Chapter 2

Methods of Indeterminate Analysis

CHAPTER HIGHLIGHTS

- *Introduction*
- *Strain energy method*
- *Work done by a force on a member*
- *Law of reciprocal deflection or Maxwell's reciprocal deflection*
- *Bettis law*
- *Castigliano's theorem*
- *Moment distribution method*
- *Sinking of supports*
- *Portal frames*
- *Slope–deflection equations*

INTRODUCTION

In this chapter, the various methods used to analyses an indeterminate structure are discussed. Broadly all these various methods are classified into force method and displacement method.

Force Method

Also known as flexibility coefficient method or compatibility method.

- In this method, redundant forces are chosen as unknowns and compatibility equations are used for solution.
- The various methods of this category are
  1. Method of consistent deformation
  2. Three moment theorem
  3. Column analogy method
  4. Elastic centre method
  5. Maxwell–Mohr equations
  6. Catigliano's theorem of minimum strain energy.
- This method is suitable when $D_S < D_K$.

Displacement Method

- Also known as stiffness coefficient method or equilibrium method.
- In this method, displacements are chosen as unknowns and equilibrium equations are used for solution.
- The various methods coming under this category are
  1. Slope deflection method
  2. Moment distribution method
  3. Kani's rotation contribution method.
- This method is suitable when $D_K < D_S$.

Methods	Unknowns	Equations Used for Solution	Coefficients of the Unknowns
Force method	Forces	Compatibility	Flexibility coefficient
Displacement method	Displacements	Equilibrium	Stiffness coefficients

STRAIN ENERGY METHOD

Definition: Strain energy is defined as the internal energy stored by external work done due to loads.

Strain energy in members:

1. Due to axial loading,

$$U = \frac{P^2 L}{2AE} \quad \text{or} \quad U = \frac{\sigma^2}{2E} \times \text{Volume}$$

2. Due to bending,

$$U = \int_0^L \frac{M_x^2\, dx}{2EI_x}$$

3. Due to a uniform bending moment,

$$U = \int_0^L \frac{MM^2 dx}{2EI_x}$$

4. Due to shear,

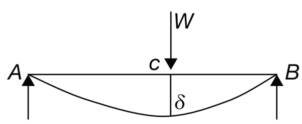
$$U = \int_V \frac{\tau^2}{2G} dv$$

5. Due to torsion,

$$U = \int_0^L \frac{T^2 dx}{2GJ}$$

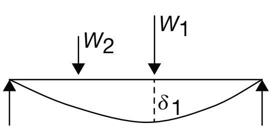
Work Done by a Force on a Member

Real Work The work done by the load is known as Real work, if a work acts on a member and produces a deflection 'δ' in its line of action by virtue of the same load.



Work done by the load (W) $= \dfrac{1}{2} W\delta$.

Virtual Work The work done by a load is known as virtual work, if a member subjected to a load 'W_1' is given a deformation δ_1 in the line of action of W_1 by virtue of other external load.

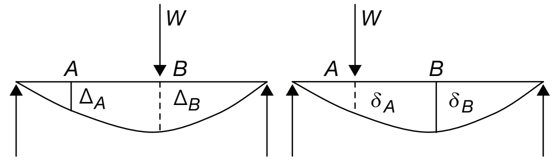


Work done by W_1 due to the displacement caused by W_2 in its direction $= W_1\delta_1$.

Law of Reciprocal Deflection or Maxwell's Reciprocal Deflection

Theorem

Statement In any structure, the deflection of any point A due to a load W at any other point B is numerically equal to the deflection of point B due to a load W applied at a point A.



$W \cdot \Delta_A = W \cdot \delta_B$

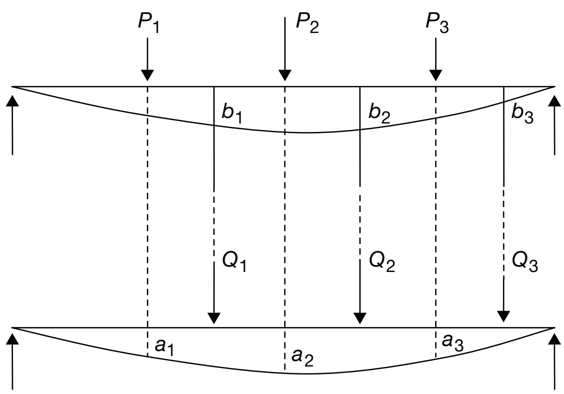
$$\Delta_A = \delta_B$$

Applicable for both determinate and indeterminate structures.

Betti's Law

- Generalized form of Maxwells's law.

Statement In any structure the material of which is elastic and follows Hooke's law and in which the supports are unyielding and the temperature is constant, the virtual work done by a system of forces P_1, P_2, P_3 during the deflection caused by a system of forces Q_1, Q_2, Q_3, ... is equal to virtual work done by the system of forces Q_1, Q_2, Q_3, ... during the deflection caused by the system of forces P_1, P_2, P_3.



∴ From Betti's theorem,

$$P_1a_1 + P_2a_2 + P_3a_3 = Q_1b_1 + Q_2b_2 + Q_3b_3$$

Castigliano's Theorem

First Theorem

In any beam or truss, the deflection at any point is given by the partial derivative of strain energy with respect to a force acting at the point in the direction in which deflection is desired.

$$\delta = \frac{\partial U}{\partial W} \qquad \theta = \frac{\partial U}{\partial M}$$

Important points [While determining deflections and slopes using Castigliano's theorem]

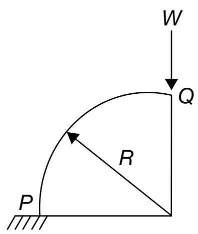
1. The material is to be elastic and follows Hookes law and in which supports are unyielding and temperature to be constant.

2. In case the deflection is to be determined at a point where no load acts introduce a fictitious force Q at the point in the direction of the desired deflection. The partial derivative of the strain energy stored with respect to Q is determined, and Q is put equal to zero.

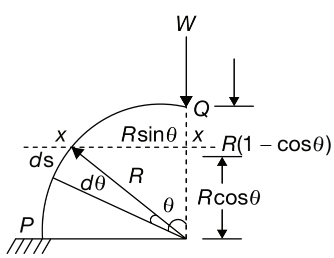
SOLVED EXAMPLES

Example 1

A uniform beam (EI = constant) PQ in the form of a quarter circle of radius R is fixed at end P and free at the end Q, where a load W is applied as shown. The vertical downward displacement, δ_q, at the loaded point Q is given by $\delta_q = \beta\left(\frac{WR^3}{EI}\right)$. Find the value of β (correct to 4 decimal places).



Solution



Vertical deflection at point Q:

The bending moment at any section X,

$$M_x = -WR\sin\theta$$

Strain energy stored

$$U = \int \frac{M_x^2\, ds}{2EI}$$

$$ds = Rd\theta, d\theta = 0 \text{ to } \frac{\pi}{2}$$

$$\therefore U = \int \frac{(-WR\sin\theta)^2\, Rd\theta}{2EI}$$

$$U = \frac{W^2R^3}{2EI}\int_0^{\pi/2} \sin^2\theta\, d\theta$$

$$U = \frac{W^2R^3}{2EI} \times \frac{\pi}{4}$$

Vertical deflection of Q,

$$\delta_q = \frac{\partial U}{\partial W} = \frac{\partial}{\partial W}\left(\frac{W^2R^3}{2EI}\right) \times \frac{\pi}{4}$$

$$\delta_q = \frac{\pi}{4}\left[\frac{WR^3}{EI}\right]$$

$$\beta = \frac{\pi}{4} = 0.7853.$$

Second Theorem

Used to analyse statically indeterminate structures based on principle of least work.

