Chapter 5

Permutations and Combinations

Contents

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The concepts of permutations and combinations can be traced back to the advent of Jainism in India and perhaps even earlier. Among the Jains, Mahavira, *(around 850 A.D.) is perhaps the world's first mathematician credited with providing the general formulae for permutations and combinations.*

Bhaskaracharya *(born* 1114 A.D.*) treated the subject matter of permutations and combinations under the name* Anka Pasha *in his famous work* Lilavati. *In addition to the general formulae for* ^nC_r *and* ^nP_r *already provided by* Mahavira,

Outside India, the subject matter of permutations and combinations had its humble beginnings in China in the famous book I-King (Book of changes). The first book which gives a comkplete treatment of the subject matter of permutations and combinations is Ars conjectandi *written by a Swiss,* Jacob Bernouli (1654-1705 A.D.*) posthumously published in* 1713 A.D. *This book contains essentially the theory of permutations and combinations as is known today.*

Permutations

5.1 The Factorial

Factorial notation: Let n be a positive integer. Then, the continued product of first n natural numbers is called factorial $\underline{n}$, to be denoted by n ! or n . Also, we define 0 ! = 1.

when n is negative or a fraction, n ! is not defined.

Thus, $n! = n(n-1)(n-2)......3.2.1.$

Deduction: $n! = n(n-1)(n-2)(n-3)......3.2.1$

$$= n[(n-1)(n-2)(n-3)......3.2.1] = n[(n-1)!]$$

Thus, $5! = 5\times(4!), 3! = 3\times(2!)$ and $2! = 2\times(1!)$

Also, $1! = 1\times(0!) \Rightarrow 0! = 1$.

5.2 Exponent of Prime *p* in *n* !

Let p be a prime number and n be a positive integer. Then the last integer amongst 1, 2, 3, ...$(n-1)$, n which is divisible by p is $\left[\frac{n}{p}\right]p$, where $\left[\frac{n}{p}\right]$ denote the greatest integer less than or equal to $\frac{n}{p}$.

For example: $\left[\frac{10}{3}\right]=3$, $\left[\frac{12}{5}\right]=2$, $\left[\frac{15}{3}\right]=5$ etc.

Let $E_p(n)$ denotes the exponent of the prime p in the positive integer n. Then,

$$E_p(n!) = E_p(1.2.3..........(n-1)n) = E_p\left(p.2p.3p.......\left[\frac{n}{p}\right]p\right) = \left[\frac{n}{p}\right] + E_p\left(1.2.3......\left[\frac{n}{p}\right]\right)$$

[$\because$ Remaining integers between 1 and n are not divisible by p]

Now the last integer amongst 1, 2, 3,..... $\left[\frac{n}{p}\right]$ which is divisible by p is

$\left[\frac{n/p}{p}\right] = \left[\frac{n}{p^2}\right] = \left[\frac{n}{p}\right] + E_p\left(p, 2p, 3p....\left[\frac{n}{p^2}\right]p\right)$ because the remaining natural numbers from 1 to $\left[\frac{n}{p}\right]$ are not divisible by p = $\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + E_p\left(1.2.3......\left[\frac{n}{p^2}\right]\right)$

Similarly we get $E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + ... \left[\frac{n}{p^S}\right]$

where S is the largest natural number. Such that $p^S \le n < p^{S+1}$.

5.3 Fundamental Principles of Counting

(1) **Addition principle :** Suppose that A and B are two disjoint events (mutually exclusive); that is, they never occur together. Further suppose that A occurs in m ways and B in n ways. Then A or B can occur in $m + n$ ways. This rule can also be applied to more than two mutually exclusive events.

Example: 1 A college offers 7 courses in the morning and 5 in the evening. The number of ways a student can select exactly one course, either in the morning or in the evening

(a) 27 (b) 15 (c) 12 (d) 35

Solution: (c) The student has seven choices from the morning courses out of which he can select one course in 7 ways.

For the evening course, he has 5 choices out of which he can select one course in 5 ways.

Hence he has total number of 7 + 5 = 12 choices.

(2) **Multiplication principle :** Suppose that an event X can be decomposed into two stages A and B. Let stage A occur in m ways and suppose that these stages are unrelated, in the sense that stage B occurs in n ways regardless of the outcome of stage A. Then event X occur in mn ways. This rule is applicable even if event X can be decomposed in more than two stages.

Note : ❑ The above principle can be extended for any finite number of operations and may be stated as under :

If one operation can be performed independently in m different ways and if second operation can be performed independently in n different ways and a third operation can be performed independently in p different ways and so on, then the total number of ways in which all the operations can be performed in the stated order is $(m \times n \times p \times .....)$

Example: 2 In a monthly test, the teacher decides that there will be three questions, one from each of exercise 7, 8 and 9 of the text book. If there are 12 questions in exercise 7, 18 in exercise 8 and 9 in exercise 9, in how many ways can three questions be selected

(a) 1944 (b) 1499 (c) 4991 (d) None of these

Solution: (a) There are 12 questions in exercise 7. So, one question from exercise 7 can be selected in 12 ways. Exercise 8 contains 18 questions. So, second question can be selected in 18 ways. There are 9 questions in exercise 9. So, third question can be selected in 9 ways. Hence, three questions can be selected in $12 \times 18 \times 9 = 1944$ ways.

5.4 Definition of Permutation

The ways of arranging or selecting a smaller or an equal number of persons or objects at a time from a given group of persons or objects with due regard being paid to the order of arrangement or selection are called the (different) *permutations*.

For example : Three different things *a, b* and *c* are given, then different arrangements which can be made by taking two things from three given things are *ab, ac, bc, ba, ca, cb*.

Therefore the number of permutations will be 6.

5.5 Number of Permutations without Repetition

(1) Arranging n objects, taken r at a time equivalent to filling r places from n things

r-places :	1	2	3	4		r
Number of choices :	n	$(n-1)$	$(n-$	$)(n-$		$n-(r-$

The number of ways of arranging = The number of ways of filling r places.

$$= n(n-1)(n-2).......(n-r+1) = \frac{n(n-1)(n-2).....(n-r+1)((n-r)!)}{(n-r)!} = \frac{n!}{(n-r)!} = {}^nP_r$$

(2) The number of arrangements of n different objects taken all at a time = ${}^nP_n = n!$

Note:
- ❑ ${}^nP_0 = \frac{n!}{n!} = 1; {}^nP_r = n.{}^{n-1}P_{r-1}$
- ❑ $0! = 1; \frac{1}{(-r)!} = 0$ or $(-r)! = \infty$ $(r \in N)$

Example: 3 If ${}^nP_4 : {}^nP_5 = 1:2$, then $n =$ **[MP PET 1987; Rajasthan PET 1996]**

(a) 4 (b) 5 (c) 6 (d) 7

Solution: (c) $\frac{{}^nP_4}{{}^nP_5} = \frac{1}{2} \Rightarrow \frac{n!}{(n-4)!} \times \frac{(n-5)!}{n!} = \frac{1}{2} \Rightarrow n-4 = 2 \Rightarrow n = 6$.

Example: 4 In a train 5 seats are vacant then how many ways can three passengers sit **[Rajasthan PET 1985; MP PET 2003]**

(a) 20 (b) 30 (c) 60 (d) 10

Solution: (c) Number of ways are = ${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{120}{2} = 60$.

Example: 5 How many words comprising of any three letters of the word "UNIVERSAL" can be formed

(a) 504 (b) 405 (c) 540 (d) 450

Solution: (a) Required numbers of words = ${}^9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!} = 504$.

Example: 6 How many numbers of five digits can be formed from the numbers 2, 0, 4, 3, 8 when repetition of digit is not allowed

[MP PET 2000]

(a) 96 (b) 120 (c) 144 (d) 14

Solution: (a) Given numbers are 2, 0, 4, 3, 8

Numbers can be formed = {Total – Those beginning with 0}

= {5 ! – 4 !} = 120 – 24 = 96.

Example: 7 How many numbers can be made with the help of the digits 0, 1, 2, 3, 4, 5 which are greater than 3000 (repetition is not allowed)

(a) 180 (b) 360 (c) 1380 (d) 1500

Solution: (c) All the 5 digit numbers and 6 digit numbers are greater than 3000. Therefore number of 5 digit numbers = ${}^6P_5 - {}^5P_5 = 600$.

{Since the case that 0 will be at ten thousand place should be omit}. Similarly number of 6 digit numbers 6 ! – 5 ! = 600.

Now the numbers of 4 digit numbers which are greater than 3000, having 3, 4 or 5 at first place, this can be done in 3 ways and remaining 3 digit may be filled from remaining 5 digits *i.e.,* required number of 4 digit numbers are $^5P_3 \times 3 = 180$.

Hence total required number of numbers = 600 + 600 + 180 = 1380.

5.6 Number of Permutations with Repetition

(1) The number of permutations (arrangements) of n different objects, taken r at a time, when each object may occur once, twice, thrice,........upto r times in any arrangement = The number of ways of filling r places where each place can be filled by any one of n objects.

r – places :	1	2	3	4	r
Number of choices :	n	n	n	n	n

The number of permutations = The number of ways of filling r places = $(n)^r$

(2) The number of arrangements that can be formed using n objects out of which p are identical (and of one kind) q are identical (and of another kind), r are identical (and of another kind) and the rest are distinct is $\frac{n!}{p!q!r!}$.

Example: 8 The number of arrangement of the letters of the word "CALCUTTA" **[MP PET 1984]**

(a) 2520 (b) 5040 (c) 10080 (d) 40320

Solution: (b) Required number of ways $= \frac{8!}{2!2!2!} = 5040$. [since here 2C's, 2T's and 2A's]

Example: 9 The number of 5 digit telephone numbers having at least one of their digits repeated is

(a) 90,000 (b) 100,000 (c) 30,240 (d) 69,760

Solution: (d) Using the digits 0, 1, 2,.......,9 the number of five digit telephone numbers which can be formed is 10^5 .

(since repetition is allowed)

The number of five digit telephone numbers which have none of the digits repeated = $^{10}P_5 = 30240$

$\therefore$ The required number of telephone numbers = $10^5 - 30240 = 69760$.

Example: 10 How many words can be made from the letters of the word 'COMMITTEE' **[MP PET 2002; RPET 1986]**

(a) $\frac{9!}{(2!)^2}$ (b) $\frac{9!}{(2!)^3}$ (c) $\frac{9!}{2!}$ (d) 9 !

Solution: (b) Number of words = $\frac{9!}{2!2!2!} = \frac{9!}{(2!)^3}$ [Since here total number of letters is 9 and 2M's, 2T's and 2E's]

5.7 Conditional Permutations

(1) Number of permutations of n dissimilar things taken r at a time when p particular things always occur = $^{n-p}C_{r-p}\, r!$

(2) Number of permutations of n dissimilar things taken r at a time when p particular things never occur = $^{n-p}C_r\, r!$

(3) The total number of permutations of n different things taken not more than r at a time, when each thing may be repeated any number of times, is $\frac{n(n^r-1)}{n-1}$.

(4) Number of permutations of n different things, taken all at a time, when m specified things always come together is $m!\times(n-m+1)!$

(5) Number of permutations of n different things, taken all at a time, when m specified things never come together is $n!-m!\times(n-m+1)!$

(6) Let there be n objects, of which m objects are alike of one kind, and the remaining $(n-m)$ objects are alike of another kind. Then, the total number of mutually distinguishable permutations that can be formed from these objects is $\frac{n!}{(m!)\times(n-m)!}$.

Note : ❑ The above theorem can be extended further *i.e.,* if there are n objects, of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of 3rd kind;......: p_r are alike of r^{th} kind such that $p_1+p_2+......+p_r=n$; then the number of permutations of these n objects is $\frac{n!}{(p_1!)\times(p_2!)\times......\times(p_r!)}$.

Important Tips

☞ ***Gap method :*** *Suppose 5 males A, B, C, D, E are arranged in a row as × A × B × C × D × E ×. There will be six gaps between these five. Four in between and two at either end. Now if three females P, Q,R are to be arranged so that no two are together we shall use gap method i.e., arrange them in between these 6 gaps. Hence the answer will be* $^6P_3$.

☞ ***Together :*** *Suppose we have to arrange 5 persons in a row which can be done in 5 ! = 120 ways. But if two particular persons are to be together always, then we tie these two particular persons with a string. Thus we have 5 – 2 + 1 (1 corresponding to these two together) = 3 +1 = 4 units, which can be arranged in 4! ways. Now we loosen the string and these two particular can be arranged in 2 ! ways. Thus total arrangements = 24 × 2 = 48.*

Never together = Total – Together = 120 – 48 = 72.

Example: 11 All the letters of the word 'EAMCET' are arranged in all possible ways. The number of such arrangement in which two vowels are not adjacent to each other is
[EAMCET 1987; DCE 2000]

(a) 360 (b) 114 (c) 72 (d) 54

Solution: (c) First we arrange 3 consonants in 3 ! ways and then at four places (two places between them and two places on two sides) 3 vowels can be placed in $^4P_3\times\frac{1}{2!}$ ways.

Hence the required ways = 3 ! × $^4P_3\times\frac{1}{2!}=72$.

Example: 12 The number of words which can be made out of the letters of the word 'MOBILE' when consonants always occupy odd places is

(a) 20 (b) 36 (c) 30 (d) 720

Solution: (b) The word 'MOBILE' has three even places and three odd places. It has 3 consonants and 3 vowels. In three odd places we have to fix up 3 consonants which can be done in ${}^3P_3$ ways. Now remaining three places we have to fix up remaining three places which can be done in ${}^3P_3$ ways.

The total number of ways = ${}^3P_3 \times {}^3P_3 = 36$.

