QUANTITATIVE ABILITY TEST 3

(INDICES, SURDS AND LOGARITHMS)

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Find the value of x, if $2^x = 8^y$ and $6^{4y} = 216^{x+y-2}$.

(A) $2\frac{1}{4}$ (B) $2\frac{1}{2}$

(C) $3\frac{1}{2}$ (D) $3\frac{1}{3}$

2. If $x = \sqrt[3]{9\sqrt[3]{9\sqrt[3]{9\sqrt[3]{9\ldots\ldots\ldots\ldots}}}}$, what is the value of x?

(A) 3 (B) –3

(C) $\sqrt{3}$ (D) 9

3. If $9^{3x-4} = 6561.\sqrt{27^{x-2}}$, then find the value of x.

(A) 1 (B) 2

(C) 3 (D) None of these

4. If $p^{\ell} = q^m = r^n$ and $\frac{1}{\ell} + \frac{1}{n} = \frac{2}{m}$, then which of the following is valid? ($p > 1$, $q > 1$, $r > 1$)

I. $p\ell = q^2$ II. $pq = \ell^2$ III. $p^2 = qr$

(A) Both I and I (B) Both II and III

(C) Both III and I (D) None of these

5. Find the value of x if $(125)^{2x-3} = (25)^{3^{(-1)^{2^{3^4}}}}$.

(A) 4.5 (B) 2.5

(C) 1.5 (D) None of these

6. If $2^{2x^2+3} = 8^{2x+1}$ and x is positive, then what is the value of x?

(A) 2 (B) 3

(C) 1 (D) 4

7. If $t_1 = \sqrt{5}$, $t_2 = \sqrt{5\sqrt{5}}$, $t_3 = \sqrt{5\sqrt{5\sqrt{5}}}$ and so on, then the product of the first ten terms $(t_1)(t_2)(t_3)(t_4)\ldots(t_{10})$ is equal to

(A) $\sqrt[512]{5^{4609}}$ (B) $\sqrt[2048]{5^{18431}}$

(C) $\sqrt[1024]{5^{9217}}$ (D) $\sqrt[512]{5^{4607}}$

8. If $x^2 \neq x$, then

$$\frac{x^{4b} + x^{2(a+b)} + x^{4a}}{\left(x^{2a} + x^{a+b} + x^{2b}\right)\left(x^{2a} - x^{a+b} + x^{2b}\right)}$$ is equal to

(A) $\frac{x^a}{x^b}$ (B) $\frac{x^{2a}}{x^{2b}}$

(C) x^{a+b} (D) None of these

9. If $3^{x+3} - 3^{x-3} = 6552$, then find x^2.

(A) 5 (B) 25

(C) 3 (D) 9

10. If x, y, z are real numbers such that $xyz = 1$, then the expression $\frac{1}{1+x+y^{-1}} + \frac{1}{1+y+z^{-1}} + \frac{1}{1+z+x^{-1}}$ is equal to

(A) 1 (B) $\frac{3}{x+y+z}$

(C) $\frac{3}{x^{-1}+y^{-1}+z^{-1}}$ (D) $\frac{x+y+z}{3}$

11. If $\frac{(81^a)^a (81^b)^b (81^c)^c}{(6561^b)^{-c}(6561^c)^{-a}(6561^a)^{-b}} = 3.$

Then $a + b + c$ could be

(A) 2 (B) $\frac{1}{3}$

(C) $-\frac{1}{2}$ (D) $-\frac{1}{3}$

12. If $x = \cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots\infty}}}}$ find x.

(A) $\sqrt{3}+\frac{3}{2}$ (B) $\sqrt{3}-\frac{3}{2}$

(C) $\sqrt{3}+\frac{1}{2}$ (D) $\sqrt{3}-\frac{1}{2}$

13. If $A = 8^{88^8}$, $B = 8^{8^{88}}$, $C = 8^{888}$ and $D = 8^{8^{8^8}}$, which of the following represents the ascending order of the values of A, B, C, D?

(A) *CDAB* (B) *CABD*

(C) *CBAD* (D) *ACBD*

14. Solve for x: $\sqrt{x} + \sqrt{x - \sqrt{1-x}} = 1$.

(A) 1 (B) $\frac{16}{25}$

(C) $\frac{4}{5}$ (D) 0

15. The arithmetic mean of two surds is $5+9\sqrt{2}$, and one of the surds is $1+12\sqrt{2}$

What is the square root of the other surd?