Statement In any and every case of statical indetermination wherein, an indefinite number of different values of the redundant forces satisfy the conditions of statical equilibrium, their actual values are given by those that render the total strain energy stored to a minimum.

$$\boxed{\frac{\partial U}{\partial X} = 0}$$

MOMENT DISTRIBUTION METHOD

- Developed by Prof. Hardy Cross in 1930.
- Used in the analysis of statically indeterminate beams and frames.

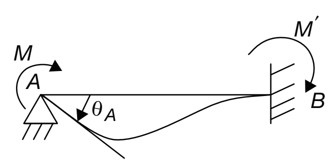
Definitions

Member Stiffness When a structural member of uniform section is subjected to a moment at one end only, then the moment required so as to rotate that end to produce unit slope, is called the stiffness of the member.

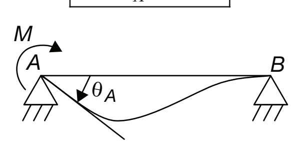
Member stiffness,

$$S = \frac{4EI}{L}, \text{ when far end fixd}$$

$$S = \frac{3EI}{L}, \text{ when far end pinned or roller supported}$$



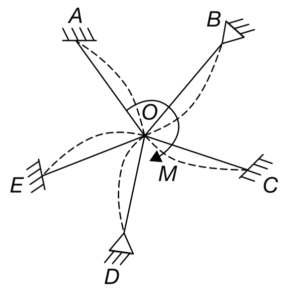
$$\boxed{S = \frac{M}{\theta_A} = \frac{4EI}{L}}$$



$$\boxed{S = \frac{M}{\theta_A} = \frac{3EI}{L}}$$

The Distribution Theorem

Statement A moment which is applied to a structural joint to produce rotation without translation gets distributed among the connecting members at the joint in the same proportion as their stiffness.



$M = M_{OA} + M_{OB} + M_{OC} + M_{OD} + M_{OE} =$ Total applied moment

$S = S_{OA} + S_{OB} + S_{OC} + S_{OD} + S_{OE} =$ Total stiffness of joint 'O'

$$M_{OA} = \left[\frac{S_{OA}}{S}\right] \times M; M_{OB} = \left[\frac{S_{OB}}{S}\right] \times M;$$

$$M_{OC} = \left[\frac{S_{OC}}{S}\right] \times M; M_{OD} = \left[\frac{S_{OD}}{S}\right] \times M;$$

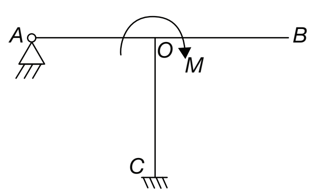
$$M_{OE} = \left[\frac{S_{OE}}{S}\right] \times M.$$

The ratio $\frac{S_{OA}}{S}$ is called distribution factor for the member OA at joint O.

Definition of Distribution Factor The distribution factor for a member at a joint is the ratio of the stiffness of the member to the total stiffness of all the members meeting at the joint.

Example 2

The frame below shows three beam elements OA, OB, OC with identical length L and flexural rigidity EI, subject to an external moment M applied at the rigid joint O. The correct set of bending moments $\{M_{OA}, M_{OB}, M_{OC}\}$ are



EI/L is constant for all three members.

(A) $\left\{\frac{3M}{8}, \frac{M}{8}, \frac{4M}{8}\right\}$ (B) $\left\{\frac{3M}{11}, \frac{4M}{11}, \frac{4M}{11}\right\}$

(C) $\left\{\frac{M}{3}, \frac{M}{3}, \frac{M}{3}\right\}$ (D) $\left\{\frac{3M}{7}, 0, \frac{4M}{7}\right\}$

Solution

Moment applied at a joint is distributed to the members based on their distribution factors.

$$M_{OA} = \left(\frac{S_{OA}}{S}\right)M; M_{OB} = \left(\frac{S_{OB}}{S}\right)M; M_{OC} = \left(\frac{S_{OC}}{S}\right)M$$

$$S_{OA} = \frac{3EI}{L}\text{[for end hinged]}$$

$$S_{OB} = 0 \text{ [for end free]}$$

$$S_{OC} = \frac{4EI}{L}\text{[for end fixed]}$$

$$S = S_{OA} + S_{OB} + S_{OC} = \frac{3EI}{L} + 0 + \frac{4EI}{L}$$

$$S = \frac{7EI}{L}$$

$$M_{OA} = \left[\frac{(3EI/L)}{(7EI/L)}\right] \times M = \frac{3M}{7}$$

$$M_{OB} = \left[\frac{0}{(7EI/L)}\right] \times M = 0$$

$$M_{OC} = \left[\frac{(3EI/L}{(7EI/L)}\right] \times M = \frac{4M}{7}$$

Hence, the correct answer is option (D).

Relative Stiffness

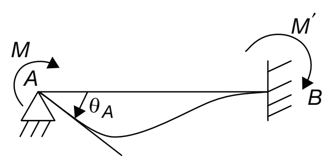
- The relative stiffness of a member of a joint whose farther end is fixed is $\frac{I}{l}$

  Where

  $I =$ Moment of inertia of section

  $l =$ Length of the member
- The relative stiffness of a member at a joint whose farther end is hinged or simply supported is $\frac{3I}{4l}$.

Carry Over Factor It is defined as an amount of moment 'M' that is carried from the pin to the wall.



Moment M at the pin induces a moment of $M' = \frac{M}{2}$ at fixed end.

$$\therefore \text{Carry over factor} = \frac{\text{Moment induced at other end}}{\text{Moment applied at one end}}$$

$$\text{Carry over factor} = \frac{M}{2}, \text{ for far end fixed}$$
$$= 0, \text{ for far end pinned.}$$

Moment Distribution Procedure for Analysis

The following procedure is adopted:

Step 1: Calculation of distribution factors:

Joint	Member	Relative stiffness = K	ΣK	Distribution factor = $\frac{K}{\Sigma K}$

NOTE

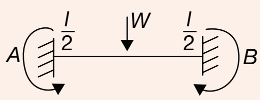
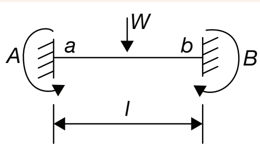
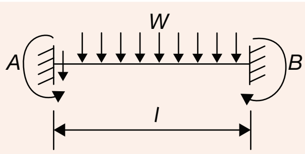
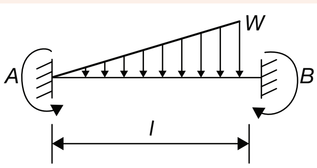
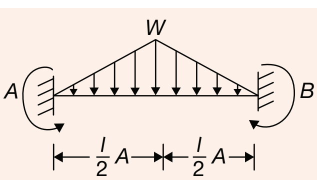
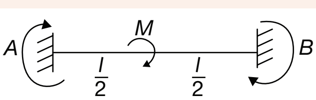
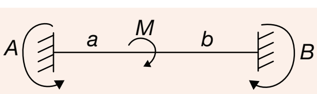
Sum of distribution factors at a joint is equal to one.

