JEE ADVANCED - 2017 (Paper 2)

PART-III

SECTION-1 : (Maximum Marks : 21)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubble is darkened.
Negative Marks : –1 In all other cases.

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37. Three randomly chosen non-negative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is

(A) $\frac{36}{55}$ (B) $\frac{6}{11}$ (C) $\frac{5}{11}$ (D) $\frac{1}{2}$

Sol. : z is even.

So, $z = 0, 2, 4, 6, 8$ or 10

$\therefore$ $x + y = 10$, $x + y = 8$, $x + y = 6$, $x + y = 4$, $x + y = 2$, $x + y = 0$

has 11, 9, 7, 5, 3, 1 non-negative solutions.

So, $n(A) = 36$

$$n(U) = \sum_{k=1}^{11} k = \frac{11 \cdot 12}{2} = 66$$

$\therefore$ $P(A) = \frac{36}{66} = \frac{6}{11}$ **Ans. (B)**

38. Let $S = \{1, 2, 3, ..., 9\}$. For $k = 1, 2, ..., 5$, let N_k be the number of subsets of S, each containing five elements out of which exactly k are old. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$ ....

(A) 125 (B) 252 (C) 210 (D) 126

Sol. : $N_1 + N_2 + N_3 + N_4 + N_5$ = Total number of ways – {when no odd number is selected}

Total number ways = ${}^9C_5$

Number of ways when no odd is selected is zero. ($\because$ only available even numbers are 2, 4, 6, 8)

${}^9C_5$ – zero = 126 **Ans. (D)**

39. If $f : R \to R$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in R$, and $f\left(\frac{1}{2}\right) = \frac{1}{2}$, $f(1) = 1$, then

(A) $0 < f'(1) \leq \frac{1}{2}$ (B) $f'(1) \leq 0$ (C) $f'(1) > 1$ (D) $\frac{1}{2} < f'(1) \leq 1$

Sol. : Using LMVT on $f(x)$ for $x \in \left[\frac{1}{2}, 1\right]$

$$\frac{f(1) - f\left(\frac{1}{2}\right)}{1 - \frac{1}{2}} = f'(c), \text{ where } c \in \left(\frac{1}{2}, 1\right)$$

$$\frac{1 - \frac{1}{2}}{\frac{1}{2}} = f'(c) \Rightarrow f'(c) = 1, \text{ where } c \in \left(\frac{1}{2}, 1\right)$$

$\because$ $f'(x)$ is an increasing function $\forall x \in$ R.

$\therefore$ $f'(1) > 1$ $\quad (f'(1) > f'(c) = 1)$ **Ans. (C)**

40. If $y = y(x)$ satisfies the differential equation

$8\sqrt{x}\,(\sqrt{9+\sqrt{x}})dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx$, $x > 0$ and $y(0) = \sqrt{7}$, then $y(256) =$ ....

(A) 80 (B) 3 (C) 16 (D) 9

Sol. : $y = \frac{1}{8}\int \frac{dx}{\sqrt{\left(4+\sqrt{9+\sqrt{x}}\right)}\cdot\sqrt{x}\cdot\left(\sqrt{9+\sqrt{x}}\right)}$

Let $\sqrt{9+\sqrt{x}} = t$, so $\frac{dx}{\sqrt{x}\cdot\sqrt{9+\sqrt{x}}} = 4\,dt$

$\therefore$ $y = \frac{4}{8}\int \frac{dt}{\sqrt{4+t}}$

$\therefore$ $y = \sqrt{4+t} + c$

$\therefore$ $y(x) = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right) + c$

at $x = 0$; $y(0) = \sqrt{7} \Rightarrow c = 0$

$\therefore$ $y(x) = \sqrt{4+\sqrt{9+\sqrt{x}}}$

$\therefore$ $y(256) = \sqrt{4+\sqrt{9+\sqrt{256}}} = \sqrt{4+5} = 3$ **Ans. (B)**

41. How many 3×3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of M^TM is 5 ?

(A) 198 (B) 126 (C) 135 (D) 162

Sol. : Let $M = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

$\therefore$ $tr(M^TM) = a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 5$, where entries are from {0, 1 2}

Only two cases are possible.

(I) five entries 1 and other four zero.

