Whole Numbers

CHAPTER - 2

2.1 Introduction

In our previous class, we learnt about counting things. While counting things, we need numbers 1, 2, 3, ... to count. These numbers are called natural numbers. We express the set of natural numbers in the form of N = {1, 2, 3, 4, ...}

While learning about natural numbers, we experienced that if we add '1' to any natural number, we get the next natural number. For example, if we add '1' to '16', then we get the number 17 which is again a natural number. In the same way if we deduct '1' from any natural number, generally we get a natural number. For example if we deduct '1' from a natural number 25, the result is 24, which is a natural number.Is this true if 1 is deducted from 1?

The next number of any natural number is called its successor and the number just before a number is called the predecessor.

for example, the successor of 9 is 10

and the predecessor of 9 is 8.

Now fill the following table with the successor and predecessor of the numbers provided:

S.No.	Natural number	Predecessor	Successor
1.	13		
2.	237		
3.	999		
4.	26		
5	9		
6	1		

Discuss with your friends

1. Which natural number has no successor?
2. Which natural number has no predecessor?

2.2 Whole Numbers

You might have come to know that the number '1' has no predecessor in natural numbers. We include zero to the collection of natural numbers. The natural numbers along with the zero form the collection of Whole numbers.

Whole numbers are represented like as follows.

W = {0, 1, 2, 3......}

Do This

Which is the smallest whole number?

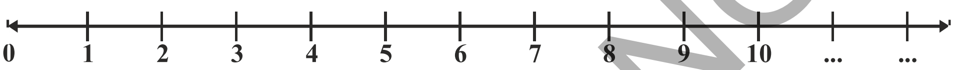
Think, Discuss and Write

1. Are all natural numbers whole numbers?
2. Are all whole numbers natural numbers?

2.3 Representation of Whole Numbers on Number Line

Draw a line. Mark a point on it. Label it as '0'. Mark as many points as you like on the line at equal distance to the right of 0. Label the points as1, 2, 3, 4, ... respectively. The distance between any two consecutive points is the unit distance. You can go to any whole number on the right.

The number line for whole numbers is:



On the number line given above you know that the successor of any number will lie to the right of that number. For example, the successor of 3 is 4. 4 is greater than 3 and lies on the right side of number 3.

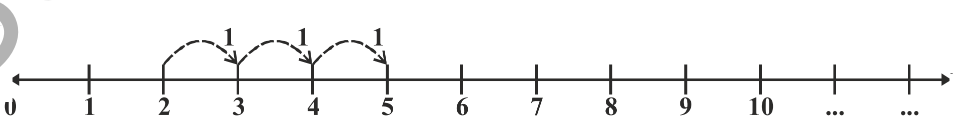
Now can we say that all the numbers that lie on the right of that number are greater than the number?

Discuss with your friends and fill the table.

S.No.	Number	Position on number line	Relation between numbers
1.	12, 8	12 lies on the right of 8	12 > 8
2.	12, 16		
3.	236, 210		
4.	1182, 9521		
5.	10046, 10960		

Addition on number line

Addition of whole numbers can be represented on number line. In the line given below, the addition of 2 and 3 is shown as below.



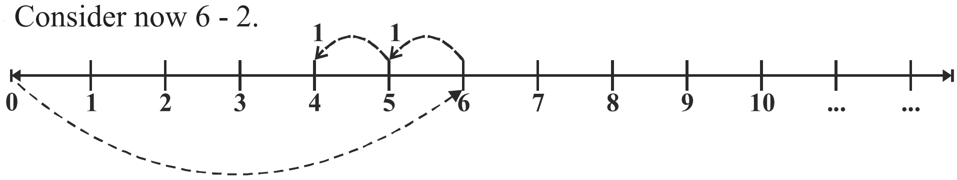
Start from 2, we add 3 to two. We make 3 jumps to the right on the number line, as shown above. We will reach at 5.

So, 2 + 3 = 5

So whenever we add two numbers we move on the number line towards right starting from any of them.

Subtraction on the Number Line

Consider now 6 - 2.