Step 2: Calculation of fixed end moments:

Sign convention:

1. Clockwise moments that act on the member are considered positive.
2. Whereas, counter clockwise moments are negative.

Fixed end moments for some standard cases

S.No.	Loading	Fixed End Moment M_{AB}	Fixed End Moment M_{BA}
1.		$\frac{-Wl}{8}$	$\frac{+Wl}{8}$
2.		$\frac{-Wab^2}{l^2}$	$\frac{+Wa^2b}{l^2}$
3.		$\frac{-Wl^2}{12}$	$\frac{+Wl^2}{12}$
4.		$\frac{-Wl^2}{30}$	$\frac{+Wl^2}{20}$
5.		$\frac{-5}{96}Wl^2$	$\frac{+5}{96}Wl^2$
6.		$\frac{+M}{4}$	$\frac{+M}{4}$
7.		$\frac{+Mb(3a-l)}{l^2}$	$\frac{+Mb(3b-l)}{l^2}$

Step 3: Moment distribution process:

1. Determine the moment that is needed to put each joint in equilibrium.
2. Release the joints and distribute the counter balancing moments into the connecting span at each joint.
3. Carry these moments in each span over to its other end by multiplying with the carry over factor.
4. These iterations are continued till the unbalanced moment at a joint is zero or negligible.

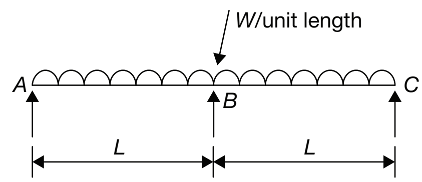
Example 3

A two span continuous beam having equal spans each of length 'L' is subjected to a uniformly distributed load 'W' per unit length. The beam has constant flexural rigidity. The bending moment at middle support is

(A) $\frac{WL^2}{4}$ (B) $\frac{WL^2}{8}$

(C) $\frac{WL^2}{12}$ (D) $\frac{WL^2}{16}$

Solution

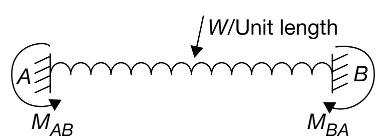


Step 1: Calculation of distribution factors:

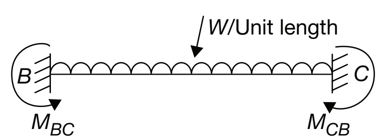
Joint	Member	Relative Stiffness = K	ΣK	Distribution Factor $\frac{K}{\Sigma K}$
B	BA	$\frac{3I}{4L}$	$\frac{3I}{2L}$	$\frac{1}{2}$
	BC	$\frac{3I}{4L}$		$\frac{1}{2}$

Step 2: Calculation of fixed end moments:

For calculating fixed moments, assume each support as fixed.



$$M_{AB} = \frac{-WL^2}{12} \quad M_{BA} = \frac{+WL^2}{12}$$



$$M_{BC} = \frac{-WL^2}{12} \quad M_{CB} = \frac{+WL^2}{12}$$

Step 3: Moment distribution process:

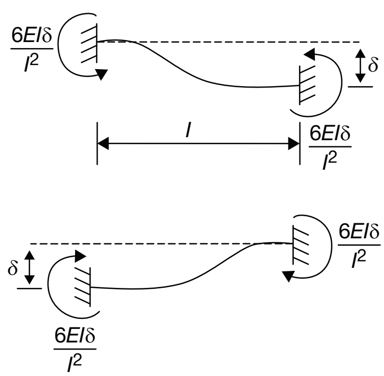
Joint A		B		C
Distribution factors		0.5		0.5
Initial fixed end moments	$\frac{-WL^2}{12}$	$\frac{+WL^2}{12}$	$\frac{-WL^2}{12}$	$\frac{+WL^2}{12}$
Release moments	$\frac{+WL^2}{12}$			$\frac{-WL^2}{12}$
Carry over		$\frac{WL^2}{24}$	$\frac{-WL^2}{24}$	
Final fixed end moment	0	$\frac{WL^2}{8}$	$\frac{-WL^2}{8}$	0

∴ Bending moment at middle support is $\frac{WL^2}{8}$

Hence, the correct answer is option (B).

Sinking of Supports

Both Ends Fixed Due to the sinking of supports, there will be an additional fixed end moments at each end whose magnitude is given by $\frac{6EI\delta}{l^2}$, where δ is the difference of the level of end supports.



- The nature of the fixed end moment is anticlockwise at each end if the left support is at a higher level.
- The nature of the fixed end moment is clockwise at each end if the right support is at a higher level.

One End Fixed–Other End Hinged

Due to the sinking of support, there will be an additional fixed end moment at fixed end only, whose magnitude is given by $\frac{3EI\delta}{l^2}$, where 'δ' is the difference of the level of end supports.

Portal Frames

Portal frames may be classified into:

1. Non-sway type of frames
2. Sway type of frames

Non-sway Type of Frames

In this case, frame is subjected to symmetrical loading, identical end conditions and two columns of frame are identical so that the frame does not sway to any side.

Sway Type of Frames

These may be classified into:

1. Pure sway frame
2. General sway frame

Pure Sway Frame

This case arises when the portal frame carried a horizontal load at the level of the beam.

General Sway Frame

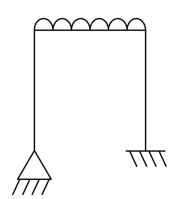
The frame will sway in the direction which has lighter moment (lesser value of I/L) when subjected to a symmetrical loading and do not have same properties, [i.e., Different conditions, Different material properties, etc].

Example 4

A single bay single storey portal frame has a hinged left support and a fixed right support. It is loaded with udl on the beam. Which one of the following statements is true with regard to the deformation of the frame?

(A) It would sway to the left side
(B) It would sway to the right side
(C) It would not sway at all
(D) None of these

Solution



It would sway to the left side since less moment in left side as compared to the right side.

Moments due to sway towards left side $= \frac{3EI\delta}{l^2}$

Moments due to sway towards right side $= \frac{6EI\delta}{l^2}$

Therefore, sway towards left side.

Hence, the correct answer is option (A).

Slope Deflection Method

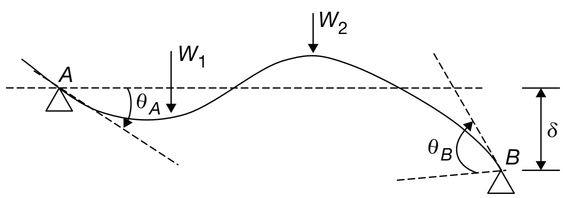
- Presented by Prof. George A. Maney in 1915.
- Used to analyse statically indeterminate beams and frames with rigid joints.

Sign Conventions The following sign conventions are used:

1. At the end of any span clockwise end moments and clockwise slopes are positive.
2. The downward deflection of the right end of span with respect to its left end is positive.
3. Deflection of the upper end towards the right relative to the lower end is positive.

Slope–Deflection Equations

Consider an intermediate span *AB* subjected to an external load system. Let θ_A and θ_B be the slopes at the ends *A* and *B*. let 'δ' be the transverse downward deflection of the right end *B* with respect to the left end *A*.



Final moments at the end *A*,

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

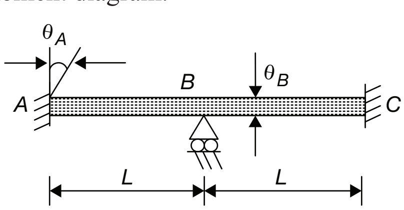
Final moments at the end *B*,

$$M_{BA} = M_{FBA} = \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

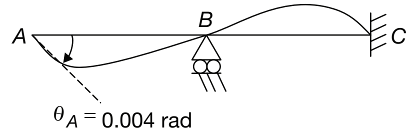
The above two equations are Slope–deflection equations.

