CBSE Class 10 Mathematics Basic

Sample Paper - 07 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. Classify 3.121221222... as rational or irrational.

OR

If sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

2. Show that $x = -3$ is a solution of $x^2 + 6x + 9 = 0$.
3. For what value of a the following pair of linear equation has infinitely many solutions?
$2x + ay = 8$
$ax + 8y = a$

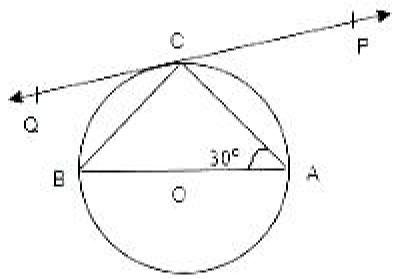
4. A line m is tangent to the circle with radius 5 cm. Find the distance between the centre and the line m.
5. Which of the term of A.P. 5, 2, -1,....is - 49?

OR

Find the common difference of the AP : $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots.$
6. How many two-digit numbers are divisible by 6?
7. Show that the equation $3x^2 + 7x + 8 = 0$ is not true for any real value of x.

OR

Form a quadratic equation whose roots are -3 and 4.
8. In the following figure, PQ is a tangent at a point C to circle with centre O. If AB is a diameter and $\angle CAB = 30^o$, then find $\angle PCA$.



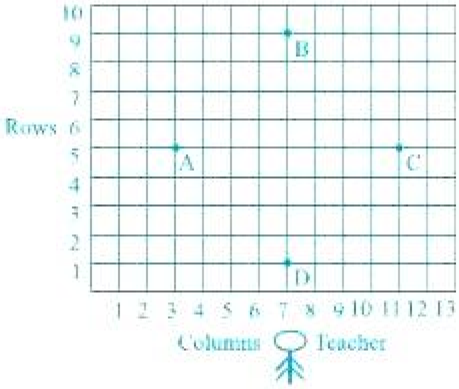
9. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle (in cm) which touches the smaller circle.

OR

If the angle between two radii of a circle is 130°, then what is the angle between the tangents at the end points of radii at their point of intersection ?
10. In ΔABC, AC = 24 cm, BC = 10 cm and AB = 26 cm. Find $\angle C$.
11. Find the sum of the first 16 terms of the A.P. 10, 6, 2,........
12. Prove the trigonometric identity:
$\operatorname{cosec}^2\theta + \sec^2\theta = \operatorname{cosec}^2\theta \sec^2\theta$
13. Evaluate $\frac{5\sin^2 30^\circ + \cos^2 45^\circ - 4\tan^2 30^\circ}{2\sin 30^\circ \cos 30^\circ + \tan 45^\circ}$.
14. The slant height of a bucket is 26 cm. The diameter of upper and lower circular ends are 36 cm and 16 cm. Find the height of the bucket.
15. What is the sum of first n terms of the AP a, 3a, 5a,.....

16. If P(E) = 0.20, then what is the probability of 'not E'?

17. Students of a school are standing/seating in rows and columns in their playground for Yoga practice. A, B, C and D are the positions of four students as shown in the figure.



i. The positions of A, B respectively are:
   a. (3, 5), (8, 7)
   b. (3, 5), (9, 7)
   c. (3, 5), (7, 9)
   d. (5, 3), (7, 9)

ii. The distance between A and B is:
   a. $\sqrt{32}$ units
   b. $\sqrt{23}$ units
   c. $\sqrt{42}$ units
   d. $\sqrt{35}$ units

iii. It is possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D then the Position of Jaspal is:
   a. (3, 7)
   b. (3, 5)
   c. (5, 7)
   d. (7, 5)

iv. The distance between A and C is
   a. 8 units
   b. 6 units
   c. 4 units
   d. $\sqrt{32}$ units

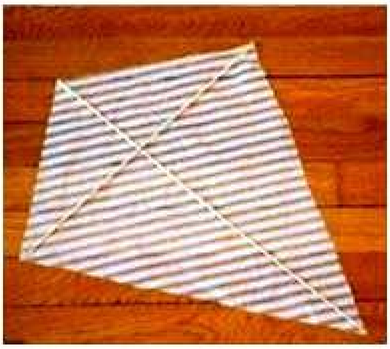
v. The positions of C and B respectively are:
   a. (11, 5), (9, 7)

b. (11, 5), (7, 9)

c. (5, 11), (7, 9)

d. (11, 7), (5, 9)

