6.6 NUCLEAR REACTIONS

6.249 Initial momentum of the α particle is $\sqrt{2\,m\,T_\alpha}\;\hat{i}$ (where $\hat{i}$ is a unit vector in the incident direction). Final momenta are respectively $\vec{p}_\alpha$ and $\vec{p}_{Li}$. Conservation of momentum reads

$$\vec{p}_\alpha + \vec{p}_{Li} = \sqrt{2\,m\,T_\alpha}\;\hat{i}$$

Squaring $$p_\alpha^2 + p_{Li}^2 + 2\,p_\alpha\,p_{Li}\cos\Theta = 2\,m\,T_\alpha \tag{1}$$

where Θ is the angle between $\vec{p}_\alpha$ and $\vec{p}_{Li}$.

Also by energy conservation $$\frac{p_\alpha^2}{2\,m} + \frac{p_{L_i}^2}{2\,M} = T_\alpha$$

(m & M are respectively the masses of α particle and Li^6 .) So

$$p_\alpha^2 + \frac{m}{M}p_{Li}^2 = 2\,m\,T_\alpha \tag{2}$$

Substracting (2) from (1) we see that

$$p_{Li}\left[\left(1 - \frac{m}{M}\right)p_{Li} + 2\,p_\alpha\cos\Theta\right] = 0$$

Thus if $$p_{Li} \neq 0$$

$$p_\alpha = -\frac{1}{2}\left(1 - \frac{m}{M}\right)p_{Li}\sec\Theta\,.$$

Since p_α , p_{Li} are both positive number (being magnitudes of vectors) we must have

$$-1 \le \cos\Theta < 0 \quad \text{if} \quad m < M\,.$$

This being understood, we write

$$\frac{p_{Li}^2}{2\,M}\left[1 + \frac{M}{4\,m}\left(1 - \frac{m}{M}\right)^2 \sec^2\Theta\right] = T_\alpha$$

Hence the recoil energy of the L_i nucleus is

$$\frac{p_{Li}^2}{2\,M} = \frac{T_\alpha}{1 + \dfrac{(M-m)^2}{4\,m\,M}\sec^2\Theta}$$

As we pointed out above $\Theta \neq 60°$. If we take $\Theta = 120°$, we get

recoil energy of $Li = 6\,\text{MeV}$

6.250 (a) In a head on collision

$$\sqrt{2\,m\,T} = p_d + p_n$$

$$T = \frac{p_d^2}{2\,M} + \frac{p_n^2}{2\,m}$$

Where p_d and p_n are the momenta of deuteron and neutron after the collision. Squaring

$$p_d^2 + p_n^2 + 2\,p_d\,p_n = 2\,m\,T$$

$$p_n^2 + \frac{m}{M} p_d^2 = 2mT$$

or since $p_d \neq 0$ in a head on collisions

$$p_n = -\frac{1}{2}\left(1 - \frac{m}{M}\right) p_d .$$

Going back to energy conservation

$$\frac{p_d^2}{2M}\left[1 + \frac{M}{4m}\left(1 - \frac{m}{M}\right)^2\right] = T$$

So
$$\frac{p_d^2}{2M} = \frac{4mM}{(m+M)^2} T$$

This is the energy lost by neutron. So the fraction of energy lost is

$$\eta = \frac{4mM}{(m+M)^2} = \frac{8}{9}$$

(b) In this case neutron is scattered by 90°. Then we have from the diagram

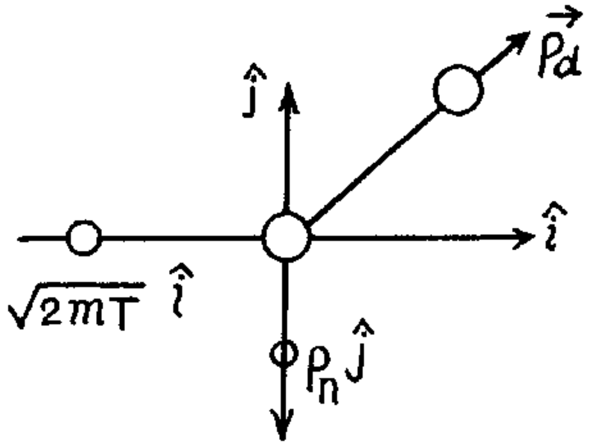
$$\vec{p_d} = p_n \hat{j} + \sqrt{2mT}\,\hat{i}$$

Then by energy conservation

$$\frac{p_n^2 + 2mT}{2M} + \frac{p_n^2}{2m} = T$$

or
$$\frac{p_n^2}{2m}\left(1 + \frac{m}{M}\right) = T\left(1 - \frac{m}{M}\right)$$

or
$$\frac{p_n^2}{2m} = \frac{M-m}{M+m} \cdot T$$



The energy lost by neutron in then

$$T - \frac{p_n^2}{2m} = \frac{2m}{M+m} T$$

or fraction of energy lost is $\eta = \dfrac{2m}{M+m} = \dfrac{2}{3}$

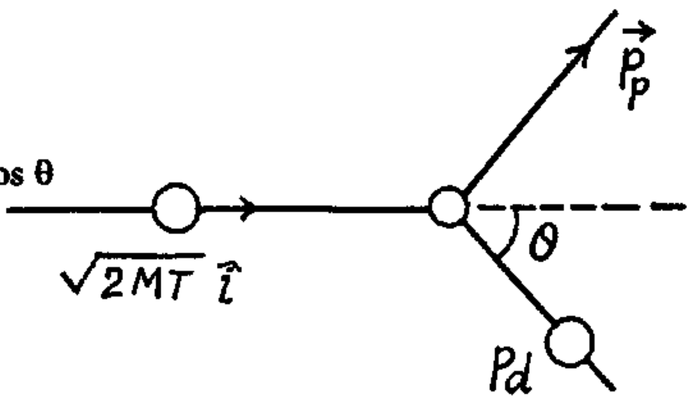
6.251 From conservation of momentum

$$\sqrt{2MT}\,\hat{i} = \vec{p_d} + \vec{p_p}$$

or $p_p^2 = 2MT + p_d^2 - 2\sqrt{2MT}\, p_d \cos\theta$

From energy conservation

$$T = \frac{p_d^2}{2M} + \frac{p_p^2}{2m}$$



(M = mass of denteron, m = mass of proton)