Example 5

The two span continuous beam shown below is subject to a clockwise rotational slip θ_A = 0.004 radian at the fixed end *A*. Applying the slope–deflection method of analysis, determine the slope θ_B at *B*. Given that flexural rigidity *EI* = 25000 kNm^2 and span *L* = 5 m. Determine the end moments (in kN/m units) in the two spans, and draw the bending moment diagram.



Solution



By using slope deflection equation for span *BA*;

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

Since no loads are acting on the span and also M_{FBA} = 0; $\delta = 0$

$$\therefore M_{BA} = \frac{2EI}{L}(2\theta_B + \theta_A)$$

By using slope–deflection equation for span *BC*,

$M_{BA} = 0$; $\delta = 0$ [no loads acting on beam and no settlement of supports]

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right)$$

$$M_{BC} = \frac{2EI}{L}(2\theta + \theta_C)$$

For equilibrium;

$$M_{BA} + M_{BC} = 0$$

$$\frac{2EI}{L}(2\theta_B + \theta_A) + \frac{2EI}{L}(2\theta_B + \theta_C) = 0$$

θ_A = 0.004 radians (given) and $\theta_C = 0$ (fixed)

$$\therefore \frac{2EI}{L}(2\theta_B + 0.004) + \frac{2EI}{L}(2\theta_B) = 0$$

$4\theta_B + 0.004 = 0$

$\theta_B = -0.001$ radians

The negative sign above indicates anticlockwise rotation.

θ_B = 0.001 radians (anticlockwise).

Support Moment's Calculation

$$M_{AB} = \frac{2EI}{L}(2\theta + \theta_B)$$

$$= \frac{2\times 25000}{5}(2\times 0.004 - 0.001)$$

M_{AB} = 70 kN/m (clockwise)

$$M_{AB} = \frac{2EI}{L}(2\theta_B + \theta_A)$$

$$= \frac{2\times 25000}{5}(2\times(-0.001) + 0.004)$$

M_{AB} = 20 kN/m (clockwise)

$$M_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C)$$

$$= \frac{2\times 25000}{5}(2\times(-0.001)+0)$$

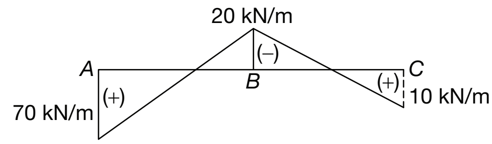
$$M_{BC} = -20 \text{ kN/m (anticlockwise)}$$

$$M_{CB} = \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$= \frac{2\times 25000}{5}(2\times 0+(-0.001))$$

$$M_{CB} = -20 \text{ kN/m (anticlockwise)}$$

Bending Moment Diagram



Exercises

1. For which of the following conditions, the virtual work should be zero according to the principle of virtual work?

I. A body moving with constant acceleration.
II. A body rotating with constant speed.
III. A body in equilibrium.
IV. A body moving with constant momentum.

Select the correct answer using the codes given below:

(A) I only (B) I and II
(C) III only (D) IV only

2. In flexibility method the unknown quantities are ______, whereas in stiffness method the unknown quantities are ______.

3. Which of the following statements is true with regard to the flexibility method of analysis?

(A) The method is used to analyze determinate structures.
(B) The method is used only for manual analysis of indeterminate structures.
(C) The method is used for analysis of flexible structures.
(D) The method is used for analysis of indeterminate structures with lesser degree of static indeterminacy.

4. Methods of indeterminate structural analysis may be grouped under either force method or displacement method. Which of the groupings given below is correct?

	Force Method	Displacement Method
(A)	Moment distribution method, consistent deformation method	Method of three moments, slope deflection method
(B)	Method of three moments, consistent deformation method	Moment distribution method, slope deflection method
(C)	Slope deflection method, consistent deformation method	Moment distribution method, method of three moments
(D)	Moment distribution. Method of three moments	Slope deflection method, consistent deformation method

5. Match List I with List II and select the correct answer using the codes given below the lists:

	List I		List II
a.	Slope deflection method	1.	Force method
b.	Moment distribution method	2.	Displacement
c.	Method of three moments		
d.	Castigliano's second theorem		

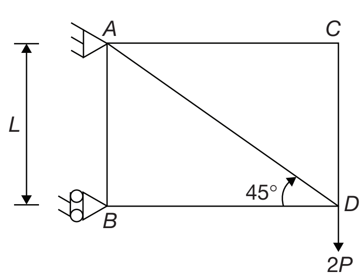
Codes:

	a	b	c	d		a	b	c	d
(A)	1	2	1	2	(B)	1	1	2	2
(C)	2	2	1	1	(D)	2	1	2	1

6. The unit load method used in structural analysis is

(A) applicable only to statically indeterminate structures.
(B) another name for stiffness method.
(C) an extension of Maxwell's reciprocal theorem.
(D) derived from Castigliano's theorem.

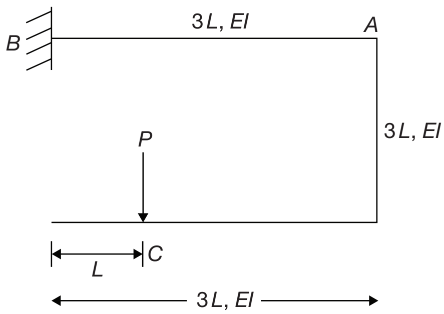
7. The strain energy stored in member '*AB*' of the pin jointed truss shown aside when '*E*' and '*A*' are same for all members is



(A) $\frac{2P^2L}{AE}$ (B) $\frac{P^2L}{AE}$

(C) $\frac{P^2L}{2AE}$ (D) zero

8. For the structure shown, the vertical deflection at point *A* is given by



(A) $\frac{PL^3}{81EI}$ (B) $\frac{2PL^3}{81EI}$

(C) zero (D) $\frac{PL^3}{72EI}$

9. A single bay portal frame of height '*h*' fixed at the base is subjected to a horizontal displacement 'δ' at the top. The base moments developed is proportional to _______, where '*I*' is the moment of inertia of the cross-section.

(A) I/h (B) I/h^2

(C) I/h^3 (D) None of these

10. The ratio of the stiffness of a beam at the near end when the far end is hinged to the stiffness of the beam at the near end when the far end is fixed is

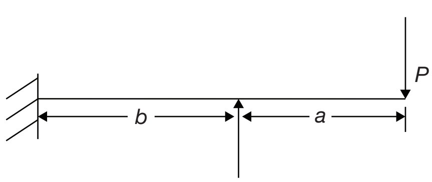
(A) $\frac{1}{2}$ (B) $\frac{3}{4}$

(C) 1 (D) $\frac{4}{3}$

11. A single bay single storey portal frame has a hinged left support and a fixed right support. It is loaded with UDL on the beam. Which one of the following statements is true with regard to the deformation of the frame?