18. Rahul is studying in X Standard. He is making a kite to fly it on a Sunday. Few questions came to his mind while making the kite. Give answers to his questions by looking at the figure.



i. Rahul tied the sticks at what angles to each other?

a. 30^{o}

b. 60^{o}

c. 90^{o}

d. 60^{o}

ii. Which is the correct similarity criteria applicable for smaller triangles at the upper part of this kite?

a. RHS

b. SAS

c. SSA

d. AAS

iii. Sides of two similar triangles are in the ratio 4:9. Corresponding medians of these triangles are in the ratio:

a. 2:3

b. 4:9

c. 81:16

d. 16:81

iv. In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle. This theorem is called:

a. Pythagoras theorem

b. Thales theorem

c. The converse of Thales theorem

d. The converse of Pythagoras theorem

v. What is the area of the kite, formed by two perpendicular sticks of length 6 cm and 8 cm?

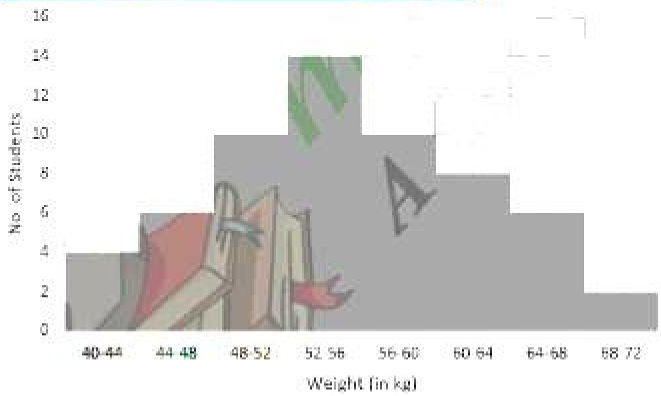
a. 48 cm^2

b. 14 cm^2

c. 24 cm^2

d. 96 cm^2

19. The DAV public school organized a free health check-up camp for the 60 students of class 10th. They have provided the BMI report to each student. This report estimates the body fat and is a good measure of risk for diseases that can occur with overweight and obesity. On the basis of this report, the following graph is made which describes the weight (in kg) of the students:



i. Identify the modal class in the given graph.

a. 48-52

b. 68-72

c. 56-60

d. 52-56

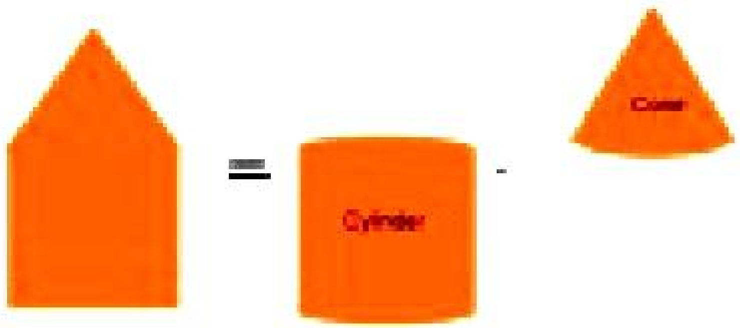
ii. Calculate the mode weight of the students.
   a. 45 kg
   b. 44 kg
   c. 56 kg
   d. 54 kg

iii. Find the median weight of the students if the mean weight is 55.2 kg.
   a. 54.8 kg.
   b. 64.8 kg.
   c. 55.8 kg.
   d. 45.8 kg.

iv. The lower limit of the modal class is:
   a. 64
   b. 56
   c. 52
   d. 48

v. The empirical relationship between mean, median and mode is:
   a. Mode = 3 Median + 2 Mean
   b. Mode = 3 Median - 2 Mean
   c. Mode = 3 Mean + 2 Median
   d. 3 Mode = Median + 2 Mean