Start from 6. Since we subtract 2 from 6, we take 2 steps to the left on the number line, as shown above. We reach 4. So, 6 - 2 = 4 .Thus moving towards left means subtraction.

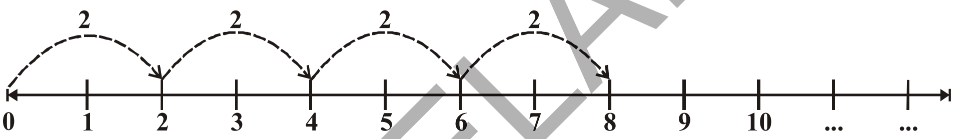
Do This

Show these on number line:

1. 5 + 3
2. 5 - 3
3. 3 + 5
4. 10 + 1

Multiplication on the Number Line

Let us now consider the multiplication of the whole numbers on the number line. Let us find 4×2. We know that 4×2 means taking 2 steps four times. 4×2 means four jumps towards right, each of 2 steps.



Start from 0, move 2 units to the right each time, making 4 such moves. We will reach 8.

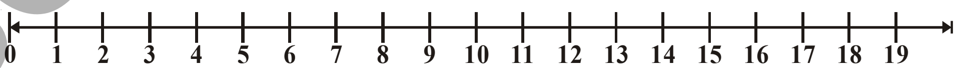
So, $4 \times 2 = 8$

Try These

Find the following by using number line:

1. What number should be deducted from 8 to get 5?
2. What number should be deducted from 6 to get 1?
3. What number should be added to 6 to get 8?
4. How many 6 are needed to get 30?

Raju and Gayatri together made a number line and played a game on it.



Raju asked "Gayatri, where will you reach if you jump thrice, taking leaps of 3, 8 and 5"? Gayatri said 'the first leap will take me to 3 and then from there I will reach 11 in the second step and another five steps from there to 16'.

Do you think Gayatri described where she would reach correctly?

Draw Gayatri's steps.

Play this game using addition and subtraction on this number line with your friend.

EXERCISE - 2.1

1. Which of the statements are true (T) and which are false (F). Correct the false statements.
   - i. There is a natural number that has no predecessor.
   - ii. Zero is the smallest whole number.
   - iii. All whole numbers are natural numbers.
   - iv. A whole number that lies on the number line lies to the right side of another number is the greater number.
   - v. A whole number on the left of another number on the number line, is greater.
   - vi. We can't show the smallest whole number on the number line.
   - vii. We can show the greatest whole number on the number line.
2. How many whole numbers are there between 27 and 46?
3. Find the following using number line.
   - i. $6 + 7 + 7$ ii. $18 - 9$ iii. 5×3
4. In each pair, state which whole number on the number line is on the right of the other number.
   - i. 895 ; 239 ii. 1001 ; 10001 iii. 10015678 ; 284013
5. Mark the smallest whole number on the number line.
6. Choose the appropriate symbol from < or >
   - i. 8 .... 7 ii. 5 2
   - iii. 0 .... 1 iv. 10 5
7. Place the successor of 11 and predecessor of 5 on the number line.

2.4 PROPERTIES OF WHOLE NUMBERS

Studying the properties of whole numbers help us to understand numbers better. Let us look at some of the properties.

Take any two whole numbers and add them.

Is the result a whole number? Think of some more examples and check.

Your additions may be like this:

$2 + 3 = 5$, a whole number

$0 + 7 = 7$, a whole number

$20 + 51 = 71$, a whole number

$0 + 1 = 1$, a whole number

$0 + 0 = 0$, a whole number

Here, we observe that the sum of any two whole numbers is always a whole number.

Can you find any pair of whole numbers, which when added will not give a whole number? We see that no such pair exists and the collection of whole numbers are closed under addition. This property is known as the closure property of addition for whole numbers.

Let us check whether the collection of whole numbers is also closed under multiplication. Try with 5 examples.

Your multiplications may be like this:

5	×	6	=	30, a whole number
11	×	0	=	0, a whole number
16	×	5	=	80, a whole number
10	×	100	=	1000, a whole number
7	×	16	=	112, a whole number

The product of any two whole numbers is found to be a whole number too. Hence, we say that the collection of whole numbers is closed under multiplication.