So
$$p_p^2 = 2\,m\,T - \frac{m}{M}p_d^2\,.$$

Hence
$$p_d^2\left(1+\frac{m}{M}\right) - 2\sqrt{2MT}\;p_d\cos\theta + 2\,(M-m)\,T = 0$$

For real roots
$$4\,(2\,M\,T)\cos^2\theta - 4\times 2\,(M-m)\,T\left(1+\frac{m}{M}\right) \ge 0$$
$$\cos^2\theta \ge \left(1-\frac{m^2}{M^2}\right)$$

Hence
$$\sin^2\theta \le \frac{m^2}{M^2}$$

i.e.
$$\theta \le \sin^{-1}\frac{m}{M}$$

For deuteron-proton scaltering θ_{max} = $30°$.

6.252 This problem has a misprint. Actually the radius R of a nucleus is given by
$$R = 1{\cdot}3\ \sqrt[3]{A}\ fm$$
where
$$fm = 10^{-15}\,\text{m}\,.$$
Then the number of nucleous per unit volume is
$$\frac{A}{\frac{4\pi}{3}R^3} = \frac{3}{4\pi}\times(1{\cdot}3)^{-3}\times 10^{+39}\,\text{cm}^{-3} = 1{\cdot}09\times 10^{38}\ \text{per cc}$$

The corresponding mass density is

$(1{\cdot}09\times 10^{38}\times$ mass of a nucleon) per cc = $1{\cdot}82\times 10^{11}$ kg/cc

6.253 (a) The particle x must carry two nucleons and a unit of positive charge. The reaction is
$$B^{10}\,(d\,,\alpha)\,B_e^8$$

(b) The particle x must contain a proton in addition to the constituents of O^{17}. Thus the reaction is
$$O^{17}\,(d\,,n)\,F^{18}$$

(c) The particle x must carry nucleon number 4 and two units of +ve charge. Thus the particle must be $x = \alpha$ and the reaction is
$$Na^{23}\,(p\,,\alpha)\,Ne^{20}$$

(d) The particle x must carry mass number 37 and have one unit less of positive charge. Thus $x = Cl^{37}$ and the reaction is
$$Cl^{37}\,(p\,,n)\,Ar^{37}$$

6.254 From the basic formula

$$E_b = Z\,m_H + (A - Z)\,m_n - M$$

We define

$$\Delta_H = m_H - 1 \text{ amu}$$

$$\Delta_n = m_n - 1 \text{ amu}$$

$$\Delta = M - A \text{ amu}$$

Then clearly $E_b = Z\,\Delta_H + (A - Z)\,\Delta_n - \Delta$

6.255 The mass number of the given nucleus must be

$$27 \Big/ \left(\frac{3}{2}\right)^3 = 8$$

Thus the nucleus is Be^8. Then the binding energy is

$$E_b = 4 \times 0{\cdot}00867 + 4 + \times 0{\cdot}00783 - 0{\cdot}00531 \text{ amu}$$

$$= 0{\cdot}06069 \text{ amu} = 56{\cdot}5 \text{ MeV}$$

On using 1 amu = 931 MeV.

6.256 (a) Total binding energy of the O^{16} nucleus is

$$E_b = 8 \times {\cdot}00867 + 8 \times {\cdot}00783 + 0{\cdot}00509 \text{ amu}$$

$$= 0{\cdot}13709 \text{ amu} = 127{\cdot}6 \text{ MeV}$$

So B.E. per nucleon is 7·98 Mev/nucleon

(b) B.E. of neutron in B^{11}nucleus

$$= \text{B.E. of } B^{11} - \text{B.E. of } B^{10}$$

(since on removing a neutron from B^{11} we get B^{10})

$$= \Delta_n - \Delta_{B_{11}} + \Delta_{B_{10}} = {\cdot}00867 - {\cdot}00930 + {\cdot}01294$$

$$= 0{\cdot}01231 \text{ amu} = 11{\cdot}46 \text{ MeV}$$

B.E. of (an α–particle in B^{11})

$$= \text{B.E. of } B^1 - \text{B.E. of } Li^7 - \text{B.E. of } \alpha$$

(since on removing an α from B^{11} we get Li^7)

$$= -\Delta_{B_{11}} + \Delta_{Li_7} + \Delta_\alpha$$

$$= -0{\cdot}00930 + 0{\cdot}01601 + 0{\cdot}00260$$

$$= 0{\cdot}00931 \text{ amu} = 8{\cdot}67 \text{ MeV}$$

(c) This energy is

[B.E. of O^{16} + 4 (B.E. of α particles)]

$$= -\Delta_{O^{16}} + 4\,\Delta_\alpha$$

$$= 4 \times 0{\cdot}00260 + 0{\cdot}00509$$

$$= 0{\cdot}01549 \text{ amu} = 14{\cdot}42 \text{ MeV}$$

6.257 B.E. of a neutron in B^{11} − B.E. of a proton in B^{11}

$$= (\Delta_n - \Delta_{B^{11}} + \Delta_{B^{10}}) - (\Delta_p - \Delta_{B^{11}} + \Delta_{B_e^{10}})$$

$$= \Delta_n - \Delta_p + \Delta_{B^{10}} - \Delta_{B_e^{10}} = 0{\cdot}00867 - 0{\cdot}00783$$

$$+ 0{\cdot}01294 - 0{\cdot}01354 = 0{\cdot}00024 \text{ amu} = 0{\cdot}223 \text{ MeV}$$

The difference in binding energy is essentially due to the coulomb repulsion between the proton and the residual nucleus Be^{10} which together constitute B^{11}.

