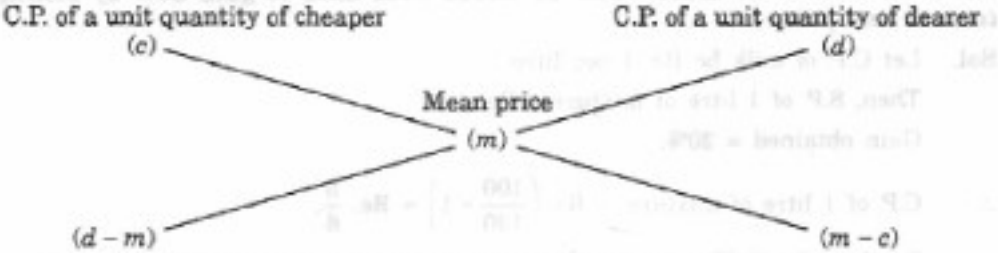
20. ALLIGATION OR MIXTURE

IMPORTANT FACTS AND FORMULAE

1. **Alligation** : It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.
2. **Mean Price** : The cost price of a unit quantity of the mixture is called the mean price.
3. **Rule of Alligation** : If two ingredients are mixed, then

$$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}}\right) = \frac{\text{(C.P. of dearer)} - \text{(Mean price)}}{\text{(Mean price)} - \text{(C.P. of cheaper)}}.$$

We present as under :



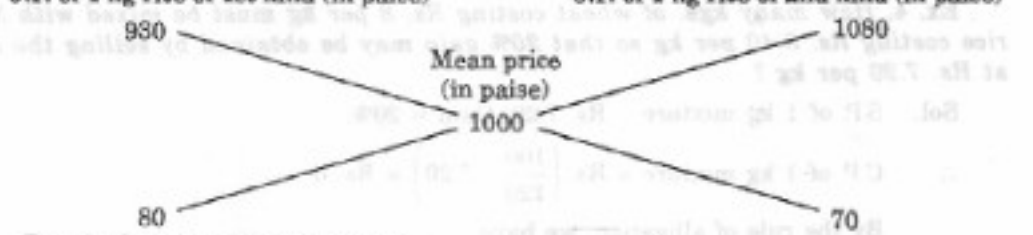
$\therefore$ (Cheaper quantity) : (Dearer quantity) = $(d - m) : (m - c)$.

4. Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid $= \left[x\left(1 - \frac{y}{x}\right)^n\right]$ units.

SOLVED EXAMPLES

Ex. 1. ***In what ratio must rice at Rs. 9.30 per kg be mixed with rice at Rs. 10.80 per kg so that the mixture be worth Rs. 10 per kg ?***

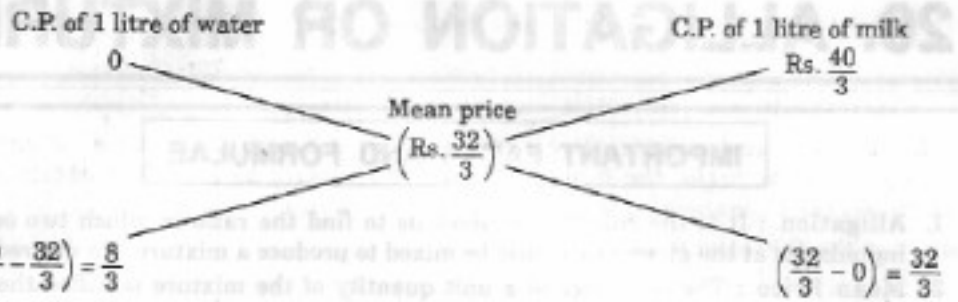
Sol. By the rule of alligation, we have :



$\therefore$ Required ratio = 80 : 70 = 8 : 7.

Ex. 2. ***How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for Rs. 20 so as to have a mixture worth Rs. $10\frac{2}{3}$ a litre ?***

Sol. C.P. of 1 litre of milk = Rs. $\left(20 \times \frac{2}{3}\right)$ = Rs. $\frac{40}{3}$.



$\therefore$ Ratio of water and milk $= \frac{8}{3} : \frac{32}{3} = 8 : 32 = 1 : 4.$

$\therefore$ Quantity of water to be added to 60 litres of milk $= \left(\frac{1}{4} \times 60\right)$ litres $= 15$ litres.