20. Due to heavy floods in a state, thousands of people were homeless. 50 schools collectively offered to the state government to provide the place and the canvas for 1500 tent to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with the conical upper part of the same base radius but of height 2.1 m. [use $\pi = \frac{22}{7}$]



i. Area of canvas used to make the tent is
   a. TSA of cylindrical portion + CSA of the conical portion
   b. CSA of cylindrical portion + CSA of the conical portion
   c. CSA of cylindrical portion + TSA of the conical portion
   d. TSA of cylindrical portion + TSA of the conical portion
ii. The volume of the tent is
   a. $\pi r^2(\frac{1}{3}r + h)$ cubic units
   b. $\frac{1}{3}\pi r^2(r + h)$ cubic units
   c. $\frac{4}{3}\pi r^2 h$ cubic units
   d. none of these
iii. If the canvas used to make the tent cost ₹120 per sq.m, find the amount to be paid by the schools for making the tents.
   a. ₹ 11098
   b. ₹ 88889
   c. ₹ 11088
   d. ₹ 99998
iv. Amount shared by each school to set-up the tents.
   a. ₹ 442640
   b. ₹ 222640
   c. ₹ 332640
   d. ₹ 552640
v. According to the given information, what is the ratio of the curved surface area of the cylindrical portion to the conical portion:
   a. 1:2
   b. 2:3
   c. 4:1
   d. 2:1

Part-B

21. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}\,x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.
22. If the co-ordinates of two points are A(3, 4), B(5, - 2) and a point P(x, 5) is such that PA = PB then find the area of ΔPAB.

OR

Show that the quadrilateral whose vertices are (2, -1), (3, 4), (-2, 3) and (-3, -2) is a rhombus.

23. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, then find the values of p and q.
24. Draw a right angled ΔABC in which BC = 12 cm, AB = 5 cm, and $\angle B = 90°$. Construct a triangle similar to it and of scale factor $\frac{2}{3}$. Is the new triangle also a right triangle?
25. If A = 60° and B = 30°, verify that sin (A - B) = sin A cos B - cos A sin B.

OR

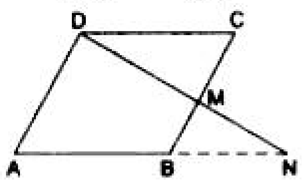
If $\sin\theta = \frac{a}{b}$, find $\sec\theta + \tan\theta$ in terms of a and b.

26. Two concentric circles are of radii 7 cm and r cm respectively where r > 7. A chord of the larger circle of the length 48 cm, touches the smaller circle. Find the value of r.
27. Prove that $\sqrt{5}$ is irrational.
28. Solve the quadratic equation by factorization: $\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = \frac{5}{2}, x \neq -\frac{1}{2}, 1$

OR

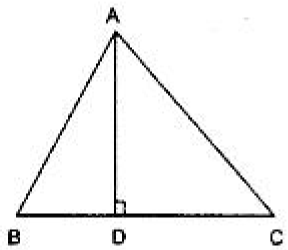
Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe would fill the cistern.

29. Find all zeros of the polynomial $3x^3 + 10x^2 - 9x - 4$ if one of its zero is 1.
30. M is a point on the side BC of a parallelogram ABCD. DM when produced meets AB at N. Prove that.
    i. $\frac{DM}{MN} = \frac{DC}{BN}$.
    ii. $\frac{DN}{DM} = \frac{AN}{DC}$



OR

In Fig. if $AD \perp BC$ and $\frac{BD}{DA} = \frac{DA}{DC}$, Prove that Δ ABC is a right triangle.



31. Three unbiased coins are tossed simultaneously. Find the probability of getting (i) exactly 2 heads, (ii) at least 2 heads, (iii) at most 2 heads.
32. A man rowing a boat away from a lighthouse 150 m high takes 2 minutes to change the angle of elevation of the top of lighthouse from 45° to 30°. Find the speed of the boat. (Use $\sqrt{3}$ = 1.732)
33. Find the mean marks per student, using assumed-mean method:

Marks	0 -10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Number of students	12	18	27	20	17	6

34. The difference between the sides at right angles in a right-angled triangle is 7 cm. The area of the triangle is 60 cm^2. Find its perimeter.
35. Abdul travelled 300 km by train and 200 km by taxi taking 5 hours 30 minutes. But, if he travels 260 km by train and 240 km by taxi, he takes 6 minutes longer. Find the speed of the train and that of the taxi.
36. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and the flag pole mounted on it.

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Solution

Part-A

1. 3.121221222... is an irrational number because it is a non-terminating and non-repeating decimal.

OR

$p(x) = 3x^2 - kx + 6$

Sum of the zeroes = $-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\Rightarrow \quad 3 = -\frac{(-k)}{3}$

$\therefore k = 9$

2. $x^2 + 6x + 9 = 0$.

