Real Number

			IIT	Foundatio	n Materi	al
0		SECTI	ON - I			
		Straight Ob	jective T	ype		
This sec (B), (C)	ction contains m , (D), out of whi	ultiple choice qu ich ONLY ONE i	estions. Each s correct. Ch	question has 4 oose the correc	1 choices (A) et option.	,
1.	A number n le What is the larg	ave the same rer gest possible valu	nainder whil 1e of n?	e dividing 5814	1, 5430, 595	58.
	(a) 96	(b) 12	(c) 48	(d) 192		
2.	Given that x_1 x_1 x_2 x_3	, X ₂ , X ₃ ,X ₁₅ X ₁₅	, x ₁₆ are pos	sitive real num	nber such th	nat
	$\frac{1}{X_2} = \frac{1}{X_3} = \frac{1}{X_4}.$	$\dots = \frac{13}{X_{16}}$				if
	$X_1 + X_2 + X_3 + X_3$	$x_4 = 20, x_5 + x_6 +$	$x_7 + x_8 = 320$	x_{13}^{3}	$+ X_{14} + X_{15} + 2$	X ₁₆
	(a) 20480	(b) 19680	(c) 40960	(d) 8192	20	
3.	What is the la	st digit in the fir	nite decimals	presentation	of the numt	ber
	$\frac{1}{5^{2003}}$?					
	(a) 2	(b) 4	(c) 6	(d) 8		
4.	If $S_n = 1 - 2$	2+3-4+5-6+	7-8+	(upto n	terms) th	en
	$S_{2002} - S_{2003} +$	S ₂₀₀₄ is				
	(a) 3005	(b) -1001	(c) -3005	(d) 1001	1	
5.	The remainder	on dividing 121	$n^{n} - 25^{n} + 190$	$00^n - (-4)^n$ by 2	2000 is	
	(a) 1000	(b) 1	(c) 0	(d) 8		
6.	The highest po	wer of 2003 that	divides 2003	3 is		
	(a) 10 (b) 9		(c) 101	(d) 1002	1	
7.	What is the smallest positive integer n such that $\sqrt{n} - \sqrt{n-1} < 0.01$?					
	(a) 2499	(b) 2500	(c) 2501	(d) 1000	00	
8.	The largest inte	eger n for which	$n^{200} < 5^{300}$ is			
	(a) 8	(b) 9	(c) 10	(d) 11		

9.	The unit digit of $3^{1001} \times 7^{1002} \times 13^{1003}$ is						
	(a) 1	(b) 3	(c) 5	(d) 7			
10.	The smallest po integer is	ositive integer x	for which 1260.	$x = N^3$ where N is an			
	(a) 1050	(b) 1260	(c) 1260^2	(d) 7350			
11.	When 2^{1000} is di	When 2^{1000} is divided by 13, the remainder is					
	(a) 1	(b) 2	(c) 3	(d) 7			
12.	The sum of all th	ne integers betwe	en 50 and 350 w	hich in 1 is			
	(a) 5684	(b) 5539	(c) 5208	(d) 4877			
13.	The number of s	solutions in positi	ve integers of 2x	+ 3y = 763 is			
	(a) 225	(b) 254	(c) 128	(d) 127			
14.	When 1000 single digit non-zero numbers are the unit place is 5. The maximum carry over in this case is						
	(a) 495	(b) 895	(c) 899	(d) 995			
15.	If $!_5 = 4 + 6 - 5$, $!_{12} = 11 + 13 - 12$ and $!_{23} = 22 + 24 - 23$ then what is the value $!40 + !41 + !42 + \dots + !50$?						
	(a) 505	(b) 495	(c) 455	(d) 465			
16.	$(a-1)^{2} + (b-2)^{2} + (c-3)^{2} + (d-4)^{2} = 0$ then $a \times b \times c \times d + 1$ is			$a \times b \times c \times d + 1$ is			
	(a) 0^2		(b) 10 ²				
	(c) 5^2		(d) $1^2 + 2^2 + 3^2$	$+4^{2}+1$			
17.	The length term of sequence (2, 5) (3, 7) (5, 11) (7, 13) (11, 17) is						
	(a) (23, 29)	(b) (19, 13)	(c) (20, 04)	(d) (29, 37)			
18.	The sum of the digits of the number $10^n - 1$ is 3798. The value of n is						
	(a) 431	(b) 673	(c) 422	(d) 501			
19.	Three people e different primes. is	each think of a The product of	number, which the three numbe	is the product of two rs which are thought of			
	(a) 120	(b) 12100	(c) 240	(d) 3000			