$\therefore \quad {}^9C_5 \times 1 = 126$ selections

(II) One entry is 2, one entry is 1 and other selections are 0.

$\therefore \quad {}^9C_2 \times 2!$ Possibilities are there. (1, 2 from 9 entries)

Total number = 126 + 72 = 198. **Ans. (A)**

42. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overrightarrow{OP} \cdot \overrightarrow{OQ} + \overrightarrow{OR} \cdot \overrightarrow{OS} = \overrightarrow{OR} \cdot \overrightarrow{OP} + \overrightarrow{OQ} \cdot \overrightarrow{OS} = \overrightarrow{OQ} \cdot \overrightarrow{OR} + \overrightarrow{OP} \cdot \overrightarrow{OS}$$

Then the triangle PQR has S as its

(A) incentre (B) orthocenter (C) circumcentre (D) centroid

Sol. : Let position vector of P($\vec{p}$), Q($\vec{q}$), R($\vec{r}$) and S($\vec{s}$)

$\overrightarrow{OP} \cdot \overrightarrow{OQ} + \overrightarrow{OR} \cdot \overrightarrow{OS} = \overrightarrow{OR} \cdot \overrightarrow{OP} + \overrightarrow{OQ} \cdot \overrightarrow{OS}$

$\Rightarrow \quad \vec{p} \cdot \vec{q} + \vec{r} \cdot \vec{s} - \vec{q} \cdot \vec{s} - \vec{p} \cdot \vec{r} = 0$

$\Rightarrow \quad (\vec{p} - \vec{s}) \cdot (\vec{q} - \vec{r}) = 0$

So, $\overrightarrow{SP} \cdot \overrightarrow{RQ} = 0$. So, $\overrightarrow{SP} \perp \overrightarrow{RQ}$

Also, $\overrightarrow{OR} \cdot \overrightarrow{OP} + \overrightarrow{OQ} \cdot \overrightarrow{OS} = \overrightarrow{OQ} \cdot \overrightarrow{OR} + \overrightarrow{OP} \cdot \overrightarrow{OS}$

$\Rightarrow \quad \vec{r} \cdot \vec{p} + \vec{q} \cdot \vec{s} - \vec{q} \cdot \vec{r} - \vec{p} \cdot \vec{s} = 0$

$\Rightarrow \quad (\vec{r} - \vec{s}) \cdot (\vec{p} - \vec{q}) = 0$

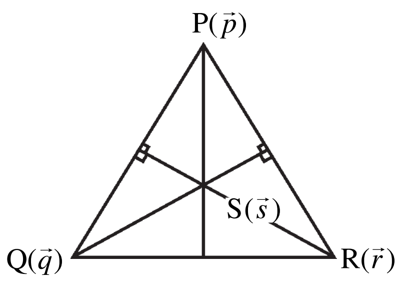
So, $\quad \overrightarrow{SR} \perp \overrightarrow{QP}$

Also, $\overrightarrow{OP} \cdot \overrightarrow{OQ} + \overrightarrow{OR} \cdot \overrightarrow{OS} = \overrightarrow{OQ} \cdot \overrightarrow{OR} + \overrightarrow{OP} \cdot \overrightarrow{OS}$

$\Rightarrow \quad \vec{p} \cdot \vec{q} + \vec{r} \cdot \vec{s} = \vec{q} \cdot \vec{r} + \vec{p} \cdot \vec{s}$

$\Rightarrow \quad (\vec{q} - \vec{s}) \cdot (\vec{p} - \vec{r}) = 0$

So, $\quad \overrightarrow{SQ} \perp \overrightarrow{RP}$



$\therefore$ Triangle PQR has S as its orthocentre.