We can say that whole numbers are closed under addition and multiplication.

Think, Discuss and Write

1. Are the whole numbers closed under subtraction?

   Your subtractions may be like this:

   7 - 5 = 2, a whole number

   5 - 7 = ?, not a whole number

   ..... - ..... = ...

   ..... - ..... = ...

   Take as many examples as possible and check.

2. Are the whole numbers closed under division?

   Now observe this table:

   6 ÷ 3 = 2, a whole number

   5 ÷ 2 = $\frac{5}{2}$ is not a whole number

   ..... ÷ ..... = ...

   ..... ÷ ..... = ...

   Confirm it by taking a few more examples.

Division by Zero

Let us find 6 ÷ 2

6 Divided by 2 means, we subtract 2 from 6 repeatedly i.e. we subtract 2 from 6 again and again till we get zero.

6 - 2 = 4 once
4 - 2 = 2 twice
2- 2 = 0 thrice
So, 6 ÷ 2 = 3

Let us consider 3 ÷ 0,

Here we have to subtract zero again and again from 3

3- 0 = 3 once
3 - 0 = 3 twice
3 - 0 = 3 thrice and so on.....

Will this ever stop? No. So, 3 ÷ 0 is not a number that we can reach.

So division of a whole number by 0 does not give a known number as answer.

i.e. Division by zero is not define.

Do This

1. Find out 12 ÷ 3 and 42 ÷ 7

2. What would 6 ÷ 0 and 9 ÷ 0 be equal to?

Commutativity of whole numbers

Observe the following additions;

2 + 3 = 5 ; 3 + 2 = 5

We see in both cases that we get 5. Look at this

7 + 8 = 15 ; 8 + 7 = 15

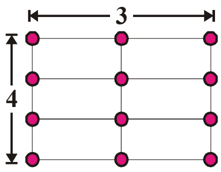
We find that 7 + 8 and 8 + 7 are also equal.

Here, the sum is same, though the order of addition of a pair of whole numbers is changed.

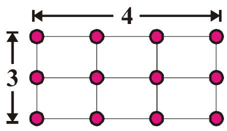
Check it for few more examples, 10 + 11, 25 + 10.

Thus it is clear that we can add two whole numbers in any order. We say that addition is commutative for whole numbers.

Observe the following figure:



4 × 3 = 12



3 × 4 = 12

We observe that, the product is same, though the order of multiplication of two whole numbers is changed.

Check it for few more examples of whole numbers, like 6 × 5, 7 × 9 etc. Do you get these to be equal too?

Thus, addition and multiplication are commutative for whole numbers.

Try These

Take a few examples and check whether -

1. Subtraction is commutative for whole numbers or not?
2. Division is commutative for whole numbers or not?

Associativity of addition and multiplication

Observe the following:

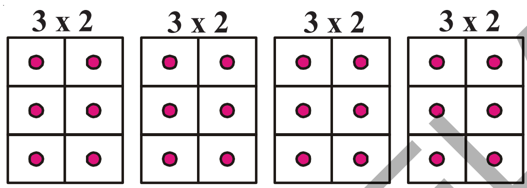
i. $(3 + 4) + 5 = 7 + 5 = 12$ ii. $3 + (4 + 5) = 3 + 9 = 12$

So, $(3 + 4) + 5 = 3 + (4 + 5)$

In (i) we add 3 and 4 first and then add 5 to the sum and in (ii) we add 4 and 5 first, and then add the sum to 3. But the result is the same.

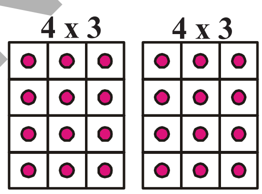
This is called associative property of addition for whole numbers. Create 10 more examples and check it for them. Could you find any example where the sums are not identical?