(A) $6-21\sqrt{2}$ (B) $4-3\sqrt{2}$

(C) $\sqrt{3}\left(\sqrt{2}+1\right)$ (D) $\sqrt{2}\left(2-\sqrt{3}\right)$

16. $\dfrac{1}{\sqrt{6}+\sqrt{7}-\sqrt{13}}+\dfrac{1}{\sqrt{6}-\sqrt{7}-\sqrt{13}} =$

(A) $\sqrt{6}$ (B) $\dfrac{1}{\sqrt{6}}$

(C) 6 (D) $\dfrac{1}{6}$

17. Find the square root of

$$\left[1+\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\text{.........} \frac{1}{\sqrt{324}+\sqrt{323}}\right]$$

(A) $3\sqrt{2}$ (B) $\dfrac{1}{\sqrt{2}}$

(C) $2\sqrt{3}$ (D) $\dfrac{\sqrt{3}-1}{2}$

18. If $x^y = y^z = z^x$ and $(x, y, z) > 0$, then

(A) $3\left[\dfrac{xy+yz+zx}{xyz}\right]$ (B) $\dfrac{xy+yz+zx}{xyz}$

(C) $\dfrac{x+y+z}{xyz}$ (D) $\dfrac{xy+yz+zx}{x+y+z}$

19. If $\log_4 3$, $\log_4 (3^m - 2)$ and $\log_4\left(3^m - \dfrac{8}{3}\right)$ are in arithmetic progression, then the number of possible values of m are

(A) 1 (B) 2

(C) 4 (D) 5

20. If $\log_x 162 = m$ and $\log_x 72 = n$, then what is the value of $\log_x 7776$ in terms of m and n?

(A) $\dfrac{m+3n}{m+5n}$ (B) $\dfrac{3m-5n}{m+2n}$

(C) $\dfrac{m+3n}{2}$ (D) $\dfrac{3m-5n}{2}$

21. Which of the following is a possible value of x if $\log_3 x^2 - \log_3 x\sqrt{x} = 8\log_x 3$?

(A) $\dfrac{1}{81}$ (B) $\dfrac{1}{243}$

(C) 243 (D) 9

22. If $a =$ $a = \log_4 31$, then ______ .

(A) $a < 2$ (B) $2 < a < 2.5$

(C) $2.5 < a < 2.8$ (D) $2.8 < a$

23. If $\log_{10}(2x + 3) - 1 = \log_{10}x$, then find x.

(A) $\dfrac{2}{7}$ (B) $\dfrac{3}{4}$

(C) $\dfrac{7}{8}$ (D) $\dfrac{3}{8}$

24. If $a^2 + 4b^2 = 12ab$, what is the value of $\log(a + 2b)$?

(A) $\log\left(\dfrac{a}{2}\right)+\log\left(\dfrac{b}{2}\right)+\log 2$

(B) $(\log a+\log b-\log 2)\dfrac{1}{2}$

(C) $\dfrac{1}{2}(\log a+\log b+4\log 2)$

(D) $\dfrac{1}{2}(\log a-\log b+4\log 2)$

25. Simplify $\dfrac{\log_m p.\log_n p}{\log_m p+\log_n p}$.

(A) 1 (B) $\log_p (m + n)$

(C) $\log_p mn$ (D) $\log_{mn} p$

26. If $a > 1$, $\log_a a + \log_{a^{\frac{1}{2}}} a + \log_{a^{\frac{1}{3}}} a + \ldots..+ \log_{a^{\frac{1}{20}}} a =$

(A) 420 (B) 210

(C) 380 (D) 190

27. If $\dfrac{\log x}{y-z}=\dfrac{\log y}{z-x}=\dfrac{\log z}{x-y}$, then find the value of $\log\left[\left(x^{y^2+yz+z^2+z}\right)\left(y^{z^2+zx+x^2+z}\right)\left(z^{x^2+xy+y^2+z}\right)\right]$