Put x = -3 in the equation

$\Rightarrow (-3)^2 + 6(-3) + 9$

$\Rightarrow 9 - 18 + 9 = 0$

Hence, it is a solution of the given equation.

3. For infinite numbers of solution

$\frac{2}{a} = \frac{a}{8} = \frac{8}{a} \Rightarrow \frac{2}{a} = \frac{a}{8}$ and $\frac{a}{8} = \frac{8}{a}$

$a^2 = 16 \; and \; a^2 = 64$

$\therefore$ The system do not have infinite solutions for any value of a.

4. A line m is tangent to the circle with radius 5 cm.

$\therefore$ Distance between the centre and the line m= Radius of the circle = 5 cm.

5. Here, $a = 5, d = 2 - 5 = -3$

$\because l = a + (n-1)d$

$-49 = 5 + (n-1)(-3)$

or, $-49 - 5 = -3n + 3$

or, $3n = 49 + 5 + 3$

or, $3n = 57$

or, n = $\frac{57}{3}$ = 19th term

OR

Common difference(d) = n^{th} term - $(n - 1)^{th}$ term

$\therefore$ d = a_2 - a_1

$$d = (\frac{1-p}{p}) - (\frac{1}{p}) = \frac{(1-p)-(1)}{p} = \frac{-p}{p} = -1$$

d = -1

6. The two-digit numbers divisible by 6 start from 12, 18, 24, ..., 96

   Here,

   a = 12

   d = 6

   $a_n = a + (n - 1)d$

   $\Rightarrow 96 = 12 + (n - 1)(6)$

   $\Rightarrow 96 = 12 + 6n - 6$

   $\Rightarrow 90 = 6n$

   $\Rightarrow n = 15$

   Thus, 15 two-digit numbers are divisible by 6.

7. We have the following equation,

   $3x^2 + 7x + 8 = 0.$

   $\therefore D = (7^2 - 4 \times 3 \times 8) = (49 - 96) = -47 < 0.$

   Since D is less than zero, so given equation has no real roots.

   Therefore, the equation is not true for any real value of x

OR

We have, x = 4 and x = -3.

Then,

$x - 4 = 0$ and $x + 3 = 0$

$\Rightarrow (x - 4)(x + 3) = 0$

$\Rightarrow x^2 + 3x - 4x - 12 = 0$

$\Rightarrow x^2 - x - 12 = 0$

This is the required quadratic equation

8. Given,PQ is a tangent at a point C to circle with centre O and $\angle CAB = 30^o$.

   Join OC.

$\therefore OA = OC$ [$\because$ radii of a circle]

$\Rightarrow \angle OCA = \angle OAC$ [$\because$ angles opposite to equal sides of a triangle are equal]

$\Rightarrow \angle OCA = 30^o$ $[\angle OAC = 30^o]$...(i)

A tangent is perpendicular to the radius at the point of contact.

$\therefore OC \perp PQ$

$\Rightarrow \angle OCP = 90^o$

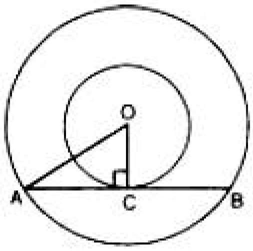
$\Rightarrow \angle OCA + \angle PCA = \angle OCP$

$\Rightarrow \angle OCA + \angle PCA = 90^o$

$\Rightarrow 30^o + \angle PCA = 90^o$ [$\because$Using Eq(i)]

$\Rightarrow \angle PCA = 90^o - 30^o$

$\Rightarrow \angle PCA = 60^o$

9. 

Now, In $rt.\triangle\ OCA$

$AO^2 = OC^2 + AC^2$

$\Rightarrow AC^2 = 5^2 - 3^2$

$\Rightarrow AC = 4$

We know, $OC \perp AB$

$\therefore AC = BC$

Hence, $AC = 2(4) = 8cm$

OR

Since, sum of the angles between radii and between intersection point of tangent is 180°.

Angle at the point of intersection of tangents

$= 180^\circ - 130^\circ = 50^\circ$

10. $AB^2 = 26 \times 26 = 676$

$AC^2 = 24 \times 24 = 576$

$BC^2 = 10 \times 10 = 100$

Therefore, $AB^2 = AC^2 + BC^2$

Hence, by Pythagoras theorem, AB is the hypotenuse. The angle opposite to hypotenuse is angle C which is equal to 90°.

