Viscosity

Viscosity

Definition. It is the property of a fluid by virtue of which an internal resistance comes into play when the fluid is in motion, and opposes the relative motion between its different layers.

Demonstration. Take some water in a glass tumbler and stir it with a spoon for a minute. Take out the spoon from water and leave the water free. After few minutes, it will come to stop due to internal friction acting between the layers of water.

Coefficient of viscosity

Introduction. When liquid flows over a flat surface, liquid layer in contact with fixed surface AB does not move. Upper and upper layers move forward with increasing velocity. Due to relative motion, a backward dragging force F acts tangentially to every layer.

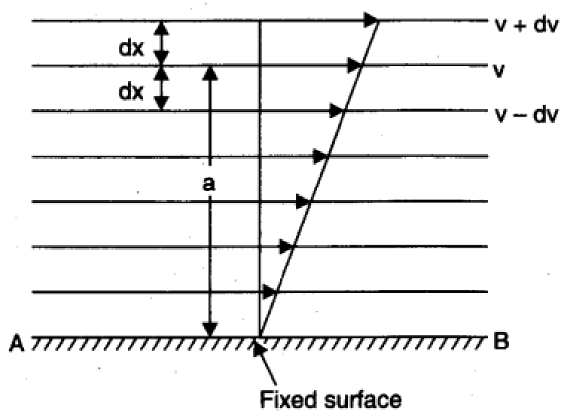

Fig. 12.01. Relative motion between different layers.

Force F depends upon the following factors :

(*i*) *Area (A) of the layer* and is directly proportional to it *i.e.*, $F \propto A$.

(*ii*) *Velocity (v) of the layer* from fixed surface and is directly proportional to it *i.e.*, $F \propto v$.

(*iii*) *Distance* (*x*) *of layer from fixed surface* and is inversely proportional to it *i.e.*, $F \propto \frac{1}{x}$.

Combining, $F \propto \frac{Av}{x}$

or $$\boxed{F = -\eta \frac{Av}{x}}$$

where η is constant of proportionality. It is called '*coefficient of viscosity*' (Negative sign means that force acts in opposite direction of velocity).

If dv represents a small change in velocity over distance dx, then

$$F = -\eta A \frac{dv}{dx}.$$

The term $\frac{dv}{dx}$ is called velocity gradient (which means rate of change of velocity with distance).

Definition. To maintain uniform flow of liquid, a force equal to viscous force is to be applied on the liquid. The external force is given by, $F = \eta A \frac{dv}{dx}$.

If $A = 1, \frac{dv}{dx} = 1$, then $F = \eta$.

Hence, coefficient of viscosity may be defined as (or is equal to) the tangential force required to maintain a unit velocity gradient between two layers each of unit area.

Unit, (i) The C.G.S. unit of coefficient of viscosity is poise (P).

One poise is coefficient of viscosity of a liquid if a force of 1 dyne is required to maintain a velocity gradient of one cm per sec per cm between two layers, each of area one cm^2.

(ii) The S.I. unit of coefficient of viscosity is a poiseuille (PI) or decapoise.

One poiseuille is coefficient of viscosity of a liquid if a force of 1 newton is required to maintain a velocity gradient of one metre per sec per metre between two layers, each of area one m^2.

1 poiseuille = 10 poise. (Poiseuille is also called decapoise)

Dimensional Formula. We have $F = \eta A \frac{dv}{dx}$

Hence, $$\eta = \frac{Fdx}{Adv}$$

$$[\eta] = \frac{[MLT^{-2}][L]}{[L]^2[LT^{-1}]}$$

$$[\eta] = [ML^{-1}T^{-1}]$$

Stoke’s law

It was shown by Stokes that if a small sphere of radius r be moving with a uniform velocity υ (terminal velocity) through an infinite homogeneous and in compressible fluid of coefficient of viscosity η, it experiences a force F given by,
F=6πηrυ.
This relation is known as Stokes’ law.

Terminal velocity

Definition. The maximum velocity acquired by the body, falling freely in a viscous medium, is called terminal velocity.
Expression. Considering a small sphere of radius r of density p falling freely in a viscous medium (liquid) of viscosity q and density a (Fig). The forces acting on it are :

The weight of the sphere acting downward

$$W = \frac{4}{3}\pi r^3 \rho g.$$

The upward thrust = Weight of the liquid displaced by the sphere

$$B = \frac{4}{3}\pi r^3 \sigma g$$

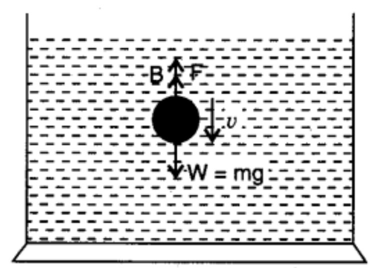

Fig. 12.02

The effective downward force,

$$= \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g.$$

$$= \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

Upward force of viscosity, $F = 6\pi\eta r v$

When the downward force is balanced by the upward force of viscosity, the body falls down with a constant velocity, called terminal velocity.

Hence, with terminal velocity,

$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

or Terminal velocity, $$v = \frac{2}{9}\frac{r^2 (\rho - \sigma) g}{\eta}$$

This is the required expression.