CHAPTER 9

Algebraic Expressions

9.1 Lets consider the given expressions

(i) $2n + 1$ (ii) 0 (iii) $5xy$ (iv) $15 + 7 + 0$ (v) $3\frac{p}{q}$ (vi) $\frac{4}{5}a^2b$

Which one of them are numeric expressions. You will find that'0', 15+7+0, are numeric expressions while $2n+1$, $5xy$, $3\frac{p}{q}$ $\frac{4}{5}a^2b$ are algebraic expressions because they are made up by the combinations of constants and variables.

As you know algebraic expressions are formed with variables and constants. The expression $3x+5$ is formed with variable x and constant 3 and 5

In the earlier class we have studied that algebraic expression can be named as monomials, binomials and trinomials according the numbers of terms contained in it in general .... Polynomial.

Monomial	Binomials	Trinomials
x, $\frac{p}{4}$, m^2	$2x + y$	$m^2 + 2m + 5$
$307q$, a^3	$5p + 4$	$a^2 + b + 3c$
	$3a^2 + 5b$	$a^3 + b^3 + c^3$

$m + 2 + p + 1$

$a_1 + a_2 + a_3 + \ldots\ldots a_m$

Polynomial

Do and learn:-

Give five different examples of numeric and algebraic Expression. Then categorize them into monomials, binomials, and trinomials.

9.2 Power of an expression

In an expression the term with the highest power is called the power of that expression like in $7x^3y$ the power of this monomial is $3 + 1 = 4$. While in $5p^2q - 3pq + 7qr$ the term $5p^2q$ hold the highest power $2 + 1 = 3$.

9.3 Like and Unlike terms

Look at the following expression given in the table.

LIKE TERMS	UNLIKE TERMS
$4x$, $-2x$, x , $-x$	$5x$, $5x^2$, x^3, xy , y^2x
pq, $5pq$, $\frac{3}{5}pq$	$\frac{3}{5}pq$, $\frac{3}{5}p$, q, p^2q, pq^2

Do and learn:-

From the following tick the essential condition for like terms
(1) Same signs (2) Same coefficient (3) Same exponents (4) Same number of variable.

We note that like terms are terms that have the same variable and exponents but they may have different numeric coefficient. In unlike terms variables are different or the exponents of the variable are different or both are different. Even if the numeric coefficient are equal but the exponents and variables are not equal the term is not called like .

Do and learn:-

(1) Find the LIKE terms from the following

$a\,x^2y$, $2n$, $5y^2$-$7x^2$, $-3n$, $7xy$, $25y^2$

(2) Write three LIKE terms for the expression $7xy^2$

9.4 Addition and Subtraction of Algebraic Expressions

In class 7, we learnt how to add and subtract algebraic expressions for example, $7x+4x=11x$ not $11x^2$ that is $x+x \neq x^2 2x^2y+3xy=?$

So in addition of like terms exponent remains the same.
For example, $7x^2y - 3x^2y = 4x^2y$

Do and learn:-

Fill in the blank by adding the following like terms	
$4n + (-3n) = \ldots\ldots\ldots$ $5pq + 12pq = \ldots\ldots\ldots$	$-5x^2y + (-3\,x^2y) = \ldots\ldots\ldots$ $2ab^2 + 11ab^2 = \ldots\ldots$

In case of unlike terms neither their exponent nor their coefficient gets added. They are represented as it is with the sign + or –

Such as in, $(7a^2b) + (3a^2b^2) = 7a^2b + 3a^2b^2$

$(-3pq) - (+p^3) = -3pq - p^3$

Try to understand this through these examples

Example 1: Add: $3x^2 + 4xy + 2y^2$ and $5y^2 - xy + 7x^2$

Solution: Writing the three expressions in separate rows, with like terms one below the other, we have

$$\begin{array}{r} 3x^2+4xy+2y^2 \\ +\ 7x^2-xy+5y^2 \\ \hline 10\,x^2+3xy+7y^2 \\ \hline \end{array}$$

or

x^2	xy	y^2
3	4	2
7	-1	5
10	3	7

That is $10x^2 + 3xy + 7y^2$

Do and learn:-

(i) Sheela says that the sum of $2pq$ and $4pq$ is $8p^2q^2$. Is she is right ?