11. Here,$a=10, d=6-10=-4, n=16$

    S = $\frac{n}{2}[2a+(n-1)d]$

    $\therefore \quad S_{16}=\frac{16}{2}[2\times 10+(16-1)(-4)]$

    $=8[20+15\times(-4)]$

    $=8[20-60]$

    $=8\times(-40)$

    $=-320$

12. We have,

    LHS = cosec$^2\theta$ + sec$^2\theta$

    $\Rightarrow \text{LHS}=\frac{1}{\sin^2\theta}+\frac{1}{\cos^2\theta}$

    $\Rightarrow \text{LHS}=\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta\sin^2\theta}=\frac{1}{\sin^2\theta\cos^2\theta}=\frac{1}{\sin^2\theta}\times\frac{1}{\cos^2\theta}=\operatorname{cosec}^2\theta\sec^2\theta$ = RHS

13. According to question,

    $=\frac{5\times(1/2)^2+(1/\sqrt{2})^2-4\times(1/\sqrt{3})^2}{2\times\frac{1}{2}\times\frac{\sqrt{3}}{2}+1}$

    $=\frac{\frac{5}{4}+\frac{1}{2}-\frac{4}{3}}{\frac{\sqrt{3}}{2}+1}$

    $=\frac{\frac{5}{12}}{\frac{\sqrt{3}+2}{2}}$

    $=\frac{5}{12}\times\frac{2}{\sqrt{3}+2}$

    $=\frac{5}{6(\sqrt{3}+2)}=\frac{5}{6(\sqrt{3})+12}$

14. Here, length of a bucket = 26 cm

    upper radius of a bucket = 18 cm

    lower radius of a bucket = 8 cm

    d = difference in radius = 18 - 8 = 10 cm

    Let h be the height of bucket

    $\therefore \quad h=\sqrt{l^2-d^2}$

    $=\sqrt{(26)^2-(10)^2}$

    $=\sqrt{676-100}$

    $=\sqrt{576}=24\text{cm}$

    Therefore height of the bucket is 24cm

15. First term= a

    common difference, $d=3a-a=2a$

Sum of n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

Putting the values

$S_n = \frac{n}{2}[2a + (n-1)(2a)]$

$S_n = \frac{n}{2}[2a + 2an - 2a]$

$S_n = \frac{n}{2}[2an] = (n)(an)$

$S_n = an^2$

So sum of n terms is an^2.

16. $P(E) = 0.20$

$\therefore P(notE) = 1 - P(E)$

$= 1 - 0.20 = 0.80$

17. i. (c) The positions of the students are A (3, 5), B(7, 9), C(11, 5) and D(7, 1).

ii. (a) To find the distance between them, we use the distance formula.

So,

$AB = \sqrt{(7-3)^2 + (9-5)^2} = \sqrt{(4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$

$BC = \sqrt{(11-7)^2 + (5-9)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$

$CD = \sqrt{(7-11)^2 + (1-5)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$

And $DA = \sqrt{(3-7)^2 + (5-1)^2} = \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$

iii. (d) We see that, AB = BC = CD = DA i.e., all sides are equal.

Now, we find the length of both diagonals;

$AC = \sqrt{(11-3)^2 + (5-5)^2} = \sqrt{(8)^2 + 0} = 8$

and $BD = \sqrt{(7-7)^2 + (1-9)^2} = \sqrt{0 + (-8)^2} = 8$

Here, AC = BD

Since AB = BC = CD = DA and AC = BD, so we can say that ABCD is a square.

Thus, it is possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D.

As we also know that diagonals of a square bisect each other, so, let P be a position of Jaspal in which he is equidistant from each of the four students A, B, C and D.