Example: 13 The number of 4 digit number that can be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7 so that each number contain digit 1 is

(a) 1225 (b) 1252 (c) 1522 (d) 480

Solution: (d) After fixing 1 at one position out of 4 places, 3 places can be filled by ${}^7P_3$ ways. But some numbers whose fourth digit is zero, so such type of ways = ${}^6P_2$

$\therefore$ Total ways = ${}^7P_3 - {}^6P_2 = 480$.

Example: 14 m men and n women are to be seated in a row, so that no two women sit together. If $m > n$, then the number of ways in which they can be seated is

(a) $\frac{m!(m+1)!}{(m-n+1)!}$ (b) $\frac{m!(m-1)!}{(m-n+1)!}$ (c) $\frac{(m-1)!(m+1)!}{(m-n+1)!}$ (d) None of these

Solution: (a) First arrange m men, in a row in m ! ways. Since $n < m$ and no two women can sit together, in any one of the m ! arrangement , there are $(m + 1)$ places in which n women can be arranged in ${}^{m+1}P_n$ ways.

$\therefore$ By the fundamental theorem, the required number of arrangement = $m!\ {}^{m+1}P_n = \frac{m!(m+1)!}{(m-n+1)!}$.

Example: 15 If the letters of the word 'KRISNA' are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word 'KRISNA' is

(a) 324 (b) 341 (c) 359 (d) None of these

Solution: (a)

Words starting from A are 5 ! = 120;
Words starting from I are 5 ! = 120
Words starting from KA are 4 ! = 24;
Words starting from KI are 4 ! = 24
Words starting from KN are 4 ! = 24;
Words starting from KRA are 3 ! = 6
Words starting from $KRIA$ are 2 ! = 2;
Words starting from $KRIN$ are 2 ! = 2
Words starting from $KRIS$ are 1 ! = 1
Words starting from $KRISNA$ are 1 ! = 1

Hence rank of the word KRISNA is 324

Example: 16 We are to form different words with the letters of the word 'INTEGER'. Let m_1 be the number of words in which I and N are never together, and m_2 be the number of words which begin with I and end with R. Then m_1/m_2 is equal to

[AMU 2000]

(a) 30 (b) 60 (c) 90 (d) 180

Solution: (a) We have 5 letters other than 'I' and 'N' of which two are identical (E's). We can arrange these letters in a line in $\frac{5!}{2!}$ ways. In any such arrangement 'I' and 'N' can be placed in 6 available gaps in ${}^6P_2$ ways, so required number = $\frac{5!}{2!}{}^6P_2 = m_1$.

Now, if word start with I and end with R then the remaining letters are 5. So, total number of ways = $\frac{5!}{2!} = m_2$.

$$\therefore \quad \frac{m_1}{m_2} = \frac{5!}{2!}.\frac{6!}{4!}.\frac{2!}{5!} = 30 .$$

Example: 17 An n digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is **[IIT 1998]**

(a) 6 (b) 7 (c) 8 (d) 9

Solution: (b) Since at any place, any of the digits 2, 5 and 7 can be used total number of such positive n-digit numbers are 3^n. Since we have to form 900 distinct numbers, hence $3^n \geq 900 \Rightarrow n = 7$.

Example: 18 The number of numbers that can be formed with the help of the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy odd places, is

(a) 24 (b) 18 (c) 12 (d) 30

Solution: (b) The 4 odd digits 1, 3, 3, 1 can be arranged in the 4 odd places, in $\frac{4!}{2!2!} = 6$ ways and 3 even digits 2, 4, 2 can be arranged in the three even places $\frac{3!}{2!} = 3$ ways. Hence the required number of ways = 6 × 3 = 18.

5.8 Circular Permutations

So far we have been considering the arrangements of objects in a line. Such permutations are known as linear permutations.

Instead of arranging the objects in a line, if we arrange them in the form of a circle, we call them, circular permutations.

In circular permutations, what really matters is the position of an object relative to the others.

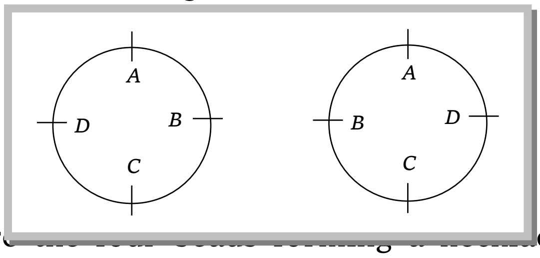
Thus, in circular permutations, we fix the position of the one of the objects and then arrange the other objects in all possible ways.

There are two types of circular permutations :

(i) The circular permutations in which clockwise and the anticlockwise arrangements give rise to different permutations, *e.g.* Seating arrangements of persons round a table.

(ii) The circular permutations in which clockwise and the anticlockwise arrangements give rise to same permutations, e.g. arranging some beads to form a necklace.

Look at the circular permutations, given below :



Suppose *A, B, C, D* are the four beads forming a necklace. They have been arranged in clockwise and anticlockwise directions in the first and second arrangements respectively.

Now, if the necklace in the first arrangement be given a turn, from clockwise to anticlockwise, we obtain the second arrangement. Thus, there is no difference between the above two arrangements.

(1) **Difference between clockwise and anticlockwise arrangement :** If anticlockwise and clockwise order of arrangement are not distinct *e.g.*, arrangement of beads in a necklace, arrangement of flowers in garland etc. then the number of circular permutations of n distinct items is $\frac{(n-1)!}{2}$

(2) **Theorem on circular permutations**

Theorem 1 : The number of circular permutations of n different objects is $(n-1)!$

Theorem 2 : The number of ways in which n persons can be seated round a table is $(n-1)!$

Theorem 3 : The number of ways in which n different beads can be arranged to form a necklace, is $\frac{1}{2}(n-1)!$.

Note : ❑ When the positions are numbered, circular arrangement is treated as a linear arrangement.

❑ In a linear arrangement, it does not make difference whether the positions are numbered or not.

Example: 19 In how many ways a garland can be made from exactly 10 flowers **[MP PET 1984]**

(a) 10 ! (b) 9 ! (c) 2 (9!) (d) $\frac{9!}{2}$

Solution: (d) A garland can be made from 10 flowers in $\frac{1}{2}(9!)$ ways [$\because$ n flower's garland can be made in $\frac{1}{2}(n-1)!$ ways]

Example: 20 In how many ways can 5 boys and 5 girls sit in a circle so that no boys sit together

(a) 5! × 5! (b) 4! × 5 ! (c) $\frac{5!\times 5!}{2}$ (d) None of these

Solution: (b) Since total number of ways in which boys can occupy any place is $(5-1)!=4!$ and the 5 girls can be sit accordingly in 5! ways. Hence required number of ways are 4 ! × 5 !.

Example: 21 The number of ways in which 5 beads of different colours form a necklace is

(a) 12 (b) 24 (c) 120 (d) 60

Solution: (a) The number of ways in which 5 beads of different colours can be arranged in a circle to form a necklace are

$=(5-1)!=4!$.

But the clockwise and anticlockwise arrangement are not different (because when the necklace is turned over one gives rise to another). Hence the total number of ways of arranging the beads = $\frac{1}{2}(4!)=12$.

Example: 22 The number of ways in which 5 male and 2 female members of a committee can be seated around a round table so that the two female are not seated together is

(a) 480 (b) 600 (c) 720 (d) 840

Solution: (a) Fix up a male and the remaining 4 male can be seated in 4! ways. Now no two female are to sit together and as such the 2 female are to be arranged in five empty seats between two consecutive male and number of arrangement will be ${}^5P_2$. Hence by fundamental theorem the total number of ways is = $4!\times {}^5P_2=24\times 20=480$ ways.

Combinations

5.9 Definition

Each of the different groups or selections which can be formed by taking some or all of a number of objects, irrespective of their arrangements, is called a combination.

Suppose we want to select two out of three persons *A, B* and *C*.

We may choose *AB* or *BC* or *AC*.

Clearly, *AB* and *BA* represent the same selection or group but they give rise to different arrangements.

Clearly, in a group or selection, the order in which the objects are arranged is immaterial.

Notation: The number of all combinations of n things, taken r at a time is denoted by $C(n,r)$ or ${}^nC_r$ or $\binom{n}{r}$.

(1) **Difference between a permutation and combination :** (i) In a combination only selection is made whereas in a permutation not only a selection is made but also an arrangement in a definite order is considered.

(ii) In a combination, the ordering of the selected objects is immaterial whereas in a permutation, the ordering is essential. For example *A, B* and *B, A* are same as combination but different as permutations.

(iii) Practically to find the permutation of n different items, taken r at a time, we first select r items from n items and then arrange them. So usually the number of permutations exceeds the number of combinations.

(iv) Each combination corresponds to many permutations. For example, the six permutations *ABC, ACB, BCA, BAC, CBA* and *CAB* correspond to the same combination *ABC*.

Note : ❑ Generally we use the word ‘arrangements’ for permutations and word “selection” for combinations.

5.10 Number of Combinations without Repetition

The number of combinations (selections or groups) that can be formed from n different objects taken $r(0 \le r \le n)$ at a time is ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

Let the total number of selections (or groups) = x. Each group contains r objects, which can be arranged in r ! ways. Hence the number of arrangements of r objects = $x \times (r!)$. But the number of arrangements = ${}^nP_r$.

$$\Rightarrow x \times (r!) = {}^nP_r \Rightarrow x = \frac{{}^nP_r}{r!} \Rightarrow x = \frac{n!}{r!(n-r)!} = {}^nC_r .$$

Important Tips

☞ ${}^nC_r$ *is a natural number.*

☞ ${}^nC_0 = {}^nC_n = 1, {}^nC_1 = n$

☞ ${}^nC_r = {}^nC_{n-r}$

☞ ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

☞ ${}^nC_x = {}^nC_y \Leftrightarrow x = y$ or $x + y = n$

☞ $n . {}^{n-1}C_{r-1} = (n-r+1)\,{}^nC_{r-1}$

☞ *If n is even then the greatest value of* ${}^nC_r$ *is* ${}^nC_{n/2}$.

☞ *If n is odd then the greatest value of* ${}^nC_r$ *is* $\dfrac{{}^nC_{n+1}}{2}$ *or* $\dfrac{{}^nC_{n-1}}{2}$.

☞ ${}^nC_r = \dfrac{n}{r} . {}^{n-1}C_{r-1}$

☞ $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$

☞ ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

☞ ${}^nC_0 + {}^nC_2 + {}^nC_4 + ... = {}^nC_1 + {}^nC_3 + {}^nC_5 + ... = 2^{n-1}$

☞ $^{2n+1}C_0 + ^{2n+1}C_1 + ^{2n+1}C_2 + ... + ^{2n+1}C_n = 2^{2n}$ ☞ $^nC_n + ^{n+1}C_n + ^{n+2}C_n + ^{n+3}C_n + ... + ^{2n-1}C_n = ^{2n}C_{n+1}$

Note : ❑ Number of combinations of n dissimilar things taken all at a time $^nC_n = \frac{n!}{n!(n-n)!} = \frac{1}{0!} = 1$, $(\because 0! = 1)$.

Example: 23 If $^{15}C_{3r} = ^{15}C_{r+3}$, then the value of r is **[IIT 1967; Rajasthan PET 1991; MP PET 1998; Karnataka CET 1996]**

(a) 3 (b) 4 (c) 5 (d) 8

Solution: (a) $^{15}C_{3r} = ^{15}C_{r+3} \Rightarrow ^{15}C_{15-3r} = ^{15}C_{r+3} \Rightarrow 15 - 3r = r + 3 \Rightarrow r = 3$.

Example: 24 $\frac{^nC_r}{^nC_{r-1}} =$ **[MP PET 1984]**

(a) $\frac{n-r}{r}$ (b) $\frac{n+r-1}{r}$ (c) $\frac{n-r+1}{r}$ (d) $\frac{n-r-1}{r}$

Solution: (c) $\frac{^nC_r}{^nC_{r-1}} = \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}} \Rightarrow \frac{n!}{r!(n-r)!} \times \frac{(r-1)!(n-r+1)!}{n!} = \frac{(n-r+1)(r-1)!(n-r)!}{r(r-1)!(n-r)!} = \frac{(n-r+1)}{r}$.

Example: 25 If $^{n+1}C_3 = 2\,^nC_2$, then $n =$ **[MP PET 2000]**

(a) 3 (b) 4 (c) 5 (d) 6

Solution: (c) $^{n+1}C_3 = 2.^nC_2$

$\Rightarrow \frac{(n+1)!}{3!(n-2)!} = 2.\frac{n!}{2!(n-2)!} \Rightarrow \frac{n+1}{3.2!} = \frac{2}{2!} \Rightarrow n+1 = 6 \Rightarrow n = 5$.

Example: 26 If $^nC_{r-1} = 36, ^nC_r = 84$ and $^nC_{r+1} = 126$ then the value of r is **[IIT 1979; Pb. CET 1993; DCE 1999; MP PET 2001]**

(a) 1 (b) 2 (c) 3 (d) None of these

Solution: (c) Here $\frac{^nC_{r-1}}{^nC_r} = \frac{36}{84}$ and $\frac{^nC_r}{^nC_{r+1}} = \frac{84}{126}$

$3n - 10r = -3$ and $4n - 10r = 6$; on solving we get $n = 9$ and $r = 3$.

Example: 27 In a conference of 8 persons, if each person shake hand with the other one only, then the total number of shake hands shall be

(a) 64 (b) 56 (c) 49 (d) 28

Solution: (d) Total number of shake hands when each person shake hands with the other once only = $^8C_2 = 28$ ways.

Example: 28 How many words of 4 consonants and 3 vowels can be formed from 6 consonants and 5 vowels.**[Rajasthan PE**

(a) 75000 (b) 756000 (c) 75600 (d) None of these

Solution: (b) Required number of words = $^6C_4 \times ^5C_3 \times 7!$ = 756000

[Selection can be made in $^6C_4 \times ^5C_3$ while the 7 letters can be arranged in 7!]