Ex. 3. ***In what ratio must water be mixed with milk to gain 20% by selling the mixture at cost price ?***

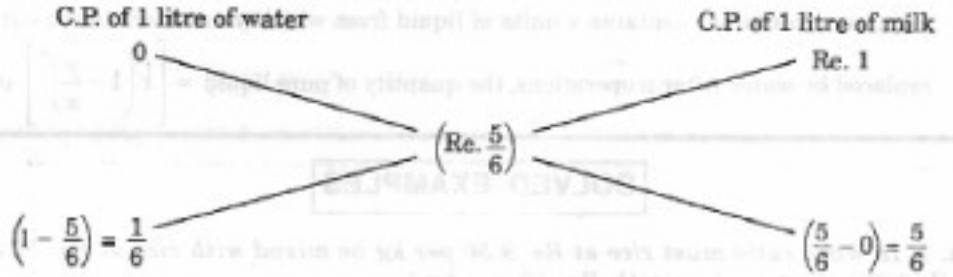
Sol. Let C.P. of milk be Re. 1 per litre.

Then, S.P. of 1 litre of mixture = Re. 1.

Gain obtained = 20%.

$\therefore$ C.P. of 1 litre of mixture $= \text{Rs.}\left(\frac{100}{120} \times 1\right) = \text{Re.}\ \frac{5}{6}.$

By the rule of alligation, we have :



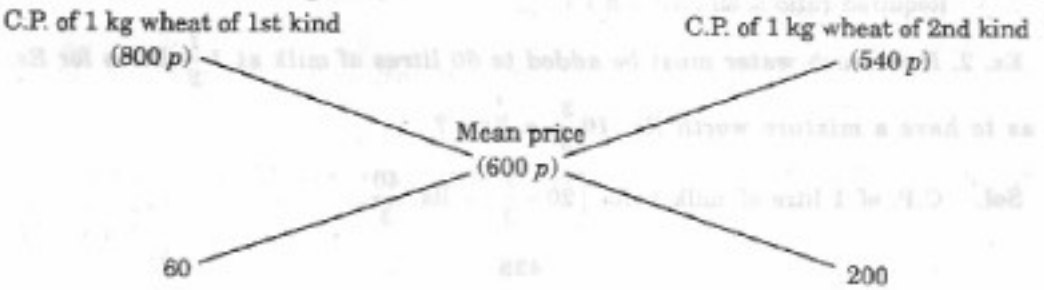
$\therefore$ Ratio of water and milk $= \frac{1}{6} : \frac{5}{6} = 1 : 5.$

Ex. 4. ***How many kgs. of wheat costing Rs. 8 per kg must be mixed with 36 kg of rice costing Rs. 5.40 per kg so that 20% gain may be obtained by selling the mixture at Rs. 7.20 per kg ?***

Sol. S.P. of 1 kg mixture = Rs. 7.20, Gain = 20%.

$\therefore$ C.P. of 1 kg mixture $= \text{Rs.}\left(\frac{100}{120} \times 7.20\right) = \text{Rs. } 6.$

By the rule of alligation, we have :



Wheat of 1st kind : Wheat of 2nd kind = 60 : 200 = 3 : 10.

Let x kg of wheat of 1st kind be mixed with 36 kg of wheat of 2nd kind.

Then, $3 : 10 = x : 36$ or $10x = 3 \times 36$ or $x = 10.8$ kg.

Ex. 5. ***The milk and water in two vessels A and B are in the ratio 4 : 3 and 2 : 3 respectively. In what ratio, the liquids in both the vessels be mixed to obtain a new mixture in vessel C containing half milk and half water ?***

Sol. Let the C.P. of milk be Re. 1 per litre.

Milk in 1 litre mixture of A $= \frac{4}{7}$ litre; Milk in 1 litre mixture of B $= \frac{2}{5}$ litre;

Milk in 1 litre mixture of C $= \frac{1}{2}$ litre.

$\therefore$ C.P. of 1 litre mixture in A = Re. $\frac{4}{7}$; C.P. of 1 litre mixture in B = Re. $\frac{2}{5}$.

Mean price = Re. $\frac{1}{2}$.

By the rule of alligation, we have :

C.P. of 1 litre mix. in A C.P. of 1 litre mix. in B

$\left(\frac{4}{7}\right)$ $\left(\frac{2}{5}\right)$

$\left(\frac{1}{2}\right)$

$\left(\frac{1}{10}\right)$ $\left(\frac{1}{14}\right)$

$\therefore$ Required ratio $= \frac{1}{10} : \frac{1}{14} = 7 : 5$.