Coordinates of P = Midpoint of AC

$= \left(\frac{3+11}{2}, \frac{5+5}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$

Hence, the required position of Jaspal is (7, 5).

iv. (a) 8 units

v. (b) (11, 5), (7, 9)

18. i. (c) 90^{o}

ii. (b) SAS

iii. (b) 4:9

iv. (d) Converse of Pythagoras theorem

v. (a) 48 cm^2

19. First, let us convert the graphical distribution in the form of a table as shown below:

Weight	**No. of Students**
40-44	4
44-48	6
48-52	10
52-56	14
56-60	10
60-64	8
64-68	6
68-72	2

i. (d) The modal class is the class with the highest frequency. As the maximum frequency is 14 and the class corresponding to it is 52-56, so

Modal class = 52-56

ii. (d) Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

where l is the lower limit of the modal class,

h is the size of the class interval,

f_1 is the frequency of the modal class,

f_0 is the frequency of the class preceding the modal class,

f_2 is the frequency of the class succeeding the modal class

So, here l = 52, f_1 = 14, f_0 = 10, f_2 = 10, h = 4

Thus, Mode = 52 + $\frac{14-10}{28-10-10} \times 4$ = 54

Hence, the mode weight of the students is 54 kg.

iii. (a) Given: Mean = 55.2 kg

Now, Mode = 3 Median - 2 Mean

Thus, Median = $\frac{1}{3}$ (Mode + 2 Mean) = $\frac{1}{3}$ (54 + 2×55.2) = 54.8

Hence, the median weight of the students is 54.8 kg.

iv. (c) 52

v. (b) Mode = 3 Median - 2 Mean

20. i. (b) CSA of cylindrical portion + CSA of the conical portion

ii. (a) $\pi r^2(\frac{1}{3}r + h)$ cubic units

iii. (c) ₹ 11088

iv. (c) ₹ 332640

v. (d) 2:1

Part-B

21. we are given that α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}x + 3$,

Let, $x^2 - 4\sqrt{3}x + 3 = 0$

If α and β are the zeroes of $x^2 - 4\sqrt{3}x + 3$.

then $\alpha + \beta = -\frac{b}{a}$

or, $\alpha + \beta = 4\sqrt{3}$

and $\alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{3}{1}$

or, $\alpha\beta = 3$

$\therefore \quad \alpha + \beta - \alpha\beta = 4\sqrt{3} - 3$

22. Given PA = PB or $PA^2 = PB^2$, using distance formula

$(x - 3)^2 + (5 - 4)^2 = (x-5)^2 + (5 + 2)^2$

On solving, we get, x = 16

$\therefore$ ar ΔPAB,

$= \frac{1}{2}$ [16(4 + 2) + 3 (-2 -5) + 5 (5 -4)]

$= \frac{1}{2}$ [96 - 21 + 5] = 40

Hence, area of triangle = 40 sq units.

OR

Let A(2, -1), B(3, 4), C(-2, 3) and D(-3, -2)

$AB = \sqrt{(3-2)^2 + (4+1)^2} = \sqrt{(1)^2 + (5)^2} = \sqrt{1+25} = \sqrt{26}$

$BC = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}$

$CD = \sqrt{(-3+2)^2 + (-2-3)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}$

$$AD = \sqrt{(-3-2)^2+(-2+1)^2} = \sqrt{(-5)^2+(-1)^2} = \sqrt{25+1} = \sqrt{26}$$

Since AB = BC = CD = AD

∴ ABCD is a rhombus.

23. Given polynomial, $2x^2 - 5x - 3$

$\Rightarrow 2x^2 - 6x + x - 3$

$\Rightarrow 2x(x - 3) + 1(x - 3)$

$\Rightarrow (x - 3)(2x + 1)$

To get zeroes of the polynomial, equate polynomial = 0

$\Rightarrow (x - 3)(2x + 1) = 0$

$\Rightarrow x = 3, x = \frac{-1}{2}$

So, zeroes of $x^2 + px + q$ are $2 \times 3 = 6$ and $2 \times \frac{-1}{2} = -1$

Let the zeroes of $x^2 + px + q$ be a = 6 and b = -1

We know that, $a + b = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

$6 + (-1) = \frac{-p}{1}$

$5 = -p$

$p = -5$

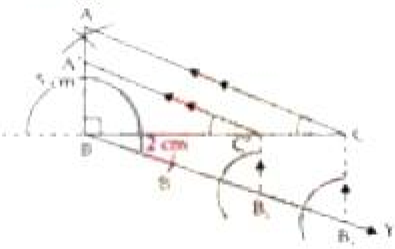
Also, $a \times b = \frac{constant\ term}{coefficient\ of\ x^2}$

$6 \times (-1) = \frac{q}{1}$

$q = -6$

Therefore, p = -5 and q = -6.

24. Here, scale factor or ratio factor is $\frac{2}{3} < 1$, so triangle to be constructed will be smaller than given ΔABC.