Example: 29 To fill 12 vacancies there are 25 candidates of which five are from scheduled caste. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, then the number of ways in which the selection can be made

[Rajasthan PET 1981]

(a) $^5C_3 \times ^{22}C_9$ (b) $^{22}C_9 - ^5C_3$ (c) $^{22}C_3 + ^5C_3$ (d) None of these

Solution: (a) The selection can be made in ${}^5C_3 \times {}^{22}C_9$ [since 3 vacancies filled from 5 candidates in ${}^5C_3$ ways and now remaining candidates are 22 and remaining seats are 9, then remaining vacancies filled by ${}^{22}C_9$ ways. Hence total number of ways ${}^5C_3 \times {}^{22}C_9$.

5.11 Number of Combinations with Repetition and All Possible Selections

(1) The number of combinations of n distinct objects taken r at a time when any object may be repeated any number of times.

= coefficient of x^r in $(1+x+x^2+.......+x^r)^n$ = coefficient of x^r in $(1-x)^{-n} = {}^{n+r-1}C_r$

(2) The total number of ways in which it is possible to form groups by taking some or all of n things at a time is $2^n - 1$.

(3) The total number of ways in which it is possible to make groups by taking some or all out of $n = (n_1 + n_2 + ...)$ things, when n_1 are alike of one kind, n_2 are alike of second kind, and so on is $\{(n_1+1)(n_2+1)......\} - 1$.

(4) The number of selections of r objects out of n identical objects is 1.

(5) Total number of selections of zero or more objects from n identical objects is $n+1$.

(6) The number of selections taking at least one out of $a_1 + a_2 + a_3 + ... + a_n +$ k objects, where a_1 are alike (of one kind), a_2 are alike (of second kind) and so on...... a_n are alike (of n^th kind) and k are distinct = $[(a_1+1)(a_2+1)(a_3+1).......(a_n+1)]2^k - 1$.

Example: 30 There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is

(a) 10^2 (b) 1023 (c) 2^{10} (d) 10 !

Solution: (b) Number of ways are = $2^{10} - 1 = 1023$

[– 1 corresponds to none of the lamps is being switched on.]

Example: 31 10 different letters of English alphabet are given. Out of these letters, words of 5 letters are formed. How many words are formed when atleast one letter is repeated

(a) 99748 (b) 98748 (c) 96747 (d) 97147

Solution: (a) Number of words of 5 letters in which letters have been repeated any times = 10^5

But number of words on taking 5 different letters out of 10 = ${}^{10}C_5 = 252$

$\therefore$ Required number of words = $10^5 - 252$ = 99748.

Example: 32 A man has 10 friends. In how many ways he can invite one or more of them to a party

(a) 10 ! (b) 2^{10} (c) $10! - 1$ (d) $2^{10} - 1$

Solution: (d) Required number of friend = $2^{10} - 1$ (Since the case that no friend be invited *i.e.,* ${}^{10}C_0$ is excluded)

Example: 33 Numbers greater than 1000 but not greater than 4000 which can be formed with the digits 0, 1, 2, 3, 4 (repetition of digits is allowed), are

(a) 350 (b) 375 (c) 450 (d) 576

Solution: (b) Numbers greater than 1000 and less than or equal to 4000 will be of 4 digits and will have either 1 (except 1000) or 2 or 3 in the first place with 0 in each of remaining places.

After fixing 1^st place, the second place can be filled by any of the 5 numbers. Similarly third place can be filled up in 5 ways and 4^th place can be filled up in 5 ways. Thus there will be 5 × 5 × 5 = 125 ways in which 1 will be in first place but this include 1000 also hence there will be 124 numbers having 1 in the first place. Similarly 125 for each 2 or 3. One number will be in which 4 in the first place and *i.e.,* 4000. Hence the required numbers are 124 + 125 + 125 +1 = 375 ways.

5.12 Conditional Combinations

(1) The number of ways in which r objects can be selected from n different objects if k particular objects are

(i) Always included = ${}^{n-k}C_{r-k}$ (ii) Never included = ${}^{n-k}C_r$

(2) The number of combinations of n objects, of which p are identical, taken r at a time is

$= {}^{n-p}C_r + {}^{n-p}C_{r-1} + {}^{n-p}C_{r-2} + ... + {}^{n-p}C_0$ if $r \le p$ and

$= {}^{n-p}C_r + {}^{n-p}C_{r-1} + {}^{n-p}C_{r-2} + ... + {}^{n-p}C_{r-p}$ if $r > p$

Example: 34 In the 13 cricket players 4 are bowlers, then how many ways can form a cricket team of 11 players in which at least 2 bowlers included

(a) 55 (b) 72 (c) 78 (d) None of these

Solution: (c) The number of ways can be given as follows:

2 bowlers and 9 other players = ${}^4C_2 \times {}^9C_9$; 3 bowlers and 8 other players = ${}^4C_3 \times {}^9C_8$

4 bowlers and 7 other players = ${}^4C_4 \times {}^9C_7$

Hence required number of ways = 6 × 1 + 4 × 9 + 1 × 36 = 78.

Example: 35 In how many ways a team of 10 players out of 22 players can be made if 6 particular players are always to be included and 4 particular players are always excluded

(a) ${}^{22}C_{10}$ (b) ${}^{18}C_3$ (c) ${}^{12}C_4$ (d) ${}^{18}C_4$

Solution: (c) 6 particular players are always to be included and 4 are always excluded, so total number of selection, now 4 players out of 12.

Hence number of ways = ${}^{12}C_4$.

Example : 36 In how many ways can 6 persons to be selected from 4 officers and 8 constables, if at least one officer is to be included

[Roorkee 1985; MP PET 2001]

(a) 224 (b) 672 (c) 896 (d) None of these

Solution: (c) Required number of ways = ${}^4C_1 \times {}^8C_5 + {}^4C_2 \times {}^8C_4 + {}^4C_3 \times {}^8C_3 + {}^4C_4 \times {}^8C_2$ = 4 × 56 + 6 × 70 + 4 × 56 + 1 × 28 = 896.

5.13 Division into Groups

Case I : (1) The number of ways in which n different things can be arranged into r different groups is ${}^{n+r-1}P_n$ or $n\,!\ {}^{n-1}C_{r-1}$ according as blank group are or are not admissible.

(2) The number of ways in which n different things can be distributed into r different group is

$r^n - {}^rC_1(r-1)^n + {}^rC_2(r-2)^n - .... + (-1)^{n-1}\ {}^nC_{r-1}$ or Coefficient of x^n is $n\,!\ (e^x - 1)^r$

Here blank groups are not allowed.

(3) Number of ways in which $m \times n$ different objects can be distributed equally among n persons (or numbered groups) = (number of ways of dividing into groups) × (number of groups) ! = $\frac{(mn)!n!}{(m!)^n n!} = \frac{(mn)!}{(m!)^n}$.

Case II : (1) The number of ways in which $(m+n)$ different things can be divided into two groups which contain m and n things respectively is, ${}^{m+n}C_m.{}^nC_n = \frac{(m+n)!}{m!n!}, m \neq n$.

Corollary: If $m = n$, then the groups are equal size. Division of these groups can be given by two types.

Type I : If order of group is not important : The number of ways in which $2n$ different things can be divided equally into two groups is $\frac{(2n)!}{2!(n!)^2}$

Type II : If order of group is important : The number of ways in which $2n$ different things can be divided equally into two distinct groups is $\frac{(2n)!}{2!(n!)^2} \times 2! = \frac{2n!}{(n!)^2}$

(2) The number of ways in which $(m + n + p)$ different things can be divided into three groups which contain m, n and p things respectively is ${}^{m+n+p}C_m.{}^{n+p}C_n.{}^pC_p = \frac{(m+n+p)!}{m!n!p!}, m \neq n \neq p$

Corollary: If $m = n = p$, then the groups are equal size. Division of these groups can be given by two types.

Type I : If order of group is not important : The number of ways in which $3p$ different things can be divided equally into three groups is $\frac{(3p)!}{3!(p!)^3}$

Type II : If order of group is important : The number of ways in which $3p$ different things can be divided equally into three distinct groups is $\frac{(3p)!}{3!(p!)^3} t3! = \frac{(3p)!}{(p!)^3}$

Note : ❑ If order of group is not important : The number of ways in which mn different things can be divided equally into m groups is $\frac{mn!}{(n!)^m m!}$

❑ If order of group is important: The number of ways in which mn different things can be divided equally into m distinct groups is $\frac{(mn)!}{(n!)^m m!} \times m! = \frac{(mn)!}{(n!)^m}$.

Example: 37 In how many ways can 5 prizes be distributed among four students when every student can take one or more prizes

[BIT Ranchi 1990; Rajasthan PET 1988, 97]

(a) 1024 (b) 625 (c) 120 (d) 60

Solution: (a) The required number of ways = $4^5 = 1024$ [since each prize can be distributed by 4 ways]

Example: 38 The number of ways in which 9 persons can be divided into three equal groups is

(a) 1680 (b) 840 (c) 560 (d) 280

Solution: (d) Total ways = $\frac{9!}{(3!)^3} = \frac{9\times 8\times 7\times 6\times 5\times 4}{3\times 2\times 3\times 2\times 3\times 2}$ = 280.

Example: 39 The number of ways dividing 52 cards amongst four players equally, are **[IIT 1979]**

(a) $\frac{52!}{(13!)^4}$ (b) $\frac{52!}{(13!)^2 4!}$ (c) $\frac{52!}{(12!)^4 4!}$ (d) None of these

Solution: (a) Required number of ways = ${}^{52}C_{13}\times {}^{39}C_{13}\times {}^{26}C_{13}\times {}^{13}C_{13}$ = $\frac{52!}{39!13!}\times\frac{39!}{26!13!}\times\frac{26!}{13!13!}\times\frac{13!}{13!} = \frac{52!}{(13!)^4}$.

Example: 40 A question paper is divided into two parts A and B and each part contains 5 questions. The number of ways in which a candidate can answer 6 questions selecting at least two questions from each part is

(a) 80 (b) 100 (c) 200 (d) None of these

Solution: (c) The number of ways that the candidate may select

2 questions from A and 4 from B = ${}^5C_2\times{}^5C_4$; 3 questions form A and 3 from B = ${}^5C_3\times{}^5C_3$

4 questions from A and 2 from B = ${}^5C_4\times{}^5C_2$. Hence total number of ways are 200.

5.14 Derangement

Any change in the given order of the things is called a derangement.

If n things form an arrangement in a row, the number of ways in which they can be deranged so that no one of them occupies its original place is $n!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+......+(-1)^n.\frac{1}{n!}\right)$.

Example: 41 There are four balls of different colours and four boxes of colurs same as those of the balls. The number of ways in which the balls, one in each box, could be placed such that a ball doesn't go to box of its own colour is **[IIT 1992]**

(a) 8 (b) 7 (c) 9 (d) None of these

Solution: (c) Number of derangement are = $4!\left\{\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right\} = 12-4+1=9$.

(Since number of derangements in such a problem is given by $n!\left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}......+(-1)^n\frac{1}{n!}\right\}$.

5.15 Some Important Results for Geometrical Problems

(1) Number of total different straight lines formed by joining the n points on a plane of which m (< n) are collinear is ${}^nC_2-{}^mC_2+1$.

(2) Number of total triangles formed by joining the n points on a plane of which m (< n) are collinear is ${}^nC_3-{}^mC_3$.

(3) Number of diagonals in a polygon of n sides is ${}^nC_2-n$.

(4) If m parallel lines in a plane are intersected by a family of other n parallel lines. Then total number of parallelograms so formed is ${}^mC_2\times{}^nC_2$ i.e $\frac{mn(m-1)(n-1)}{4}$

(5) Given n points on the circumference of a circle, then

(i) Number of straight lines = ${}^nC_2$ (ii) Number of triangles = ${}^nC_3$ (iii) Number of quadrilaterals = ${}^nC_4$.

(6) If n straight lines are drawn in the plane such that no two lines are parallel and no three lines are concurrent. Then the number of part into which these lines divide the plane is = $1+\Sigma n$.

(7) Number of rectangles of any size in a square of $n \times n$ is $\sum_{r=1}^{n} r^3$ and number of squares of any size is $\sum_{r=1}^{n} r^2$.

(8) In a rectangle of $n \times p$ $(n < p)$ number of rectangles of any size is $\frac{np}{4}(n+1)(p+1)$ and number of squares of any size is $\sum_{r=1}^{n}(n+1-r)(p+1-r)$.

Example: 42 The number of diagonals in a octagon will be **[MP PET 1984; Pb. CET 1989, 2000]**

(a) 28 (b) 20 (c) 10 (d) 16

Solution: (b) Number of diagonals = ${}^8C_2 - 8$ = $28 - 8$ = 20.

Example: 43 The number of straight lines joining 8 points on a circle is

(a) 8 (b) 16 (c) 24 (d) 28

Solution: (d) Number of straight line = ${}^8C_2$ = 28.

Example: 44 The number of triangles that can be formed by choosing the vertices from a set of 12 points, seven of which lie on the same straight line, is **[Roorkee 1989, 2000; BIT Ranchi 1989; MP PET 1995; Pb. CET 1997; DCE 20**

(a) 185 (b) 175 (c) 115 (d) 105

Solution: (a) Required number of ways = ${}^{12}C_3 - {}^7C_3$ = 220 – 35 = 185.

Example: 45 Out of 18 points in a plane, no three are in the same straight line except five points which are collinear. The number of (i) straight lines (ii) triangles which can be formed by joining them

(i) (a) 140 (b) 142 (c) 144 (d) 146

(ii) (a) 816 (b) 806 (c) 800 (d) 750

Solution: (c, b) Out of 18 points, 5 are collinear

(i) Number of straight lines = ${}^{18}C_2 - {}^5C_2 + 1 = 153 - 10 + 1 = 144$

(ii) Number of triangles $= {}^{18}C_3 - {}^5C_3 = 816 - 10 = 806$.