20.	The largest positive integer which cannot be written in the form $5m +$ when m and n are positive integer is			en in the form 5m + 7n	
	(a) 25		(b) 35		
	(c) grater than 2	100	(d) greater than	350	
21.	Let n be the lea number.	ast positive intege	r such that 1260r	n is the cube of a natural	
	(a) 1 < n < 50		(b) 50 < n < 10	00	
	(c) 100 < n <	1000	(d) 1000 < n <	10000	
22.	For how many	integers n is $\sqrt{9}$ -	$\overline{-(n+2)^2}$ a real	number?	
	(a) 3	(b) 5	(c) 7	(d) infinitely many	
23.	How many sol divisible by 99?	utions are there t	for (a, b) if 7ab7	3 is a five digit number	
	(a) 3	(b) 2	(c) 0	(d) 1	
24.	The number 1($07^{90} - 76^{90}$ is divi	sible by		
	(a) 61	(b) 62	(c) 64	(d) none of these	
25.	When $1^{2003} + 3$ remainder is	3^{3003} + + 200	3 ²⁰⁰³ is divided	l by 2004, then the	
	(a) 0	(b) 1	(c) 1002	(d) 2003	
26.	Each number f and their produ the end of this p	rom 1, 2, 3, uct is also written product is	. 100 (decimal sc in base 6. Then	ale) is written in base 6 the number of zeros at	
	(a) 24	(b) 48	(c) 18	(d) 97	
27.	The three last digits of 7^{9999} are				
	(a) 263	(b) 143	(c) 343	(d) 523	
28.	The natural nu order to form t of times the dig	mber 1, 2, 3, he single numbe it 1 occurs in this	upto 100 are r 1234 number is	e written in that natural 979899100 the number	
	(a) 11	(b) 12	(c) 20	(d) 21	
29.	If $n!=1\times 2\times (n+1)!=n \ltimes (n+1)!$	$3 \times \dots \times n$ has $n+1$ has six at the	as four zeros he end, then the v	at the end and value of n is	

(a) 20 (b) 24 (c) 25 (d) 2

30. Given $2x^2 + 3y^2 = 35$, then the number of ordered pairs (x, y) where (x, y) are integers satisfying the above equation is

(a) 2 (b) 4 (c) 8 (d) 16

SECTION - II Assertion - Reason Questions

This section contains certain number of questions. Each question contains STATEMENT-1 (Assertion) and STATEMENT - 2 (Reason). Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct. Choose the correct option.

31. STATEMENT-1: $17^8 + 13^7 - 5^8 + 2^7$ is divisible by 3

because

STATEMENT - 2: If sum of the digit of a number is divisible by 3 then the number is divisible by 3.

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

32. STATEMENT-1: If the number A3640548981270644B is divisible by 99 then A -= 9, B = 1

because

STATEMENT - 2: A six digit number of the form ABC. ABC is always divisible by 7 or 13.

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

- (c) Statement 1 is True, Statement 2 is False
- (d) Statement 1 is False, Statement 2 is True
- 33. STATEMENT-1: 2222⁵⁵⁵⁵ + 5555²²²² is divisible by 7

because

STATEMENT - 2: $x^n + y^n$ is divisible by x + y if n is odd positive integer.

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

- (c) Statement 1 is True, Statement 2 is False
- (d) Statement 1 is False, Statement 2 is True
- 34. STATEMENT-1: $5^{10} 3^{10}$ is divisible by 11

because

STATEMENT - 2: If P is a prime number G.C.D. of (a, p) = 1 then $a^{p-1} \equiv 1 \pmod{p}$.