(ii) Raees adds $4p$ and $7q$ and gets $11pq$ as its answer. Do you agree with his answer ?

Example 2: Subtract $3xy + 9y^2$ from $15xy + 7x^2 - 3y^2$.

Solution: Writing the three expressions in separate rows, with like terms one below the other, change the sign of the subtracted expression and solve

$$\begin{array}{r} 15xy + 7x^2 - 3y^2 \\ \underset{-}{}3xy \underset{-}{+}9y^2\ \underset{-}{7} \\ \hline 18xy + 7x^2 + 6y^2 \\ \hline \end{array}$$

or

x^2	xy	y^2
	15	-3
0	-3	-9
- 7	-12	-12

That is $7x^2 + 12xy - 12y^2$

Note that subtraction of a number is the same as addition of its additive inverse. Thus subtracting –5 is the same as adding +5. Similarly, subtracting $-3xy$ is the same as adding $+3xy$:

SECOND METHOD

$\Rightarrow 15xy + 7x^2 - 3y^2 - (3xy + 9y^2)$

$\Rightarrow 15xy + 7x^2 - 3y^2 - 3xy - 9y^2$

$\Rightarrow 15xy - 3xy + 7x^2 - 3y^2 - 9y^2$

$\Rightarrow 12xy + 7x^2 - 12y^2$

Do and learn:-

1. If $A = 2y^2 + 3x - x^2$ and $B = 3x^2 - x^2$ then find A+B and A-B

9.5 Multiplication of Algebraic Expressions:

Rakesh and Leela are playing a game of arranging the stars.

```
*  *  *  *  *
*  *  *  *  *
*  *  *  *  *
```

The stars are arranged in such a way that each row contains 5 stars and there are 3 such rows.
Then total number of stars = $5 \times 3 = 15$

```
*  *  *  *  *  *  *  *
*  *  *  *  *  *  *  *
*  *  *  *  *  *  *  *
- - - - - - - - - - - - - - - -
- - - - - - - - - - - - - - - -
*  *  *  *  *  *  *  *
```

If there are 8 stars in each row and there are n such rows then total number of stars = $8 \times n$

```
*  *  * - - - - - - - - *
*  *  * - - - - - - - - *
- - - - - - - - - - - - - -
- - - - - - - - - - - - - -
*  *  * - - - - - - - - *
```

If there are p stars in each row and there are q such rows
Then total number of stars = $p \times q$

```
*  *  * - - - - - *  *  *
*  *  * - - - - - - *  *  *
- - - - - - - - - - - - - - - - -
- - - - - - - - - - - - - - - -
*  *  * - - - - - - - *  *  *
*  *  * - - - - - - - *  *  *
*  *  * - - - - - - - *  *  *
```

If there are $q+3$ stars in each row and there are $p+2$ such rows then Total number of stars = $(p+2) \times (q+3)$

(i) Now you think of other similar situations in which two algebraic expressions have to be multiplied?

(ii) Kareena gets up and says , when we find the area of a square of side $(x+5)$ then area is given by $(x+5) \times (x+5)$

(iii) Raju says that in the same way we can find the area of the triangle. If base and height of the triangle is $(m+3)$ and 4x then

$$\text{Its area} = \frac{1}{2} \times 4x \times (m+3)$$

(iv) Sarita points out that when we buy things, we have to carry out multiplication. For example, if price of bananas per dozen = Rs x, then cost of z dozen bananas = Rs (xz).

Suppose, the price per dozen was increased by 3 rupees and the bananas needed were less by 2 dozen then, price of bananas per dozen = $(x+3)$ and bananas needed = $(z-2)$ dozens,

Therefore, we would have to pay = Rs $(x+3) \times (z-2)$

In the entire above example we have to multiply two or more than two quantities. If the quantities are given in the form of expressions then we need to find their product. Let's learn the multiplication of algebraic expression in a systematic way.