EXERCISE 20

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. In what ratio must a grocer mix two varieties of pulses costing Rs. 15 and Rs. 20 per kg respectively so as to get a mixture worth Rs. 16.50 per kg ? **(R.R.B. 2003)**
   (*a*) 3 : 7 (*b*) 5 : 7 (*c*) 7 : 3 (*d*) 7 : 5
2. Find the ratio in which rice at Rs. 7.20 a kg be mixed with rice at Rs. 5.70 a kg to produce a mixture worth Rs. 6.30 a kg. **(IGNOU, 2003)**
   (*a*) 1 : 3 (*b*) 2 : 3 (*c*) 3 : 4 (*d*) 4 : 5
3. In what ratio must tea at Rs. 62 per kg be mixed with tea at Rs. 72 per kg so that the mixture must be worth Rs. 64.50 per kg ?
   (*a*) 3 : 1 (*b*) 3 : 2 (*c*) 4 : 3 (*d*) 5 : 3
4. In what ratio must water be mixed with milk costing Rs. 12 per litre to obtain a mixture worth of Rs. 8 per litre ?
   (*a*) 1 : 2 (*b*) 2 : 1 (*c*) 2 : 3 (*d*) 3 : 2
5. The cost of Type 1 rice is Rs. 15 per kg and Type 2 rice is Rs. 20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of rice is : **(M.B.A. 2002)**
   (*a*) Rs. 18 (*b*) Rs. 18.50 (*c*) Rs. 19 (*d*) Rs. 19.50

6. In what ratio must a grocer mix two varieties of tea worth Rs. 60 a kg and Rs. 65 a kg so that by selling the mixture at Rs. 68.20 a kg he may gain 10% ?
(a) 3 : 2 (b) 3 : 4 (c) 3 : 5 (d) 4 : 5
(S.S.C. 2004)

7. How many kilograms of sugar costing Rs. 9 per kg must be mixed with 27 kg of sugar costing Rs. 7 per kg so that there may be a gain of 10% by selling the mixture at Rs. 9.24 per kg ?
(a) 36 kg (b) 42 kg (c) 54 kg (d) 63 kg

8. In what ratio must water be mixed with milk to gain $16\frac{2}{3}\%$ on selling the mixture at cost price ?
(L.I.C.A.A.O. 2003)
(a) 1 : 6 (b) 6 : 1 (c) 2 : 3 (d) 4 : 3

9. A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is :
(a) 4% (b) $6\frac{1}{4}\%$ (c) 20% (d) 25%

10. Two vessels A and B contain spirit and water mixed in the ratio 5 : 2 and 7 : 6 respectively. Find the ratio in which these mixture be mixed to obtain a new mixture in vessel C containing spirit and water in the ratio 8 : 5 ?
(a) 4 : 3 (b) 3 : 4 (c) 5 : 6 (d) 7 : 9

11. Two vessels A and B contain milk and water mixed in the ratio 8 : 5 and 5 : 2 respectively. The ratio in which these two mixtures be mixed to get a new mixture containing $69\frac{3}{13}\%$ milk, is :
(a) 2 : 7 (b) 3 : 5 (c) 5 : 2 (d) 5 : 7

12. A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contains 50% water. How much milk should he mix from each of the containers so as to get 12 litres of milk such that the ratio of water to milk is 3 : 5 ?
(a) 4 litres, 8 litres (b) 6 litres, 6 litres
(c) 5 litres, 7 litres (d) 7 litres, 5 litres

13. One quality of wheat at Rs. 9.30 per kg is mixed with another quality at a certain rate in the ratio 8 : 7. If the mixture so formed be worth Rs. 10 per kg, what is the rate per kg of the second quality of wheat ?
(a) Rs. 10.30 (b) Rs. 10.60 (c) Rs. 10.80 (d) Rs. 11

14. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be :
(S.S.C. 1999)
(a) Rs. 169.50 (b) Rs. 170 (c) Rs. 175.50 (d) Rs. 180

15. A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is :
(a) 400 kg (b) 560 kg (c) 600 kg (d) 640 kg

16. A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is :
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

17. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container ?
(a) 26.34 litres (b) 27.36 litres (c) 28 litres (d) 29.16 litres

18. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 : 65. How much wine did the cask hold originally ? (N.I.F.T. 2003)

(a) 18 litres (b) 24 litres (c) 32 litres (d) 42 litres

19. A can contains a mixture of two liquids A and B in the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7 : 9. How many litres of liquid A was contained by the can initially ?