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

35. STATEMENT-1: The number of +ve integers ** 3600 that co-prime to 3600 is 960

because

STATEMENT - 2: The number of integers $\leq n$ and co-prime to n is called Eula's function for n and denoted by (n)

$$(n)\left[1-\frac{1}{P_1}\right]\left[1-\frac{1}{P_2}\right]\dots\left[1-\frac{1}{P_r}\right]$$

6

Where P_1, P_2, \dots, P_r distinct prime numbers.

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

36. STATEMENT-1: The product of $n(n^2 - 1)$ is multiple of 6

because

STATEMENT - 2: Product n consecutive integers is divisible by n!.

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

37. STATEMENT-1: The number of divisible of 9504 is 48

because

STATEMENT - 2: If number $N = p^a \times q^b \times r^c$.

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Where p, q, r are primes then the number of factors of N are (a + 1) (b + 1) (c + 1) including 1 and itself.
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(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

38. STATEMENT-1: The sum of divisors of 2160 is 7440

because

STATEMENT - 2: The smallest integers with exactly divisors is 360.

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

39. STATEMENT-1: The highest power of 7 contains in 1000 ! is 164 because

STATEMENT - 2: The highest power of 3 contains in 1000 ! is 498

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

40. STATEMENT-1: The number of zeros at the end of 400 ! is 99 because

STATEMENT - 2: The number of factor at the end of n ! is = $\sum_{k=1}^{\alpha} \left[\frac{n!}{5^k} \right]$

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

Section - III

Linked Comprehension Type

This section contains paragraphs. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct. Choose the correct option.

Paragraph for Question Nos. 41 to 43

If a and b are two integers and n is a positive integer, then a is said to be congruent to b modulo m is m |a - b| in notation form we express it as $a \equiv b \pmod{m}$ or $a - b \equiv 0 \pmod{m}$.

41.	The remai	nder in the divisic	on of $\frac{20}{2}$ by 7	
	(a) 4	(b) 2	(c) 0	(d) 3
42.	The remair	nder in the division	n of $\frac{40}{5}$ by 23	
	(a) 4	(b) 2	(c) 0	(d) 3
43.	The remair	nder when $\frac{57}{2}$ is a	livided by 8	
	(a) 4	(b) 2	(c) 0	(d) 3

Paragraph for Question Nos. 44 to 46

Let $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ $a + c \equiv b + d \pmod{m}$ $a - c \equiv b - d \pmod{m}$ $ac \equiv bd \pmod{m}$ $pa + qc \equiv pb + qd \pmod{m}$ $an \equiv bn \pmod{m}$ for every positive integer n.



Paragraph for Question Nos. 47 to 49

If P is a prime number G.C.D. of (a, b) = 1Then $a^{P-1} \ge \pmod{p}$ If P is prime then $(a+b)^{P} \circ (a^{P}+b^{P}) \pmod{p}$ If P is prime then $(P-1)! + 1 \circ O \pmod{p}$ The remainder when the number $10^{10} + 10^{10^2} + \dots + 10^{(10^{10})}$ is divided 47. by 7 is (a) 4 (b) 5 (c) 6 (d) 7 The remainder when $13^{73} + 14^3$ is divided by 11 is 48. (b) 4 (a) 2 (c) 5 (d) 6 The remainder 2000^{1000} is divided by 13 is 49.

(a) 2 (b) 4 (c) 3 (d) 7

Paragraph for Question Nos. 50 to 52

The expression $a^n - b^n$ is always divisible by a – b if n is any positive integer.

The expression $a^n - b^n$ is divisible by a + b if n is any even positive integer.

The expression $a^n + b^n$ is exactly divisible by a + b if n is odd, but not if n be even.

50.	$4^{4n} - 5^{3n}$ is divisible by				
	(a) 131	(b) 15	(c) 8	(d) 16	
51.	$2^{4n} - 1$ is divisible by				
	(a) 131	(b) 15	(c) 8	(d) 16	
52.	$3^{2n} + 7$ is divisible by				
	(a) 131	(b) 15	(c) 8	(d) 16	

Paragraph for Question Nos. 53 to 55

If is any real number, then the largest integer, which does not exceed x is called the integer part of x, the integer [x] is after called the bracket function/step sunction.