(A) 2 (B) 0

(C) 3 (D) None of these

28. If $abc = 1$, then find the value of $\dfrac{1}{\log_{bc} a^3}+\dfrac{1}{\log_{ac} b^3}+\dfrac{1}{\log_{ab} c^3}$

(A) $\dfrac{-1}{3}$ (B) $\dfrac{1}{3}\log abc$

(C) -1 (D) $\log_{a+b+c} abc$

29. If $\log_6 27 = t$, then find the value of $\log_{18} 4$ in terms of t.

(A) $3\left(\dfrac{2-t}{2+t}\right)$ (B) $2\left(\dfrac{3-t}{3+t}\right)$

(C) $\dfrac{6+t}{3+t}$ (D) $\dfrac{4-t}{3+t}$

30. For $a \geq b$, $b > 1$ the value of the expression $\log_a\left(\dfrac{a}{b}\right)+\log_b\left(\dfrac{b}{a}\right)$ can never be

(A) 0 (B) 1

(C) -2 (D) -0.5

31. If $\log_4(x^2+x)-\log_4(x+1)=2$, then $\sqrt{x}$ =

(A) 2 (B) 4

(C) 8 (D) 16

32. If $\log_{10} 3 = 0.4771$, then find the number of digits in $(243)^{50}$.

(A) 200 (B) 205

(C) 120 (D) 210

33. If (log 16) (log 27) = (log x) (log y) and (log 4096) [log x – log 27] = [log16 – log y] (log 512), which of the following can be the value of (x – y)?

(A) –11
(B) 73
(C) –73
(D) More than one options

34. What is the value of?

$$\log_{64}\sqrt{512\sqrt{512\sqrt{512\sqrt{512.................\infty}}}}.$$

(A) 2.5 (B) 3
(C) 1.5 (D) 1

35. If $\log_{bc}a = \frac{1}{p}$, $\log_{ca}b = \frac{1}{q}$ and $\log_{ab}c = \frac{1}{r}$, find the value of $\frac{1}{p+1}+\frac{1}{q+1}+\frac{1}{r+1}$

(A) 1 (B) $\frac{3}{2}$
(C) 2 (D) None of these

Answer Keys

1. A	**2.** A	**3.** D	**4.** D	**5.** B	**6.** B	**7.** C	**8.** D	**9.** B	**10.** A
11. C	**12.** B	**13.** B	**14.** B	**15.** C	**16.** B	**17.** A	**18.** A	**19.** B	**20.** C
21. A	**22.** B	**23.** D	**24.** C	**25.** D	**26.** B	**27.** B	**28.** C	**29.** B	**30.** B
31. B	**32.** C	**33.** D	**34.** C	**35.** A					

Hints and Explanations

1. Given $2^x = 8^y \Rightarrow 2^x = (2^3)^y$

$\Rightarrow \quad x = 3y$

Also $6^{4y} = 216^{x+y-2}$

$\Rightarrow \quad 6^{4y} = 6^{3(3y+y-2)}$

$\Rightarrow \quad 6^{4y} = 6^{12y-6}$

$\Rightarrow \quad 4y = 12y - 6$

$\Rightarrow \quad 6 = 8y \Rightarrow y = \frac{3}{4}$

$\therefore \quad x = 3y = \frac{9}{4} = 2\frac{1}{4}$

Choice (A)

2. Given $x = \sqrt[3]{9\sqrt[3]{9\sqrt[3]{9\sqrt[3]{9................}}}}$

$\Rightarrow \quad x^3 = 9\sqrt[3]{9\sqrt[3]{9\sqrt[3]{9\sqrt[3]{9................}}}} = 9x$

$\Rightarrow x = 0, 3, –3$. Since x is positive, $x = 3$. Choice (A)

3. $9^{3x-4} = 3^8.(27)^{\frac{(x-2)}{2}}$

$9^{3x-4} = 3^8.(3^3)^{\frac{(x-2)}{2}}$

$3^{2(3x-4)} = 3^8.3^{3\frac{(x-2)}{2}}$

$\frac{3x}{2} = 8+8-3$

$6x - 8 = 8 + \frac{(3x-6)}{2}$

$\frac{9x}{2} = 13 \Rightarrow x = \frac{26}{9}$ Choice (D)

4. Given that $p^\ell = q^m = r^n$

Let each be equal to k.

Hence $p^\ell = k$; $\log_p k = \ell$

$\log_q k = m$; $\log_r k = n$ given that

$\frac{1}{\ell} + \frac{1}{n} = \frac{2}{m}$

$\Rightarrow \quad \frac{1}{\log_p k} + \frac{1}{\log_r k} = \frac{2}{\log_q k}$

$\Rightarrow \quad \log_k p + \log_k r = 2 \cdot \log_k q$

$\Rightarrow \quad \log_k (pr) = \log_k q^2 \quad pr = q^2$ Choice (D)

5. 2^{3^4} is always even, as 2 raised to any power is even.