(a) 10 (b) 20 (c) 21 (d) 25

20. A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup ?

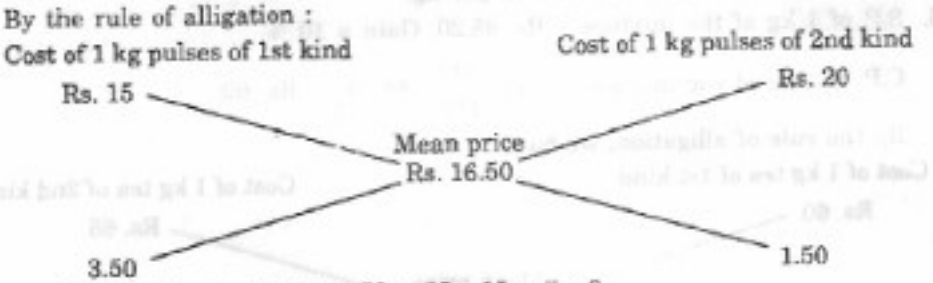
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{7}$

ANSWERS

1. (c) 2. (b) 3. (a) 4. (a) 5. (a) 6. (a) 7. (d) 8. (a) 9. (c) 10. (d)
11. (a) 12. (b) 13. (c) 14. (c) 15. (c) 16. (b) 17. (d) 18. (b) 19. (c) 20. (c)

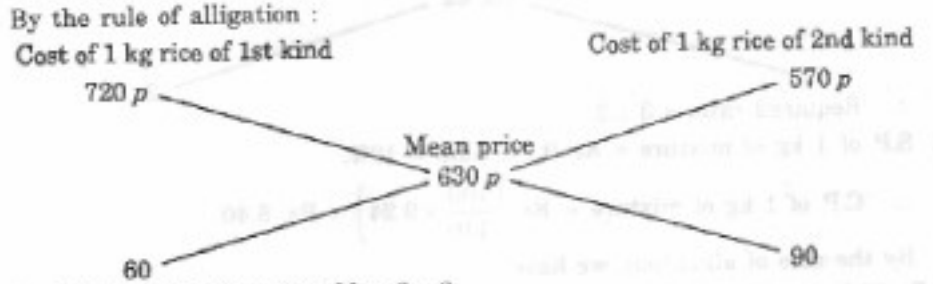
SOLUTIONS

1. By the rule of alligation :



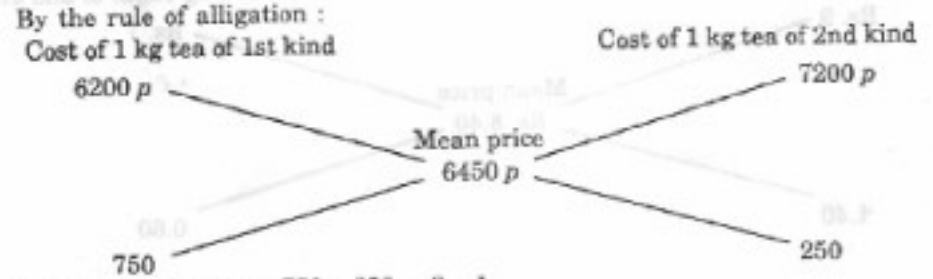
∴ Required rate = 3.50 : 1.50 = 35 : 15 = 7 : 3.

2. By the rule of alligation :



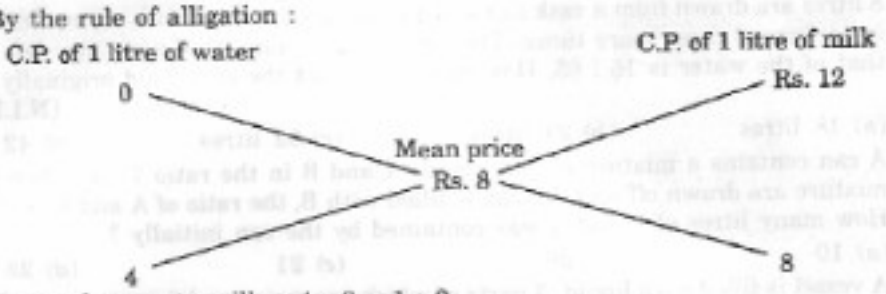
∴ Required ratio = 60 : 90 = 2 : 3.

3. By the rule of alligation :



∴ Required ratio = 750 : 250 = 3 : 1.