(A) It would sway to the left side.
(B) It would sway to the right side.
(C) It would not sway at all.
(D) None of these

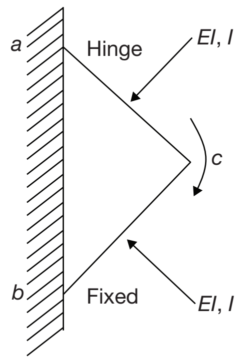
12. The magnitude of the bending moment at the fixed support of the beam is equal to



(A) $P \cdot a$ (B) $\frac{P \cdot a}{2}$

(C) $P \cdot b$ (D) $P(a + b)$

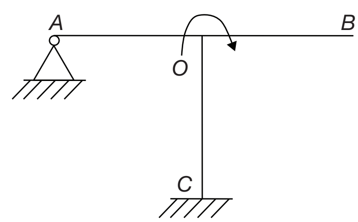
13. Rotational stiffness-coefficient, K_{11} for the frame having two members of equal $\frac{EI}{l}$ is given by



(A) $\frac{5EI}{l}$ (B) $\frac{6EI}{l}$

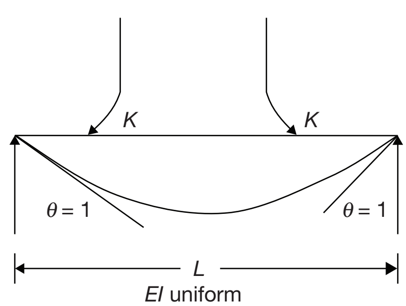
(C) $\frac{7EI}{l}$ (D) $\frac{8EI}{l}$

14. The frame shows three beam elements *OA*, *OB* and *OC*, with identical length *L* and flexural rigidity *EI*, subject to an external moment *M* applied at the rigid joint *O*. The correct set of bending moments $\{M_{OA}, M_{OB}, M_{OC}\}$ is



(A) $\{3M/8, M/8, 4M/8\}$
(B) $\{3M/11, 4M/11, 4M/11\}$
(C) $\{M/3, M/3, M/3\}$
(D) $\{3M/7, 0, 4M/7\}$

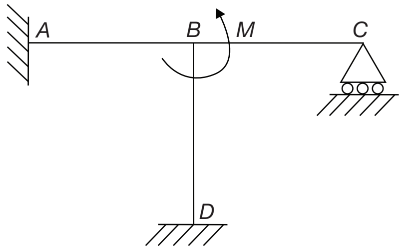
15. The stiffness *K* of a beam deflecting in a symmetric mode, as shown in the figure, is



(A) $\frac{EI}{L}$ (B) $\frac{2EI}{L}$

(C) $\frac{4EI}{L}$ (D) $\frac{6EI}{L}$

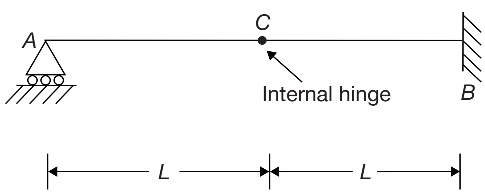
16. All members of the frame shown in the figure have the same flexural rigidity EI and length L. If a moment M is applied at joint B, the rotation of the joint is



(A) $\frac{ML}{12EI}$ (B) $\frac{ML}{11EI}$

(C) $\frac{ML}{8EI}$ (D) $\frac{ML}{7EI}$

17. Cary-over factor C_{AB} for the beam shown in the figure is



(A) 1/4 (B) 1/2

(C) 3/4 (D) 1

18. The number of simultaneous equations to be solved in the slope deflection method is equal to

(A) static indeterminacy.

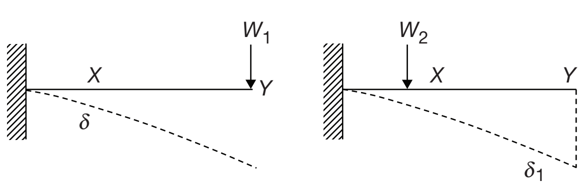
(B) kinematic indeterminacy.

(C) number of joint displacements in the structure.

(D) None of these

19. Flexibility of structure may be defined as the displacement caused by _______ force and stiffness of a structure may be defined as the force required for _______ displacement.

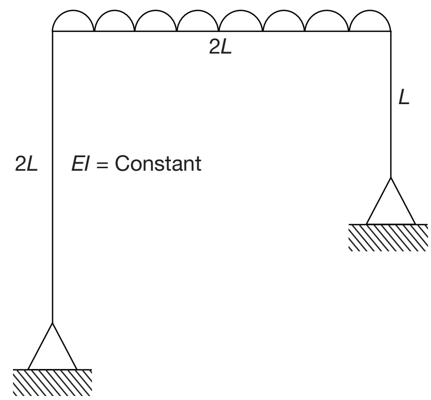
20. In the cantilever beam shown in the given figure, δ_2 is the deflection under X due to load W_1 at Y and δ_1 is the deflection under Y due to load W_2 at X. The ratio of $\frac{\delta_1}{\delta_2}$ is



(A) $\frac{W_1}{W_2}$ (B) $\frac{W_2}{W_1+W_2}$

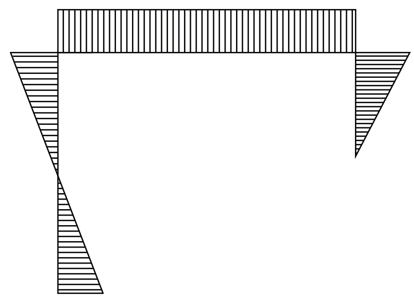
(C) $\frac{W_2}{W_1}$ (D) $\frac{W_1}{W_1+W_2}$

21. The given figure shows a portal frame with loads.

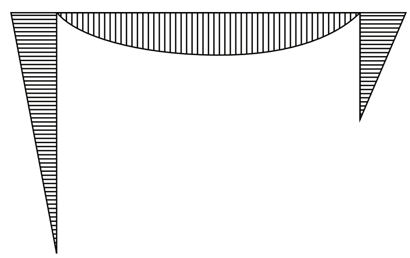


The bending moment diagram for this frame will be

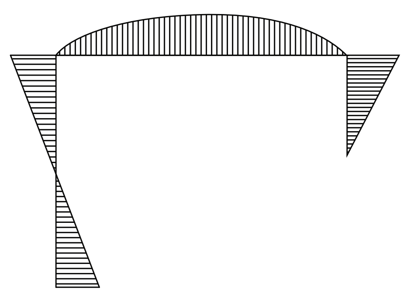
(A)



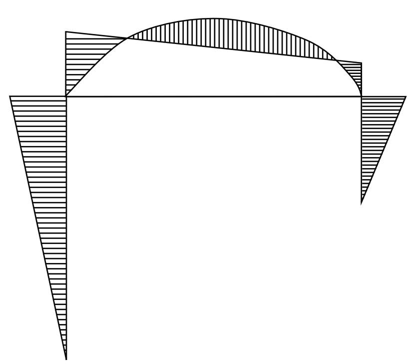
(B)



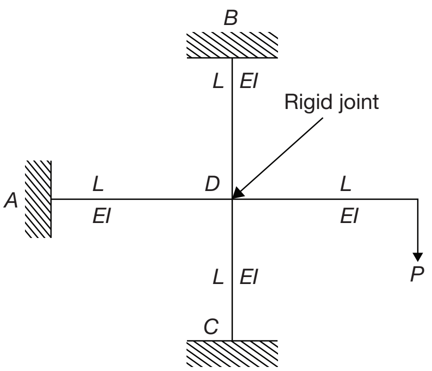
(C)



(D)



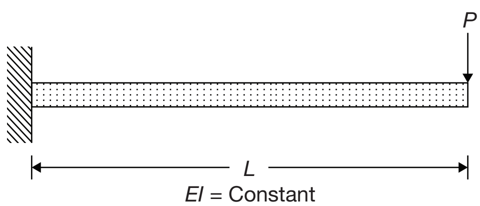
22. The given figure shows a frame loaded with a single concentrated load P. The fixed-end moment developed at support A will be



(A) $\frac{PL}{8}$ (B) $\frac{PL}{6}$

(C) $\frac{PL}{4}$ (D) $\frac{PL}{3}$

23. The strain energy due to bending in the cantilever beam shown in the figure is

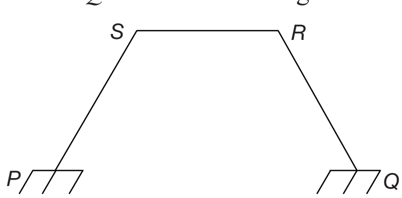


(A) $\frac{PL}{3EI}$ (B) $\frac{P^2L^3}{6EI}$

(C) $\frac{P^2L^3}{EI}$ (D) $\frac{P^2L^3}{2EI}$

24. Clapeyron's theorem is applied to
(A) simply supported beam.
(B) propped cantilever beam.
(C) fixed and continuous beam.
(D) continuous beam only.