53. For a real number x, [x] denotes the integer part of x the value of

$$\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} + \frac{1}{100} \\ \frac{1}{2} + \begin{bmatrix} \frac{1}{2} + \frac{2}{100} \\ \frac{1}{2} + \frac{2}{100} \end{bmatrix} + \dots + \begin{bmatrix} \frac{1}{2} + \frac{99}{100} \\ \frac{1}{2} + \frac{99}{100} \end{bmatrix} \text{ is}$$
(a) 50 (b) 47 (c) 45 (d) 52
54. The number of zeros at the end of 400 !
(a) 67 (b) 99 (c) 78 (d) 85
55. If 'n' is any positive integer then $\begin{bmatrix} \frac{n+1}{2} \\ \frac{1}{2} \end{bmatrix} + \begin{bmatrix} \frac{n+2}{4} \\ \frac{1}{8} \end{bmatrix} + \begin{bmatrix} \frac{n+4}{8} \\ \frac{1}{8} \end{bmatrix} + \dots$ is
(a) n (b) $\frac{n}{2}$ (c) $\frac{n}{4}$ (d) $\frac{n}{8}$

SECTION - IV Matrix - Match Type

This section contains Matrix-Match type questions. Each question contains statements given in two columns which have to be matched. Statements (a, b, c, d) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are a-p, a-s, b-q, b-r, c-p, c-q and d-s, then the correctly bubbled 4 x 4 matrix should be as follows:

	р	q	r	s
A	۲	9	\bigcirc	\odot
B	P	9	\odot	\odot
С	Ø		\bigcirc	\bigcirc
D	P	(9)	\bigcirc	۲

(a) $\frac{1}{2} - \frac{2}{4} + \frac{3}{8} - \frac{4}{16} + \frac{5}{32} + \dots + \frac{n}{2^n} (-1)^{n+1} + \dots$ (p) $\frac{2}{9}$ (b) $1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \dots + \frac{1}{16}$ (q) 76 (1+2+3+....+16)(c) $2 2^2 + 3 2^3 + 4 2^4 + 513 2^{513}$ (r) 2^{613} (s) $\frac{2007}{4014}$ (d) $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\dots\left(1-\frac{1}{2007^2}\right)$

(a) G.C.D. of $2^{100} - 1, 2^{120} - 1$	(p) $2^{20} - 1$
(b) G.C.D. of (2n + 13, n + 7)	(q) 1

12

56.

57.

Column I

1.....

Column II

(c) If
$$\frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$$
 (r) (1, 5, 2)

(d) $2^{11} + 2^8 + 2^n$ is a perfect square then (s) 12 the value of n

58.

Column I

Column II

(a) $n^2 + n + 41$ is a perfect square (b) $2^{200} - 2^{192} \cdot 31 + 2^n$ is perfect square (c) $S_n = 1 - 2 + 3 - 4 + 5 - 6 + 7 \dots$ (c) $S_{n} = 1 - 2 + 3 - 4 + 5 - 6 + 7 \dots$ (c) $S_{2002} - S_{2003}$ (c) $S_{2002} - S_{2003}$

(d) If
$$3^{\frac{a}{b}}, 3^{\frac{b}{a}} = 3^{\frac{5}{2}}$$
 the $\left|9^{\frac{a}{b}} - \frac{b}{a}\right|$ is (s) 27

59.

Column I

Column II

(a) Unit digit of 777^{777} (p) 7(b) Unit digit of 23^{562} (q) 9(c) Unit of 2^{2^n} , n > 2(r) 6(d) Unit digit of $333^{333} + 777^{777}$ (s) 0

60.

Column I

Column II

(a) 3124a6 is divisible by 6 then a is(p) 2, 5, 8(b) 71a is divisible by 4 then a is(q) 2, 6(c) 3A55B is divisible by 792 then A is(r) 2(d) A356 is divisible by 99 then A is(s) 4