

$$\Rightarrow 5a - 4b = -21$$

Substituting the value of a in $5a - 4b = -21$

$$\Rightarrow 5(-b - 6) - 4b = -21$$

$$\Rightarrow -5b - 30 - 4b = -21$$

$$\Rightarrow 9b = -9$$

$$\Rightarrow b = -1$$

As $a = -6 - b$

$$\Rightarrow a = -6 + 1 = -5$$

Hence the given system of equation will have infinitely many solution if $a = -5$ and $b = -1$.

30. Let $f(x) = (3x^4 + 6x^3 - 2x^2 - 10x - 5)$

Two zeroes of $f(x)$ are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Hence $\left(x - \sqrt{\frac{5}{3}}\right) \left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ is a factor of $f(x)$.

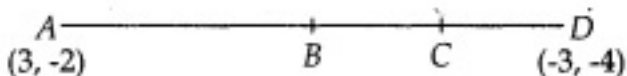
Applying Division Algorithm to find more factors we get:

$$\begin{array}{r}
 3x^2 + 6x + 3 \\
 x^2 - \frac{5}{3} \overline{) 3x^4 + 6x^3 - 2x^2 - 10x - 5} \\
 \underline{\pm 3x^4 \quad \mp 5x^2} \\
 +6x^3 + 3x^2 - 10x - 5 \\
 \underline{\pm 6x^3 \quad \mp 10x} \\
 +3x^2 - 5 \\
 \underline{\pm 3x^2 \quad \mp 5} \\
 0
 \end{array}$$

Hence we get

$$f(x) = (3x^2 - 5)(x^2 + 2x + 1) = (3x^2 - 5)(x + 1)(x + 1)$$

Therefore, other two zeroes of $f(x)$ are -1 and -1 .

31. 

Let the co-ordinates be (x, y) and (x', y') respectively.

$$\frac{AB}{AD} = \frac{1}{3} \text{ or, } \frac{BD}{AD} = \frac{2}{3}$$

or, $AB : BD = 1 : 2$

Using intersection formula

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

$$x = \frac{(1 \times -3) + 2 \times 3}{1+2} \text{ and } y = \frac{1 \times (-4) + 2 \times -2}{1+2}$$

$$x = 1 \text{ and } y = -\frac{8}{3}$$

C is at the mid-point of BD.

So using mid-point formula

$$x' = \frac{1-3}{2}$$

$$\text{and } y' = \frac{-\frac{8}{3} + (-4)}{2}$$

$$\text{Therefore, } x' = -1 \text{ and } y' = -\frac{10}{3}$$

Hence, the co-ordinate of B and C = $\left(1, -\frac{8}{3}\right)$ and $\left(-1, -\frac{10}{3}\right)$.

32. LHS

$$= \sin^6 \theta + \cos^6 \theta$$

$$= (\sin^2 \theta)^3 + (\cos^2 \theta)^3$$

$$= (\sin^2 \theta + \cos^2 \theta)^3 - 3\sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)$$

$$\left[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b) \right]$$

$$= (1)^3 - 3\sin^2 \theta \cos^2 \theta \times 1 \left[\because \sin^2 \theta + \cos^2 \theta = 1 \right]$$

$$= 1 - 3\sin^2 \theta \cos^2 \theta$$

$$\text{RHS} = \frac{4-3(x^2-1)^2}{4}$$

$$= \frac{4-3\{(\sin \theta + \cos \theta)^2 - 1\}^2}{4} \left[\text{given } x = \sin \theta + \cos \theta \right]$$

$$= \frac{4-3\{\sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta - 1\}^2}{4}$$

$$= \frac{4-3\{1+2\sin \theta \cos \theta - 1\}^2}{4} \left[\because \sin^2 \theta + \cos^2 \theta = 1 \right]$$

$$= \frac{4-3(2\sin \theta \cos \theta)^2}{4}$$

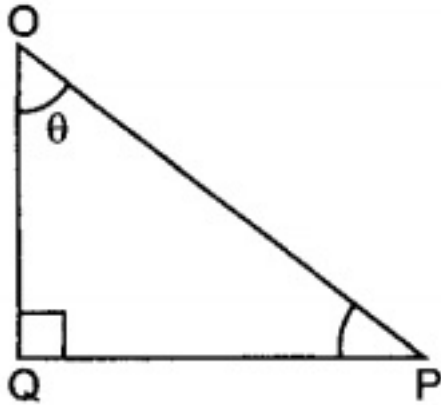
$$= \frac{4-3 \times 4\sin^2 \theta \cos^2 \theta}{4}$$

$$= \frac{4(1-3\sin^2 \theta \cos^2 \theta)}{4}$$

LHS = RHS

Hence proved.