Observe the following:



$4 \times (3{\times}2)$ = four times $(3{\times}2)$

Fig. (a)



$2 \times (4{\times}3)$ = twice of $(4{\times}3)$

Fig. (b)

Count the number of blocks in fig. (a), and in fig. (b). What do you get? The number of blocks is the same in fig. (a) we have 3×2 blocks in each box. So the total number of blocks is $4 \times (3{\times}2) = 24$

In fig. (b) each box has 4×3 blocks. So the total number of blocks is $2 \times (4 \times 3) = 24$

Thus, $4 \times (3{\times}2) = 2 \times (4 \times 3)$

In multiplication also, we see that the result is same, whichever order of multiplication you follow the result is the same.

This is associative property for multiplication of whole numbers.

We see that addition and multiplication are associative over whole numbers.

Do This

Verify the following:

i. $(5 \times 6) \times 2 = 5 \times (6 \times 2)$

ii. $(3 \times 7) \times 5 = 3 \times (7 \times 5)$

Example-1. Find 196 + 57 + 4.

Solution: 196 + (57 + 4)

= 196 + (4 + 57) [Commutative property]

= (196 + 4) + 57 [Associative property]

= 200 + 57 = 257

Here we used a combination of commutative and associative properties for addition.

Do you think using the commutative and associative properties made the calculations easier?

Example-2. Find 5 × 9 × 2 × 2 × 3 × 5

Solution: 5 × 9 × 2 × 2 × 3 × 5

= 5 × 2 × 9 × 2 × 5 × 3 [Commutative property]

= (5 × 2) × 9 × (2 × 5) × 3 [Associative property]

= 10 × 9 × 10 × 3

= 90 × 30 = 2700

Here we used a combination of commutative and associative properties for multiplication.

Do you think using the commutative and associative properties made the calculations easier?

Do This

Use the commutative and associative properties to simplify the following:

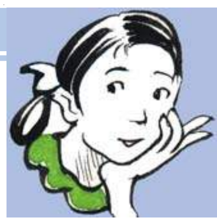
i. 319 + 69 + 81

ii. 431 + 37 + 69 + 63

iii. 2 × (71 × 5)

iv. 50 × 17 × 2

Think, Discuss and Write



Is (16 ÷ 4) ÷ 2 = 16 ÷ (4 ÷ 2)?

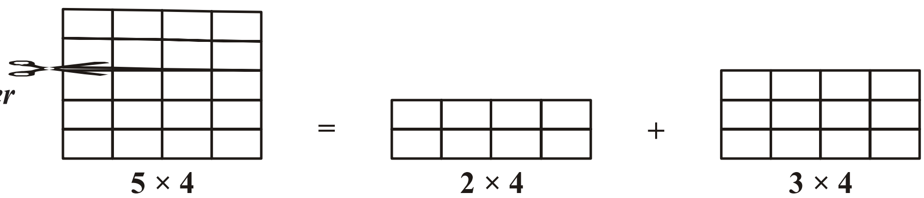
Does the associative property for division hold for the set of whole numbers?

Check if the property holds for subtraction of whole numbers too.

Give 5 examples each for substantiate your answer.

Observe the following

Cut the number grid as shown



The grid paper 5 × 4 has been divided into two pieces 2 × 4 and 3 × 4

Thus, $5 \times 4 = (2 \times 4) + (3 \times 4)$

$= 8 + 12 = 20$

also since $5 = 2 + 3$, we have

$5 \times 4 = (2 + 3) \times 4$ Thus we can say $(2 + 3) \times 4 = (2 \times 4) + (3 \times 4)$

In the same way, $(5 + 6) \times 7 = 11 \times 7 = 77$ and

$(5 \times 7) + (6 \times 7) = 35 + 42 = 77$

We see that both are equal.

This is known as distributive property of multiplication over addition.

Using the distributive property find value of ; $2 \times (5 + 6)$; $5 \times (7 + 8)$, $19 \times 7 + 19 \times 3$

Example-3. Find 12×75 using distributive property.

Solution: $12 \times 75 = 12 \times (70 + 5)$ $\qquad = 12 \times (80 - 5)$

$= (12 \times 70) + (12 \times 5)$ or $\qquad = (12 \times 80) - (12 \times 5)$

$= 840 + 60 = 900$ $\qquad = 960 - 60 = 900$

Do This

Find 25×78; 17×26; $49 \times 68 + 32 \times 49$ using distributive property

Identity (for addition and multiplication)

When you add 7 and 5, you get a new whole number 12. Addition of two whole numbers gives a new whole number. But is this always so for all whole numbers?