5.16 Multinomial Theorem

Let $x_1, x_2, ..., x_m$ be integers. Then number of solutions to the equation $x_1 + x_2 + ... + x_m = n$ .....(i)

Subject to the condition $a_1 \le x_1 \le b_1, a_2 \le x_2 \le b_2, ..., a_m \le x_m \le b_m$ .....(ii)

is equal to the coefficient of x^n in

$(x^{a_1} + x^{a_1+1} + ... + x^{b_1})(x^{a_2} + x^{a_2+1} + ... + x^{b_2})......(x^{a_m} + x^{a_{m+1}} + ... + x^{b_m})$ ......(iii)

This is because the number of ways, in which sum of *m* integers in (i) equals *n*, is the same as the number of times x^n comes in (iii).

(1) **Use of solution of linear equation and coefficient of a power in expansions to find the number of ways of distribution :** (i) The number of integral solutions of $x_1 + x_2 + x_3 + ... + x_r = n$ where $x_1 \ge 0, x_2 \ge 0, ...x_r \ge 0$ is the same as the number of ways to distribute *n* identical things among *r* persons.

This is also equal to the coefficient of x^n in the expansion of $(x^0 + x^1 + x^2 + x^3 + ...)^r$

= coefficient of x^n in $\left(\frac{1}{1-x}\right)^r$ = coefficient of x^n in $(1-x)^{-r}$

$= \text{coefficient of } x^n \text{ in } \left\{1 + rx + \frac{r(r+1)}{2!}x^2 + ... + \frac{r(r+1)(r+2)......(r+n-1)}{n!}x^n + ...\right.$

$= \frac{r(r+1)(r+2)....(r+n-1)}{n!} = \frac{(r+n-1)!}{n!(r-1)!} = {}^{n+r-1}C_{r-1}$

(ii) The number of integral solutions of $x_1 + x_2 + x_3 + ... + x_r = n$ where $x_1 \geq 1, x_2 \geq 1, ...x_r \geq 1$ is same as the number of ways to distribute n identical things among r persons each getting at least 1. This also equal to the coefficient of x^n in the expansion of $(x^1 + x^2 + x^3 + ...)^r$

$= \text{coefficient of } x^n \text{ in } \left(\frac{x}{1-x}\right)^r = \text{coefficient of } x^n \text{ in } x^r(1-x)^{-r}$

$= \text{coefficient of } x^n \text{ in } x^r\left\{1 + rx + \frac{r(r+1)}{2!}x^2 + ... + \frac{r(r+1)(r+2).....(r+n-1)}{n!}x^n + ...\right\}$

$= \text{coefficient of } x^{n-r} \text{ in } \left\{1 + rx + \frac{r(r+1)}{2!}x^2 + ... + \frac{r(r+1)(r+2).....(r+n-1)}{n!}x^n + ...\right\}$

$= \frac{r(r+1)(r+2)......(r+n-r-1)}{(n-r)!} = \frac{r(r+1)(r+2).....(n-1)}{(n-r)!} = \frac{(n-1)!}{(n-r)!(r-1)!} = {}^{n-1}C_{r-1}$.

Example: 46 A student is allowed to select utmost n books from a collection of $(2n+1)$ books. If the total number of ways in which he can select one book is 63, then the value of n is **[IIT 1987; Rajasthan PET 1999]**

(a) 2 (b) 3 (c) 4 (d) None of these

Solution: (b) Since the student is allowed to select utmost n books out of $(2n+1)$ books. Therefore in order to select one book he has the choice to select one, two, three,......., n books.

Thus, if T is the total number of ways of selecting one book then $T = {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n = 63$.

Again the sum of binomial coefficients

${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + ... + {}^{2n+1}C_{2n+1} = (1+1)^{2n+1} = 2^{2n+1}$

or, ${}^{2n+1}C_0 + 2({}^{2n-1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n) + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow 1 + 2(T) + 1 = 2^{2n+1} \Rightarrow 1 + T = \frac{2^{2n+1}}{2} = 2^{2n} \Rightarrow 1 + 63 = 2^{2n} \Rightarrow 2^6 = 2^{2n} \Rightarrow n = 3$.

Example: 47 If x, y and r are positive integers, then ${}^xC_r + {}^xC_{r-1}{}^yC_1 + {}^xC_{r-2}\,{}^yC_2 + ... + {}^yC_r =$

[Karnataka CET 1993; Rajasthan PET 2001]

(a) $\frac{x!y!}{r!}$ (b) $\frac{(x+y)!}{r!}$ (c) ${}^{x+y}C_r$ (d) ${}^{xy}C_r$

Solution: (c) The result ${}^{x+y}C_r$ is trivially true for $r = 1, 2$ it can be easily proved by the principle of mathematical induction that the result is true for r also.

5.17 Number of Divisors

Let $N = p_1^{\alpha_1}.p_2^{\alpha_2}.p_3^{\alpha_3}......p_k^{\alpha_k}$, where $p_1, p_2, p_3, ...p_k$ are different primes and $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_k$ are natural numbers then :

(1) The total number of divisors of N including 1 and N is = $(\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1)....(\alpha_k + 1)$

(2) The total number of divisors of N excluding 1 and N is = $(\alpha_1+1)(\alpha_2+1)(\alpha_3+1).....(\alpha_k+1)-2$

(3) The total number of divisors of N excluding 1 or N is = $(\alpha_1+1)(\alpha_2+1)(\alpha_3+1).....(\alpha_k+1)-1$

(4) The sum of these divisors is $=(p_1^0+p_2^1+p_3^2+......+p_1^{\alpha_1})(p_2^0+p_2^1+p_2^2+...+p_2^{\alpha_2}).....(p_k^0+p_k^1+p_k^2+....+p_k^{\alpha_k})$

(5) The number of ways in which N can be resolved as a product of two factors is

$$\begin{cases} \frac{1}{2}(\alpha_1+1)(\alpha_2+1)....(\alpha_k+1), \text{If } N \text{ is not a perfect square} \\ \frac{1}{2}[(\alpha_1+1)(\alpha_2+1).....(\alpha_k+1)+1], \text{If } N \text{ is a perfect square} \end{cases}$$

(6) The number of ways in which a composite number N can be resolved into two factors which are relatively prime (or co-prime) to each other is equal to 2^{n-1} where n is the number of different factors in N.

Important Tips

☞ *All the numbers whose last digit is an even number 0, 2, 4, 6 or 8 are divisible by 2.*

☞ *All the numbers sum of whose digits are divisible by 3,is divisible by 3 e.g. 534. Sum of the digits is 12, which are divisible by 3, and hence 534 is also divisible by 3.*

☞ *All those numbers whose last two-digit number is divisible by 4 are divisible by 4 e.g. 7312, 8936, are such that 12, 36 are divisible by 4 and hence the given numbers are also divisible by 4.*

☞ *All those numbers, which have either 0 or 5 as the last digit, are divisible by 5.*

☞ *All those numbers, which are divisible by 2 and 3 simultaneously, are divisible by 6. e.g., 108, 756 etc.*

☞ *All those numbers whose last three-digit number is divisible by 8 are divisible by 8.*

☞ *All those numbers sum of whose digit is divisible by 9 are divisible by 9.*

☞ *All those numbers whose last two digits are divisible by 25 are divisible by 25 e.g., 73125, 2400 etc.*

Example: 48 The number of divisors of 9600 including 1 and 9600 are

(a) 60 (b) 58 (c) 48 (d) 46

Solution: (c) Since 9600 = $2^7\times3^1\times5^2$

Hence number of divisors = $(7+1)(1+1)(2+1)=48$.

Example: 49 Number of divisors of $n=38808$ (except 1 and n) is

(a) 70 (b) 68 (c) 72 (d) 74

Solution: (a) Since 38808 = 8 × 4851 = 8 × 9 × 539 = 8 × 9 × 7 × 7 × 11 = $2^3\times3^2\times7^2\times11$

So, number of divisors = (3 + 1) (2 + 1) (2 + 1) (1 + 2) – 2 = 72 – 2 = 70.

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Assignment

Fundamental concept and Permutations without repetition

Basic Level

1. $(n-r+1)\,^nP_{r-1} =$

(a) $^{n-1}P_r$ (b) $^{n+1}P_r$ (c) ^nP_r (d) $^nP_{r-1}$

2. If $^5P_r = 120$, then the value of r is

(a) 2 (b) 3+ (c) 5 (d) 4

3. If $^nP_5 : {}^nP_3 = 2:1$, then the value of n is **[Rajasthan PET 1989]**

(a) 2 (b) 3 (c) 4 (d) 5

4. The value of $n.{}^{n-1}P_{r-1}$ is **[DCE 1998]**

(a) ^nP_r (b) $^{n-1}P_{r-1}$ (c) $^{n+1}P_{r+1}$ (d) $^{n-1}P_r$

5. If $^{m+n}P_2 = 56$ and $^{m-n}P_2 = 12$, then m, n are equal to

(a) 5, 1 (b) 6, 2 (c) 7, 3 (d) 9, 6

6. If $^{K+5}P_{K+1} = \frac{11(K-1)}{2}\,^{K+3}P_K$ then the values of K are

(a) 2 and 6 (b) 2 and 11 (c) 7 and 11 (d) 6 and 7

7. There are 5 roads leading to a town from a village. The number of different ways in which a villager can go to the town and return back, is

(a) 25 (b) 20 (c) 10 (d) 5

8. How many words can be formed from the letters of the word BHOPAL

(a) 124 (b) 240 (c) 360 (d) 720

9. How many numbers can be formed from the digits1, 2, 3, 4 when the repetition is not allowed

(a) $^4P_4$ (b) $^4P_3$ (c) $^4P_1 + {}^4P_2 + {}^4P_3$ (d) $^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$

10. How many numbers lying between 500 and 600 can be formed with the help of the digits 1, 2, 3, 4, 5, 6 when the digits are not to be repeated

(a) 20 (b) 40 (c) 60 (d) 80

11. 4 buses runs between Bhopal and Gwalior. If a man goes from Gwalior to Bhopal by a bus and comes back to Gwalior by another bus, then the total possible ways are

(a) 12 (b) 16 (c) 4 (d) 8

12. In how many ways can 10 true-false questions be replied

(a) 20 (b) 100 (c) 512 (d) 1024

13. There are 8 gates in a hall. In how many ways a person can enter in the hall and come out from a different gate

(a) 7 (b) 8 × 8 (c) 8 + 7 (d) 8 × 7

14. *P*, *Q*, *R* and *S* have to give lectures to an audience. The organiser can arrange the order of their presentation in **[BIT Ranchi 1991; Pb. CET 1991]**

(a) 4 ways (b) 12 ways (c) 256 ways (d) 24 ways

15. The product of any r consecutive natural numbers is always divisible by

(a) $r\,!$ (b) r^2 (c) r^n (d) None of these

16. The number of ways in which first, second and third prizes can be given to 5 competitors is

(a) 10 (b) 60 (c) 15 (d) 125

17. In a railway compartment there are 6 seats. The number of ways in which 6 passengers can occupy these 6 seats is

[Karnataka CET 2001]

(a) 36 (b) 30 (c) 720 (d) 120

18. If any number of flags are used, how many signals can be given with the help of 6 flags of different colours

(a) 1956 (b) 1958 (c) 720 (d) None of these

19. The number of ways of painting the faces of a cube with six different colours is

(a) 1 (b) 6 (c) 6 ! (d) None of these

Advance Level

20. The value of $2^n\{1.3.5.....(2n-3)(2n-1)\}$ is

(a) $\frac{(2n)!}{n\,!}$ (b) $\frac{(2n)!}{2^n}$ (c) $\frac{n!}{(2n)!}$ (d) None of these

21. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, then r = **[Roorkee 1983; Kurukshetra CEE 1998]**

(a) 31 (b) 41 (c) 51 (d) None of these

22. The value of ${}^nP_r$ is equal to

(a) ${}^{n-1}P_r + r\ {}^{n-1}P_{r-1}$ (b) $n.{}^{n-1}P_r + {}^{n-1}P_{r-1}$ (c) $n({}^{n-1}P_r + {}^{n-1}P_{r-1})$ (d) ${}^{n-1}P_{r-1} + {}^{n-1}P_r$

23. The exponent of 3 in 100 ! is

(a) 33 (b) 44 (c) 48 (d) 52

24. The number of positive integral solutions of $abc = 30$ is **[UPSEAT 2001]**

(a) 30 (b) 27 (c) 8 (d) None of these

25. The number of 4 digit even numbers that can be formed using 0, 1, 2, 3, 4, 5, 6 without repetition is

(a) 120 (b) 300 (c) 420 (d) 20

26. The number of five digits numbers that can be formed without any restriction is

(a) 990000 (b) 100000 (c) 90000 (d) None of these

27. How many numbers less than 1000 can be made from the digits 1, 2, 3, 4, 5, 6 (repetition is not allowed)

(a) 156 (b) 160 (c) 150 (d) None of these

28. How many even numbers of 3 different digits can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 (repetition is not allowed)

(a) 224 (b) 280 (c) 324 (d) None of these

29. A five digit number divisible by 3 has to formed using the numerals 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways in which this can be done is

(a) 216 (b) 240 (c) 600 (d) 3125

30. In a circus there are ten cages for accommodating ten animals. Out of these four cages are so small that five out of 10 animals cannot enter into them. In how many ways will it be possible to accommodate ten animals in these ten cages **[Roorkee 1989]**

(a) 66400 (b) 86400 (c) 96400 (d) None of these

31. How many numbers can be made with the digits 3, 4, 5, 6, 7, 8 lying between 3000 and 4000 which are divisible by 5 while repetition of any digit is not allowed in any number