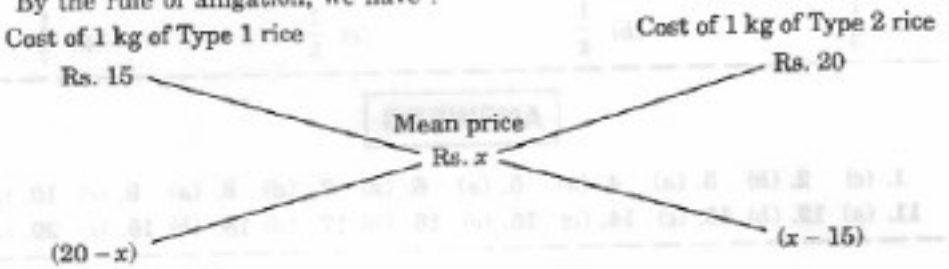
4. By the rule of alligation :



Ratio of water to milk = 4 : 8 = 1 : 2.

5. Let the price of the mixed variety be Rs. x per kg.

By the rule of alligation, we have :



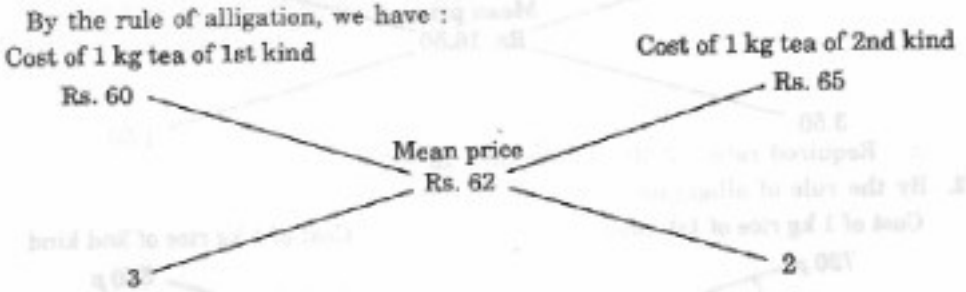
$$\therefore \quad \frac{(20-x)}{(x-15)} = \frac{2}{3} \Rightarrow 60 - 3x = 2x - 30 \Rightarrow 5x = 90 \Rightarrow x = 18.$$

So, price of the mixture is Rs. 18 per kg.

6. S.P. of 1 kg of the mixture = Rs. 68.20, Gain = 10 %.

C.P. of 1 kg of the mixture = Rs. $\left(\frac{100}{110} \times 68.20\right)$ = Rs. 62.

By the rule of alligation, we have :

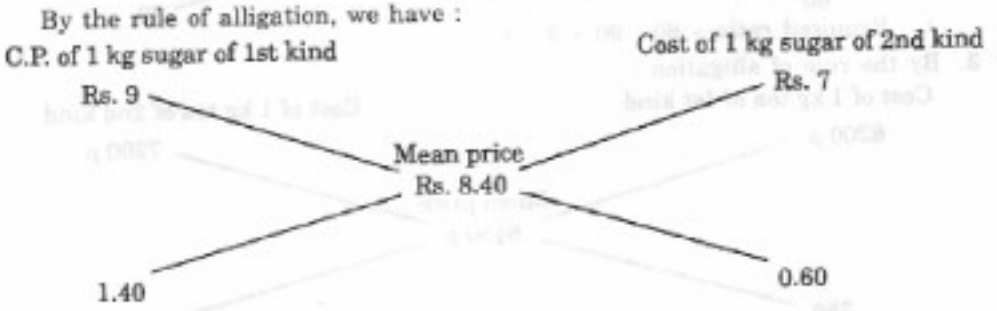


$\therefore$ Required ratio = 3 : 2.

7. S.P. of 1 kg of mixture = Rs. 9.24, Gain = 10%.

$\therefore$ C.P. of 1 kg of mixture = Rs. $\left(\frac{100}{110} \times 9.24\right)$ = Rs. 8.40.

By the rule of alligation, we have :



$\therefore$ Ratio of quantities of 1st and 2nd kind = 14 : 6 = 7 : 3.

Let x kg of sugar of 1st kind be mixed with 27 kg of 2nd kind.

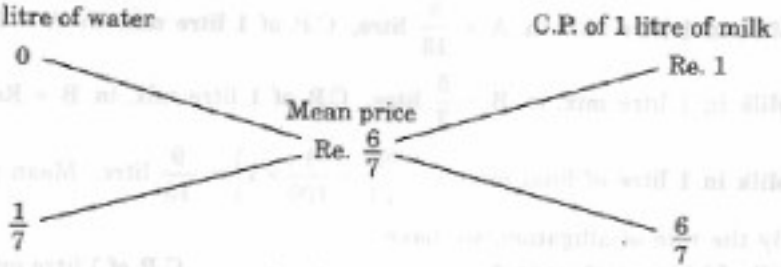
Then, $7 : 3 = x : 27$ or $x = \left(\frac{7 \times 27}{3}\right) = 63$ kg.