6.258 Required energy is simply the difference in total binding energies-

$$= \text{B.E. of } Ne^{20} - 2\,(\text{B.E. of } He^4) - \text{B.E. of } C^{12}$$

$$= 20\,\varepsilon_{Ne} - 8\,\varepsilon_\alpha - 12\,\varepsilon_C$$

(ε is binding energy per unit nucleon.)

Substitution gives $\quad 11{\cdot}88 \text{ MeV}$.

6.259 (a) We have for Li^8

$$41{\cdot}3 \text{ MeV} = 0{\cdot}044361 \text{ amu} = 3\,\Delta_H + 5\,\Delta_n - \Delta$$

Hence $\quad \Delta = 3 \times 0{\cdot}00783 + 5 \times 0{\cdot}00867 - 0{\cdot}09436 = 0{\cdot}02248 \text{ amu}$

(b) For C^{10}

$$10 \times 6{\cdot}04 = 60{\cdot}4 \text{ MeV}$$
$$= 0{\cdot}06488 \text{ amu}$$
$$= 6\,\Delta_H + 4\,\Delta_n - \Delta$$

Hence $\quad \Delta = 6 \times 0{\cdot}00783 + 4 \times 0{\cdot}00867 - 0{\cdot}06488 = 0{\cdot}01678 \text{ amu}$

Hence the mass of C^{10} is $\quad 10{\cdot}01678 \text{ amu}$

6.260 Suppose M_1, M_2, M_3, M_4 are the rest masses of the nuclei A_1, A_2, A_3 and A_4 perticipating in the reaction

$$A_1 + A_2 \rightarrow A_3 + A_4 + Q$$

Here Q is the energy released. Then by conservation of energy.

$$Q = c^2 (M_1 + M_2 - M_3 - M_4)$$

Now $\quad M_1 c^2 = c^2 (Z_1 m_H + (A_1 - Z) m_n) - E_1$ etc. and

$$Z_1 + Z_2 = Z_3 + Z_4 \text{(conservation of change)}$$

$$A_1 + A_2 = A_3 + A_4 \text{ (conservation of heavy particles)}$$

Hence $\quad Q = (E_3 + E_4) - (E_1 + E_2)$

6.261 (a) the energy liberated in the fission of 1 kg of U^{235} is

$$\frac{1000}{235} \times 6{\cdot}023 \times 10^{23} \times 200 \text{ MeV} = 8{\cdot}21 \times 10^{10} \text{ kJ}$$

The mass of coal with equivalent calorific value is

$$\frac{8{\cdot}21 \times 10^{10}}{30000} \text{ kg} = 2{\cdot}74 \times 10^6 \text{ kg}$$

(b) The required mass is

$$\frac{30\times10^9\times4{\cdot}1\times10^3}{200\times1{\cdot}602\times10^{-13}\times6{\cdot}023\times10^{23}}\times\frac{235}{1000}\,\text{kg} = 1{\cdot}49\text{ kg}$$

6.262 The reaction is (in effect).

$$H^2+H^2\rightarrow He^4+Q$$

Then
$$Q = 2\,\Delta_{H^2}-\Delta_{He^4}+Q$$
$$= 0{\cdot}02820-0{\cdot}00260$$
$$= 0{\cdot}02560\text{ amu} = 23{\cdot}8\text{ MeV}$$

Hence the energy released in 1 gm of He^4 is

$$\frac{6{\cdot}023\times10^{23}}{4}\times23{\cdot}8\times1\,6{\cdot}02\times10^{-13}\text{ Joule} = 5{\cdot}75\times10^8\text{ kJ}$$

This energy can be derived from

$$\frac{5{\cdot}75\times10^8}{30000}\text{kg} = 1{\cdot}9\times10^4\text{ kg of Coal.}$$

6.263 The energy released in the reaction

$$Li^6+H^2\rightarrow2\,He^4$$

is
$$\Delta_{Li^6}+\Delta_{H^2}-2\,\Delta_{He^4}$$
$$= 0{\cdot}01513+0{\cdot}01410-2\times0{\cdot}00\,260\text{ amu}$$
$$= 0{\cdot}02403\text{ amu} = 22{\cdot}37\text{ MeV}$$

(This result for change in B.E. is correct because the contribution of Δ_n & Δ_H cancels out by conservation law for protons & neutrons.)-

Energy per nucleon is then

$$\frac{22{\cdot}37}{8} = 2{\cdot}796\text{ MeV/nucleon}.$$

This should be compared with the value $\frac{200}{235} = 0{\cdot}85$ MeV/nucleon

6.264 The energy of reaction

$$Li^7+p\rightarrow2\,He^4$$

is,
$$2\times\text{B.E. of }He^4-\text{B.E. of }Li^7$$
$$= 8\,\varepsilon_\alpha-7\,\varepsilon_{Li} = 8\times7{\cdot}06-7\times5{\cdot}60 = 17{\cdot}3\text{ MeV}$$

6.265 The reaction is $N^{14}(\alpha,p)\,O^{17}$.

It is given that (in the Lab frame where N^{14} is at rest) $T_\alpha = 4{\cdot}0$ MeV.