(a) 60 (b) 12 (c) 120 (d) 24

32. All possible four digit numbers are formed using the digits 0, 1, 2, 3 so that no number has repeated digits. The number of even numbers among them is

(a) 9 (b) 18 (c) 10 (d) None of these

33. The total number of seven digit numbers the sum of whose digits is even is

(a) 9000000 (b) 4500000 (c) 8100000 (d) None of these

34. The sum of all 4 digit numbers that can be formed by using the digits 2, 4, 6, 8 (repetition of digits not allowed) is

(a) 133320 (b) 533280 (c) 53328 (d) None of these

35. How many numbers greater than 24000 can be formed by using digits 1, 2, 3, 4, 5 when no digit is repeated **[Rajasthan**

(a) 36 (b) 60 (c) 84 (d) 120

36. How many numbers greater than hundred and divisible by 5 can be made from the digits 3, 4, 5, 6, if no digit is repeated **[AMU 1999]**

(a) 6 (b) 12 (c) 24 (d) 30

37. The sum of all numbers greater than 1000 formed by using the digits 1, 3, 5, 7 no digit is repeated in any number is **[AMU 1997]**

(a) 106656 (b) 101276 (c) 117312 (d) 811273

38. 3 copies each of 4 different books are available. The number of ways in which these can be arranged on the shelf is

[Karnataka CET 1996]

(a) 12 ! (b) $\frac{12!}{3!4!}$ (c) $\frac{12!}{(3!)4}$ (d) 369,000

39. Eleven books consisting of 5 Mathematics, 4 Physics and 2 Chemistry are placed on a shelf. The number of possible ways of arranging them on the assumption that the books of the same subject are all together is

(a) 4 ! 2! (b) 11! (c) 5! 4! 3! 2! (d) None of these

40. The number of positive integers which can be formed by using any number of digits from 0, 1, 2, 3, 4, 5 but using each digit not more than once in each number is

(a) 1200 (b) 1500 (c) 1600 (d) 1630

41. Let A be a set of $n(\geq 3)$ distinct elements. The number of triplets (x, y, z) of the elements of A in which at least two coordinates are equal is

(a) ${}^nP_3$ (b) $n^3 - {}^nP_3$ (c) $3n^2 - 2n$ (d) $3n^2(n-1)$

42. The number of distinct rational numbers x such that $0 < x < 1$ and $x = \frac{p}{q}$, where $p, q \in \{1, 2, 3, 4, 5, 6\}$ is

(a) 15 (b) 13 (c) 12 (d) 11

43. The total number of 5 digit numbers of different digits in which the digit in the middle is the largest is

(a) $\sum_{n=4}^{9} {}^nP_4$ (b) 33 (3!) (c) 30 (3 !) (d) None of these

44. Two teams are to play a series of 5 matches between them. A match ends in a win or loss or draw for a team. A number of people forecast the result of each match and no two people make the same forecast for the series of matches. The smallest group of people in which one person forecasts correctly for all matches will contain n people, where n is

(a) 81 (b) 243 (c) 486 (d) None of these

Number of Permutations with Repetition

Basic Level

45. The number of permutations of the letters $x^2y^4z^3$ will be

(a) $\frac{9!}{2!4!}$ (b) $\frac{9!}{2!4!3!}$ (c) $\frac{9!}{4!3!}$ (d) $9!$

46. How many numbers consisting of 5 digits can be formed in which the digits 3, 4 and 7 are used only once and the digit 5 is used twice

(a) 30 (b) 60 (c) 45 (d) 90

47. The number of different arrangements which can be made from the letters of the word SERIES taken all together is

(a) $\frac{6!}{2!2!}$ (b) $\frac{6!}{4!}$ (c) $6!$ (d) None of these

48. How many words can be formed with the letters of the word MATHEMATICS by rearranging them

(a) $\frac{11!}{2!2!}$ (b) $\frac{11!}{2!}$ (c) $\frac{11!}{2!2!2!}$ (d) $11!$

49. How many words can be made out from the letters of the word INDEPENDENCE, in which vowels always come together

[Roorkee 1989]

(a) 16800 (b) 16630 (c) 1663200 (d) None of these

50. In how many ways 5 red, 4 blue and 1 green balls can be arranged in a row

(a) 1260 (b) 2880 (c) $9!$ (d) $10!$

51. Using 5 conveyances, the number of ways of making 3 journeys is

(a) 3×5 (b) 3^5 (c) 5^3 (d) $5^3 - 1$

52. The total number of permutations of the letters of the word "BANANA" is **[Rajasthan PET 1997, 2000]**

(a) 60 (b) 120 (c) 720 (d) 24

53. The number of 7 digit numbers which can be formed using the digits 1, 2, 3, 2, 3, 3, 4 is

(a) 420 (b) 840 (c) 2520 (d) 5040

54. The number of 3 digit odd numbers, that can be formed by using the digits 1, 2, 3, 4, 5, 6 when the repetition is allowed, is

[Pb. CET 1999]

(a) 60 (b) 108 (c) 36 (d) 30

55. How many different nine-digit numbers can be formed from the digits of the number 223355888 by rearrangement of the digits so that the odd digits occupy even places **[IIT Screening 2000; Karnataka CET 2002]**

(a) 16 (b) 36 (c) 60 (d) 180

56. Using all digits 2, 3, 4, 5, 6 how many even numbers can be formed

(a) 24 (b) 48 (c) 72 (d) 120

57. Let S be the set of all functions from the set A to the set A. If $n(A) = k$ then $n(S)$ is

(a) $k!$ (b) k^k (c) $2^k - 1$ (d) 2^k

58. The number of ways in which 6 rings can be worn on the four fingers of one hand is **[AMU 1983]**

(a) 4^6 (b) ${}^6C_4$ (c) 6^4 (d) None of these

59. In how many ways can 4 prizes be distributed among 3 students, if each student can get all the 4 prizes

(a) $4!$ (b) 3^4 (c) $3^4 - 1$ (d) 3^3

60. In how many ways 3 letters can be posted in 4 letter-boxes, if all the letters are not posted in the same letter-box

(a) 63 (b) 60 (c) 77 (d) 81

61. There are 4 parcels and 5 post-offices. In how many different ways the registration of parcel can be made

(a) 20 (b) 4^5 (c) 5^4 (d) $5^4 - 4^5$

Advance Level

62. How many numbers lying between 10 and 1000 can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 (repetition is allowed)

(a) 1024 (b) 810 (c) 2346 (d) None of these

63. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is

[IIT 1980; MNR 1998, 99; DCE 2001]

(a) 69760 (b) 30240 (c) 99748 (d) None of these

64. Six identical coins are arranged in a row. The number of ways in which the number of tails is equal to the number of heads is

(a) 20 (b) 9 (c) 120 (d) 40

65. The total number of permutations of $n(>1)$ different things taken not more than r at a time, when each thing may be repeated any number of times is

(a) $\frac{n(n^n-1)}{n-1}$ (b) $\frac{n^r-1}{n-1}$ (c) $\frac{n(n^r-1)}{n-1}$ (d) None of these

66. How many number less than 10000 can be made with the eight digits 1, 2, 3, 4, 5, 6, 7, 0 (digits may repeat)

(a) 256 (b) 4095 (c) 4096 (d) 4680

67. The total number of natural numbers of six digits that can be made with digits 1, 2, 3, 4, if the all digits are to appear in the same number at least once, is

(a) 1560 (b) 840 (c) 1080 (d) 480

68. A library has *a* copies of one book, *b* copies of each of two books, *c* copies of each of three books and single copies of *d* books. The total number of ways in which these books can be distributed is

(a) $\frac{(a+b+c+d)!}{a!b!c!}$ (b) $\frac{(a+2b+3c+d)!}{a!(b!)^2(c!)^3}$ (c) $\frac{(a+2b+3c+d)!}{a!b!c!}$ (d) None of these

69. The number of ways of arranging 2*m* white counters and 2*n* red counters in a straight line so that the arrangement is symmetrical with respect to a central mark

(a) $(m+n)!$ (b) $\frac{(m+n)!}{m!n!}$ (c) $\frac{2(m+n)!}{m!n!}$ (d) None of these

70. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 are **[AIEEE 2002]**

(a) 216 (b) 375 (c) 400 (d) 720

71. The number of ways of arranging the letter AAAAA BBB CCC D EE F in a row when no two C's are together is

(a) $\frac{15!}{5!3!3!2!}-3!$ (b) $\frac{15!}{5!3!3!2!}-\frac{13!}{5!3!2!}$ (c) $\frac{12!}{5!3!2!}\times\frac{^{13}P_3}{3!}$ (d) $\frac{12!}{5!3!2!}\times {^{13}P_3}$

72. The number of 4 digit numbers that can be made with the digits 1, 2, 3, 4 and 5 in which at least two digits are identical, is

(a) $4^5-5!$ (b) 505 (c) 600 (d) None of these

Conditional Permutations

Basic Level

73. The number of words which can be formed from the letters of the word MAXIMUM, if two consonants cannot occur together, is

(a) 4 ! (b) 3 ! × 4 ! (c) 7 ! (d) None of these

74. The number of ways in which the letters of the word TRIANGLE can be arranged such that two vowels do not occur together is

(a) 1200 (b) 2400 (c) 14400 (d) None of these

75. How many words can be formed form the letters of the word COURTESY, whose first letter is C and the last letter is *Y*

(a) 6 ! (b) 8 ! (c) 2 (6) ! (d) 2 (7) !

76. How many words can be made from the letters of the word DELHI, if L comes in the middle in every word

(a) 12 (b) 24 (c) 60 (d) 6

77. The number of ways in which the letters of the word ARRANGE can be arranged such that both R do not come together is

[MP PET 1993]

(a) 360 (b) 900 (c) 1260 (d) 1620

78. How many words can be made from the letters of the word BHARAT in which *B* and *H* never come together **[IIT 1977]**

(a) 360 (b) 300 (c) 240 (d) 120

79. How many words can be made from the letters of the word INSURANCE, if all vowels come together

(a) 18270 (b) 17280 (c) 12780 (d) None of these

80. There are three girls in a class of 10 students. The number of different ways in which they can be seated in a row such that no two of the three girls are together is

(a) $7!\times {}^6P_3$ (b) $7!\times {}^8P_3$ (c) $7!\times 3!$ (d) $\frac{10!}{3!7!}$

81. In how many ways can 5 boys and 5 girls stand in a row so that no two girls may be together

(a) $(5!)^2$ (b) $5!\times 4!$ (c) $5!\times 6!$ (d) $6\times 5!$

82. The number of arrangements of the letters of the word BANANA in which two N's do not appear adjacently is **[IIT Scre**

(a) 40 (b) 60 (c) 80 (d) 100

83. The number of ways in which 5 boys and 3 girls can be seated in a row so that each girl in between two boys **[Kerala (**

(a) 2880 (b) 1880 (c) 3800 (d) 2800

84. The number of words that can be formed out of the letters of the word ARTICLE so that the vowels occupy even places is

[Karnataka CET 2003]

(a) 36 (b) 574 (c) 144 (d) 754

85. The number of ways in which three students of a class may be assigned a grade of *A, B, C* or *D* so that no two students receive the same grade, is

(a) 3^4 (b) 4^3 (c) ${}^4P_3$ (d) ${}^4C_3$

86. The number of ways lawn tennis mixed double can be made up from seven married couples if no husband and wife play in the same set is

(a) 210 (b) 420 (c) 840 (d) None of these

Advance Level

87. How many numbers greater 40000 can be formed from the digits 2, 4, 5, 5, 7

(a) 12 (b) 24 (c) 36 (d) 48

88. In how many ways *n* books can be arranged in a row so that two specified books are not together

(a) $n!-(n-2)!$ (b) $(n-1)!(n-2)$ (c) $n!-2(n-1)$ (d) $(n-2)n!$

89. How many numbers between 5000 and 10,000 can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit appearing not more than once in each number

(a) $5\times {}^8P_3$ (b) $5\times {}^8C_3$ (c) $5!\times {}^8P_3$ (d) $5!\times {}^8C_3$

90. Find the total number of 9 digit numbers which have all the digits different **[IIT 1982]**

(a) 9 × 9 ! (b) 9 ! (c) 10 ! (d) None of these

91. Four dice (six faced) are rolled. The number of possible outcomes in which at least one die shows 2 is

(a) 1296 (b) 625 (c) 671 (d) None of these

92. How many numbers, lying between 99 and 1000 be made from the digits 2, 3, 7, 0, 8, 6 when the digits occur only once in each number **[MP PET 1984]**

(a) 100 (b) 90 (c) 120 (d) 80

93. The sum of the digits in the unit place of all numbers formed with the help of 3, 4, 5, 6 taken all at a time is **[Pb. CET 19**

(a) 18 (b) 432 (c) 108 (d) 144

94. All letters of the word AGAIN are permuted in all possible ways and the words so formed (with or without meaning) are written as in dictionary, then the 50^{th} word is

(a) NAAGI (b) IAANG (c) NAAIG (d) INAGA

95. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4 and then men select the chairs from amongst the remaining. The number of possible arrangements is

(a) ${}^6C_3 \times {}^4C_2$ (b) ${}^4C_2 \times {}^4P_3$ (c) ${}^4P_2 \times {}^4P_3$ (d) None of these

96. If a denotes the number of permutations of $x+2$ things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of $x-11$ things taken all at a time such that $a = 182bc$, then the value of x is

(a) 15 (b) 12 (c) 10 (d) 18

97. The number of ways in which ten candidates $A_1, A_2, ..., A_{10}$ can be ranked such that A_1 is always above A_{10} is

(a) 5 ! (b) 2 (5 !) (c) 10 ! (d) $\frac{1}{2}(10\,!)$

98. A dictionary is printed consisting of 7 lettered words only that can be made with a letter of the word CRICKET. If the words are printed at the alphabetical order, as in an ordinary dictionary, the number of word before the word CRICKET is **[Orissa JEE 2003]**

(a) 530 (b) 480 (c) 531 (d) 481

99. Seven different lecturers are to deliver lectures in seven periods of a class on a particular day. A, B and C are three of the lecturers. The number of ways in which a routine for the day can be made such that A delivers his lecture before B, and B before C, is

(a) 420 (b) 120 (c) 210 (d) None of these

100. Let $A = \{x : x$ is a prime number and $x < 30\}$. The number of different rational numbers whose numerator and denominator belong to A is

(a) 90 (b) 180 (c) 91 (d) None of these

101. The number of numbers of 9 different non-zero digits such that all the digits in the first four places are less than the digit in the middle and all the digits in the last four places are greater than that in the middle is

(a) 2 (4 !) (b) $(4!)^2$ (c) 8 ! (d) None of these

102. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order**[AIEEE 200**

(a) 480 (b) 240 (c) 360 (d) 120

Circular Permutations

Basic Level

103. If eleven members of a committee sit at a round table so that the president and secretary always sit together, then the number of arrangement is

(a) 10 ! × 2 (b) 10 ! (c) 9 ! × 2 (d) None of these

104. In how many ways can 5 keys be put in a ring

(a) $\frac{4!}{2}$ (b) $\frac{5!}{2}$ (c) 4 ! (d) 5!