8. Let C.P. of 1 litre milk be Re. 1.

S.P. of 1 litre of mixture = Re. 1, Gain $= \frac{50}{3}\%$.

$\therefore$ C.P. of 1 litre of mixture $= \left(100 \times \frac{3}{350} \times 1\right) =$ Re. $\frac{6}{7}$.

By the rule of alligation, we have :

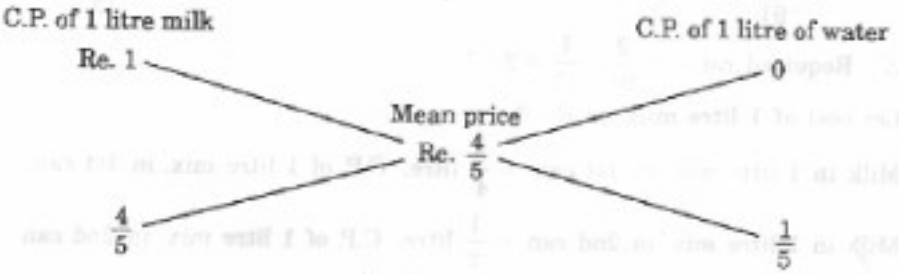


$\therefore$ Ratio of water and milk $= \frac{1}{7} : \frac{6}{7} = 1 : 6$.

9. Let C.P. of 1 litre milk be Re. 1.

Then, S.P. of 1 litre of mixture = Re. 1, Gain = 25%.

C.P. of 1 litre mixture = Re. $\left(\frac{100}{125} \times 1\right) =$ Re. $\frac{4}{5}$.



$\therefore$ Ratio of milk to water $= \frac{4}{5} : \frac{1}{5} = 4 : 1$.

Hence, percentage of water in the mixture $= \left(\frac{1}{5} \times 100\right)\% = 20\%$.

10. Let the C.P. of spirit be Re. 1 per litre.

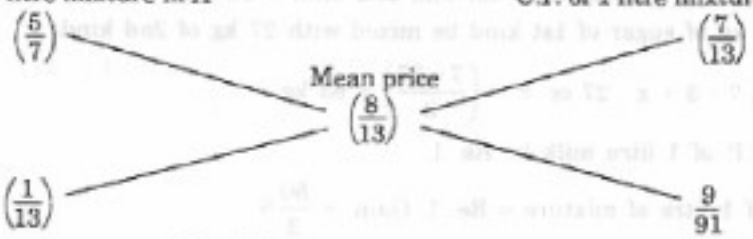
Spirit in 1 litre mix. of A $= \frac{5}{7}$ litre; C.P. of 1 litre mix. in A = Re. $\frac{5}{7}$.

Spirit in 1 litre mix. of B $= \frac{7}{13}$ litre; C.P. of 1 litre mix. in B = Re. $\frac{7}{13}$.

Spirit in 1 litre mix. of C $= \frac{8}{13}$ litre; Mean price = Re. $\frac{8}{13}$.

By the rule of alligation, we have :

C.P. of 1 litre mixture in A C.P. of 1 litre mixture in B



$\therefore$ Required ratio $= \frac{1}{13} : \frac{9}{91} = 7 : 9.$

11. Let cost of 1 litre milk be Re. 1.

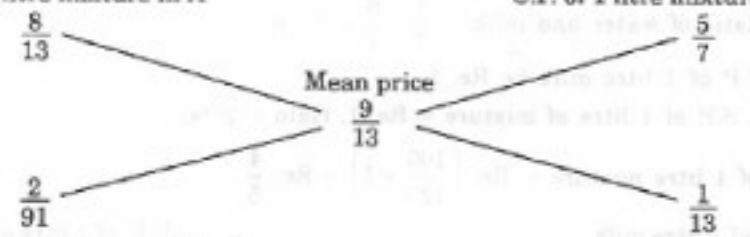
Milk in 1 litre mix. in A $= \frac{8}{13}$ litre, C.P. of 1 litre mix. in A = Re. $\frac{8}{13}$.

Milk in 1 litre mix. in B $= \frac{5}{7}$ litre, C.P. of 1 litre mix. in B = Re. $\frac{5}{7}$.