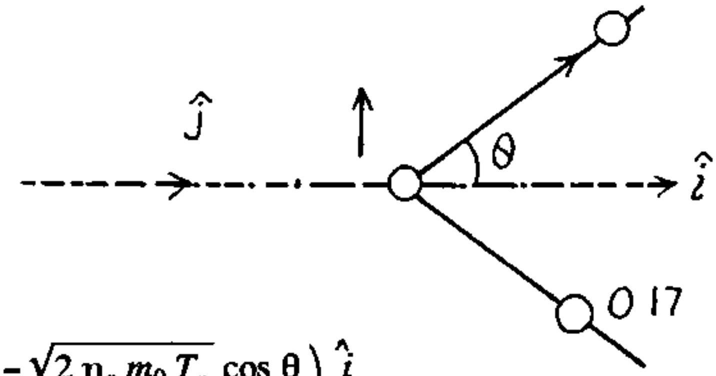
The momentum of incident α particle is

$$\sqrt{2\,m_\alpha T_\alpha}\,\hat{i} = \sqrt{2\,\eta_\alpha m_0 T_\alpha}\,\hat{i}$$

The momentum of outgoing proton is

$$\sqrt{2\, m_p\, T_p}\,(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$= \sqrt{2\,\eta_p\, m_0\, T_p}\,(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$



where $\eta_p = \frac{m_p}{m_0}$, $\eta_\alpha = \frac{m_\alpha}{m_0}$,

and m_0 is the mass of O^{17}.

The momentum of O^{17} is

$$\left(\sqrt{2\,\eta_\alpha\, m_0\, T_\alpha} - \sqrt{2\,\eta_p\, m_0\, T_p}\,\cos\theta\right)\hat{i} - \sqrt{2\, m_0\,\eta_p\, T_p}\,\sin\theta\,\hat{j}$$

By energy conservation (conservation of energy including rest mass energy and kinetic energy)

$$M_{14}\, c^2 + M_\alpha\, c^2 + T_\alpha$$

$$= M_p\, c^2 + T_p + M_{17}\, c^2$$

$$+ \left[\left(\sqrt{\eta_\alpha\, T_\alpha} - \sqrt{\eta_p\, T_p}\,\cos\theta\right)^2 + \eta_p\, T_p \sin^2\theta + \eta_p\, T_p \sin^2\theta\right]$$

Hence by definition of the Q of reaction

$$Q = M_{14}\, c^2 + M_\alpha\, c^2 - M_p\, c^2 - M_{17}\, c^2$$

$$= T_p + \eta_\alpha\, T_\alpha + \eta_p\, T_p - 2\sqrt{\eta_p\,\eta_\alpha\, T_\alpha\, T_p} \times \cos\theta - T_\alpha$$

$$= (1 + \eta_p)\, T_p + T_\alpha (1 - \eta_\alpha)$$

$$- 2\sqrt{\eta_p\,\eta_\alpha\, T_\alpha\, T_p}\,\cos\theta = -1{\cdot}19\ \text{MeV}$$

6.266 (a) The reaction is $Li^7\,(p\,,n)\,Be^7$ and the energy of reaction is

$$Q = (M_{Be^7} + M_{Li^7})\, c^2 + (M_p - M_n)\, c^2$$

$$= (\Delta_{Li_7} - \Delta_{Be^7})\, c^2 + \Delta_p - \Delta_n$$

$$= [0{\cdot}01601 + 0{\cdot}00783 - 0{\cdot}01693 - 0{\cdot}00867]\ \text{amu} \times c^2$$

$$= -1{\cdot}64\ \text{MeV}$$

(b) The reaction is $Be^9\,(n\,,\gamma)\,Be^{10}$.

Mass of γ is taken zero. Then

$$Q = (M_{Be^9} + M_n - M_{Be^{10}})\, c^2$$

$$= (\Delta_{Be^9} + \Delta_n - \Delta_{Be^{10}})\, c^2$$

$$= (0{\cdot}01219 + 0{\cdot}00867 - 0{\cdot}01354)\ \text{amu} \times c^2$$

$$= 6{\cdot}81\ \text{MeV}$$

(c) The reaction is $Li^7(\alpha, n)B^{10}$. The energy is

$$Q = (\Delta_{Li^7} + \Delta_\alpha - \Delta_n - \Delta_{B^{10}})c^2$$
$$= (0{\cdot}01601 + 0{\cdot}00260 - 0{\cdot}00867 - {\cdot}01294)\text{ amu} \times c^2$$
$$= -2{\cdot}79\text{ MeV}$$

(d) The reaction is $O^{16}(d, \alpha)N^{14}$. The energy of reaction is

$$Q = (\Delta_{O^{16}} + \Delta_d - \Delta_\alpha - \Delta_N^{14})c^2$$
$$= (-0{\cdot}00509 + 0{\cdot}01410 - 0{\cdot}00260 - 0{\cdot}00307)\text{ amu} \times c^2$$
$$= 3{\cdot}11\text{ MeV}$$

6.267 The reaction is $B^{10}(n, \alpha)Li^7$. The energy of the reaction is

$$Q = (\Delta_{B^{10}} + \Delta_n - \Delta_\alpha - \Delta_{Li^7})c^2$$
$$= (0{\cdot}01294 + 0{\cdot}00867 - 0{\cdot}00260 - 0{\cdot}01601)\text{ amu} \times c^2$$
$$= 2{\cdot}79\text{ MeV}$$