25. The degrees of freedom of the rigid frame with clamped ends at P and Q as shown in the figure is



(A) 2 (B) 3
(C) 4 (D) zero

26. The cantilever beam AB of length 'l' fixed at A and free at other end is subjected to a concentrated load W at its free end. The strain energy (U) stored in a beam is (EI constant)

(A) $\frac{W^2l^2}{4EI}$ (B) $\frac{Wl^3}{6EI}$

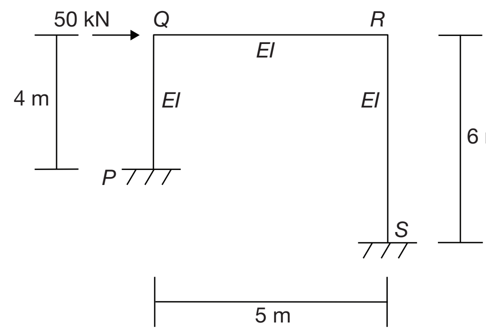
(C) $\frac{W^3l^3}{6EI}$ (D) $\frac{Wl}{EI}$

27. The bending moment induced at fixed end of cantilever beam of span 'l' if the free end undergoes a unit displacement without rotation is

(A) $\frac{3EI}{l^2}$ (B) $\frac{5EI}{l^2}$

(C) $\frac{6EI}{l^2}$ (D) $\frac{4EI}{l^2}$

28. The slope deflection equation at the end Q of member QR for the frame shown in the given figure is



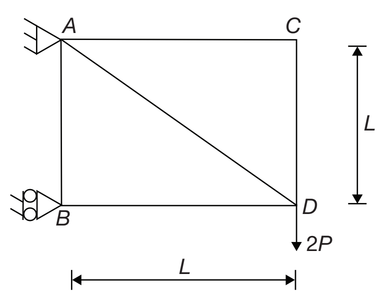
(A) $M_{QR} = \frac{2EI}{5}(2\theta_Q + \theta_R)$

(B) $M_{QR} = \frac{2EI}{5}(2\theta_Q - \theta_R)$

(C) $M_{QR} = \frac{2EI}{5}(2\theta_R - \theta_R)$

(D) $M_{QR} = \frac{EI}{5}(2\theta_Q + \theta_R)$

29. The strain energy stored in the member AB of the pin jointed truss shown in the figure is (A and E is same for all members)



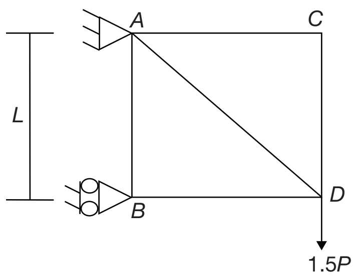
(A) $\frac{4P^2L}{2AE}$ (B) zero

(C) $\frac{P^2L}{AE}$ (D) $\frac{2P^2L}{3AE}$

30. The unit load method used in structural analysis is
(A) applicable only to indeterminate structures.
(B) derived from castigliano's, theorem.
(C) another name for stiffness method.
(D) None of these

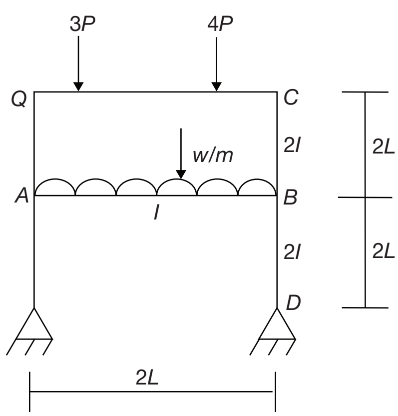
31. For linear elastic systems, the type of displacement function for strain energy is
(A) quadratic (B) linear
(C) cubic (D) quartic

32. The strain energy started in the member AB of the pin jointed truss shown in the figure when E and A are same for all members is



(A) $\frac{1.5P^2L}{AE}$ (B) $\frac{P^2L}{AE}$
(C) zero (D) $\frac{2P^2L}{AE}$

33. In the portal frame shown below, what are the distribution factors for member BA, BC, BD respectively?



(A) 1/5, 2/5, 2/5 (B) 2/5, 1/5, 1/5
(C) 1/3, 1/3, 1/3 (D) None of these

34. A propped cantilever beam PQ with fixed edge 'P' is propped at 'Q' and carries a UDL of w/m over the entire span. If the prop displaces upward by 2 mm, which one of the following is true? (If prop reaction = R_Q, moment at $P = M_P$)
(A) Both R_Q and M_P increase
(B) R_Q increases, and M_P decreases
(C) R_Q decreases and M_P increases
(D) both R_Q and M_P decreases

35. A propped cantilever beam of span 'L' is loaded with UDL of intensity w/unit length, all through the span. Bending moment at fixed end is _______.
(A) $\frac{WL^2}{8}$ (B) $\frac{WL^2}{2}$
(C) $\frac{WL^2}{12}$ (D) $\frac{WL^2}{24}$

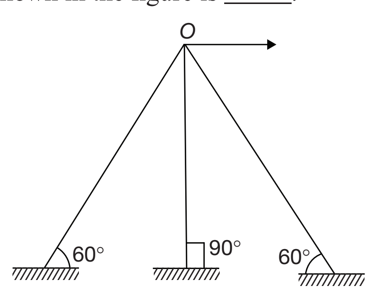
36. A homogeneous, simply supported prismatic beam of width B, depth D and span 'L' is subjected to a concentrated load of magnitude P. The load can be placed anywhere along the span of beam. The maximum flexural stress developed in the beam is
(A) $\frac{3}{4}\frac{PL}{BD^2}$ (B) $\frac{4}{3}\frac{PL}{BD^2}$
(C) $\frac{3}{2}\frac{PL}{BD^2}$ (D) $\frac{2}{3}\frac{PL}{BD^2}$

37. Sum of the distribution factors of the members meeting at a joint is ______.
(A) 0 (B) < 1
(C) $= 1$ (D) > 1

38. Which of the following is not a force method?
(A) The theorem of three moments
(B) Castigliano's theorem
(C) Moment distribution method
(D) Method of consistent deformation

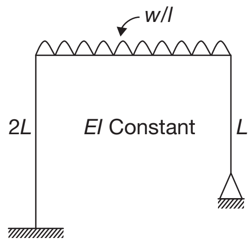
39. A beam is hinged at A and fixed at B. A moment $\frac{M}{2}$ is applied at end 'A'. What is the moment developed at B?
(A) $-\frac{M}{2}$ (B) M
(C) $\frac{M}{4}$ (D) $+\frac{M}{2}$

40. The most appropriate method for analysis of a skeletal frame shown in the figure is _____.

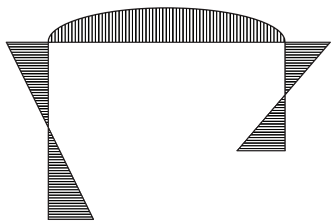


(A) Slope–deflection method
(B) Moment distribution method
(C) Kani's rotation method
(D) Strain energy method

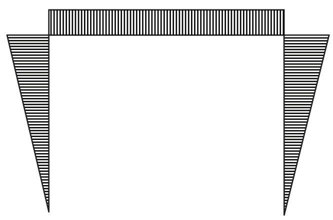
41. The bending moment diagram for the given frame is _____.