9.5.1 Multiplying a Monomial by a Monomial

We know that

$$5 \times x = x + x + x + x + x = 5x$$

And $3 \times (5x) = 5x + 5x + 5x = 15x$

Now look at the following product

(I) $x \times 3y = x \times 3 \times y = 3 \times x \times y = 3xy$

(ii) $5x \times 3y = 5 \times x \times 3 \times y = 5 \times 3 \times x \times y = 15xy$

(iii) $5x \times (-3y) = 5 \times x \times (-3y) = (5) \times (-3) \times x \times y = -15xy$

(iv) $5x \times 4x^2 = 5 \times x \times 4 \times x^2 = 5 \times 4 \times x \times x^2 = 20 \times x^3 = 20x^3$

(v) $5x \times (-4xyz) = (5 \times -4) \times (x \times xyz) = -20x \times xyz = -20x^2yz$

Multiplying three or more monomials

(i) $3x \times 5y \times 4z$

$= (3x \times 5y) \times 4z$

$= 15xy \times 4z$

$= 60xyz$

(ii) $2x^2y \times (-4y^2z) \times (-7z^2x) \times (2x^2yz)$

$= [2x^2y \times (-4y^2z)] \times [(-7z^2x) \times (2x^2yz)]$

$= (-8x^2y^3z) \times (-14x^3yz^3)$

$= 112x^5y^4z^4$

EXERCISE 9.1

1. Find the product of the following pairs of monomials.

   (i) $3 \times 5x$ (ii) $-5p, -2q$ (iii) $7t^2, -3n^2$
   (iv) $6m, 3n$ (v) $-5x^2, -2x$

2. Complete the table.

$\times$	7	x	y	$2z$	z	$-5b$	c
7	49						
x							
$2y$							
$-3a$			$-3ay$				
b							
y							
$2x^3$							
a^4					a^4z		
z^2							

3. Multiply the following monomial?

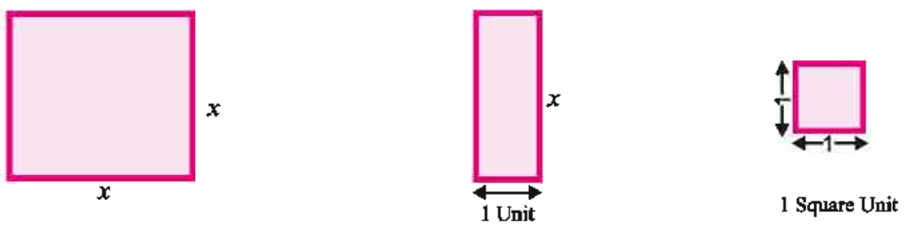
   (i) xy, x^2y, xy, x (ii) m, n, mn, m^3n, mn^3
   (iii) kl, lm, km, klm

4. Find the simple interest $= \frac{PTR}{100}$, If principal (P) $= 4x^2$, time (T) $= 5x$ and rate of interest (R) $= 5y$

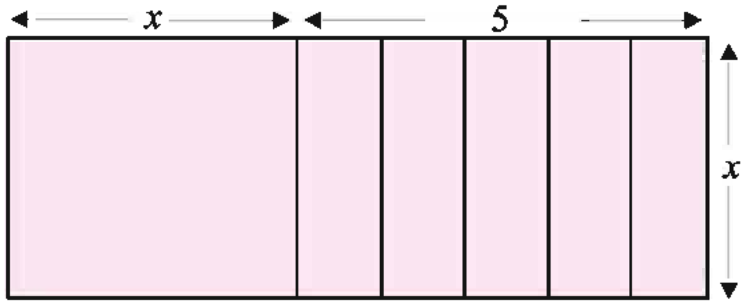
9.5.2 Multiplying a Binomial or a Trinomial by a Monomial

ACTIVITY 1

Take a square shape cardboard. let its length be x. cut rectangular shape strip of breadth 1 unit (i.e. 1cm or 1 inch) and length of x unit from it and also cut five pieces of 1 unit of square shapes.



Take a cardboard of length x and breadth $(x+5)$, i.e having area equal to $x(x+5)$. Cut it into five strips each of length x and breadth of unit length as shown in the figure.