Observe the table;

When we add zero to a whole number, we get the same whole number again.

2	+	0	=	2
9	+	0	=	9
0	+	11	=	11
.....	+	25	=	25

Zero is called as the additive identity for whole numbers.

Consider the following table now:

1	×	9	=	9
6	×	5	=	30
6	×	4	=	24
5	×	1	=	5
11	×	1	=	11
2	×	3	=	6

We see that when one of the two numbers being multiplied by 1, the result of multiplication is equal to the other number.

We see when we multiply a whole number with 1, the product will be the same whole number .One is called the multiplicative identity for whole numbers.

Exercise - 2.2

1. Give the results without actually performing the operations using the given information.

i.	28	×	19	=	532	then	19	×	28	=
ii.	1	×	47	=	47	then	47	×	1	=
iii.	a	×	b	=	c	then	b	×	a	=
iv.	58	+	42	=	100	then	42	+	58	=
v.	85	+	0	=	85	then	0	+	85	=
vi.	a	+	b	=	d	then	b	+	a	=

2. Find the sum by suitable rearrangement:
   i. 238 + 695 + 162 ii. 154 + 197 + 46 + 203
3. Find the product by suitable rearrangement.
   i. 25 × 1963 × 4 ii. 20 × 255 × 50 × 6
4. Find the value of the following:
   i. 368 × 12 + 18 × 368 ii. 79 × 4319 + 4319 × 11
5. Find the product using suitable properties:
   i. 205 × 1989 ii. 1991 × 1005
6. A milk vendor supplies 56 liters of milk in the morning and 44 liters of milk in the evening to a hostel. If the milk costs ₹ 30 per liter, how much money he gets per day?
7. Chandana and Venu purchased 12 note books and10 note books respectively. The cost of each note book is ₹ 15,then how much amount should they pay to the shop keeper?
8. Match the following

i. 1991+7 = 7+1991	[ ]	a. Additive identity
ii. 68×50 = 50×68	[ ]	b. Multiplicative identity
iii. 1	[ ]	c. Commutative under addition
iv. 0	[ ]	d. Distributive property of multiplication over addition
v. 879×(100+30) = 879×100+879×30	[ ]	e. Commutative under multiplication

2.4 Patterns in Whole Numbers

We shall try to arrange numbers in elementary shapes made up of dots. The dots would be placed on a grid with equidistant points along the two axes. The shapes we would make are (i) a line (ii) a rectangle (iii) a square and (iv) a triangle. Every number should be arranged in one of these shapes. No other irregular shape is allowed.

Whole numbers can be shown in elementary shapes made up of dots, observe the following.

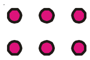
- Every number can be arranged as a line

  The number 2 is shown as 

  The number 3 is shown as  and so on.

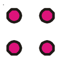
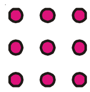
- Some numbers can also be shown as rectangle.

  For example,

  The number 6 can be shown as 

  In this rectangle observe that there are 2 rows and 3 columns.

- Some numbers like 4 or 9 can also be arranged as squares.

4  9 

What are the other numbers that form squares like this? We can see a pattern here.

$4 = 2 \times 2$ this is a perfect square.

$9 = 3 \times 3$ this is also a perfect square.

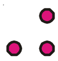
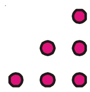
What will be the next number which can be arranged like a square?

Easily we can observe that $4 \times 4 = 16$ and 16 is the next number which is also a perfect square.

Find the next 3 numbers that can be arranged as squares?

Give 5 numbers that can be arranged as rectangles that are not squares.

- Some numbers can also be arranged as triangles.

3  6 

Note that the arrangement as a triangle would have its two sides equal. The number of dots from the bottom row can be like 4, 3, 2, 1. The top row always contains only one dot, so as to make one vertex.

What is the next possible triangle? And the next.