Milk in 1 litre of final mix. $= \left(\frac{900}{13} \times \frac{1}{100} \times 1\right) = \frac{9}{13}$ litre; Mean price = Re. $\frac{9}{13}$.

By the rule of alligation, we have :

C.P. of 1 litre mixture in A C.P. of 1 litre mixture in B



$\therefore$ Required ratio $= \frac{2}{91} : \frac{1}{13} = 2 : 7.$

12. Let cost of 1 litre milk be Re. 1.

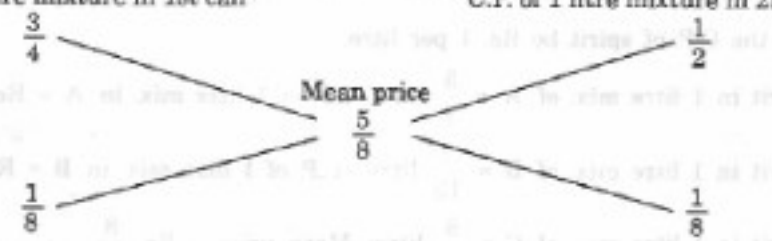
Milk in 1 litre mix. in 1st can $= \frac{3}{4}$ litre, C.P. of 1 litre mix. in 1st can = Re. $\frac{3}{4}$.

Milk in 1 litre mix. in 2nd can $= \frac{1}{2}$ litre, C.P. of 1 litre mix. in 2nd can = Re. $\frac{1}{2}$.

Milk in 1 litre of final mix. $= \frac{5}{8}$ litre, Mean price = Re. $\frac{5}{8}$.

By the rule of alligation, we have :

C.P. of 1 litre mixture in 1st can C.P. of 1 litre mixture in 2nd can

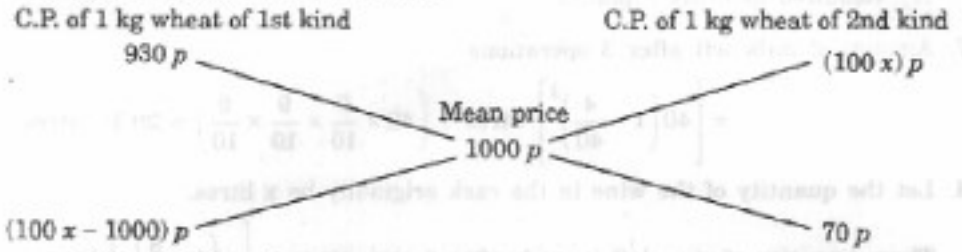


$\therefore$ Ratio of two mixtures $= \frac{1}{8} : \frac{1}{8} = 1 : 1.$

So, quantity of mixture taken from each can $= \left(\frac{1}{2} \times 12\right) = 6$ litres.

13. Let the rate of the second quality be Rs. x per kg.

By the rule of alligation, we have :



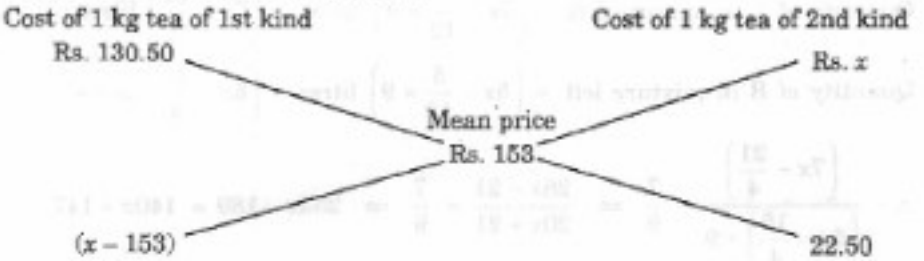
$$\therefore \quad \frac{100x - 1000}{70} = \frac{8}{7} \Rightarrow 700x - 7000 = 560 \Rightarrow 700x = 7560 \Rightarrow x = \text{Rs. } 10.80.$$

14. Since first and second varieties are mixed in equal proportions, so their average price

$$= \text{Rs.} \left(\frac{126 + 135}{2} \right) = \text{Rs. } 130.50.$$

So, the mixture is formed by mixing two varieties, one at Rs. 130.50 per kg and the other at say, Rs. x per kg in the ratio 2 : 2, *i.e.*, 1 : 1. We have to find x.