Length of new rectangle = $x + 5$ unit
Breadth of new rectangle = x unit
Area of new rectangle = $x(x + 5)$ square unit

So , $x(x+5)$ = area of square shaped cardboard of length x + area of five strips added

$= x^2 + x + x + x + x + x$

$= x^2 + 5x$

Once again

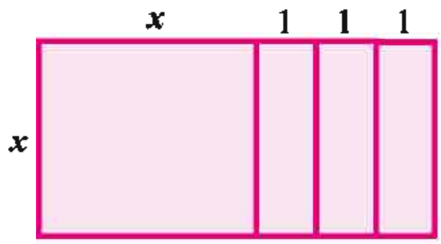
$x\ (x + 5) = x \times x + x \times 5$

$= x^2 + 5x$

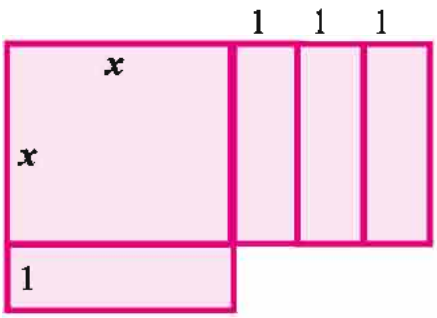
9.5.3 Multiplying a binomial by a binomial

$(x + 1)\ (x + 3)$

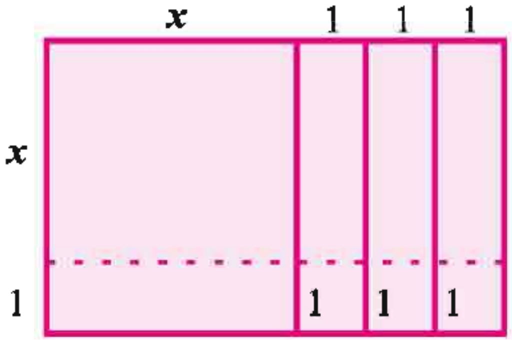
Take a cardboard of length x
Add three strips of length x and breadth 1 unit to it.



Now add one strip of same dimension below it



Now add three square of length 1 unit to the above figure so that it becomes a rectangle



Length of new rectangle = $(x+3)$ unit
Breadth of new rectangle = $(x+1)$ unit
Area of new rectangle = $(x+3)(x+1)$ square unit

So, $(x+3)(x+1)$ = area of square shaped cardboard of side x + area of four stripe + area of three square
$= x^2 + 4x + 3$

Once again $(x+3)(x+1)$
$= x(x+1) + 3(x+1)$
$= x^2 + x + 3x + 3$
$= x^2 + 4x + 3$

To multiply two binomials, multiply each term of the first binomial with the second binomial and then apply distributive property to multiply monomial with binomial. Then add the like terms to get the solution.

9.5.4 Multiplying a binomial by a trinomial

Algebraic expression $(2x + 3y)$ is binomial and $(3x + 4y - 5z)$ is trinomial. Now we multiply a binomial $(2x+3y)$ by a trinomial $(3x + 4y - 5z)$

$(2x+3y)(3x+4y-5z) = 2x(3x + 4y - 5z) + 3y(3x + 4y - 5z)$

(multiply each term of the binomial to the trinomial)

$= 2x \times 3x + 2x \times 4y - 2x \times 5z + 3y \times 3x + 3y \times 4y - 3y \times 5z$
$= 6x^2 + 8xy - 10xz + 9xy + 12y^2 - 15yz$

Now by adding or subtracting like terms we get,

$= 6x^2 + (8xy + 9xy) - 10xz + 12y^2 - 15yz$
$= 6x^2 + 17xy - 10xz + 12y^2 - 15yz$

EXERCISE 9.2

1. Multiply the given binomials.
   (i) $(2x + 5)$ and $(3x - 7)$
   (ii) $(x - 8)$ and $(3y + 5)$
   (iii) $(1.5p - 0.5q)$ and $(1.5p + 0.5q)$
   (iv) $(a + 3b)$ and $(x + 5)$
   (v) $(2lm + 3l^2)$ and $(3lm - 5l^2)$
   (vi) $(\frac{3}{4}a^2 + 3b^2)$ and $(4a^2 - \frac{5}{3}b^2)$