Hence $(-1)^{2^{3^4}} = (-1)^{\text{even number}} = +1$

Hence, $(25)^{3(-1)^{2^{3^4}}} = (25)^{3^1} = 5^6$.

$(125)^{2x-3} = 5^6. \Rightarrow 5^{3(2x-3)} = 5^6. \Rightarrow 2x - 3 = 2;\ x = 2.5$ Choice (B)

6. By equating the index of 2 on both sides we get $2x^2 + 3 = 6x + 3 \Rightarrow x = 3$ (as $x > 0$) Choice (B)

7. $t_1 = 5^{\frac{1}{2}} = 5^{1-\frac{1}{2}}$

$t_2 = 5^{\frac{3}{4}} = 5^{1-\frac{1}{2^2}}$

$t_3 = 5^{\frac{7}{8}} = 5^{1-\frac{1}{2^3}}$

$t_{10} = 5^{1-\frac{1}{2^{10}}}$

$= (t_1)(t_2)(t_3).....(t_{10}) = \left(5^{1-\frac{1}{2}}\right)\left(5^{1-\frac{1}{2^2}}\right)\left(5^{1-\frac{1}{2^3}}\right)$

$.....\left(5^{1-\frac{1}{2^{10}}}\right)$

$$= 5^{10-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+....+\frac{1}{2^{10}}\right)}$$

$$= 5^{10-\frac{1}{2}\left[1+\frac{1}{2}+\frac{1}{2^2}+....+\frac{1}{2^9}\right]}$$

$$= 5^{10-\frac{1}{2}\frac{\left[1-\frac{1}{2^{10}}\right]}{1-\frac{1}{2}}}$$

$$= 5^{10-1+\frac{1}{2^{10}}} = \sqrt[1024]{5^{9217}}$$

Choice (C)

8. Let $E =$ $E = \dfrac{x^{4b}+x^{2(a+b)}+x^{4a}}{\left(x^{2a}+x^{a+b}+x^{2b}\right)\left(x^{2a}-x^{a+b}+x^{2b}\right)}$.

Den $(E) = x^{2b}\left(x^{2a-2b}+x^{a-b}+1\right).\ x^{2b}\left(x^{2a-2b}-x^{a-b}+1\right)$

$$E=\frac{x^{4b}\left(1+x^{2(a\text{-}b)}+x^{4(a\text{-}b)}\right)}{x^{4b}\left[1\text{-}x^{(a\text{-}b)}+x^{2(a\text{-}b)}\right]\left[1+x^{a\text{-}b}+x^{2(a\text{-}b)}\right]}$$

Considering $x^{a\text{-}b} = t$, we get

$$\frac{1+\ t^2+t^4}{\left(1\text{-}t+t^2\right)\left(1+t+t^2\right)}=\frac{1+t^2+t^4}{1+\ t^2+t^4}=1$$

$(1 - t + t^2)\ (1 + t + t^2) = (1 + t^2 - t)\ (1 + t^2 + t)$

$=(\ (1 + t^2)^2 - t^2)$

Note: The condition $x^2 \neq x$ means $x \neq 0$, $x \neq 1$. If $x = 0$, E is not defined. If $x = 1$, $E = 1$.

∴ This condition need not be imposed. But imposing the condition does not make the statement (that $E = 1$) false. Choice (D)

9. $3^{x+3} - 3^{x-3} = 6552$

$$3^x\left[3^3-\frac{1}{3^3}\right]=6552$$

$$3^x\left[\frac{728}{27}\right]=6552$$

$3^x = 243 = 3^5$

⇒ $x = 5$

∴ $x^2 = 5^2 = 25$ Choice (B)

10. Given $xyz = 1$

⇒ $qxy = \dfrac{1}{z}, \dfrac{1}{xy} = z$ -- (1)

Given expression,

$$\frac{1}{1+x+y^{-1}}+\frac{1}{1+y+z^{-1}}+\frac{1}{1+z+x^{-1}}$$

$$=\frac{y}{y+xy+1}+\frac{1}{1+y+xy}+\frac{1}{1+\frac{1}{xy}+\frac{1}{x}} \quad \text{(from (1))}$$

$$=\frac{y}{y+xy+1}+\frac{1}{1+y+xy}+\frac{xy}{xy+1+y}$$

$$=\frac{y+1+xy}{1+xy+y}=1$$

Choice (A)