$\therefore$ Option (B) is correct. **Ans. (B)**

43. The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is

(A) $14x + 2y + 15z = 31$ (B) $14x + 2y - 15z = 1$

(C) $-14x + 2y + 15z = 3$ (D) $14x - 2y + 15z = 27$

Sol. : The normal to the required plane is parallel to the vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = -14\hat{i} - 2\hat{j} - 15\hat{k}$$

$\therefore$ The equation of the required plane passing through (1, 1, 1) will be

$-14(x - 1) - 2(y - 1) - 15(z - 1) = 0$

$\therefore$ $14x + 2y + 15z = 31$

$\therefore$ Option (A) is correct. **Ans. (A)**

SECTION-2 : (Maximum Marks : 28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is (are) correct.
- For each question, darken the bubble(s) corresponding to the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option,** Provided NO incorrect option is darkened.
Zero Marks	:	0	If non of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

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44. If $I = \sum_{k=1}^{98} \int_{k}^{k+1} \frac{k+1}{x(x+1)}\, dx$, then

(A) $1 < \frac{49}{50}$ (B) $1 < \log_e 99$ (C) $1 > \frac{49}{50}$ (D) $1 > \log_e 99$

Sol. : $I = \sum_{k=1}^{98}\left(\int_{k}^{k+1} \frac{(k+1)}{x(x+1)}dx\right)$

$$= \sum_{k=1}^{98}(k+1)\left(\int_{k}^{k+1}\left(\frac{1}{x}-\frac{1}{x+1}\right)dx\right)$$

$$= \sum_{k=1}^{98}(k+1)\left(\left(\log x-\log(x+1)\right)_k^{k+1}\right)$$

$$= \sum_{k=1}^{98} (k+1)(\log(k+1) - \log(k+2) - \log k + \log(k+1))$$

$$= \sum_{k=1}^{98} ((k+1)\log(k+1) - k\cdot\log k) - \sum_{k=1}^{98} ((k+1)\cdot\log(k+2) - k\cdot\log(k+1) +$$

$$\sum_{k=1}^{98}(\log(k+1) - \log k) \qquad \text{(Difference series)}$$

$$1 = (99\log 99) + (-99\log 100 + \log 2) + (\log 99) = \log\left(\frac{2\times(99)^{100}}{(100)^{99}}\right) \qquad ...(1)$$

Consider option (B) :

Now, consider $(100)^{99} = (1 + 99)^{99}$

$$= {}^{99}C_0 + {}^{99}C_1(99) + {}^{99}C_2(99)^2 + ...+ {}^{99}C_{97}(99)^{97} + \underbrace{{}^{99}C_{98}(99)^{98}}_{(\text{value }(99)^{99})} + \underbrace{{}^{99}C_{99}(99)^{99}}_{(\text{value }(99)^{99})}$$

$\therefore \quad (100)^{99} > 2\cdot(99)^{99}$

So, $\quad \dfrac{2\times(99)^{99}}{(100)^{99}} < 1$

$\therefore \quad 2\,\dfrac{(99)^{100}}{(100)^{99}} < 99$

$\therefore \quad I = \log\dfrac{2\times(99)^{100}}{(100)^{99}} < \log 99$

$\therefore$ Option (B) is true.

Now, $\displaystyle\sum_{k=1}^{98}\int_k^{k+1}\frac{k+1}{(x+1)^2}\,dx < \sum_{k=1}^{98}\int_k^{k+1}\frac{(k+1)dx}{x(x+1)}$

$\therefore \quad \displaystyle\sum_{k=1}^{98}\left(-\frac{k+1}{x+1}\right)_k^{k+1} < I$ (on integration)

$\therefore \quad \displaystyle\sum_{k=1}^{98}\left(-\frac{k+1}{k+2}+1\right) < I$

$\therefore \quad \displaystyle\sum_{k=1}^{98}\frac{1}{k+2} < I$

$\therefore \quad \left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+....+\frac{1}{100}\right) < I$

$\therefore \quad \dfrac{98}{100} < \dfrac{1}{3}+\dfrac{1}{4}+\dfrac{1}{5}+....+\dfrac{1}{100} < I$

$\therefore \quad I > \dfrac{49}{50}$

Hence option (C) is correct. **Ans. (B), (C)**

45. If $f : \mathrm{R} \to \mathrm{R}$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in \mathrm{R}$, and $f(0) = 1$, then

(A) $f(x) > e^{2x}$ in $(0, \infty)$ (B) $f(x)$ is decreasing in $(0, \infty)$

(C) $f(x)$ is increasing in $(0, \infty)$ (D) $f'(x) < e^{2x}$ in $(0, \infty)$

Sol. :