Step of construction:

i. Draw BC = 12 cm.

ii. Draw ∠CBA = 90° with scale and compass.

iii. Cut BA = 5 cm such that ∠ABC = 90°.

iv. Join AC. ΔABC is the given triangle.

v. Draw an acute ∠CBY such that A and Y are in opposite direction with respect to BC.

vi. Divide BY in 3 equal segments by marking arc at same distance at B_1, B_2, and B_3.

vii. Join B_3C.

viii. Draw B_2C' || B_2C by making equal alternate angles at B_2 and B_3.

ix. From point C', draw C' A' || CA by making equal alternate angles at C and C'.

ΔA'BC' is the required triangle of scale factor $\frac{2}{3}$. This triangle is also a right triangle.

25. According to the question, A = 60° and B = 30°

LHS = sin (A - B) = sin (60° - 30°) = sin 30° $= \frac{1}{2}$ ...(i)

RHS = sin A. cos B - cos A. sin B = sin 60° . cos 30° - cos 60° . sin 30°

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$ ...(ii)

∴ From (i) and (ii),

sin (A - B) = sin A . cos B - cos A . sin B

OR

According to the question, $\sin\theta = \frac{a}{b}$

Perpendicular = a,

hypotenuse = b

So, base $= \sqrt{b^2 - a^2}$

$\sec\theta = \frac{b}{\sqrt{b^2-a^2}}$

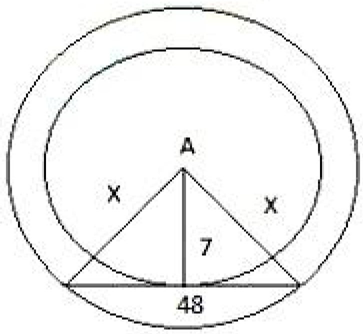
$\tan\theta = \frac{a}{\sqrt{b^2-a^2}}$

Now, $\sec\theta + \tan\theta = \frac{b}{\sqrt{b^2-a^2}} + \frac{a}{\sqrt{b^2-a^2}}$

$= \frac{a+b}{\sqrt{b^2-a^2}}$

$= \frac{a+b}{\sqrt{(b+a)(b-a)}}$

$\sec\theta + \tan\theta = \sqrt{\frac{a+b}{b-a}}$

26. 

Let us take r = x

Now using Pythagoras theorem

$(x)^2 = 24^2 + 7^2$

$(x)^2 = 576 + 49$

$(x)^2 = 625$

Therefore, x = 25 cm.

r = 25 cm.

27. According to the question, we have to prove that $\sqrt{5}$ is irrational.

Let us prove $\sqrt{5}$ irrational by contradiction.

Suppose $\sqrt{5}$ is rational.

It means that we have co-prime integers a and b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

$\Rightarrow b\sqrt{5} = a$

Squaring both sides, we get

$\Rightarrow 5b^2 = a^2$ .... (1)

It means that 5 is factor of a^2

Hence, 5 is also factor of a by Theorem. ..... (2)

If, 5 is factor of a, it means that we can write $a = 5c$ for some integer c.

Substituting value of a in (1),

$5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

It means that 5 is factor of b^2.

Hence, 5 is also factor of b by Theorem. ..... (3)

From (2) and (3), we can say that 5 is factor of both a and b. Means, b is divisble by 5 thus a and b have a common factor 5.

But, a and b are co-prime.

Therefore, our assumption was wrong.

So, $\sqrt{5}$ cannot be rational.

Hence, $\sqrt{5}$ is irrational.

28. We have,

$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = \frac{5}{2}, x \neq -\frac{1}{2}, 1$

$\Rightarrow (x - 1)(x - 1) + (2x + 1)(2x + 1) = \frac{5}{2}(2x + 1)(x - 1)$ [Multiplying both sides by (2x + 1)(x - 1)]

$\Rightarrow x^2 + 1 - 2x + (2x)^2 + 2x + 2x + 1 = \frac{5}{2}$ [2x(x - 1) + 1(x - 1)]

$\Rightarrow x^2 + 1 - 2x + 4x^2 + 4x + 1 = \frac{5}{2}$ [$2x^2$ - 2x + x - 1]

$\Rightarrow 5x^2 + 2x + 2 = \frac{5}{2}$ [$2x^2$ - x - 1]

$\Rightarrow 10x^2 + 4x + 4 = 5[2x^2 - x - 1]$

$\Rightarrow$ 4x + 5x + 4 + 5 = 0

$\Rightarrow$ 9x + 9 = 0

$\Rightarrow$ 9(x + 1) = 0

So, either x + 1 = 0 or, 9 = 0 which is not possible.