2. Find the product.

(i) $(3x + 8)(5 - 2x)$ (ii) $(x + 3y)(3x - y)$
(iii) $(a^2 + b)(a + b^2)$ (iv) $(p^2 - q^2)(2p + q)$

3. Simplify:

(i) $(x + 5)(x - 7)+35$ (ii) $(a^2 - 3)(b^2 + 3) + 5$
(iii) $(t + s^2)(t^2 - s)$ (iv) $(a+b)(c-d) + (a-b)(c+d) + 2(ac+bd)$
(v) $(a + b)(a^2 - ab + b^2)$ (vi) $(a + b + c)(a +b - c)$
(vii) $(a + b)(a - b) - a^2 + b^2$

9.6 What is an Identity?

Consider the equation, $x(x+5) = x^2+5x$

We shall evaluate both sides of this equation for some value of x-

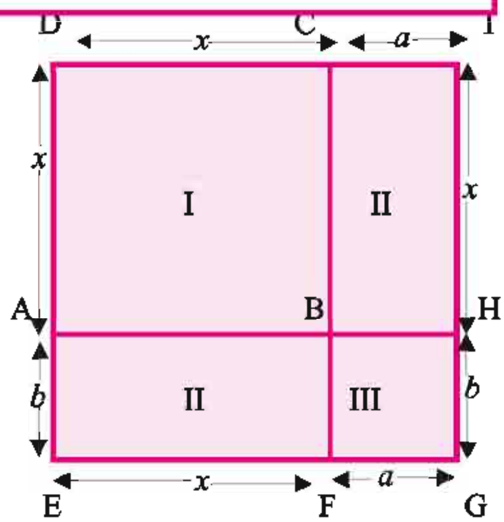
	LHS	RHS
$x = 1$	$1(1+5) = 1 \times 6=6$	$(1)^2 + 5\times1 = 1+5 = 6$
$x = 2$	$2(2+5) = 2\times7 = 14$	$(2)^2 + 5\times2 = 4+10 = 14$

Similarly you can verify the given expression for some other values also.
We have found that for any value of x, LHS = RHS.

Such an equality, true for every value of the variable is called an identity

ACTIVITY $(x + a)(x + b)$

(i) Area of square ABCD = x^2
(ii) Area of rectangle AEFB = $x \times b = bx$
(iii) Area of rectangle FGHB = $a \times b = ab$
(iv) Area of rectangle BHIC = $a \times x = ax$



Area of rectangle DEGI = I +II+III+IV

$$(x+a)(x+b) = x^2 + ax + bx + ab$$
$$= x^2 + x(a+b) + ab$$

9.6.1 Standard Identities

Let us first consider the product $(a + b)\ (a + b)$ or $(a + b)^2$

$$(a+b)^2 = (a+b)(a+b)$$
$$= a(a+b) + b(a+b)$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

Because $ab = ba$

Hence

$$(a+b)^2 = a^2 + 2ab + b^2 \qquad \text{- I}$$

Clearly, this is an identity, since the expression on the RHS is obtained from the LHS by actual multiplication. One may verify that for any value of a and any value of b, the values of the two sides are equal.

Next we consider, $(a - b)(a - b)$ or $(a - b)^2$

$$(a-b)^2 = (a-b)(a-b)$$
$$= a(a-b) - b(a-b)$$
$$= a^2 - ab - ba + b^2$$
$$= a^2 - 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2 \qquad \text{- II}$$

Finally, consider (a + b) (a – b).

$$(a+b)(a-b) = a(a-b) + b(a-b)$$
$$= a^2 - ab + ab - b^2$$
$$= a^2 - b^2$$

Hence

$$(a+b)(a-b) = a^2 - b^2 \qquad \text{- III}$$

Do and learn:-

1. Put $-b$ in place of b in Identity (I). Do you get Identity (II) ?

9.6.2 Application of Identities

We shall now see how, for many problems on multiplication of binomial expressions and also of numbers, use of the identities gives a simple alternative method of solving them.