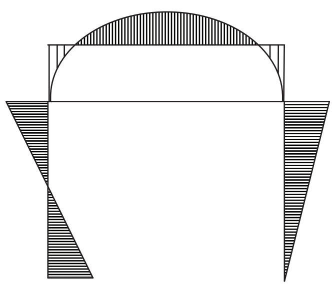
(A)



(B)

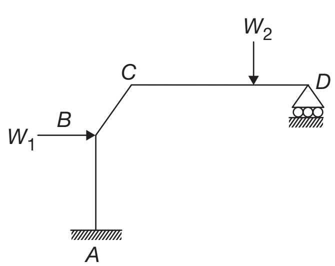


(C)



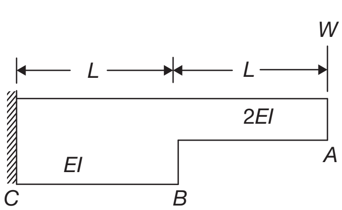
(D) None of these

42. The rigid plane frame *ABCD* has to be analyzed by slope deflection method. What is the number of unknown displacements/rotations for the frame shown in the figure?



(A) 4 (B) 3
(C) 5 (D) 2

43.

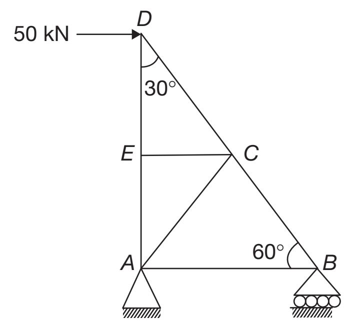


Find the vertical deflection at '*A*' due to concentrated load *W*.

(A) $\frac{WL^3}{48EI}$ (B) $\frac{12WL^3}{296EI}$

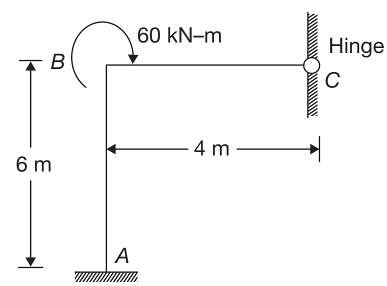
(C) $\frac{15WL^3}{6EI}$ (D) $\frac{7WL^3}{12EI}$

44. Members of the frame shown below which carries zero force are ______.



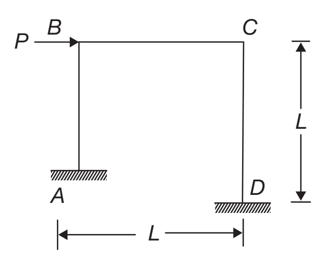
(A) *EC* only
(B) *EC* and *AB*
(C) *EC* and *AC*
(D) *EC*, *AC* and *AB*

45. For a rigid frame shown in the figure, what is the moment reaction at *A*?



(A) 28 kN-m (B) 14 kN-m
(C) 38 kN-m (D) 19 kN-m

46. What is the shear equation in slope–deflection method for the portal frame shown in the figure?



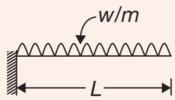
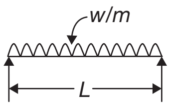
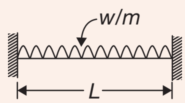
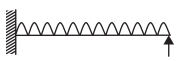
(A) $\frac{M_{AB}+M_{BA}}{L}+\frac{M_{BC}+M_{CB}}{L}+P=0$

(B) $\frac{M_{BC}+M_{CB}}{L}+\frac{M_{CD}+M_{DC}}{L}+P=0$

(C) $\dfrac{M_{AB}+M_{BA}}{L}+\dfrac{M_{CD}+M_{DC}}{L}+P=0$

(D) $\dfrac{M_{BC}+M_{CB}}{L}+P=0$

47. Match the List I with List II

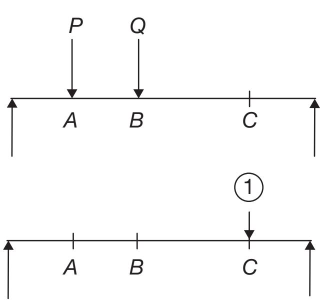
	List I		List II
	Load Condition		**Strain Energy**
1.		**a.**	$\frac{w^2L^5}{240EI}$
2.		**b.**	$\frac{w^2L^5}{640EI}$
3.		**c.**	$\frac{w^2L^5}{40EI}$
		**d.**	$\frac{w^2L^5}{1440EI}$

(A) 1 – c, 2 – b, 3 – a, 4 – d
(B) 1 – b, 2 – d, 3 – a, 4 – c
(C) 1 – a, 2 – b, 3 – d, 4 – c
(D) 1 – c, 2 – a, 3 – d, 4 – d

48. A uniformly distributed load of length 8 m crosses a simply supported girder of span 20 m. The maximum bending moment at the left quarter span point occurs when the distance between the point of CG of the total load and mid span is

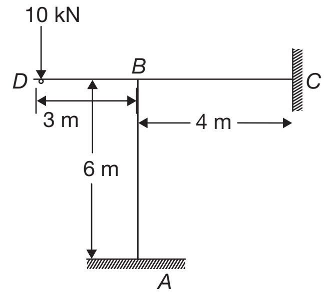
(A) 0 (B) 2 m
(C) 3 m (D) 4 m

49. In the following figure, x, y, z are the deflections under A, B, C due to loads P and Q. x', y', z' are the deflections under A, B, C due to unit load at C. The deflection z would be equal to ______.