105. In how many ways can 12 gentlemen sit around a round table so that three specified gentlemen are always together

(a) 9 ! (b) 10 ! (c) 3 ! 10! (d) 3 ! 9 !

106. n gentlemen can be made to sit on a round table in **[MP PET 1982]**

(a) $\frac{1}{2}(n+1)!$ ways (b) $(n-1)!$ ways (c) $\frac{1}{2}(n-1)!$ ways (d) $(n+1)!$ ways

107. In how many ways 7 men and 7 women can be seated around a round table such that no two women can sit together

[EAMCET 1990; MP PET 2001; DCE 2001; UPSEAT 2002]

(a) $(7!)^2$ (b) $7! \times 6!$ (c) $(6!)^2$ (d) 7 !

108. The number of circular permutations of n different objects is

(a) n ! (b) n (c) $(n-2)!$ (d) $(n-1)!$

Advance Level

109. In how many ways can 15 members of a council sit along a circular table, when the Secretary is to sit on one side of the Chairman and the Deputy secretary on the other side

(a) $2 \times 12!$ (b) 24 (c) 2 × 15 ! (d) None of these

110. 20 persons are invited for a party. In how many different ways can they and the host be seated at a circular table, if the two particular persons are to be seated on either side of the host

(a) 20! (b) 2 . 18 ! (c) 18 ! (d) None of these

111. 12 persons are to be arranged to a round table. If two particular persons among them are not to be side by side, the total number of arrangements is

(a) 9(10 !) (b) 2 (10 !) (c) 45 (8 !) (d) 10 !

112. The number of ways that 8 beads of different colours be string as a necklace is **[EAMCET 2002]**

(a) 2520 (b) 2880 (c) 5040 (d) 4320

113. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by

[AIEEE 2003; Rajasthan PET 2003]

(a) 6 ! × 5 ! (b) 30 (c) 5 ! × 4 ! (d) 7 ! × 5 !

114. In how many ways can 10 persons sit, when 6 persons sit on one round table and 4 sit on the other round table

(a) 5 ! × 3 ! (b) 10 × 5! × 3 ! (c) ${}^{10}C_6$ × 5 !× 3! (d) ${}^{10}C_6$ × 5! × 3! × 2!

115. There are 20 persons among whom two are brothers . The number of ways in which we can arrange them round a circle so that there is exactly one person between the two brothers , is

(a) 18 ! (b) 2 (18!) (c) 2 (19 !) (d) None of these

116. A family has 8 members. Four members take food two times a day on two identical round tables. For how many months 8 members can take food by sitting in different orders (1 month = 30 days)

(a) 42 months (b) 21 months (c) $\frac{21}{2}$ months (d) None of these

Fundamental concept and Number of Combinations without

Basic Level

117. If n is even and the value of ${}^nC_r$ is maximum, then r =

(a) $\frac{n}{2}$ (b) $\frac{n+1}{2}$ (c) $\frac{n-1}{2}$ (d) None of these

118. ${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3 =$ **[IIT 1980; Rajasthan PET 2002; UPSEAT 2000]**

(a) ${}^{47}C_6$ (b) ${}^{52}C_5$ (c) ${}^{52}C_4$ (d) None of these

119. If ${}^nC_3 = 220$, then $n =$

(a) 10 (b) 12 (c) 15 (d) 8

120. If $2 \times {}^nC_5 = 9 \times {}^{n-2}C_5$, then the value of n will be

(a) 7 (b) 10 (c) 9 (d) 5

121. The number of combinations of n different objects taken r at a time will be

(a) ${}^nP_r$ (b) ${}^nP_r\, r!$ (c) $\frac{{}^nP_r}{r!}$ (d) None of these

122. ${}^{n^2-n}C_2 = {}^{n^2-n}C_{10}$, then $n =$

(a) 12 (b) 4 only (c) – 3 only (d) 4 or – 3

123. ${}^nC_r + {}^nC_{r-1}$ is equal to **[MP PET 1984; Kerala (Engg.) 2002]**

(a) ${}^{n+1}C_r$ (b) ${}^nC_{r+1}$ (c) ${}^{n+1}C_{r+1}$ (d) ${}^{n-1}C_{r-1}$

124. If ${}^8C_r = {}^8C_{r+2}$, then the value of ${}^rC_2$ is **[MP PET 1984; Rajasthan PET 1987]**

(a) 8 (b) 3 (c) 5 (d) 2

125. If ${}^{20}C_{n+2} = {}^nC_{16}$, then the value of n is **[MP PET 1984]**

(a) 7 (b) 10 (c) 13 (d) No value

126. The value of ${}^{15}C_3 + {}^{15}C_{13}$ is

(a) ${}^{16}C_3$ (b) ${}^{30}C_{16}$ (c) ${}^{15}C_{10}$ (d) ${}^{15}C_{15}$

127. If ${}^{10}C_r = {}^{10}C_{r+2}$, then ${}^5C_r$ equals

(a) 120 (b) 10 (c) 360 (d) 5

128. If ${}^nC_r = 84, {}^nC_{r-1} = 36$ and ${}^nC_{r+1} = 126$, then n equals **[Rajasthan PET 1997; MP PET 2001]**

(a) 8 (b) 9 (c) 10 (d) 5

129. If ${}^nC_3 + {}^nC_4 > {}^{n+1}C_3$, then **[Rajasthan PET 1999]**

(a) $n > 6$ (b) $n > 7$ (c) $n < 6$ (d) None of these

130. Value of r for which ${}^{15}C_{r+3} = {}^{15}C_{2r-6}$ is **[Pb. CET 1999]**

(a) 2 (b) 4 (c) 6 (d) – 9

131. For $2 \le r \le n, \binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2}$ is equal to **[IIT Screening 2000]**

(a) $\binom{n+1}{r-1}$ (b) $2\binom{n+1}{r+1}$ (c) $2\binom{n+2}{r}$ (d) $\binom{n+2}{r}$

132. ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$ then the value of n is **[Rajasthan PET 2000]**

(a) 7 (b) < 7 (c) > 7 (d) None of these

133. $\binom{n}{n-r} + \binom{n}{r+1}$, whenever $0 \le r \le n-1$ is equal to **[AMU 2000]**

(a) $\binom{n}{r-1}$ (b) $\binom{n}{r}$ (c) $\binom{n}{r+1}$ (d) $\binom{n+1}{r+1}$

134. If ${}^{43}C_{r-6} = {}^{43}C_{3r+1}$, then the value of r is **[Kerala (Engg.) 2002]**

(a) 12 (b) 8 (c) 6 (d) 10

135. The least value of natural number n satisfying $C(n,5)+C(n,6)>C(n+1,5)$ is **[EAMCET 2002]**

(a) 11 (b) 10 (c) 12 (d) 13

136. If ${}^nC_r$ denotes the number of combinations of n things taken r at a time, then the expression ${}^nC_{r+1}+{}^nC_{r-1}+2\times{}^nC_r$, equals

[AIEEE 2003]

(a) ${}^{n+2}C_r$ (b) ${}^{n+2}C_{r+1}$ (c) ${}^{n+1}C_r$ (d) ${}^{n+1}C_{r+1}$

137. ${}^5C_1+{}^5C_2+{}^5C_3+{}^5C_4+{}^5C_5$ is equal to **[Rajasthan PET 1989]**

(a) 30 (b) 31 (c) 32 (d) 33

138. If $C(n,12)=C(n,8)$, then the value of $C(22,n)$ is **[Rajasthan PET 1993]**

(a) 924 (b) 308 (c) 462 (d) 231

139. If ${}^{20}C_r={}^{20}C_{r-10}$, then ${}^{18}C_r$ is equal to **[Rajasthan PET 1988]**

(a) 816 (b) 1632 (c) 4896 (d) None of these

140. If ${}^nC_4, {}^nC_5, {}^nC_6$ are in A.P. then the value of n is **[AMU 1989]**

(a) 14 or 7 (b) 11 (c) 17 (d) 8

141. There are 12 volleyball players in all in a college, out of which a team of 9 players is to be formed. If the captain always remains the same, then in how many ways can the team be formed

(a) 36 (b) 108 (c) 99 (d) 165

142. There are 16 vacancies for clerks in a certain office, 20 applications are received. In how many ways can the clerks be appointed

(a) 3800 (b) 3876 (c) 969 (d) 4845

143. In how many ways a committee of 5 members can be formed out of 8 gentlemen and 4 ladies, if one particular lady is always to be taken

(a) 140 (b) 330 (c) 560 (d) None of these

144. How many words can be formed by taking 3 consonants and 2 vowels out of 5 consonants and 4 vowels

(a) ${}^5C_3\times{}^4C_2$ (b) $\frac{{}^5C_3\times{}^4C_2}{5}$ (c) ${}^5C_3\times{}^4C_3$ (d) $({}^5C_3\times{}^4C_2)(5)!$

145. A male and a female typist are needed in an institution. If 10 ladies and 15 gentlemen apply, then in how many ways can the selection be made

(a) 125 (b) 145 (c) 150 (d) None of these

146. Everybody in a room shakes hand with everybody else. The total number of hand shakes is 66. The total number of persons in the room is

(a) 11 (b) 12 (c) 13 (d) 14

147. There are 9 chairs in a room on which 6 persons are to be seated, out of which one is guest with one specific chair. In how many ways they can sit

(a) 6720 (b) 60480 (c) 30 (d) 346

148. On the occasion of Deepawali festival each student of a class sends greeting cards to the others. If there are 20 students in the class, then the total number of greeting cards exchanged by the students is

(a) ${}^{20}C_2$ (b) $2.{}^{20}C_2$ (c) $2.{}^{20}P_2$ (d) None of these

149. A father with 8 children takes them 3 at a time to the Zoological gardens, as often as he can without taking the same 3 children together more than once. The number of times he will go to the garden is

(a) 336 (b) 112 (c) 56 (d) None of these

150. In how many ways can 5 red and 4 white balls be drawn from a bag containing 10 red and 8 white balls

[EAMCET 1991; Pb. CET 2000]

(a) ${}^8C_5 \times {}^{10}C_4$ (b) ${}^{10}C_5 \times {}^8C_4$ (c) ${}^{18}C_9$ (d) None of these

151. There are 15 persons in a party and each person shake hand with another, then total number of hand shakes is **[Rajast**

(a) ${}^{15}P_2$ (b) ${}^{15}C_2$ (c) $15!$ (d) $2(15!)$

152. A fruit basket contains 4 oranges, 5 apples and 6 mangoes. The number of ways person make selection of fruits from among the fruits in the basket is

(a) 210 (b) 209 (c) 208 (d) None of these

153. In a cricket championship there are 36 matches. The number of teams if each plays one match with other are **[Karnata**

(a) 8 (b) 9 (c) 10 (d) None of these

Advance Level

154. If ${}^{2n}C_3 : {}^nC_2 = 44 : 3$, then for which of the following values of r, the values of ${}^nC_r$ will be 15

(a) $r = 3$ (b) $r = 4$ (c) $r = 6$ (d) $r = 5$

155. ${}^nC_r + {}^{n-1}C_r + ... + {}^rC_r =$ **[AMU 2002]**

(a) ${}^{n+1}C_r$ (b) ${}^{n+1}C_{r+1}$ (c) ${}^{n+2}C_r$ (d) 2^n

156. The solution set of ${}^{10}C_{x-1} > 2 \, . \, {}^{10}C_x$ is

(a) {1, 2, 3} (b) {4, 5, 6} (c) {8, 9, 10} (d) {9, 10, 11}

157. $\sum_{r=0}^{m} {}^{n+r}C_n =$

(a) ${}^{n+m+1}C_{n+1}$ (b) ${}^{n+m+2}C_n$ (c) ${}^{n+m+3}C_{n-1}$ (d) None of these

158. If $\alpha = {}^mC_2$, then ${}^{\alpha}C_2$ is equal to

(a) ${}^{m+1}C_4$ (b) ${}^{m-1}C_4$ (c) $3\,{}^{m+2}C_4$ (d) $3\,{}^{m+1}C_4$

159. ${}^{14}C_4 + \sum_{j=1}^{4} {}^{18-j}C_3$ is equal to

(a) ${}^{18}C_3$ (b) ${}^{18}C_4$ (c) ${}^{14}C_7$ (d) None of these

160. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals **[IIT 1998]**

(a) $(n-1)a_n$ (b) na_n (c) $\frac{1}{2}na_n$ (d) None of these

161. In a football championship, there were played 153 matches. Every team played one match with each other. The number of teams participating in the championship is **[West Bengal JEE 1992; Kurukshetra CEE 1998]**

