

## POLYNOMIAL EXPRESSIONS

A polynomial expression  $S(x)$  in one variable  $x$  is an algebraic expression in  $x$  term as

$$S(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + ax + a_0$$

Where  $a_n, a_{n-1}, \dots, a, a_0$  are constant and real numbers and  $a_n$  is not equal to zero.

*Some Important points to Note:*

S.no	Points
1	$a_n, a_{n-1}, a_{n-2}, \dots, a_1, a_0$ are called the coefficients for $x^n, x^{n-1}, \dots, x^1, x^0$
2	$n$ is called the degree of the polynomial
3	when $a_n, a_{n-1}, a_{n-2}, \dots, a_1, a_0$ all are zero, it is called zero polynomial
4	A constant polynomial is the polynomial with zero degree, it is a constant value polynomial
5	A polynomial of one item is called monomial, two items binomial and three items as trinomial
6	A polynomial of one degree is called linear polynomial, two degree as quadratic polynomial and degree three as cubic polynomial

*Important Concepts on Polynomial:*

Concept	Description
<b>Zero's or roots of the polynomial</b>	It is a solution to the polynomial equation $S(x)=0$ i.e. a number "a" is said to be a zero of a polynomial if $S(a) = 0$ . If we draw the graph of $S(x) = 0$ , the values where the curve cuts the X-axis are called Zeroes of the polynomial
<b>Remainder Theorem's</b>	If $p(x)$ is an polynomial of degree greater than or equal to 1 and $p(x)$ is divided by the expression $(x-a)$ , then the remainder will be $p(a)$
<b>Factor's Theorem's</b>	If $x-a$ is a factor of polynomial $p(x)$ then $p(a)=0$ or if $p(a) = 0, x-a$ is the factor the polynomial $p(x)$

**Geometric Meaning of the Zeroes of the Polynomial:**

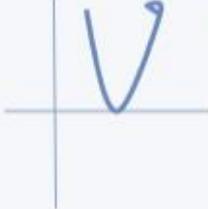
Let's us assume

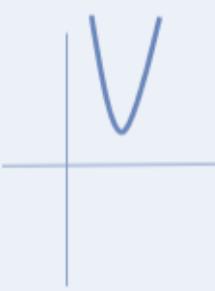
$Y = p(x)$  where  $p(x)$  is the polynomial of any form.

Now we can plot the equation  $y = p(x)$  on the Cartesian plane by taking various values of  $x$  and  $y$  obtained by putting the values. The plot or graph obtained can be of any shapes.

The zeroes of the polynomial are the points where the graph meet  $x$  axis in the Cartesian plane. If the graph does not meet  $x$  axis, then the polynomial does not have any zero's.

Let us take some useful polynomial and shapes obtained on the Cartesian plane

S.no	$y=p(x)$	Graph obtained	Name of the graph	Name of the equation
1	$y = ax + b$ where $a$ and $b$ can be any values ( $a \neq 0$ )  Example $y = 2x + 3$		Straight line.  It intersect the $x$ -axis at $(-b/a, 0)$  Example $(-3/2, 0)$	Linear polynomial
2	$y = ax^2 + bx + c$ where $b^2 - 4ac > 0$ and $a \neq 0$ and $a > 0$ Example $y = x^2 - 7x + 12$		Parabola  It intersect the $x$ -axis at two points  Example $(3, 0)$ and $(4, 0)$	Quadratic polynomial
3	$y = ax^2 + bx + c$ where $b^2 - 4ac > 0$ and $a \neq 0$ and $a < 0$ Example $y = -x^2 + 2x + 8$		Parabola  It intersect the $x$ -axis at two points  Example $(-2, 0)$ and $(4, 0)$	Quadratic polynomial
4	$y = ax^2 + bx + c$ where $b^2 - 4ac = 0$ and $a \neq 0$ $a > 0$  Example $y = (x - 2)^2$		Parabola  It intersect the $x$ -axis at one points	Quadratic polynomial

<p><b>5</b></p>	<p><math>y=ax^2+bx+c</math></p> <p>where</p> <p><math>b^2-4ac &lt; 0</math> and <math>a \neq 0</math> <math>a &gt; 0</math></p> <p>Example</p> <p><math>y=x^2-2x+6</math></p>		<p>Parabola</p> <p>It does not intersect the x-axis</p> <p>It has no zero's</p>	<p>Quadratic polynomial</p>
<p><b>6</b></p>	<p><math>y=ax^2+bx+c</math></p> <p>where</p> <p><math>b^2-4ac &lt; 0</math> and <math>a \neq 0</math> <math>a &lt; 0</math></p> <p>Example</p> <p><math>y=-x^2-2x-6</math></p>		<p>Parabola</p> <p>It does not intersect the x-axis</p> <p>It has no zero's</p>	<p>Quadratic polynomial</p>
<p><b>7</b></p>	<p><math>y=ax^3+bx^2+cx+d</math></p> <p>where <math>a \neq 0</math></p>	<p>It can be of any shape</p> 	<p>It will cut the x-axis at the most 3 times</p>	<p>Cubic Polynomial</p>
<p><b>8</b></p>	<p><math>a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0</math></p> <p>Where <math>a_n \neq 0</math></p>	<p>It can be of any shape</p> 	<p>It will cut the x-axis at the most n times</p>	<p>Polynomial of n degree</p>

**Relation between coefficient and zeros of the polynomial:**

S.no	Type of Polynomial	General form	Zero's	Relationship between Zero's and coefficients
1	Linear polynomial	$ax+b, a \neq 0$	1	$k = \frac{-\text{constant term}}{\text{Coefficient of } x}$
2	Quadratic	$ax^2+bx+c, a \neq 0$	2	$k_1 + k_2 = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$  $k_1 k_2 = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$
3	Cubic	$ax^3+bx^2+cx+d, a \neq 0$	3	$k_1 + k_2 + k_3 = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$  $k_1 k_2 k_3 = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$  $k_1 k_2 + k_2 k_3 + k_1 k_3 = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

**Formation of polynomial when the zeroes are given:**

Type of polynomial	Zero's	Polynomial Formed
Linear	$k=a$	$(x-a)$
Quadratic	$k_1=a$ and $k_2=b$	$(x-a)(x-b)$ Or $x^2-(a+b)x+ab$ Or $x^2-(\text{Sum of the zero's})x+\text{product of the zero's}$
Cubic	$k_1=a, k_2=b$ and $k_3=c$	$(x-a)(x-b)(x-c)$

**Division algorithm for polynomial:**

Let's  $p(x)$  and  $q(x)$  are any two polynomial with  $q(x) \neq 0$ , then we can find polynomial  $s(x)$  and  $r(x)$  such that

$$P(x) = s(x) q(x) + r(x)$$

Where  $r(x)$  can be zero or degree of  $r(x) < \text{degree of } g(x)$

$$\boxed{\text{Dividend} = \text{Quotient} \times \text{Divisor} + \text{Remainder}}$$