Since the incident neutron is very slow and B^{10} is stationary, the final total momentum must also be zero. So the reaction products must emerge in opposite directions. If their speeds are, repectively, v_α and v_{Li}

then $$4v_\alpha = 7v_{Li}$$

and $$\frac{1}{2}(4v_\alpha^2 + 7v_{Li}^2) \times 1{\cdot}672 \times 10^{-24} = 2{\cdot}79 \times 1{\cdot}602 \times 10^{-6}$$

So $$\frac{1}{2} \times 4v_\alpha^2\left(1 + \frac{4}{7}\right) = 2{\cdot}70 \times 10^{18}\text{ cm}^2/\text{s}^2$$

or $$v_\alpha = 9{\cdot}27 \times 10^6\text{ m/s}$$

Then $$v_{Li} = 5{\cdot}3 \times 10^6\text{ m/s}$$

6.268 Q of this reaction $(Li^7(p, n)Be^7)$ was calculated in problem 266 (a). If is $-1{\cdot}64$ MeV. We have by conservation of momentum and energy $p_p = p_{Be}$ (since initial Li and final neutron are both at rest)

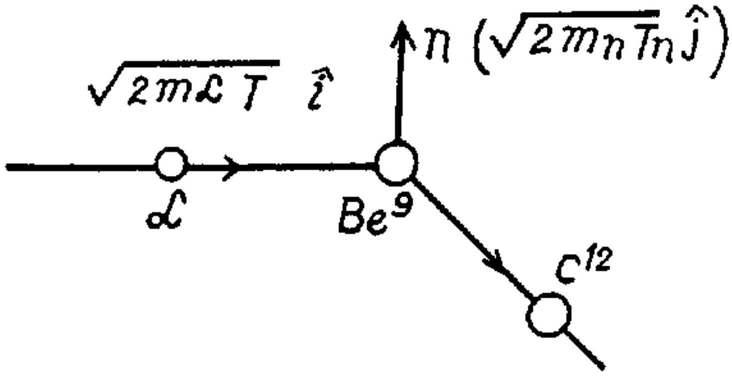
$$\frac{p_p^2}{2m_p} = \frac{p_{Be}^2}{2m_{Li}} + 1{\cdot}64$$

Then $$\frac{p_p^2}{2m_p}\left(1 - \frac{m_p}{m_{Be}}\right) = 1{\cdot}64$$

Hence $$T_p = \frac{p_p^2}{2m_p} = \frac{7}{6} \times 1{\cdot}64\text{ MeV} = 1{\cdot}91\text{ MeV}$$

6.269 It is understood that Be^9 is initially at rest. The moment of the outgoing neutron is $\sqrt{2m_n T_n}\,\hat{j}$. The momenturm of C^{12} is

$\sqrt{2\, m_\alpha T}\ \hat{i} - \sqrt{2\, m_n T_n}\ \hat{j}$



Then by energy conservation

$$T + Q = T_n + \frac{2\, m_\alpha T + 2\, m_n T_n}{2\, m_{ci}}$$

(m_c is the mass of C^{12})

Thus $$T_n = \frac{m_c (T + Q) - m_\alpha T}{m_c + m_n}$$

$$= \frac{(m_c - m_\alpha) T + m_c Q}{m_c + m_n} = \frac{Q + \left(1 - \frac{m_\alpha}{m_c}\right) T}{1 + \frac{m_n}{m_c}} = 8{\cdot}52 \text{ MeV}$$

6.270 The Q value of the reaction $Li^7 (p, \alpha) He^4$ is

$$Q = (\Delta_{Li^7} + \Delta_H - 2\, \Delta_{He^4})\, c^2$$

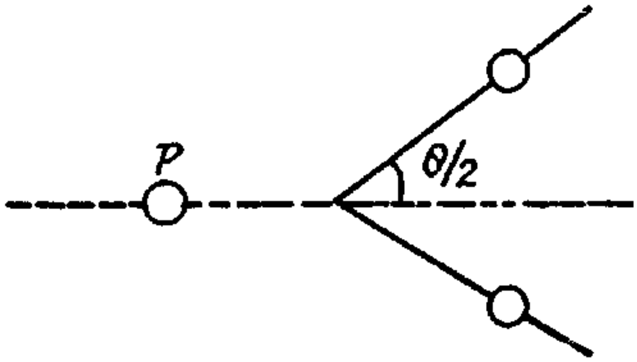
$$= (0{\cdot}01601 + 0{\cdot}00783 - 0{\cdot}00520) \text{ amu} \times c^2$$

$$= 0{\cdot}01864 \text{ amu} \times c^2 = 17{\cdot}35 \text{ MeV}$$

Since the direction of He^4 nuclei is symmetrical, their momenta must also be equal. Let T be the K.E. of each He^4. Then

$$p_p = 2\sqrt{2\, m_{He} T} \cos\frac{\theta}{2}$$

(p_p is the momentum of proton). Also



$$\frac{p_p^2}{2\, m_p} + Q = 2\, T = T_p + Q$$

Hence $$T_p + Q = 2\, \frac{p_p^2 \sec^2 \frac{\theta}{2}}{8\, m_{He}}$$

$$= T_p \frac{m_p}{2\, m_{He}} \sec^2 \frac{\theta}{2}$$

Hence $$\cos\frac{\theta}{2} = \sqrt{\frac{m_p}{2\, m_{He}} \frac{T_p}{T_p + Q}}$$