11. $\dfrac{81^{a^2+b^2+c^2}}{81^{[-2bc-2ca-2ab]}}=81^{a^2+b^2+c^2+2ab+2bc+2ca}$

$= 81^{(a+b+c)^2} = 3 = 81^{\frac{1}{4}}$

⇒ $a+b+c=\pm\dfrac{1}{2}$

Choice (C)

12. Given $x = \cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+......\infty}}}}$

⇒ $x=\dfrac{1}{4+\dfrac{1}{3+x}}$ ⇒ $x=\dfrac{3+x}{4\left(3+x\right)+1}$

⇒ $x=\dfrac{3+x}{4x+13}$

⇒ $4x^2 + 12x - 3 = 0$

⇒ $x=\dfrac{-12\pm\sqrt{144+48}}{8}$

⇒ $x=\dfrac{4\left(-3\pm\sqrt{12}\right)}{8}$

⇒ $x=\dfrac{-3\pm2\sqrt{3}}{2}$

Since $x > 0$,

$x =$ $x=\dfrac{-3}{2}+\sqrt{3}$ Choice (B)

13. $A = 8^{88^8}$ $B = 8^{8^{88}}$ $C = 8^{888}$ $D = 8^{8^{8^8}}$

Since the base of all the numbers is 8, the number power with highest index is the greatest number. Clearly 'C' has the lowest value.

Consider $A = 8^{88^8}$

and $B = 8^{888}$.

Consider the indices is 88^8

and 8^{88}

$(88)^8$ and $(8^{11})^8$

Since $8^{11} > 88$

$8^{88} > 88^8$

∴ $B > A$

Also, among the four powers the greatest power is 8^{8^8}. Hence D is the largest number.

∴ the ascending order is $CABD$. Choice (B)

14. $\sqrt{x}+\sqrt{x-\sqrt{1-x}}=1$

Squaring both sides we get, $x-\sqrt{1-x}=1+x-2\sqrt{x}$

Squaring again, we get, $1 - x = 1 + 4x - 4\sqrt{x}$

$16x = 25x^2$

$x = \dfrac{16}{25}$ Choice (B)

15. Let the other surd be "a".

$a + 1 + 12\sqrt{2}$

$= 10 + 18\sqrt{2}$

$a = 9 + 6\sqrt{2}$

$= 9 + 2\sqrt{18} = \left(\sqrt{6}+\sqrt{3}\right)^2$

$\therefore \quad \sqrt{a} = \left(\sqrt{6}+\sqrt{3}\right)$ Choice (C)

16. $$\frac{1}{\sqrt{6}+\sqrt{7}-\sqrt{13}} = \frac{\left(\sqrt{6}+\sqrt{7}+\sqrt{13}\right)}{\left(\sqrt{6}+\sqrt{7}+\sqrt{13}\right)}\frac{(1)}{\left(\sqrt{6}+\sqrt{7}-\sqrt{13}\right)}$$

$$= \frac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{\left(\sqrt{6}+\sqrt{7}\right)^2-\left(\sqrt{13}\right)^2} = \frac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{13+2\sqrt{42}-13}$$

$$= \frac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{2\sqrt{42}}$$

$$\frac{1}{\sqrt{6}-\sqrt{7}-\sqrt{13}} = \frac{1\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{\left(\sqrt{6}-\sqrt{7}-\sqrt{13}\right)\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}$$

$$= \frac{\sqrt{6}-\sqrt{7}+\sqrt{13}}{\left(\sqrt{6}-\sqrt{7}\right)^2-\left(\sqrt{13}\right)^2} = \frac{\sqrt{6}-\sqrt{7}+\sqrt{13}}{13-2\sqrt{42}-13}$$

$$= \frac{-\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{2\sqrt{42}}$$

Required value $= \dfrac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{2\sqrt{42}} + \dfrac{-\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{2\sqrt{42}}$

$= \dfrac{2\sqrt{7}}{2\sqrt{42}} = \dfrac{1}{\sqrt{6}}$

Choice (B)