(a) 17 (b) 18 (c) 9 (d) 13

162. Ten persons, amongst whom are A, B and C to speak at a function. The number of ways in which it can be done if A wants to speak before B and B wants to speak before C is

(a) $\frac{10!}{6}$ (b) $3!\,7!$ (c) ${}^{10}P_3.7!$ (d) None of these

163. The number of times the digit 5 will be written when listing the integers from 1 to 1000 is

(a) 271 (b) 272 (c) 300 (d) None of these

164. All possible two factors products are formed from numbers 1, 2, 3, 4....., 200. The number of factors out of the total obtained which are multiples of 5 is

(a) 5040 (b) 7180 (c) 8150 (d) None of these

165. A car will hold 2 in the front seat and 1 in the rear seat. If among 6 persons 2 can drive, then the number of ways in which the car can be filled is

(a) 10 (b) 20 (c) 30 (d) None of these

166. The expression ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + ... + {}^nC_2)$ can be reduced to

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{n(n+1)(2n+1)}{6}$ (d) $\frac{n(2n+1)}{3}$

167. The value of $({}^7C_0 + {}^7C_1) + ({}^7C_1 + {}^7C_2) + ... + ({}^7C_6 + {}^7C_7)$ is **[AMU 1990, 92]**

(a) $2^7 - 1$ (b) $2^8 - 2$ (c) $2^8 - 1$ (d) 2^8

168. The expression ${}^nC_r + 4.{}^nC_{r-1} + 6.{}^nC_{r-2} + 4.{}^nC_{r-3} + {}^nC_{r-4}$ equals **[AMU 1993, 91]**

(a) ${}^{n+4}C_r$ (b) $2.{}^{n+4}C_{r-1}$ (c) $4.{}^nC_r$ (d) $11.{}^nC_r$

Number of Combinations with Repetition and All possible Selections

Basic Level

169. Ramesh has 6 friends. In how many ways can he invite one or more of them at a dinner

(a) 61 (b) 62 (c) 63 (d) 64

170. Out of 10 white, 9 black and 7 red balls, the number of ways in which selection of one or more balls can be made, is

(a) 881 (b) 891 (c) 879 (d) 892

171. Out of 6 books, in how many ways can a set of one or more books be chosen **[MP PET 1984]**

(a) 64 (b) 63 (c) 62 (d) 65

172. In an examination there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct, is **[Pb. CET 1990; UPSEAT 2001]**

(a) 11 (b) 12 (c) 27 (d) 63

173. The total number of different combinations of one or more letters which can be made from the letters of the word 'MISSISSIPPI' is

(a) 150 (b) 148 (c) 149 (d) None of these

174. The total number of ways of selecting six coins out of 20 one rupee coins, 10 fifty paise coins and 7 twenty five paise coins is

(a) 28 (b) 56 (c) ${}^{37}C_6$ (d) None of these

Advance Level

175. In an election there are 8 candidates, out of which 5 are to be chosen. If a voter may vote for any number of candidates but not greater then the number to be chosen, then in how many ways can a voter vote

(a) 216 (b) 114 (c) 218 (d) None of these

176. In an election the number of candidates is 1 greater than the persons to be elected. If a voter can vote in 254 ways, then the number of candidates is

(a) 7 (b) 10 (c) 8 (d) 6

177. The number of ways of dividing 52 cards amongst four players so that three players have 17 cards each and the fourth player just one card, is

(a) $\frac{52!}{(17!)^3}$ (b) 52 ! (c) $\frac{52!}{17!}$ (d) None of these

178. In a city no two persons have identical set of teeth and there is no person without a tooth. Also no person has more than 32 teeth. If we disregard the shape and size of tooth and consider only the positioning of the teeth, then the maximum population of the city is

(a) 2^{32} (b) $(32)^2 - 1$ (c) $2^{32} - 1$ (d) 2^{32-1}

179. The number of ways in which four letters of the word 'MATHEMATICS' can be arranged is given by **[Kurukshetra CEE 1996; Pb. CET 1995]**

(a) 136 (b) 192 (c) 1680 (d) 2454

180. A person is permitted to select at least one and at most n coins from a collection of $2n+1$ (distinct) coins. If the total number of ways in which he can select coins is 255, then n equals

(a) 4 (b) 8 (c) 16 (d) 32

181. The total number of ways of selecting five letters from the letters of the word 'INDEPENDENT' is

(a) 70 (b) 3320 (c) 120 (d) None of these

182. There are n different books and p copies of each in a library. The number of ways in which one or more books can be selected is

(a) $p^n + 1$ (b) $(p+1)^n - 1$ (c) $(p+1)^n - p$ (d) p^n

Conditional Combinations, Derangement, Division into groups

Basic Level

183. In how many ways can 21 English and 19 Hindi books be placed in a row so that no two Hindi books are together

(a) 1540 (b) 1450 (c) 1504 (d) 1405

184. The number of ways in which five identical balls can be distributed among ten identical boxes such that no box contains more than one ball, is

(a) 10 ! (b) $\frac{10!}{5!}$ (c) $\frac{10!}{(5!)^2}$ (d) None of these

185. In how many ways can two balls of the same colour be selected out of 4 black and 3 white balls

(a) 5 (b) 6 (c) 9 (d) 8

186. Ten persons are arranged in a row. The number of ways of selecting four persons so that no two persons sitting next to each other are selected is

(a) 34 (b) 36 (c) 35 (d) None of these

187. In a touring cricket team there are 16 players in all including 5 bowlers and 2 wicket-keepers. How many teams of 11 players from these, can be chosen, so as to include three bowlers and one wicket-keeper

(a) 650 (b) 720 (c) 750 (d) 800

188. A total number of words which can be formed out of the letters *a, b, c, d, e, f* taken 3 together such that each word contains at least one vowel, is

(a) 72 (b) 48 (c) 96 (d) None of these

189. Out of 6 boys and 4 girls, a group of 7 is to be formed. In how many ways can this be done if the group is to have a majority of boys **[MP PET 1994]**

(a) 120 (b) 90 (c) 100 (d) 80

190. Let A be a set containing 10 distinct elements. Then the total number of distinct functions from A to A, is

(a) 10 ! (b) 10^{10} (c) 2^{10} (d) $2^{10} - 1$

191. A lady gives a dinner party for six guests. The number of ways in which they may be selected from among ten friends, if two of the friends will not attend the party together is

(a) 112 (b) 140 (c) 164 (d) None of these

192. The number of ways in which mn students can be distributed equally among n sections is

(a) $(mn)^n$ (b) $\frac{(mn)!}{(m!)^n}$ (c) $\frac{mn}{m!}$ (d) $\frac{mn}{m!n!}$

193. There are 3 candidates for a post and one is to be selected by the votes of 7 men. The number of ways in which votes can be given is

(a) 7^3 (b) 3^7 (c) ${}^7C_3$ (d) None of these

194. In how many ways can 10 balls be divided between two boys, one receiving two and the other eight balls

(a) 45 (b) 75 (c) 90 (d) None of these

195. The number of ways in which thirty five apples can be distributed among 3 boys so that each can have any number of apples, is

(a) 1332 (b) 666 (c) 333 (d) None of these

196. The number of ways in which six different prizes can be distributed among three children each receiving at least one prize is

[JMIEEE 1997]

(a) 270 (b) 540 (c) 1080 (d) 2160

Advance Level

197. In how many ways can Rs. 16 be divided into 4 person when none of them get less than Rs. 3

(a) 70 (b) 35 (c) 64 (d) 192

198. Two packs of 52 cards are shuffled together. The number of ways in which a man can be dealt 26 cards so that he does not get two cards of the same suit and same denomination is

(a) ${}^{52}C_{26}.2^{26}$ (b) ${}^{104}C_{26}$ (c) $2.{}^{52}C_{26}$ (d) None of these

199. Choose the correct number of ways in which 15 different books can be divided into five heaps of equal number of books

[MP PET 1982]

(a) $\frac{15!}{5!(3!)^5}$ (b) $\frac{15!}{(3!)^5}$ (c) ${}^{15}C_5$ (d) ${}^{15}P_5$

200. There were two women participating in a chess tournament. Every participant played two games with the other participants. The number of games that the men played between themselves proved to exceed by 66 the number of games that the men played with the women. The number of participants is

(a) 6 (b) 11 (c) 13 (d) None of these

201. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many ways can we place the balls so that no box remains empty

(a) 50 (b) 100 (c) 150 (d) 200

202. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw

(a) 64 (b) 45 (c) 46 (d) None of these

203. In how many ways can a committee be formed of 5 members from 6 men and 4 women if the committee has at least one women

[Rajasthan PET 1987; IIT 1968]

(a) 186 (b) 246 (c) 252 (d) None of these

204. Six '+' and four '–' signs are to placed in a straight line so that no two '–' signs come together, then the total number of ways are

[IIT 1988]

(a) 15 (b) 18 (c) 35 (d) 42

205. The number of groups that can be made from 5 different green balls, 4 different blue balls and 3 different red balls, if at least 1 green and 1 blue ball is to be included

(a) 3700 (b) 3720 (c) 4340 (d) None of these

206. In an election there are 5 candidates and three vacancies. A voter can vote maximum to three candidates, then in how many ways can he vote

(a) 125 (b) 60 (c) 10 (d) 25

207. A committee of 12 is to be formed from 9 women and 8 men in which at least 5 women have to be included in a committee. Then the number of committees in which the women are in majority and men are in majority are respectively **[IIT 1994]**

(a) 4784, 1008 (b) 2702, 3360 (c) 6062, 2702 (d) 2702, 1008

208. The number of ways in which 10 persons can go in two boats so that there may be 5 on each boat, supposing that two particular persons will not go in the same boat is

(a) $\frac{1}{2}({}^{10}C_5)$ (b) $2({}^8C_4)$ (c) $\frac{1}{2}({}^8C_5)$ (d) None of these

209. There are 10 persons named *A, B,....J*. We have the capacity to accommodate only 5. In how many ways can we arrange them in a line if *A* is must and *G* and *H* must not be included in the team of 5

(a) ${}^8P_5$ (b) ${}^7P_5$ (c) ${}^7C_3(4!)$ (d) ${}^7C_3(5!)$

210. The number of ways in which we can select three numbers from 1 to 30 so as to exclude every selection of all even numbers is

(a) 4060 (b) 3605 (c) 455 (d) None of these

211. In a steamer there are stalls for 12 animals and there are horses, cows and calves (not less than 12 each) ready to be shipped. They can be loaded in

(a) $3^{12}-1$ (b) 3^{12} (c) $(12)^3-1$ (d) None of these

212. There are $(n+1)$ white and $(n+1)$ black balls each set numbered 1 to $n+1$. The number of ways in which the balls can be arranged in a row so that the adjacent balls are of different colours is

(a) $(2n+2)!$ (b) $(2n+2)!\times 2$ (c) $(n+1)!\times 2$ (d) $2\{(n+1)!\}^2$

213. Sixteen men compete with one another in running, swimming and riding. How many prize lists could be made if there were altogether 6 prizes of different values one for running, 2 for swimming and 3 for riding

(a) $16^3\times 15\times 14^2$ (b) $16^3\times 15^2\times 14$ (c) $16\times 15\times 14$ (d) None of these

214. The number of ways in which a committee of 6 members can be formed from 8 gentlemen and 4 ladies so that the committee contains at least 3 ladies is

(a) 252 (b) 672 (c) 444 (d) 420

215. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is

(a) 140 (b) 196 (c) 280 (d) 346

216. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is **[AIEEE 20**

(a) ${}^8C_3$ (b) 21 (c) 3^8 (d) 5

217. In the next World Cup of cricket there will be 12 teams, divided equally in two groups. Teams of each group will play a match against each other. From each group 3 top teams will qualify for the next round. In this round each team will play against others once. Four top teams of this round will qualify for the semifinal round, where each team will play against the others once. Two top teams of this round will go to the final round, where they will play the best of three matches. The minimum number of matches in the next World Cup will be

(a) 54 (b) 53 (c) 38 (d) None of these

218. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{r}$ be a variable vector such that $\vec{r}.\hat{i},\vec{r}.\hat{j}$ and $\vec{r}.\hat{k}$ are positive integers. If $\vec{r}.\vec{a}\le 12$ then the number of values of $\vec{r}$ is

(a) ${}^{12}C_9-1$ (b) ${}^{12}C_3$ (c) ${}^{12}C_9$ (d) None of these

219. A man has 7 relatives, 4 women and 3 men. His wife also has 7 relatives, 3 women and 4 men. In how many ways can they invite 3 women and 3 men so that 3 of them are the man's relatives and 3 his wife's

(a) 485 (b) 484

(c) 468 (d) None of these

220. A person wishes to make up as many different parties as he can out of his 20 friends such that each party consists of the same number of persons. The number of friends he should invite at a time is

(a) 5 (b) 10 (c) 8 (d) None of these

Geometrical Problems

Basic Level

221. The number of triangles that can be formed by 5 points in a line and 3 points on a parallel line is
(a) ${}^8C_3$ (b) ${}^8C_3 - {}^5C_3$ (c) ${}^8C_3 - {}^5C_3 - 1$ (d) None of these

222. The maximum number of points of intersection of 20 straight lines will be
(a) 190 (b) 220 (c) 200 (d) None of these

223. If a polygon has 44 diagonals, then the number of its sides are **[MP PET 1998; Pb. CET 1996]**
(a) 7 (b) 11 (c) 8 (d) None of these