Substitution gives

$$\theta = 170{\cdot}53°$$

Also $$T = \frac{1}{2}(T_p + Q) = 9{\cdot}18 \text{ MeV}.$$

6.271 Energy required is minimum when the reaction products all move in the direction of the incident particle with the same velocity (so that the combination is at rest in the centre of mass frame). We then have

$$\sqrt{2\,m\,T_{th}} = (m+M)\,v$$

(Total mass is constant in the nonrelativistic limit).

$$T_{th} - |Q| = \frac{1}{2}(m+M)\,v^2 = \frac{m\,T_{th}}{m+M}$$

or
$$T_{th}\frac{M}{m+M} = |Q|$$

Hence
$$T_{th} = \left(1+\frac{m}{M}\right)|Q|$$

6.272 The result of the previous problem applies and we find that energy required to split a deuteron is
$$T \geq \left(1+\frac{M_p}{M_d}\right)E_b = 3{\cdot}3\text{ MeV}$$

6.273 Since the reaction $Li^7(p,n)Be^7$ ($Q = -1{\cdot}65$ MeV) is initiated, the incident proton energy must be

$$\geq \left(1+\frac{M_p}{M_{Li}}\right)\times 1{\cdot}65 = 1{\cdot}89\text{ MeV}$$

since the reaction $Be^9(p,n)B^9$ ($Q = -1{\cdot}85$ MeV) is not initiated,

$$T \leq \left(1+\frac{M_p}{M_{Ve}}\right)\times 1{\cdot}85 = 2{\cdot}06\text{ MeV} \quad \text{Thus} \quad 1{\cdot}89\text{ MeV} \leq T_p \leq 2{\cdot}06\text{ MeV}$$

6.274 We have $4{\cdot}0 = \left(1+\dfrac{m_n}{M_{B^{11}}}\right)|Q|$

or
$$Q = -\frac{11}{12}\times 4\text{ MeV} = -3{\cdot}67\text{ MeV}$$

6.275 The Q of the reaction $Li^7(p,n)Be^7$ was calculated in problem 266 (a). It is $-1{\cdot}64$ MeV Hence, the threshold K.E. of protons for initiating this reaction is

$$T_{th} = \left(1+\frac{m_p}{m_{Li}}\right)|Q| = \frac{8}{7}\times 1{\cdot}64 = 1{\cdot}87\text{ MeV}$$

For the reaction $Li^7(p,d)Li^6$

we find
$$\begin{aligned} Q &= (\Delta_{Li^7} + \Delta_M - \Delta_d - \Delta_{Li^6})\,c^2 \\ &= (0{\cdot}01601 + 0{\cdot}00783 - 0{\cdot}01410 - 0{\cdot}01513)\text{ amu}\times c^2 \\ &= -5{\cdot}02\text{ MeV} \end{aligned}$$

The threshold proton energy for initiating this reaction is

$$T_{th} = \left(1+\frac{m_p}{m_{Li^7}}\right)\times|Q| = 5{\cdot}73\text{ MeV}$$

6.276 The Q of $Li^7(\alpha, n) B^{10}$ was calculated in problem 266 (c). It is $Q = 2{\cdot}79$ MeV

Then the threshold energy of α-particle is

$$T_{th} = \left(1 + \frac{m_\alpha}{m_{Li}}\right)|Q| = \left(1 + \frac{4}{7}\right) 2{\cdot}79 = 4{\cdot}38 \text{MeV}$$

The velocity of B^{10} in this case is simply the volocity of centre of mass :-

$$v = \frac{\sqrt{2\, m_\alpha T_{th}}}{m_\alpha + m_{Li}} = \frac{1}{1 + \dfrac{m_{Li}}{m_\alpha}} \sqrt{\frac{2\, T_{th}}{m_\alpha}}$$

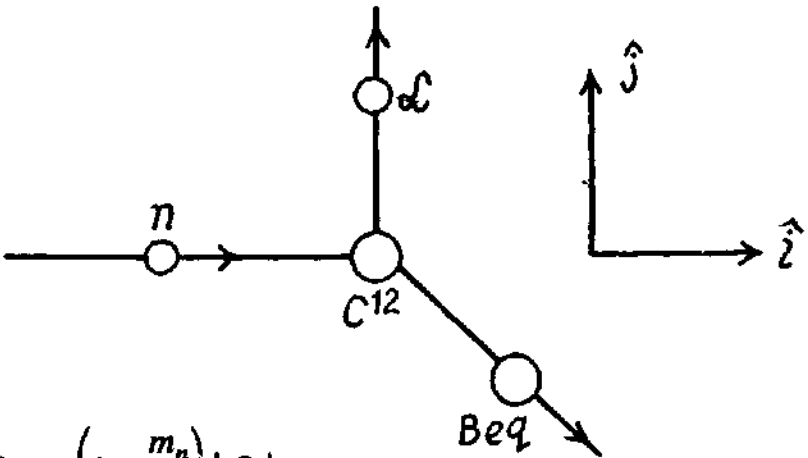
This is because both B^{10} and n are at rest in the CM frame at theshold.