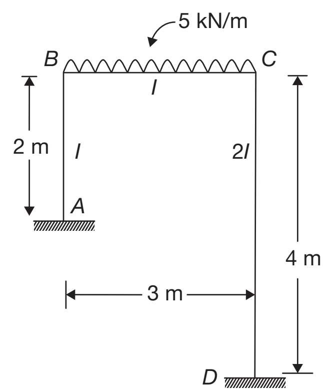
(A) $Px + Qy$
(B) $Px' + Qy'$
(C) $Py + Qx$
(D) $Py' + Qx'$

50. What is the value of θ_B for the beam shown in the figure?



(A) 0 (B) $\frac{18}{EI}$ ACW
(C) $\frac{30}{EI}$ ACW (D) $\frac{30}{EI}$ CW

51. In the portal frame shown in the figure, the ratio of sway moments in column AB and CD will be equal to ______.



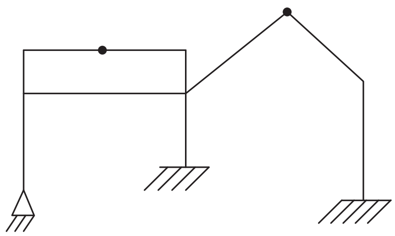
(A) 1 : 4 (B) 1 : 2
(C) 2 : 1 (D) 4 : 1

52. Consider the following statements:

Williot–Mohr diagram is used to determine the deflection in
I. a truss.
II. on arch.
III. a rigid frame.
Which of these statements is/are correct?
(A) Only I
(B) Only II
(C) Only III
(D) I, II and III

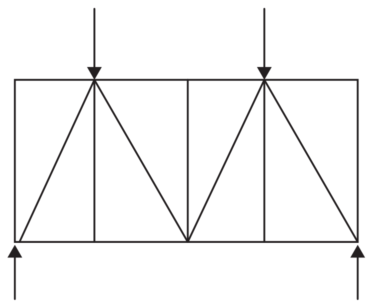
53. The moment distribution method in structural analysis falls in the category of
(A) displacement method.
(B) flexibility method.
(C) force method.
(D) first order approximate method.

54. The statical indeterminacy for the given 2-D frame is



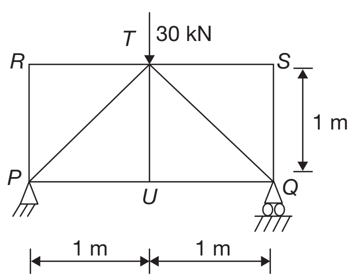
(A) 3 (B) 6
(C) 5 (D) 4

55. In the plane truss shown below, how many members have zero force?



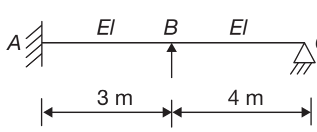
(A) 7 (B) 5
(C) 3 (D) 9

56. A simply supported truss shown in the given figure carries a load of 30 kN at *T*, the forces in members *UT* and *QU* are respectively



(A) zero and 15 kN (Tensile)
(B) 30 kN and 15 kN (Tensile)
(C) zero and 15 kN (Compressive)
(D) 30 kN and 15 kN (Compressive)

57. What are the distribution factors at joint *B* for the members *BA* and *BC* respectively, as shown in the figure?



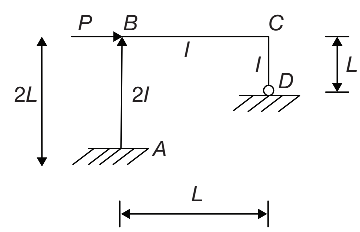
(A) 0.36, 0.64
(B) 0.50, 0.50
(C) 0.64, 0.36
(D) 0.75, 0.25

58. A fixed beam *AB* is subjected to a triangular load varying from zero at *B* to *W* per unit length at end *A*. The ratio of fixed end moment at *B* to *A* will be

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$

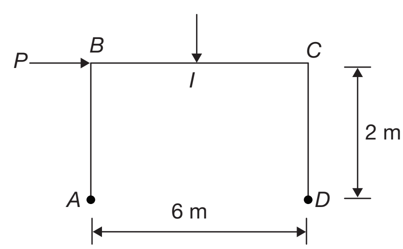
(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

59. The given figure shows a portal frame with one end fixed and other end hinged. The ratio of the fixed end moment $\frac{M_{BA}}{M_{CD}}$ due to side away will be



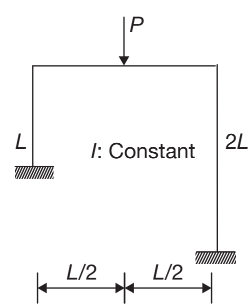
(A) 4 (B) 0.75
(C) 1 (D) 2

60. A portal frame is shown in the figure. If $\theta_B = \theta_C = \frac{250}{EI}$ radian, the value of moment at *B* will be



(A) 120 kN-m
(B) 250 kN-m
(C) 300 kN-m
(D) 400 kN-m

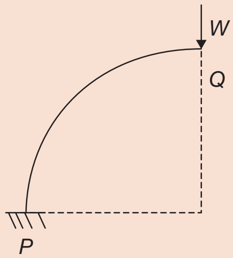
61. A rigid-jointed plane frame shown in the figure.



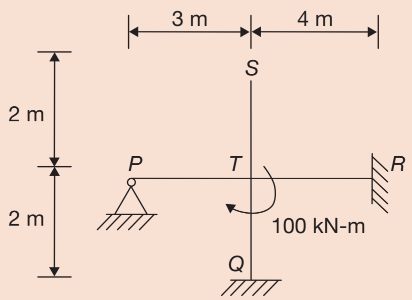
(A) will sway to right
(B) will sway to left
(C) will not sway
(D) None of these

Previous Years' Questions

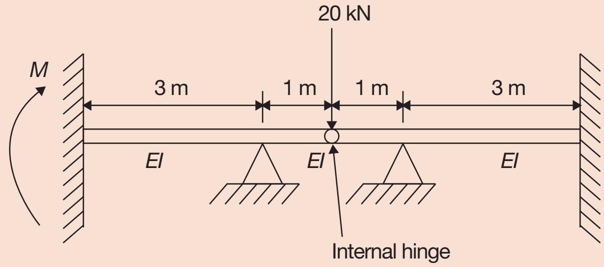
1. A uniform beam (EI = constant) PQ in the form of a quarter-circle of radius R is fixed at end P and free at the end Q, where a load W is applied as shown. The vertical downward displacement, δ_q, at the loaded point Q is given by $\delta_q = \beta\left(\frac{WR^3}{EI}\right)$. Find the value of β (correct to 4 decimal places). **[GATE, 2013]**



2. All members in the rigid-jointed frame shown are prismatic and have the same flexural stiffness EI. Find the magnitude of the bending moment at Q (In kN-m) due to the given loading. **[GATE, 2013]**



3. For the beam shown below, the value of the support moment M is ________ kN-m. **[GATE, 2015]**



4. In a system two connected rigid bars AC and BC are of identical length, L with pin supports at A and B. The bars are interconnected at C by a frictionless hinge. The rotation of the hinge is restrained by a rotational spring of stiffness, k. The system initially assumes a straight line configuration, ACB. Assuming both the bars as weightless, the rotation at supports, A and B, due to a transverse load, P applied at C is **[GATE, 2015]**

(A) $\frac{PL}{4k}$ (B) $\frac{PL}{2k}$

(C) $\frac{P}{4k}$ (D) $\frac{Pk}{4L}$

Answer Keys

Exercises

1. C	**2.** forces, displacements			**3.** D	**4.** B	**5.** C	**6.** D	**7.** D	**8.** C
9. B	**10.** B	**11.** A	**12.** B	**13.** C	**14.** D	**15.** B	**16.** B	**17.** D	**18.** C
19. unit, unit		**20.** C	**21.** D	**22.** B	**23.** B	**24.** C	**25.** B	**26.** C	**27.** C
28. A	**29.** B	**30.** B	**31.** B	**32.** C	**33.** A	**34.** B	**35.** A	**36.** C	**37.** C
38. C	**39.** C	**40.** D	**41.** C	**42.** C	**43.** C	**44.** C	**45.** B	**46.** C	**47.** D
48. C	**49.** B	**50.** B	**51.** C	**52.** A	**53.** A	**54.** B	**55.** A	**56.** A	**57.** C
58. A	**59.** C	**60.** B	**61.** A						

Previous Years' Questions

1. C **2.** 25 **3.** 5 **4.** A