$f'(x) > 2f(x), \quad \forall x \in \mathrm{R}$

$\therefore \quad f'(x) - 2f(x) > 0, \quad \forall x \in \mathrm{R}$

$\therefore \quad e^{-2x}(f'(x) - 2f(x)) > 0, \quad \forall x \in \mathrm{R}$

$\therefore \quad \frac{d}{dx}(e^{-2x} f(x)) > 0, \quad \forall x \in \mathrm{R}$

Let $g(x) = e^{-2x} f(x)$

$\therefore$ $g(x)$ is strictly increasing $\forall x \in \mathrm{R}$

Also $g(0) = 1$

$\therefore \quad \forall x > 0, g(x) > g(0) = 1$

$\therefore \quad e^{-2x} \cdot f(x) > 1, \forall x \in (0, \infty) \Rightarrow f(x) > e^{2x}, \forall x \in (0, \infty)$

$\therefore$ option (A) is correct.

As, $f'(x) > 2f(x) > 2e^{2x} > 2, \quad \forall x \in (0, \infty)$

$\therefore$ $f(x)$ is strictly increasing on $x \in (0, \infty)$

$\therefore$ option (C) is correct.

As, we have proved above that

$\therefore \quad f'(x) > 2 \cdot e^{2x} \quad \forall x \in (0, \infty)$

$\therefore$ option (D) is incorrect.

$\therefore$ options (A) and (C) are correct. **Ans. (A), (C)**

46. If $f(x) = \begin{vmatrix} cos(2x) & cos(2x) & sin(2x) \\ -cos\,x & cos\,x & -sin\,x \\ sin\,x & sin\,x & cos\,x \end{vmatrix}$, then

(A) $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$

(B) $f(x)$ attains its maximum at $x = 0$

(C) $f(x)$ attains its minimum at $x = 0$

(D) $f'(x) = 0$ at more than three points in $(-\pi, \pi)$

Sol. : Expansion of determinant is $cos2x + cos^2 2x - sin^2 2x$

$\therefore \quad f(x) = cos2x + cos4x$

$f'(x) = -2sin2x - 4sin4x = -2sin2x(1 + 4cos2x)$

$f'(x) = 0 \Rightarrow x = \frac{n\pi}{2}, cos2x = -\frac{1}{4}, x \in (-\pi, \pi)$

Also, $f'(x) < 0$ in *nbhd* of 0 for $x > 0$

$f'(x) > 0$ in *nbhd* of 0 for $x < 0$

$\therefore$ f has local maximum at $x = 0$.

$\therefore$ $x = 0, \pm\frac{\pi}{2}$ and two solutions of $cos2x = -\frac{1}{4}$ in $(-\pi, \pi)$.

Option (B) is true.

or

$f''(x) = -4cos\ 2x\ (1 + 4cos2x) - 2sin\ 2x\ (-8sin2x)$

$\therefore$ $f''(0) = -20$

$\therefore$ $f(0)$ is maximum. So, Option (B) is true. **Ans. (B), (D)**

47. Let α and β be nonzero real numbers such that $2(cos\beta - cos\alpha) + cos\alpha\ cos\beta = 1$. Then which the following is/are true ?

(A) $tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\,tan\left(\frac{\beta}{2}\right) = 0$

(B) $\sqrt{3}\,tan\left(\frac{\alpha}{2}\right) - tan\left(\frac{\beta}{2}\right) = 0$

(C) $tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\,tan\left(\frac{\beta}{2}\right) = 0$

(D) $\sqrt{3}\,tan\left(\frac{\alpha}{2}\right) + tan\left(\frac{\beta}{2}\right) = 0$

Sol. : $2(cos\beta - cos\alpha) + cos\alpha\ cos\beta - 1 = 0$

$$2\left[\left(\frac{1-tan^2\frac{\beta}{2}}{1+tan^2\frac{\beta}{2}}\right)-\left(\frac{1-tan^2\frac{\alpha}{2}}{1+tan^2\frac{\alpha}{2}}\right)\right] + \left[\left(\frac{1-tan^2\frac{\alpha}{2}}{1+tan^2\frac{\alpha}{2}}\right)\left(\frac{1-tan^2\frac{\beta}{2}}{1+tan^2\frac{\beta}{2}}\right)-1\right] = 0$$

$\therefore$ $4\left(tan^2\frac{\alpha}{2} - tan^2\frac{\beta}{2}\right) + 1 - tan^2\frac{\alpha}{2} - tan^2\frac{\beta}{2} + tan^2\frac{\alpha}{2}\ tan^2\frac{\beta}{2} -$