Substituting the values of various quantities

we get $$v = 5{\cdot}27 \times 10^6 \text{ m/s}$$

6.277 The momentum of incident neutron is $\sqrt{2\, m_n T}\, \hat{i}$, that of α particle is $\sqrt{2\, m_\alpha T_\alpha}\, \hat{j}$ and of Be^9 is

$-\sqrt{2\, m_\alpha T_\alpha}\, \hat{j} + \sqrt{2\, m_n T}\, \hat{i}$



By conservation of energy

$$T = T_\alpha + \frac{m_\alpha T_\alpha + m_n T}{M} + |Q|$$

(M is the mass of Be^9). Thus

$$T_\alpha = \left[T\left(1 - \frac{m_n}{M}\right) - |Q|\right] \frac{M}{M + m_\alpha}.$$

Using $$T_{th} = \left(1 + \frac{m_n}{M}\right)|Q|$$

we get $$T_\alpha = \frac{M}{M + m_\alpha}\left[\left(1 - \frac{m_n}{M}\right) T - \frac{T_{th}}{1 + \dfrac{m_n}{M}}\right]$$

M' is the mass of C^{12} nucleus.

or $$T_\alpha = \frac{1}{M + m_\alpha}\left[(M - m_n)\, T - \frac{M M'}{M' + m_n} T_{th}\right] = 2{\cdot}21 \text{ MeV}$$

6.278 The formula of problem 6.271 does not apply here because the photon is always reletivistic. At threshold, the energy of the photon E_γ implies a momentum $\dfrac{E_\gamma}{c}$. The velocity of centre of mass with respect to the rest frame of initial H^2 is

$$\frac{E_\gamma}{(m_n + m_p)\, c}$$

Since both n & p are at rest in CM frame at threshold, we write

$$E_\gamma = \frac{E_\gamma^2}{2(m_n+m_p)c^2} + E_b$$

by conservation of energy. Since the first term is a small correction, we have

$$E_\gamma \approx E_b + \frac{E_b^2}{2(m_n+m_p)c^2}$$

Thus $$\frac{\delta E}{E_b} = \frac{E_b}{2(m_n+m_p)c^2} = \frac{2{\cdot}2}{2\times 2\times 938} = 5{\cdot}9\times 10^{-4}$$

or nearly 0·06 %.

6.279 The reaction is

$$p + d \to He^3$$

Excitation energy of He^3 is just the energy available in centre of mass. The velocity of the centre of mass is

$$\frac{\sqrt{2\,m_p T_p}}{m_p+m_d} = \frac{1}{3}\sqrt{\frac{2\,T_p}{m_p}}$$

In the CM frame, the kinetic energy available is ($m_d = 2\,m_p$)

$$\frac{1}{2}m_p\left(\frac{2}{3}\sqrt{\frac{2\,T_p}{m_p}}\right)^2 + \frac{1}{2}2\,m_p\left(\frac{1}{3}\sqrt{\frac{2\,T}{m_p}}\right)^2 = \frac{2\,T}{3}$$

The total energy available is then $Q + \frac{2\,T}{3}$

where
$$\begin{aligned} Q &= c^2(\Delta_n + \Delta_d - \Delta_{He}^3) \\ &= c^2\times(0{\cdot}00783 + 0{\cdot}01410 - 0{\cdot}01603)\,\text{amu} \\ &= 5{\cdot}49\ \text{MeV} \end{aligned}$$

Finally $$E = 6{\cdot}49\ \text{MeV}.$$

6.280 The reaction is

$$d + C^{13} \to N^{15*} \to n + N^{14}$$

Maxima of yields determine the energy levels of N^{15*}. As in the previous problem the excitation energy is

$$E_{exc} = Q + E_K$$

where E_K = available kinetic energy. This is found is as in the previous problem. The velocity of the centre of mass is

$$\frac{\sqrt{2\,m_d T_i}}{m_d+m_c} = \frac{m_d}{m_d+m_c}\sqrt{\frac{2\,T_i}{m_d}}$$

So $$E_K = \frac{1}{2} m_d \left(1 - \frac{m_d}{m_d + m_c}\right)^2 \frac{2\,T_i}{m_d} + \frac{1}{2} m_c \left(\frac{m_d}{m_d + m_c}\right)^2 \frac{2\,T_i}{m_d} = \frac{m_c}{m_d + m_c} T_i$$

Q is the Q value for the ground state of N^{15} : We have

$$Q = c^2 \times (\Delta_d + \Delta_{C^{13}} - \Delta_{N^{15}})$$
$$= c^2 \times (0{\cdot}01410 + 0{\cdot}00335 - 0{\cdot}00011) \text{ amu}$$
$$= 16{\cdot}14 \text{ MeV}$$

The excitation energies then are

16·66 MeV , 16·92 MeV

17·49 MeV and 17·70 MeV.

6.281 We have the relation

$$\frac{1}{\eta} = e^{-n \sigma d}$$

Here $\frac{1}{\eta}$ = attenuation factor

n = no. of Cd nuclei per unit volume

σ = effective cross section

d = thickness of the plate

Now $$n = \frac{\rho N_A}{M}$$

(ρ = density, M = Molar weight of Cd, N_A = Avogadro number.)

Thus $$\sigma = \frac{M}{\rho N_A d} \ln \eta = 2{\cdot}53 \text{ kb}$$

6.282 Here

$$\frac{1}{\eta} = e^{-(n_2 \sigma_2 + n_1 \sigma_1) d}$$

where 1 refers to O^1 and 2 to D nuclei

Using $n_2 = 2n$, $n_1 = n$ = concentration of O nuclei in heavy water we get

$$\frac{1}{\eta} = e^{-(2\sigma_2 + \sigma_1) n d}$$

Now using the data for heavy water

$$n = \frac{1{\cdot}1 \times 6{\cdot}023 \times 10^{23}}{20} = 3{\cdot}313 \times 10^{22} \text{ per cc}$$

Thus substituting the values

$$\eta = 20{\cdot}4 = \frac{I_0}{I}.$$

6.283 In traversing a distance d the fraction which is either scattered or absorbed is clearly

$$1 - e^{-n(\sigma_s + \sigma_a)d}$$

by the usual definition of the attenuation factor. Of this, the fraction scattered is (by definition of scattering and absorption cross section)

$$w = \left\{1 - e^{-n(\sigma_s + \sigma_a)d}\right\} \frac{\sigma_s}{\sigma_s + \sigma_a}$$