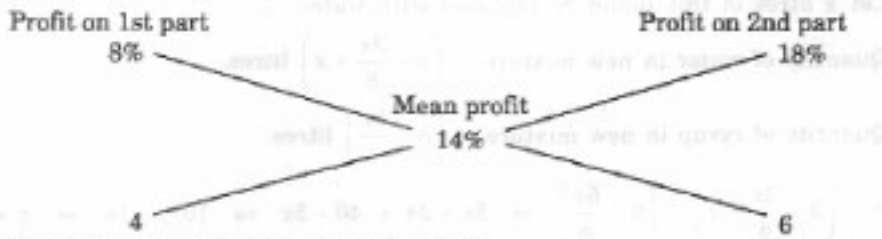
By the rule of alligation, we have :



$$\therefore \quad \frac{x - 153}{22.50} = 1 \Rightarrow x - 153 = 22.50 \Rightarrow x = 175.50.$$

Hence, price of the third variety = Rs. 175.50 per kg.

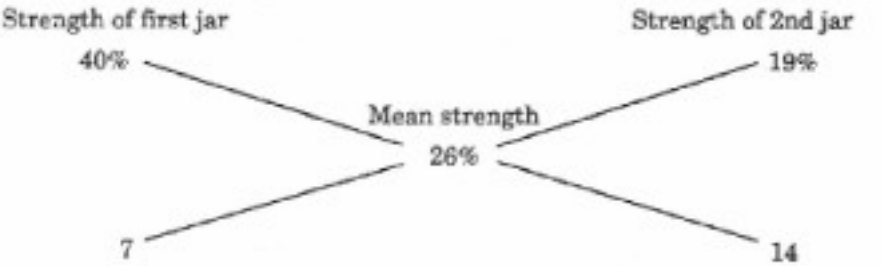
15. By the rule of alligation, we have :



Ratio of 1st and 2nd parts = 4 : 6 = 2 : 3.

$$\therefore \quad \text{Quantity of 2nd kind} = \left(\frac{3}{5} \times 1000 \right) \text{kg} = 600 \text{ kg}.$$

16. By the rule of alligation, we have :



So, ratio of 1st and 2nd quantities = 7 : 14 = 1 : 2.

$\therefore$ Required quantity replaced $= \frac{2}{3}$.

17. Amount of milk left after 3 operations

$$= \left[40\left(1-\frac{4}{40}\right)^3\right] \text{ litres} = \left(40\times\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}\right) = 29.16 \text{ litres.}$$

18. Let the quantity of the wine in the cask originally be x litres.

Then, quantity of wine left in cask after 4 operations $= \left[x\left(1-\frac{8}{x}\right)^4\right]$ litres.

$$\therefore \frac{x\left(1-\frac{8}{x}\right)^4}{x} = \frac{16}{81} \Rightarrow \left(1-\frac{8}{x}\right)^4 = \left(\frac{2}{3}\right)^2 \Rightarrow \left(\frac{x-8}{x}\right) = \frac{2}{3}$$

$$\Rightarrow 3x - 24 = 2x \Rightarrow x = 24.$$

19. Suppose the can initially contains $7x$ and $5x$ litres of mixtures A and B respectively.

Quantity of A in mixture left $= \left(7x - \frac{7}{12}\times 9\right)$ litres $= \left(7x - \frac{21}{4}\right)$ litres.

Quantity of B in mixture left $= \left(5x - \frac{5}{12}\times 9\right)$ litres $= \left(5x - \frac{15}{4}\right)$ litres.

$$\therefore \frac{\left(7x-\frac{21}{4}\right)}{\left(5x-\frac{15}{4}\right)+9} = \frac{7}{9} \Rightarrow \frac{28x-21}{20x+21} = \frac{7}{9} \Rightarrow 252x - 189 = 140x + 147$$

$$\Rightarrow 112x = 336 \Rightarrow x = 3.$$

So, the can contained 21 litres of A.

20. Suppose the vessel initially contains 8 litres of liquid.

Let x litres of this liquid be replaced with water.

Quantity of water in new mixture $= \left(3 - \frac{3x}{8} + x\right)$ litres.

Quantity of syrup in new mixture $= \left(5 - \frac{5x}{8}\right)$ litres.

$$\therefore \left(3 - \frac{3x}{8} + x\right) = \left(5 - \frac{5x}{8}\right) \Rightarrow 5x + 24 = 40 - 5x \Rightarrow 10x = 16 \Rightarrow x = \frac{8}{5}.$$

So, part of the mixture replaced $= \left(\frac{8}{5}\times\frac{1}{8}\right) = \frac{1}{5}$.