OR



In $\triangle POQ$, right angled at Q.

$$\sec \theta = \frac{OP}{OQ} = \frac{25}{7}$$

So, $OP = 25k$ and $OQ = 7k$

$$PQ^2 = OP^2 - OQ^2$$

$$= (25k)^2 - (7k)^2$$

$$= 625k^2 - 49k^2 = 576k^2$$

$$PQ = \sqrt{576k^2} = 24k$$

$$\tan \theta = \frac{PQ}{OQ} = \frac{24}{7} \text{ and } \operatorname{cosec} \theta = \frac{OP}{PQ} = \frac{25}{24}.$$

33. Length of rectangle ABCD

$$= AB = 80\text{cm}$$

Breadth of rectangle ABCD

$$= BC = 70\text{cm}$$

\therefore Area of rectangle ABCD

$$= AB \times BC$$

$$= 80 \times 70$$

$$= 5600 \text{ cm}^2$$

In right-angled $\triangle AED$,

$$AE^2 = (AD^2 - DE^2)$$

$$= (70^2 - 42^2)$$

$$= (70 + 42) (70 - 42)$$

$$= 112 \times 28$$

$$= 4 \times 28 \times 28$$

$$= 2 \times 28$$

$$= 56 \text{ cm}$$

\therefore Area of $\triangle AED$

$$= \frac{1}{2} \times DE \times AE$$

$$= \frac{1}{2} \times 42 \times 56$$

$$= 1176 \text{ cm}^2$$

$$\text{Area of semi-circle} = \frac{1}{2} \pi \times \left(\frac{70}{2} \right)^2$$

$$= \left\{ \frac{1}{2} \times \frac{22}{7} \times 35 \times 35 \right\} \text{ cm}^2$$

$$= 1925 \text{ cm}^2$$

Thus, Area of the shaded region

$$= \text{Area of rectangle ABCD} - (\text{Area of } \triangle AED + \text{Area of semi-circle})$$

$$= 5600 - (1176 + 1925)$$

$$= 5600 - 3101$$

$$= 2499 \text{ cm}^2$$

34.

Calculation of Mean

Class-interval	Mid-values (x)	Frequency f_i	$d_i = x_i - 25$	$u_i = \frac{x_i - 25}{10}$	$f_i u_i$
0-10	5	7	-20	-2	-14
10-20	15	10	-10	-1	-10
20-30	25	15	0	0	0
30-40	35	8	10	1	8
40-50	45	10	20	2	20
		$N = \Sigma f_i = 50$			$\Sigma f_i u_i = 4$

We have,

$$A = 25, h = 10, N = 50 \text{ and } \Sigma f_i u_i = 4$$

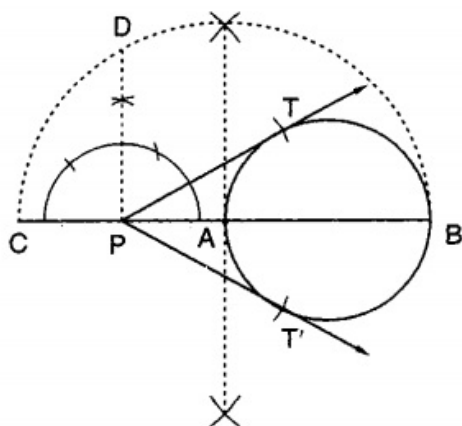
$$\therefore \text{Mean} = A + h \left\{ \frac{1}{N} \sum f_i u_i \right\}$$

$$\Rightarrow \text{Mean} = 25 + 10 \times \frac{4}{50} = 25.8$$

Section D

35. Steps of construction

STEP I Draw a line segment 4 cm.



STEP II Take a point P outside the circle and draw a secant PAB, intersecting the circle at A and B.

STEP III Produce AP to C such that AP = CP.

STEP IV Draw a semi-circle with CB as diameter.

STEP V Draw $PD \perp CB$, intersecting the semi-circle at D.