17. The given function is $1 + \dfrac{1}{\sqrt{2}+1} + \dfrac{1}{\sqrt{3}+\sqrt{2}} +$

$= 1 + \dfrac{\sqrt{2}-1}{2-1} + \dfrac{\sqrt{3}-\sqrt{2}}{3-2} + \dfrac{\sqrt{4}-\sqrt{3}}{4-3} + \ldots +$

(on rationalizing the denominator of each term)

$= 1 + \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \ldots + \sqrt{324} - \sqrt{323}$

$= \sqrt{324} = 18$ ($\because$ all terms cancel off except $\sqrt{324}$)

Hence, the square root of the given expression is $\sqrt{18}$

$= 3\sqrt{2}$. Choice (A)

18. Let $x^y = y^z = z^x = k$

$\Rightarrow \quad x = k^{\frac{1}{y}}, y = k^{\frac{1}{z}}, z = k^{\frac{1}{x}}$

consider $\dfrac{1}{x}\log_z xyz$

$= \dfrac{1}{x}\log_{k^{\frac{1}{x}}}\left(k^{\frac{1}{y}}.k^{\frac{1}{z}}.k^{\frac{1}{x}}\right)$

$= \dfrac{1}{x}\log_{k^{\frac{1}{x}}}\left(k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}\right) = \dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}$

Similarly $\dfrac{1}{y}\log_x xyz = \dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}$ and $\dfrac{1}{z}\log_y xyz$

$= \dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}$

Hence the given expression is equal to

$3\left[\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right] = 3\left(\dfrac{xy+yz+zx}{xyz}\right)$

Choice (A)

19. Given $\log_4 3 + \log_4\left(3^m - \dfrac{8}{3}\right) = 2\log_4\left(3^m - 2\right)$

$\Rightarrow \quad 3\left(3^m - \dfrac{8}{3}\right) = \left(3^m - 2\right)^2$

$\Rightarrow \quad 3^{m+1} - 8 = 3^{2m} + 4 - 4(3^m)$

$\Rightarrow \quad 3^{2m} - 7(3^m) + 12 = 0$

Let $3^m = x$

$\Rightarrow \quad x^2 - 7x + 12 = 0$

$x^2 - 4x - 3x + 12 = 0$

$x(x-4) - 3(x-4) = 0$

$\Rightarrow \quad (x-4)(x-3) = 0$

$\Rightarrow \quad x = 4$ or 3

$\Rightarrow \quad 3^m = 4$ or $3^m = 3$

$\Rightarrow \quad m = \log_3 4$ (or) $m = 1$

Hence m can take two values. Choice (B)

20. Given $\log_x 162 = m$

$\Rightarrow \quad \log_x 3^4 \ (2) = m$

$\therefore \quad m = 4\log_x 3 + \log_x 2$

Given $\log_x 72 = n$

$\Rightarrow \quad \log_x 3^2\, 2^3 = n$

$\therefore \quad n = 2\log_x 3 + 3\log_x 2$

Let $\log_x 3 = 1$ and $\log_x 2 = b$

$\Rightarrow m = 4a + b$ --- (1)

$n = 2a + 3b$ --- (2)

2 (2) – (1) gives

$5b = 2n - m \Rightarrow b = \dfrac{2n-m}{5}$

similarly $a = \dfrac{3m-n}{10}$

Now consider $\log_x 7776$

$= \log_x 3^5.2^5$

$= 5[\log_x 3 + \log_x 2]$

$= 5\left[\dfrac{3m-n}{10} + \dfrac{2n-m}{5}\right]$

$= 5\left[\dfrac{m+3n}{10}\right] = \dfrac{m+3n}{2}$

Choice (C)

21. $\log_3 x^2 - \log_3 x\sqrt{x}$
$= 8\log_x 3$

$\Rightarrow \quad \log_3 \frac{x^2}{x\sqrt{x}} = 8\log_x 3$

$\Rightarrow \quad \log_3 \sqrt{x} = 8\log_x 3$

$\Rightarrow \quad \frac{1}{2}\log_3 x = \frac{8}{\log_3 x}$

$\Rightarrow \quad (\log_3 x)^2 = 16$

$\Rightarrow \quad \log_3 x = 4$

$\Rightarrow \quad x = 3^4 = 81$ or $x = 3^{-4} = \frac{1}{81}$

Choice (A)