Do you observe any pattern here? Observe the number of dots in each row and think about it. Now complete the following table:

Number	Line	Rectangle	Square	Triangle
2	Yes	No	No	No
3	Yes	No	No	No
4	Yes	No	Yes	No
5				
.....				
25				

Is 1 a square or not? why?

TRY THESE

1. Which numbers can be shown as a line only?
2. Which numbers can be shown as rectangles?
3. Which numbers can be shown as squares?
4. Which numbers can be shown as triangles? eg. 3, 6, ...

Patterns of numbers

We can use patterns to guide us in simplifying processes. Study the following:

1. 296 + 9 = 296 + 10 - 1 = 306 - 1 = 305
2. 296 - 9 = 296 - 10 + 1 = 286 + 1 = 287
3. 296 + 99 = 296 + 100 - 1 = 396 - 1 = 395
4. 296 - 99 = 296 - 100 + 1 = 196 + 1 = 197

Let us see one more pattern:

1. 65 × 99 = 65 (100 - 1) = 6500 - 65 = 6435
2. 65 × 999 = 65 (1000 - 1) = 65000 - 65 = 64935
3. 65 × 9999 = 65 (10000 - 1) = 650000 - 65 = 649935
4. 65 × 99999 = 65 (100000 - 1) = 6500000 - 65 = 6499935

and so on.

Here, we can see a shortcut to multiply a number by numbers of the form 9, 99, 999, ... This type of shortcuts enable us to do sums mentally.

Observe the following pattern: It suggests a way of multiplying a number by 5, 15, 25, ...

(You can think of extending it further).

a. $46 \times 5 = 46 \times \frac{10}{2} = \frac{460}{2} = 230 = 230 \times 1$

b. $46 \times 15 = 46 \times (10 + 5)$
$= 46 \times 10 + 46 \times 5 = 460 + 230 = 690 = 230 \times 3$

c. $46 \times 25 = 46 \times (20 + 5)$
$= 46 \times 20 + 46 \times 5 = 920 + 230 = 1150 = 230 \times 5$ .....

Can you think of some more examples of using such processes to simplify calculations.

EXERCISE - 2.3

1. Study the pattern:

1 × 8 + 1 = 9
12 × 8 + 2 = 98
123 × 8 + 3 = 987

$1234 \times 8 + 4 = 9876$

$12345 \times 8 + 5 = 98765$

Write the next four steps. Can you find out how the pattern works?

2. Study the pattern:

$91 \times 11 \times 1 = 1001$

$91 \times 11 \times 2 = 2002$

$91 \times 11 \times 3 = 3003$

Write next seven steps. Check, whether the result is correct.

Try the pattern for $143 \times 7 \times 1$, $143 \times 7 \times 2$ .....

3. How would we multiply the numbers 13680347, 35702369 and 25692359 with 9 mentally? What is the pattern that emerges.

WHAT HAVE WE DISCUSSED?

1. The numbers 1, 2, 3, ... which we use for counting are known as natural numbers.
2. Every natural number has a successor. Every natural number except 1 has a predecessor.
3. If we add the number zero to the collection of natural numbers, we get the collection of whole numbers W = {0, 1, 2, ...}
4. Every whole number has a successor. Every whole number except zero has a predecessor.
5. All natural numbers are whole numbers, and all whole numbers except zero are natural numbers.
6. We can make a number line with whole numbers represented on it. We can easily perform the number operations of addition, subtraction and multiplication on such a number line.
7. Addition corresponds to moving to the right on the number line, where as subtraction corresponds to moving to the left. Multiplication corresponds to making jumps of equal distance from zero.
8. Whole numbers are closed under addition and multiplication. But whole numbers are not closed under subtraction and division.
9. Division by zero is not defined.
10. Zero is the additive identity and 1 is the multiplicative identity of whole numbers.
11. Addition and multiplication are commutative for whole numbers.
12. Addition and multiplication are associative for whole numbers.
13. Multiplication is distributive over addition for whole numbers.
14. Commutativity, associativity and distributivity of whole numbers are useful in simplifying calculations and we often use them without being aware of them.
15. Pattern with numbers are not only interesting, but are useful especially for mental calculations. They help us to understand properties of numbers better.