STEP VI With P as centre and PD as radius draw arcs to intersect the given circle at T and Y.

STEP VII Join PT and PT'. Then, PT and PT' are the required tangents.

OR

The steps of construction :

1. The rhombus ABCD is drawn in which $AB = 4$ cm and $\angle ABC = 60^\circ$.
2. Join AC. ABCD is divided into two triangles ABC and ADC.
3. Construct triangle AB'C' similar to ABC with scale factor $\frac{2}{3}$.
4. Draw the line segment C'D' parallel to CD.

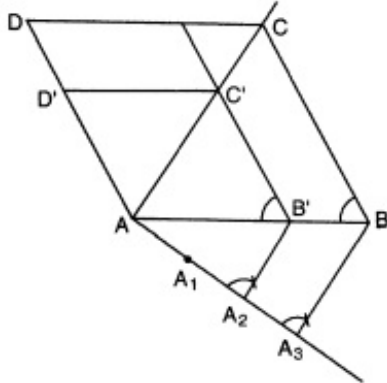
It can be observed that:

$$\frac{AB'}{AB} = \frac{2}{3} = \frac{AC'}{AC}$$

Also, $\frac{AC'}{AC} = \frac{C'D'}{CD}$

$$= \frac{AD'}{AD} = \frac{2}{3}$$

Therefore, $AB' = B'C = CD' = AD' = \frac{2}{3} AB$



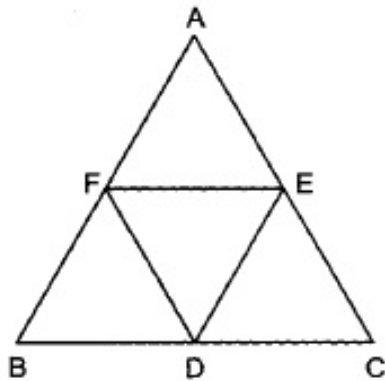
36. According to the question, we are given $\triangle ABC$ in which D, E, F are the mid-points of sides BC, CA and AB respectively.

We are required to prove that, each of the triangles AFE, FBD, EDC and DEF is similar to $\triangle ABC$.

To prove the above result, we Consider triangles AFE and ABC.

Since F and E are mid-points of AB and AC respectively.

Therefore, by mid-point theorem, we have,



$$FE \parallel BC$$

$$\Rightarrow \angle AFE = \angle B \text{ [Corresponding angles]}$$

Thus, in $\triangle AFE$ and $\triangle ABC$, we have

$$\angle AFE = \angle B$$

$$\text{and, } \angle A = \angle A \text{ [Common]}$$

Therefore, by AA criteria of similarity of similar triangles, we have,

$$\triangle AFE \sim \triangle ABC.$$

Similarly, we have

$\triangle FBD \sim \triangle ABC$ and $\triangle EDC \sim \triangle ABC$.

Now, we shall show that $\triangle DEF \sim \triangle ABC$.

Clearly, $ED \parallel AF$ and $DF \parallel EA$.

Therefore, AFDE is a parallelogram.

$\Rightarrow \angle EDF = \angle A$ [\because Opposite angles of a parallelogram are equal]

Similarly, BDEF is a parallelogram.

$\therefore \angle DEF = \angle B$ [\because Opposite angles of a parallelogram are equal]

Therefore, in triangles DEF and ABC, we have,

$\angle EDF = \angle A$ and $\angle DEF = \angle B$

Therefore, by AA-criterion of similarity, we have

$\triangle DEF \sim \triangle ABC$.

Therefore, each one of the triangles AFE, FBD, EDC and DEF is similar to $\triangle ABC$.

37. Given equations, $2x - 3y + 13 = 0$ and $3x - 2y + 12 = 0$.

Now, $2x - 3y + 13 = 0$

$$\Rightarrow y = \frac{13+2x}{3}$$

When $x=1$ then, $y=5$

When $x=4$ then, $y=7$

Thus, we have the following table giving points on the line $2x - 3y + 13 = 0$.