22. $\log_4 31 = \log_{2^2} 31 = \frac{1}{2}\log_2 31$

$2^4 < 31 < 2^5$

$\Rightarrow \quad \log_2 2^4 < \log_2 31 < \log_2 2^5$

$\Rightarrow \quad 4\log_2 2 < \log_2 31 < 5\log_2 2$

$\Rightarrow \quad \frac{4}{2} < \frac{1}{2}\log_2 31 < \frac{5}{2}$

$\Rightarrow \quad 2 < \frac{1}{2}\log_2 31 < 2.5.$ Choice (B)

23. $\log(2x + 3) - 1 = \log x$

$\log(2x + 3) - \log 10 = \log x$

$\Rightarrow \quad \log\left(\frac{2x+3}{10}\right) = \log x$

$\Rightarrow \quad \frac{2x+3}{10} = x$

$\Rightarrow \quad 2x + 3 = 10x$

$\Rightarrow \quad x = \frac{3}{8}$

Choice (D)

24. $a^2 + 4b^2 = 12ab$; adding $4ab$ to both sides of the equation, we get $(a + 2b)^2 = 16ab$

$2\log(a + 2b) = 4\log 2 + \log a + \log b$

$\log(a + 2b) = \frac{1}{2}$

$[\log a + \log b + 4\log 2]$ Choice (C)

25. $\frac{\log_m p.\log_n p}{\log_m p + \log_n p}$

$= \frac{1}{\frac{\log_m p + \log_n p}{\log_m p.\log_n p}} = \frac{1}{\frac{1}{\log_n p} + \frac{1}{\log_m p}}$

$= \frac{1}{\log_p n + \log_p m} = \frac{1}{\log_p mn}$

$= \log_{mn} p$ Choice (D)

26. $\log_a a + \log a\frac{1}{2}a + \log a\frac{1}{3}a + \ldots + \log a\frac{1}{20}$ a

$= \frac{\log a}{\log a} + \frac{\log a}{\log a^{\frac{1}{2}}} + \frac{\log a}{\log a^{\frac{1}{3}}} + \ldots + \frac{\log a}{\log a^{\frac{1}{20}}}$

$= 1 + 2 + 3 + \ldots + 20 = \frac{20\times 21}{2} = 210$ Choice (B)

27. Let $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = k$

$\log x = k(y - z)$

$\log y = k(z - x)$

$\log z = k(x - y)$

$\log x^{y^2+yz+z^2+z} = (y^2 + yz + z^2 + z)(\log x)$

$= k(y-z)(y^2 + yz + z^2 + z)$

$= k(y^3 - z^3) + k(y-z)z$ --- (1)

$\log y^{z^2+xz+x^2+z} = k(z-x)(z^2 + xz + x^2 + z)$

$= k(z^3 - x^3) + k(z-x)z$ --- (2)

$\log z^{x^2+xy+y^2+2} = k(x-y)(x^2 + xy + y^2 + z)$

$= k(x^3 - y^3) + k(x\ y)z$ --- (3)

Adding (1), (2) and (3),

$\log x^{y^2-yz+z^2+z} + \log y^{z^2+xz+x^2+z} + \log z^{x^2+xy+y^2+z}$

$= k(y^3 - z^3) + kz(y - z) + k(z^3 - x^3) + kz(z - x) + k(x^3 - y^3) + kz(x - y)$

$= k(y^3 - z^3 + z^3 - x^3 + x^3 - y^3) + kz(y - z + z - x + x - y)$

$= 0 + 0 = 0$ Choice (B)

28. $\frac{1}{\log_{bc} a^3} + \frac{1}{\log_{ac} b^3} + \frac{1}{\log_{ab} c^3}$

$= \frac{1}{\log_{1/a} a^3} + \frac{1}{\log_{1/b} b^3} + \frac{1}{\log_{1/c} c^3}$

$\left[\because bc = \frac{1}{a}, ac = \frac{1}{b} \text{ and } ab = \frac{1}{c}\right]$

$= \frac{1}{\log_{a^{-1}} a^3} + \frac{1}{\log_{b^{-1}} b^3} + \frac{1}{\log_{c^{-1}} c^3}$

$= \frac{1}{-3} + \frac{1}{-3} + \frac{1}{-3} = -1$ Choice (C)

29. $t = \log_6 27 = \frac{\log_{10} 27}{\log_{10} 6} = \frac{3\log_{10} 3}{\log_{10} 2 + \log_{10} 3}$