$\left(1 + tan^2\frac{\alpha}{2} + tan^2\frac{\beta}{2} + tan^2\frac{\alpha}{2}\ tan^2\frac{\beta}{2}\right) = 0$

$\therefore$ $2tan^2\frac{\alpha}{2} = 6tan^2\frac{\beta}{2}$

$\therefore$ $tan^2\frac{\alpha}{2} = 3tan^2\frac{\beta}{2}$

$\therefore$ $tan\frac{\alpha}{2} = \pm\sqrt{3}\,tan\frac{\beta}{2}$

$\therefore$ Solution is (A), (C) **Ans. (A), (C)**

48. If $g(x) = \int_{sin\,x}^{sin(2x)} sin^{-1}(t)\ dt$, then

(A) $g'\left(\frac{\pi}{2}\right) = -2\pi$ (B) $g'\left(-\frac{\pi}{2}\right) = 2\pi$ (C) $g'\left(\frac{\pi}{2}\right) = 2\pi$ (D) $g'\left(-\frac{\pi}{2}\right) = -2\pi$

Sol. : $g(x) = \int_{sin\,x}^{sin(2x)} sin^{-1}(t)\ dt$

So, $g'(x) = 2sin^{-1}(sin\ 2x) \times cos\ 2x - sin^{-1}(sin\,x)\ cosx$

$\therefore$ $g'\left(\frac{\pi}{2}\right) = 2sin^{-1}(sin\,\pi)\ cos\,\pi - sin^{-1}\left(sin\,\frac{\pi}{2}\right)\ cos\frac{\pi}{2}$

$= 0 - 0 = 0$

$g'\left(\frac{\pi}{2}\right) = 0$. Similarly, $g'\left(-\frac{\pi}{2}\right) = 0$.

No option matches the result.

49. If the line $x = \alpha$ divides the arc of region $R = \{(x, y) \in R^2 : x^3 \le y \le x, 0 \le x \le 1\}$ into two equal parts, then

(A) $\frac{1}{2} < \alpha < 1$ (B) $\alpha^4 + 4\alpha^2 - 1 = 0$ (C) $0 < \alpha \le \frac{1}{2}$ (D) $2\alpha^4 - 4\alpha^2 + 1 = 0$

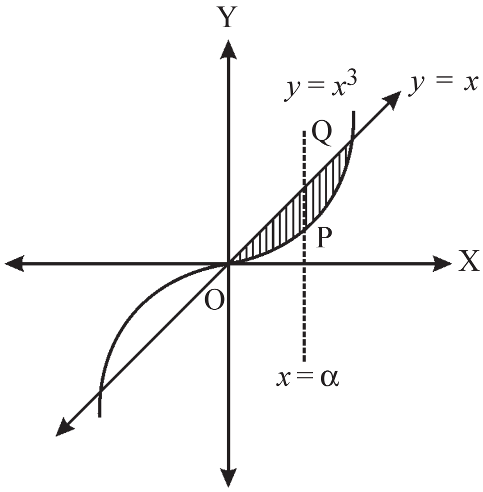
Sol. : Area between $y = x^3$ and $y = x$

in $x \in (0, 1)$ is

$$A = \int_0^1 (x - x^3)\, dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$\therefore$ Area of region OPQ $= \frac{A}{2} = \frac{1}{8}$

So, $\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^\alpha = \frac{1}{8}$



Hence, $2\alpha^4 - 4\alpha^2 + 1 = 0$. Option (D) is true.