x	1	4
y	5	7

Now, $3x - 2y + 12 = 0$

$$\Rightarrow y = \frac{12+3x}{2}$$

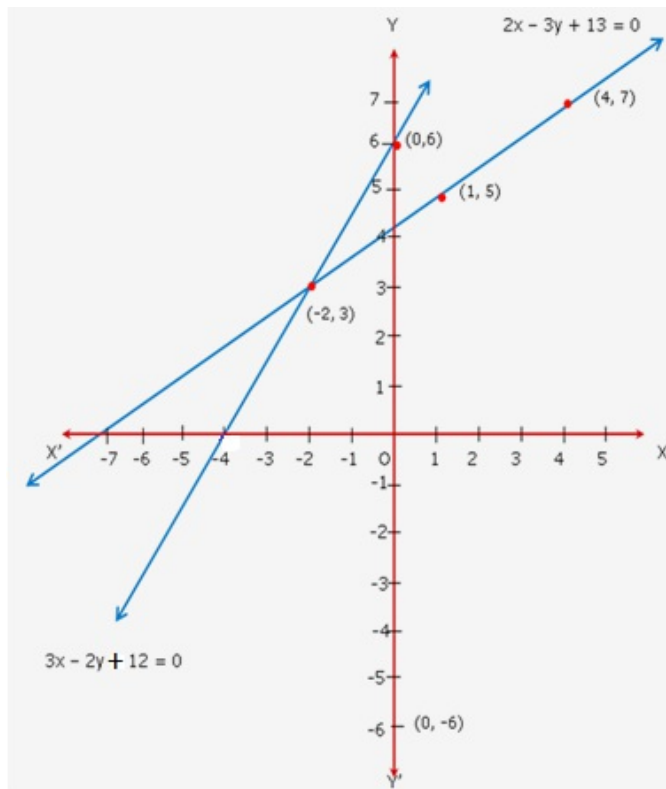
When $x=0$ then, $y=6$

When $x=-2$ then, $y=3$

Thus, we have the following table giving points on the line $3x - 2y + 12 = 0$.

x	0	-2
y	6	3

Graph:



Since, the two graphs intersect at $(-2, 3)$.

Hence, $x = -2$ and $y = 3$.

OR

Let the numerator and the denominator of the fraction be x and y respectively.

Hence, the fraction is $\frac{x}{y}$

Given, the numerator of the fraction is 4 less than the denominator.

So, $x = y - 4$

$\Rightarrow x - y = -4$ (i)

Also given, If the numerator is decreased by 2 and the denominator is increased by 1, then the denominator becomes 8 times of the numerator.

So, $y + 1 = 8(x - 2)$

$\Rightarrow y + 1 = 8x - 16$

$\Rightarrow 8x - y = 1 + 16$

$\Rightarrow 8x - y = 17$ (ii)

So, we have formed two linear equations in x & y as following:-

$x - y = -4$

$8x - y = 17$

Here x and y are unknowns.

We have to solve the above equations for x and y.

Now subtracting equation (ii) from equation (i), we get:

$$(x - y) - (8x - y) = -4 - 17$$

$$\Rightarrow x - y - 8x + y = -21$$

$$\Rightarrow -7x = -21$$

$$\Rightarrow x = \frac{-21}{-7}$$

$$\Rightarrow x = 3$$

Substituting $x = 3$ in equation (i),

$$3 - y = -4$$

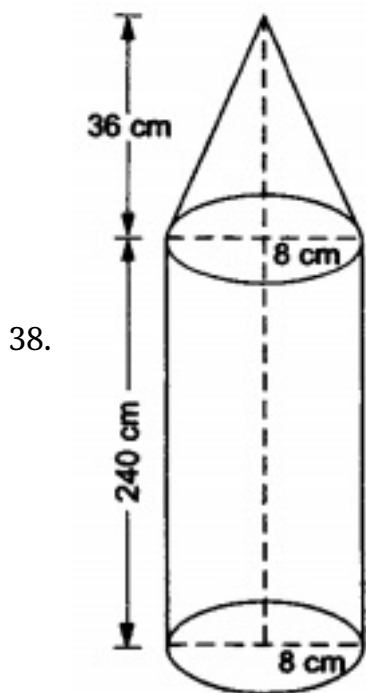
$$\Rightarrow -y = -3 - 4$$

$$\Rightarrow -y = -7$$

$$\Rightarrow y = 7$$

So, we get $x = 3$ & $y = 7$

Hence the fraction is $\frac{x}{y} = \frac{3}{7}$



Radius of the cylinder, $r = 8$ cm.

Radius of the cone, $r = 8$ cm.

Height of the cylinder, $h = 240$ cm.

Height of the cone, $H = 36$ cm.