224. How many triangles can be drawn by means of 9 non-collinear points
(a) 84 (b) 72 (c) 144 (d) 126

225. The number of diagonals in a polygon of m sides is **[BIT Ranchi 1992; MP PET 1999; UPSEAT 1999; DCE 1999]**
(a) $\frac{1}{2!}m(m-5)$ (b) $\frac{1}{2!}m(m-1)$ (c) $\frac{1}{2!}m(m-3)$ (d) $\frac{1}{2!}m(m-2)$

226. In a plane there are 10 points out of which 4 are collinear, then the number of triangles that can be formed by joining these points are
[Rajasthan PET 1990]
(a) 60 (b) 116 (c) 120 (d) None of these

227. There are 16 points in a plane out of which 6 are collinear, then how many lines can be drawn by joining these points
[Rajasthan PET 1986; MP PET 1987]
(a) 106 (b) 105 (c) 60 (d) 55

228. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is
[West Bengal JEE 1993; Rajasthan PET 2001]
(a) 6 (b) 18 (c) 12 (d) 9

229. The greatest possible number of points of intersection of 8 straight lines and 4 circles is
(a) 32 (b) 64 (c) 76 (d) 104

230. There are 16 points in a plane, no three of which are in a straight line except 8 which are all in a straight line. The number of triangles that can be formed by joining them equals
(a) 504 (b) 552 (c) 560 (d) 1120

231. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$ then n equals
(a) 5 (b) 7 (c) 6 (d) 4

232. Out of 10 points in a plane 6 are in a straight line. The number of triangles formed by joining these points are **[Rajastl**
(a) 100 (b) 150 (c) 120 (d) None of these

233. The number of straight lines that can be formed by joining 20 points no three of which are in the same straight line except 4 of them which are in the same line
(a) 183 (b) 186 (c) 197 (d) 185

234. There are n distinct points on the circumference of a circle. The number of pentagons that can be formed with these points as vertices is equal to the number of possible triangles. Then the value of n is
(a) 7 (b) 8 (c) 15 (d) 30

235. Given six line segments of lengths 2, 3, 4, 5, 6, 7 units, the number of triangle that can be formed by these lines is **[AMU 2002]**
(a) ${}^6C_3 - 7$ (b) ${}^6C_3 - 6$ (c) ${}^6C_3 - 5$ (d) ${}^6C_3 - 4$

236. A polygon has 35 diagonals, then the number of its sides is
(a) 8 (b) 9 (c) 10 (d) 11

237. If 5 parallel straight lines are intersected by 4 parallel straight lines, then the number of parallelograms thus formed is

[Kurukshetra CEE 1999]

(a) 20 (b) 60 (c) 101 (d) 126

238. The maximum number of points of intersection of 8 circles, is

(a) 16 (b) 24 (c) 28 (d) 56

239. There are 10 points in a plane of which no three points are collinear and 4 points are concyclic. The number of different circles that can be drawn through at least 3 points of these points is

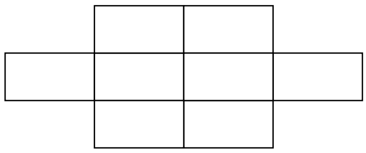
(a) 116 (b) 120 (c) 117 (d) None of these

Advance Level

240. The sides AB, BC, CA of a triangle ABC have respectively 3, 4 and 5 points lying on them. The number of triangles that can be constructed using these points as vertices is

(a) 205 (b) 220 (c) 210 (d) None of these

241. Six '×'s have to be placed in the square of the figure such that each row contains at least one ×. In how many different ways can this be done



(a) 28 (b) 27 (c) 26 (d) None of these

242. The straight lines I_1, I_2, I_3 are parallel and lie in the same plane. A total number of m points are taken on I_1, n points on I_2, k points on I_3. The maximum number of triangles formed with vertices at these points are **[IIT 1993; UPSEAT 2001]**

(a) ${}^{m+n+k}C_3$ (b) ${}^{m+n+k}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$ (c) ${}^mC_3 + {}^nC_3 + {}^kC_3$ (d) None of these

243. Six points in a plane be joined in all possible ways by indefinite straight lines, and if no two of them be coincident or parallel, and no three pass through the same point (with the exception of the original 6 points). The number of distinct points of intersection is equal to

(a) 105 (b) 45 (c) 51 (d) None of these

244. There are m points on a straight line AB and n points on another line AC, none of them being the point A. Triangles are formed from these points as vertices when (i) A is excluded (ii) A is included. Then the ratio of the number of triangles in these two cases is

(a) $\frac{m+n-2}{m+n}$ (b) $\frac{m+n-2}{2}$ (c) $\frac{m+n-2}{m+n+2}$ (d) None of these

245. There are n straight lines in a plane, no two of which are parallel and no three pass through the same point. Their points of intersection are joined. Then the number of fresh lines thus obtained is

(a) $\frac{n(n-1)(n-2)}{8}$ (b) $\frac{n(n-1)(n-2)(n-3)}{6}$ (c) $\frac{n(n-1)(n-2)(n-3)}{8}$ (d) None of these

246. A parallelogram is cut by two sets of m lines parallel to its sides. The number of parallelograms thus formed is **[Karnat**

(a) $({}^mC_2)^2$ (b) $({}^{m+1}C_2)^2$ (c) $({}^{m+2}C_2)^2$ (d) None of these

247. In a plane there are 37 straight lines of which 13 pass through the point A and 11 pass through the point B. Besides no three lines pass through one point, no line passes through both points A and B and no two are parallel. Then the number of intersection points the lines have is equal to

(a) 535 (b) 601 (c) 728 (d) None of these

248. There are n points in a plane of which p points are collinear. How many lines can be formed from these points**[Karnata**

(a) ${}^{(n-p)}C_2$ (b) ${}^nC_2 - {}^pC_2$ (c) ${}^nC_2 - {}^pC_2 + 1$ (d) ${}^nC_2 - {}^pC_2 - 1$

249. ABCD is a convex quadrilateral. 3, 4, 5 and 6 points are marked on the sides AB, BC, CD and DA respectively. The number of triangles with vertices on different sides is

(a) 270 (b) 220 (c) 282 (d) 342

250. The number of triangles that can be formed joining the angular points of decagon, is

(a) 30 (b) 45 (c) 90 (d) 120

251. The number of triangles whose vertices are at the vertices of an octagon but none of whose sides happen to come from the sides of the octagon is

(a) 24 (b) 52 (c) 48 (d) 16

252. In a polygon no three diagonals are concurrent. If the total number of points of intersection of diagonals interior to the polygon be 70, then the number of diagonals of the polygon is

(a) 20 (b) 28 (c) 8 (d) None of these

253. There are $n(>2)$ points in each of two parallel lines. Every point on one line is joined to every point on the other line by a line segment drawn within the lines. The number of points (between the lines) in which these segments intersect is

(a) ${}^{2n}C_2 - 2.{}^nC_1 + 2$ (b) ${}^{2n}C_2 - 2.{}^nC_2$ (c) ${}^nC_2 \times {}^nC_2$ (d) None of these

254. m parallel lines in a plane are intersected by a family of n parallel lines. The total number of parallelograms so formed is

(a) $\frac{(m-1)(n-1)}{4}$ (b) $\frac{mn}{4}$ (c) $\frac{m(m-1)n(n-1)}{2}$ (d) $\frac{mn(m-1)(n-1)}{4}$

255. There are three coplanar parallel lines. If any p points are taken on each of the lines, the maximum number of triangles with vertices at these points

(a) $3p^2(p-1)+1$ (b) $3p^2(p-1)$ (c) $p^2(4p-3)$ (d) None of these

Multinomial Theorem, Number of Divisors and Miscellaneous

Basic Level

256. If ${}^nP_r = 720.{}^nC_r$, then r is equal to **[Kerala (Engg.) 2001]**

(a) 6 (b) 5 (c) 4 (d) 7

257. If ${}^nP_4 = 24.{}^nC_5$, then the value of n is **[Karnataka CET 2001]**

(a) 10 (b) 15 (c) 9 (d) 5

258. If ${}^nP_3 + {}^nC_{n-2} = 14n$, then n =

(a) 5 (b) 6 (c) 8 (d) 10

259. If ${}^nP_4 = 30\ {}^nC_5$, then n = **[MP PET 1995]**

(a) 6 (b) 7 (c) 8 (d) 9

260. If ${}^nP_r = 840, {}^nC_r = 35$, then n is equal to **[EAMCET 1986]**

(a) 1 (b) 3 (c) 5 (d) 7

261. If ${}^nC_r = {}^nC_{r-1}$ and ${}^nP_r = {}^nP_{r+1}$, then the value of n is

(a) 3 (b) 4 (c) 2 (d) 5

262. ${}^nP_r \div {}^nC_r =$ **[MP PET 1984]**

(a) $n!$ (b) $(n-r)!$ (c) $\frac{1}{r!}$ (d) $r!$

263. If a, b, c, d, e are prime integers, then the number of divisors of ab^2c^2de excluding 1 as a factor, is

(a) 94 (b) 72 (c) 36 (d) 71

264. The number of proper divisors of 1800 which are also divisible by 10, is

(a) 18 (b) 34 (c) 27 (d) None of these

265. The number of odd proper divisors of $3^p.6^m.21^n$ is

(a) $(p+1)(m+1)(n+1)-2$ (b) $(p+m+n+1)(n+1)-1$ (c) $(p+1)(m+1)(n+1)-1$ (d) None of these

266. The number of proper divisors of $2^p.6^q.15^r$ is

(a) $(p+q+1)(q+r+1)\ (r+1)$ (b) $(p+q+1)(q+r+1)(r+1)-2$

(c) $(p+q)(q+r)r-2$ (d) None of these

267. The number of even proper divisors of 1008 is

(a) 23 (b) 24 (c) 22 (d) None of these

Advance Level

268. The number of numbers of 4 digits which are not divisible by 5 are

(a) 7200 (b) 3600 (c) 14400 (d) 1800

269. A set contains $(2n+1)$ elements. The number of subsets of the set which contain at most n elements is

(a) 2^n (b) 2^{n+1} (c) 2^{n-1} (d) 2^{2n}

270. The number of ways in which an examiner can assign 30 marks to 8 questions, awarding not less than 2 marks to any question is

(a) ${}^{21}C_7$ (b) ${}^{30}C_{16}$ (c) ${}^{21}C_{16}$ (d) None of these

271. In a certain test a_i students gave wrong answers to at least i questions where $i = 1, 2\ 3, ...k$. No student gave more than k wrong answers. The total numbers of wrong answers given is

(a) $a_1 + 2a_2 + 3a_3 + ... + ka_k$ (b) $a_1 + a_2 + a_3 + ... + a_k$

(c) Zero (d) None of these

272. Number of ways of selection of 8 letters from 24 letters of which 8 are *a*, 8 are *b* and the rest unlike is given by

(a) 2^7 (b) 8.2^8 (c) 10.2^7 (d) None of these

273. The number of ordered triplets of positive integers which are solutions of the equation $x+y+z=100$ is

(a) 6005 (b) 4851 (c) 5081 (d) None of these

274. A person goes in for an examination in which there are four papers with a maximum of m marks from each paper. The number of ways in which one can get $2m$ marks is

(a) ${}^{2m+3}C_3$ (b) $\frac{1}{3}(m+1)(2m^2+4m+1)$ (c) $\frac{1}{3}(m+1)(2m^2+4m+3)$ (d) None of these

275. The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, $\left(\text{where } \binom{p}{q} = 0 \text{ if } p < q\right)$, is maximum when m is **[IIT Screening 2002]**

(a) 5 (b) 15 (c) 10 (d) 20

276. The number of divisors of the form $4n+2 (n \ge 0)$ of the integer 240 is

(a) 4 (b) 8 (c) 10 (d) 3

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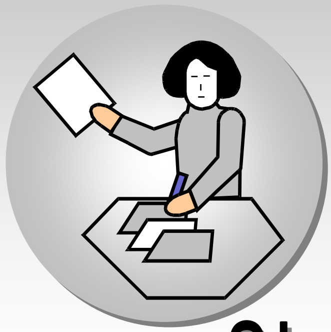

Answer Sheet

Permutations and Assignment (Basic and Advance Level)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
c	c	d	a	b	d	a	d	d	a	a	d	d	d	a	b	c	a	a	a
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
b	a	c	b	c	c	a	a	a	b	b	c	b	a	c	b	a	c	c	d
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
c	d	d	b	b	b	a	c	a	a	c	a	a	b	c	c	b	a	b	b
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
c	b	a	a	c	c	a	b	b	d	c	b	a	c	a	b	b	c	d	b
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
c	a	a	c	c	b	d	b	a	a	c	a	c	c	d	b	d	a	d	c
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
b	c	c	a	d	b	b	d	a	b	a	a	a	c	b	b	a	c	b	b
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
c	d	a	b	d	a	d	b	a	c	d	c	d	a	a	b	b	d	a	a
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
d	d	b	d	c	b	a	b	c	b	b	b	b	b	b	c	a	d	b	c
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
b	a	c	b	b	c	b	a	c	c	b	d	c	a	c	c	a	c	d	a
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
a	b	a	c	c	c	b	c	c	b	b	b	b	c	b	b	b	a	a	c
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
c	a	b	c	b	d	d	b	d	b	b	d	b	a	b	b	b	b,c	a	b
221	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**	**239**	**240**
c	a	b	a	c	b	a	b	d	a	b	a	d	b	b	c	b	d	c	a
241	**242**	**243**	**244**	**245**	**246**	**247**	**248**	**249**	**250**	**251**	**252**	**253**	**254**	**255**	**256**	**257**	**258**	**259**	**260**
c	b	c	a	c	c	a	c	d	d	d	a	c	d	c	a	c	a	c	d
261	**262**	**263**	**264**	**265**	**266**	**267**	**268**	**269**	**270**	**271**	**272**	**273**	**274**	**275**	**276**				
a	d	d	a	b	b	a	a	d	a	b	c	b	c	b	a				