$\therefore$ $\alpha^2 = \frac{4 - \sqrt{8}}{4}$

$= 1 - \frac{1}{\sqrt{2}} < 1$

Also, $\alpha^2 > 1 - \frac{3}{4} = \frac{1}{4}$

$\therefore$ $\frac{1}{2} < \alpha < 1$

Hence, (A), (D) **Ans. (A), (D)**

50. Let $f(x) = \frac{1 - x(1 + |1 - x|)}{|1 - x|} \cos\left(\frac{1}{1 - x}\right)$ for $x \ne 1$. Then

(A) $\lim_{x \to 1+} f(x)$ does not exist (B) $\lim_{x \to 1-} f(x)$ does not exist

(C) $\lim_{x \to 1-} f(x) = 0$ (D) $\lim_{x \to 1+} f(x) = 0$

Sol. : For $x < 1$, $|1 - x| = 1 - x$

$\therefore$ $f(x) = \frac{1 - x(1 + 1 - x)}{(1 - x)} \cos\frac{1}{1 - x}$

$= \frac{1 - 2x + x^2}{1 - x} \cos\frac{1}{1 - x}$

$\therefore$ $f(x) = (1 - x) \cos\frac{1}{1 - x}$ $x < 1$

If $x > 1$, $|1 - x| = x - 1$

$\therefore$ $f(x) = \frac{1 - x(1 + x - 1)}{x - 1} \cos\frac{1}{1 - x}$

$= \frac{1 - x^2}{-(1 - x)} \cos\frac{1}{1 - x}$

$= -(x + 1) \cos\frac{1}{1 - x}$ $x > 1$

$\therefore$ $\lim_{x \to 1-} f(x) = 0$, $\lim_{x \to 1+} f(x)$ does not exist. **Ans. (A), (C)**

SECTION-3 : (Maximum Marks : 12)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four option is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

  Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

  Zero Marks : 0 If all other cases.

PARAGRAPH 1

Let O be the origin, and $\overrightarrow{OX}$, $\overrightarrow{OY}$, $\overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}$, $\overrightarrow{RP}$, $\overrightarrow{PQ}$ respectively, of a triangle PQR.

51. $|\overrightarrow{OX} \times \overrightarrow{OY}| =$

(A) $\sin(Q + R)$ (B) $\sin(P + R)$ (C) $\sin 2R$ (D) $\sin(P + Q)$

Sol. : $\overrightarrow{OX} = \dfrac{\overrightarrow{QR}}{QR}$

$\overrightarrow{OY} = \dfrac{\overrightarrow{RP}}{RP}$

$|\overrightarrow{OX} \times \overrightarrow{OY}| = \sin R = \sin(P + Q)$ **Ans. (D)**

52. If the triangle PQR varies, then the minimum value of $\cos(P + Q) + \cos(Q + R) + \cos(R + P)$ is,

(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $\frac{5}{3}$ (D) $-\frac{5}{3}$

Sol. : $-(\cos P + \cos Q + \cos R) \geq -\frac{3}{2}$ as we know $\cos P + \cos Q + \cos R$ will take its maximum value when $P = Q = R = \frac{\pi}{3}$. ($\cos(P + Q) = -\cos R$ etc.) **Ans. (B)**

PARAGRAPH 2

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2,....$ let, $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

53. If $a_4 = 28$, then $p + 2q =$

(A) 14 (B) 7 (C) 12 (D) 21

Sol. : $\alpha^2 = \alpha + 1 \Rightarrow \alpha^4 = \alpha^2 + 2\alpha + 1 = \alpha + 1 + 2\alpha + 1 = 3\alpha + 2$

$\therefore \quad a_4 = 28 \Rightarrow p\alpha^4 + q\beta^4 = p(3\alpha + 2) + q(3\beta + 2) = 28$

$\therefore \quad p(3\alpha + 2) + q(3 - 3\alpha + 2) = 28$ $\qquad (\alpha + \beta = 1)$

$\therefore \quad \alpha(3p - 3q) + 2p + 5q = 28$

$\therefore \quad p = q,\ 2p + 5q = 28$. So, $p = q = 4$

$\therefore \quad p + 2q = 12$ **Ans. (C)**

54. $a_{12} =$

(A) $2a_{11} + a_{10}$ (B) $a_{11} - a_{10}$ (C) $a_{11} + a_{10}$ (D) $a_{11} + 2a_{10}$

Sol. : $\alpha^2 = \alpha + 1 \Rightarrow \alpha^n = \alpha^{n-1} + \alpha^{n-2}$

$\therefore \quad p\alpha^n + q\beta^n = p(\alpha^{n-1} + \alpha^{n-2}) + q(\beta^{n-1} + \beta^{n-2})$

$\therefore \quad a_n = a_{n-1} + a_{n-2}$

$\therefore \quad a_{12} = a_{11} + a_{10}$ **Ans. (C)**

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