Total volume of the iron pillar = volume of the cylinder + volume of the cone

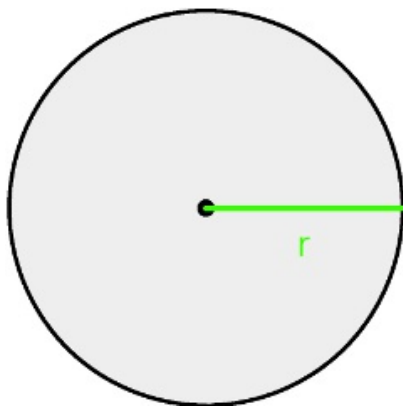
$$\begin{aligned}
&= \pi r^2 h + \frac{1}{3} \pi r^2 H = \pi r^2 \left(h + \frac{1}{3} H \right) \\
&= \frac{22}{7} \times 8 \times 8 \times \left(240 + \frac{1}{3} \times 36 \right) \\
&= 50688 \text{ cm}^3
\end{aligned}$$

$$\begin{aligned}
\therefore \text{Weight of the pillar} &= \text{Volume in cm}^3 \times \text{Weight per cm}^3 \\
&= \left(\frac{50688 \times 7.5}{1000} \right) \text{ kg} = 380.16 \text{ kg}
\end{aligned}$$

Hence, the weight of the pillar is 380.16 kg.

OR

Let n spherical shots can be obtained
diameter of spherical shots = 4.2 cm
 \Rightarrow radius of spherical shots = 2.1 cm

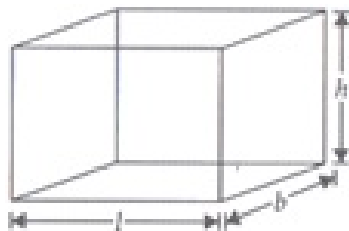


It is given that

length of cuboid = 66 cm

breadth of cuboid = 42 cm

height of cuboid = 21 cm



Spherical lead shots are recasted from cuboid of lead.

So, volume of n spherical lead shots is equal to volume of cuboid.

\therefore Volume of n spherical lead shots = Volume of lead cuboid

$$\Rightarrow n \times \frac{4}{3} \pi r^3 = l \times b \times h$$

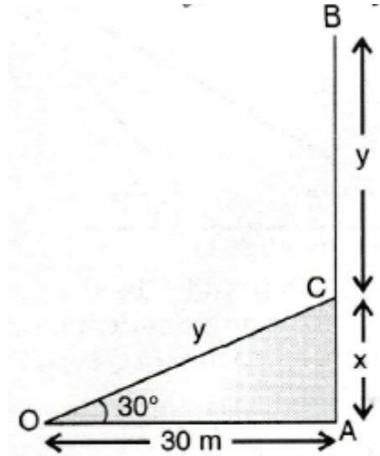
$$\Rightarrow n \times \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 = 66 \times 42 \times 21$$

$$\Rightarrow n = \frac{66 \times 42 \times 21 \times 3 \times 7 \times 1000}{4 \times 22 \times 21 \times 21 \times 21}$$

$$\Rightarrow n = 3 \times 500 = 1500$$

Hence, the number of lead shots are = 1500.

39. Let AB be the tree broken at a point C such that the broken part CB takes the position CO and strikes the ground at O. It is given that OA = 30 metres and $\angle AOC = 30^\circ$. Let AC = x and CB = y. Then, CO = y.



In $\triangle OAC$, we have

$$\tan 30^\circ = \frac{AC}{OA}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{30}$$

$$\Rightarrow x = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

Again in $\triangle OAC$, we have

$$\cos 30^\circ = \frac{OA}{OC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{30}{y}$$

$$\Rightarrow y = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

\therefore Height of the tree = (x + y) metres

$$= (10\sqrt{3} + 20\sqrt{3})$$

$$= 30\sqrt{3} \text{ metres} = 30 \times 1.732 \text{ metres} = 51.96 \text{ metres.}$$

40. We may prepare the 'more than' series as shown below:

C.I.	c.f.
More than 65	24
More than 60	54

More than 55	74
More than 50	90
More than 45	96
More than 40	100

On a graph paper, we plot the points A(40, 100), B(45, 96), C(50,90), D(55,74), E(60,54) and F(65,24).

Join all points freehand to get a 'More Than Ogive'.