Example 3: Solve $(2x + 3y)^2$ and $(103)^2$ with the help of identities

(I) $(2x+3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2$
$= 4x^2 + 12xy + 9y^2$

(i) $(103)^2 = (100+3)^2$
$= (100)^2 + 2(100)(3) + (3)^2$
$= 10000 + 600 + 9$
$= 10609$

We can directly multiply 103 by 103 to get the answer. Did you notice that we can solve $(103)^2$ by identity (1) ?

Example 4. Find the value of $(7.9)^2$

$=(7.9)^2 = (8.0 - 0.1)^2$
$= (8.0)^2 - 2 \times 8.0 \times 0.1 + (0.1)^2$
$= 64 - 1.6 + 0.01$
$= 62.41$

EXERCISE 9.3

1. Find the products of the following using suitable identity.

(i) $(x + 5)(x + 5)$ (ii) $(3x + 2)(3x + 2)$
(iii) $(5a - 7)(5a - 7)$ (iv) $(3p - \frac{1}{2})(3p - \frac{1}{2})$
(v) $(1.2m - 0.3)(1.2m + 0.3)$ (vi) $(x^2 + y^2)(x^2 - y^2)$
(vii) $(6y+7)(-6y+7)$ (viii) $(7a - 9b)(7a - 9b)$

2. Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products:

(i) $(x + 1)(x + 2)$ (ii) $(3x + 5)(3x + 1)$
(iii) $(4x - 5)(4x - 1)$ (iv) $(3a + 5)(3a - 8)$
(v) $(xyz - 1)(xyz - 2)$

3. Find the following squares by using the identities.

(i) $(b - 7)^2$ (ii) $(xy + 3z)^2$ (iii) $(6m^2 - 5n)^2$ (iv) $(\frac{3}{2}x + \frac{2}{3}y)^2$

4. Simplify

(i) $(a^2 - b^2)^2$ (ii) $(2n + 5)^2 - (2n - 5)^2$
(iii) $(7m - 8n)^2 - (7m + 8n)^2$ (iv) $(m^2 - n^2m)^2 + 2m^3n^2$

5. Show that.

(i) $(2a + 3b)^2 - (2a - 3b)^2 = 24ab$

(ii) $(4x + 5)^2 - 80x = (4x - 5)^2$

(iii) $(3x - 2y) + 24xy = (3x + 2y)^2$

(iv) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

6. Using identities, evaluate the following.

(i) 99^2 (ii) 103^2 (iii) 297×303 (iv) 78×82

7. Using $a^2 - b^2 = (a + b)(a - b)$, find

(i) $10^2 - 99^2$ (ii) $(10.3)^2 - (9.7)^2$ (iii) $153^2 - 147^2$

8. Using $a^2 - b^2 = (a + b)(a - b)$, find

(i) 103×102 (ii) 7.1×7.3 (iii) 102×99 (iv) 9.8×9.6

We Learnt

1. Expressions are formed with the help of **variables** and **constants**.
2. Expressions that contain exactly one, two and three terms are called **monomials**, **binomials** and **trinomials** respectively. In general, any expression containing one or more terms with none zero coefficients (and with variable having non –negative exponent) is called a **polynomial.**
3. In an expression the term with the highest power is called the power of that expression.
4. **Like** terms are formed with the same variables and the powers of these variables are the same, too. Coefficients of like terms need not be the same.
5. While adding (or subtracting) polynomials, first look for like terms and add (or subtract) them; then handle the unlike terms.
6. A monomial multiplied by a monomial always gives a monomial.
7. While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.
8. In carrying out the multiplication of a polynomial by a binomial (or trinomial), we multiply term by term, i.e., every term of the polynomial is multiplied by every term in the binomial (or trinomial). Note that in such multiplication, we may get terms in the product which are like and have to be combined.
9. An **identity** is an equality, which is true for all values of the variables in the equality
10. The following are the standard identities

(i) $(a + b)^2 = a^2 + 2ab + b^2$ (I)

(ii) $(a - b)^2 = a^2 - 2ab + b^2$ (II)

(iii) $(a + b)(a - b) = a^2 - b^2$ (III)