Now $3 - t = 3 - \frac{3\log 3}{\log 2 + \log 3} = \frac{3\log 2}{\log 2 + \log 3}$

$3 + t = 3 + \frac{3\log 3}{\log 2 + \log 3} = \frac{3\log 2 + 6\log 3}{\log 2 + \log 3}$

$\frac{3-t}{3+t} = \frac{(3\log 2)/(\log 2 + \log 3)}{(3\log 2 + 6\log 3)/(\log 2 + \log 3)}$

$$=\frac{\log 2}{\log 2+2\log 3}=\log_{18} 2$$

$\therefore \quad 2\frac{(3-t)}{3+t}=2\log_{18} 2=\log_{18} 4$

Choice (B)

30. Given: $\log_a\left(\frac{a}{b}\right)+\log_b\left(\frac{b}{a}\right)$

$= \log_a a-\log_a b+\log_b b-\log_b a = 2-(\log_a b+\log_b a)$

But $\log_a b + \log_b a$ is in the form of $x+\frac{1}{x}$ which is ≥ 2

$\therefore \log_a b + \log_b a \geq 2$

$\therefore$ The given expression can not be positive

$\therefore$ It can not be 1

Choice (B)

31. $\log_4\left[\frac{(x^2+x)}{(x+1)}\right]=2$

$\frac{(x^2+x)}{(x+1)}=4^2$

$\Rightarrow \quad x^2+x=16x+16$

$= x^2-15x-16=0$

$x^2-16x+x-16=0$

$x(x-16)+1(x-16)=0$

$\Rightarrow \quad (x+1)(x-16)=0$

$x=-1$ or 16

$\sqrt{x}=\sqrt{16}=4$

Choice (B)

32. Let $k=(243)^{50}=3^{250}$

Taking log on both sides, we get

$\log k = 250 \log 3$

$= 250\,(0.4771)$

$=25(4.771)=\frac{100}{4}(4.771)=119.4$

The characteristic of $\log k$ is 119.

Number of digits in $(243)^{50}$ are $119+1=120$

Choice (C)

33. Given $(\log 16)(\log 27)=(\log x)(\log(y))$

let $\log x = X$ and $\log y = Y$ then $(\log 4096)(\log x-\log 27)$

$= \log 512\,(\log 16-\log y)$

becomes

$12 \log 2\,(X-3\log 3)=9\log 2(4\log 2-Y)$

$4(X-3\log 3)=3(4\log 2-Y)$ ———— (1)

and $(\log 16)(\log 27) = (\log x)(\log y)$ becomes $12\log 2\log 3 = XY$ ———— (2)

eliminating Y using (1) and (2) we have

$4(X-3\log 3)=3(4\log 2-12(\log 2)(\log 3)\div X)$

$4X(X-3\log 3)=3(4X\log 2-12\log 2.\log 3)$

$4X^2-12X\log 3=12X\log 2-36\log 2.\log 3$

$X^2-3X\log 3-3X\log 2+9\log 2\log 3=0$

$(X-3\log 3)(X-3\log 2)=0$

$\Rightarrow \quad X=\log 27$ or $X=\log 8$

i.e., $\log x=\log 27$ or $\log x=\log 8$

$\Rightarrow \quad x=27$ or 8

when $x=27$ then $y=16$ and $x-y=11$ and when $x=8$, then $y=81$ and $x-y=-73$

Choice (D)

34. Let $\sqrt{512\sqrt{512\sqrt{512\sqrt{512\ldots\ldots\infty}}}}=x$

$x^2=512\sqrt{512\sqrt{512\sqrt{512\ldots\ldots\infty}}}$

$x^2=512x, \Rightarrow x=0$ or 512.

As x is clearly not zero, $x=512$.

Hence the required quantity is $\log_{64} 512$.

$=\log_{8^2} 8^3=\frac{3}{2}=1.5$

Choice (C)

35. $\log_{bc} a = 1/p \Rightarrow p=\log_a bc \Rightarrow p+1=\log_a abc$

Similarly $\log_{ca} b = 1/q \Rightarrow q+1=\log_b abc$

and $\log_{ab} c = 1/r \Rightarrow r+1=\log_c abc$

$\therefore \quad \frac{1}{p+1}+\frac{1}{q+1}+\frac{1}{r+1}=\log_{abc} a+\log_{abc} b+\log_{abc} c$

$=\log_{abc} abc=1$

Choice (A)