In iron $$n = \frac{\rho \times N_A}{M} = 8{\cdot}39 \times 10^{22} \text{ per cc}$$

Substitution gives $$w = 0{\cdot}352$$

6.284 (a) Assuming of course, that each reaction produces a radio nuclide of the same type, the decay constant α of the radionuclide is k/w. Hence $T = \frac{\ln 2}{\lambda} = \frac{w}{k}\ln 2$

(b) number of bombarding particles is : $\frac{It}{e}$

(e = charge on proton). Then the number of Be^7 produced is : $\frac{It}{e}w$

If λ = decay constant of $Be^7 = \frac{\ln 2}{T}$, then the activity is $A = \frac{It}{e} w \cdot \frac{\ln 2}{T}$

Hence $$w = \frac{eAT}{It \ln 2} = 1{\cdot}98 \times 10^{-3}$$

6.285 (a) Suppose N_0 = no. of Au^{197} nuclei in the foil. Then the number of Au^{197} nuclei transformed in time t is

$$N_0 \cdot J \cdot \sigma \cdot t$$

For this to equal ηN_0, we must have

$$t = \eta / (J \cdot \sigma) = 323 \text{ years}$$

(b) Rate of formation of the Au^{198} nuclei is $N_0 \cdot J \cdot \sigma$ per sec
and rate of decay is λn, where n is the number of Au^{198} at any instant.

Thus $$\frac{dn}{dt} = n_0 \cdot J \cdot \sigma - \lambda n$$

The maximum number of Au^{198} is clearly

$$n_{max} = \frac{N_0 J \sigma}{\lambda} = \frac{N_0 J \sigma T}{\ln 2}$$

because if n is smaller, $\frac{dn}{dt} > 0$ and n will increase further and if n is larger $\frac{dn}{dt} < 0$ and n will decrease. (Actually n_{max} is approached steadily as $t \to \infty$)

Substitution gives using $N_0 = 3{\cdot}057 \times 10^{19}$, $n_{max} = 1{\cdot}01 \times 10^{13}$

6.286 Rate of formation of the radionuclide is $n{\cdot}J{\cdot}\sigma$ per unit area per sec. Rate of decay is λN. Thus

$$\frac{dN}{dt} = n{\cdot}J{\cdot}\sigma - \lambda N \text{ per unit area per second}$$

Then $$\left(\frac{dN}{dt} + \lambda N\right) e^{\lambda t} = n{\cdot}J{\cdot}\sigma\, e^{\lambda t} \quad \text{or} \quad \frac{d}{dt}(N e^{\lambda t}) = n{\cdot}J{\cdot}\sigma{\cdot}e^{\lambda t}$$

Hence $$N e^{\lambda t} = \text{Const} + \frac{n{\cdot}J{\cdot}\sigma}{\lambda} e^{\lambda t}$$

The number of radionuclide at $t = 0$ when the process starts is zero. So constant $= -\dfrac{n{\cdot}J{\cdot}\sigma}{\lambda}$

Then $$N = \frac{n{\cdot}J{\cdot}\sigma}{\lambda}(1 - e^{-\lambda t})$$

6.287 We apply the formula of the previous problem except that have
N = no. of radio nuclide and no. of host nuclei originally.

Here $$n = \frac{0{\cdot}2}{197} \times 6{\cdot}023 \times 10^{23} = 6{\cdot}115 \times 10^{20}$$

Then after time t $$N = \frac{n{\cdot}J{\cdot}\sigma{\cdot}T}{\ln 2}\left(1 - e^{-\frac{t \ln 2}{T}}\right)$$

T = half life of the radionuclide.

After the source of neutrons is cut off the activity after time T will be

$$A = \frac{n{\cdot}J{\cdot}\sigma{\cdot}T}{\ln 2}(1 - e^{-t \ln 2/T})\, e^{-\tau \ln 2/T \times \frac{\ln 2}{T}} = n{\cdot}J{\cdot}\sigma\,(1 - e^{-t \ln 2/T})\, e^{-\tau \ln 2/T}$$

Thus $$J = A\, e^{\tau \ln 2/T} \Big/ n\,\sigma\,(1 - e^{-t \ln 2/T}) = 5{\cdot}92 \times 10^{9} \text{ part/cm}^2\cdot\text{s}$$

6.288 No. of nuclei in the first generation = No. of nuclei initially = N_0
N_0 in the second generation = $N_0 \times$ multiplication factor $= N_0 \cdot k$

N_0 in the the 3rd generation $= N_0 \cdot k \cdot k = N_0 k^2$

N_0 in the nth generation $= N_0 k^{n-1}$

Substitution gives $1{\cdot}25 \times 10^5$ neutrons

6.289 N_0. of fissions per unit time is clearly P/E. Hence no. of neutrons produced per unit time to $\dfrac{\nu P}{E}$. Substitution gives $7{\cdot}80 \times 10^{18}$ neutrons/sec

6.290 (a) This number is k^{n-1} where n = no. of generations in time $t = t/T$
Substitution gives 388.

(b) We write $$k^{n-1} = e^{\left(\frac{T}{\tau} - 1\right)\ln k} = e$$

or $$\frac{T}{\tau} - 1 = \frac{1}{\ln k} \quad \text{and} \quad T = \tau\left(1 + \frac{1}{\ln k}\right) = 10{\cdot}15 \text{ sec}$$