Thus, x = -1.

Hence, the given quadratic equation has repeated (equal) roots and are given by -1 and -1

OR

Let the faster pipe takes x minutes to fill the cistern and the slower pipe will take (x + 3) minutes.

According to question,

$\frac{1}{x} + \frac{1}{x+3} = \frac{1}{\frac{40}{13}}$

$\Rightarrow \frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$

$\Rightarrow \frac{x+3+x}{x^2+3x} = \frac{13}{40}$

$\Rightarrow 13x^2 - 41x - 120 = 0$

$\Rightarrow 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow$ 13x(x - 5) + 24(x - 5) = 0

$\Rightarrow$ x = 5 or $x = \frac{-24}{13}$ (Neglected)

Faster pipe will take 5 minutes and slower pipe will take 8 minutes to fill the cistern

29. Let p(x) = $3x^3 + 10x^2 - 9x - 4$ & zero = 1

Thus, one factor of p(x) = (x - 1)

We get another factor of p(x) by dividing it with (x - 1)

$$
\begin{array}{r|l}
 & 3x^2 + 13x + 4 \\
\hline
x - 1 & 3x^3 + 10x^2 - 9x - 4 \\
 & 3x^3 - 3x^2 \\
\hline
 & 13x^2 - 9x - 4 \\
 & 13x^2 - 13x \\
\hline
 & 4x - 4 \\
 & 4x - 4 \\
\hline
 & 0
\end{array}
$$

$\Rightarrow p(x) = (x - 1)(3x^2 + 13x + 4)$

$= (x - 1)(3x^2 + 12x + x + 4)$

$= (x - 1)[3x(x + 4) + (x + 4)]$

$= (x - 1)(x + 4)(3x + 1)$

For zeroes put p(x) = 0

$\Rightarrow (x - 1)(x + 4)(3x + 1) = 0$

$\Rightarrow x = 1, x = -4 \; \& \; x = -1/3$

30. Given: ABCD is a parallelogram

To Prove:

i. $\frac{DM}{MN} = \frac{DC}{BN}$.

ii. $\frac{DN}{DM} = \frac{AN}{DC}$

<u>**Proof:**</u> In $\triangle DMC$ and $\triangle NMB$, we have

$\angle \mathrm{DMC} = \angle \mathrm{NMB}$ (Vertically opposite angle)

$\angle DCM = \angle NBM$ (Alternate angles)

By AA - Similarity criteria, we have

$\triangle \mathrm{DMC} \sim \triangle \mathrm{NMB}$

$\therefore \frac{DM}{MN} = \frac{DC}{BN}$

which completes the proof of part (i).

Now, $\frac{MN}{DM} = \frac{BN}{DC}$

Adding 1 to both sides, we obtain

$\frac{MN}{DM} + 1 = \frac{BN}{DC} + 1$

$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+DC}{DC}$

$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+AB}{DC}$ [$\because$ ABCD is a parallelogram]$\Rightarrow \frac{DN}{DM} = \frac{AN}{DC}$

OR

In Δ's BDA and ADC, we have

$\frac{DB}{DA} = \frac{DA}{DC}$ [Given]

and, ∠ BDA = ∠ ADC [Each equal to 90°]

So, by SAS-criterion of similarity, we have

$\Delta BDA \sim \Delta ADC$

⇒ ∠ ABD = ∠ CAD and ∠ BAD= ∠ ACD

⇒ ∠ ABD + ∠ ACD = ∠ CAD + ∠ BAD

⇒ ∠ B + ∠ C = ∠ A

⇒ ∠ A + ∠ B + ∠ C = 2 ∠A [Adding ∠ A on both sides]

⇒ 2 ∠ A = 180°

⇒ ∠ A = 90°

⇒ ΔABC is a right triangle.

31. When 3 coins are tossed simultaneously, all possible outcomes are

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.

Total number of possible outcomes = 8.

i. Let E_1 be the event of getting exactly 2 heads.

Then, the favourable outcomes are HHT, HTH, THH.

Number of favourable outcomes = 3.

∴ P(getting exactly 2 heads) $= P(E_1) = \frac{3}{8}$.

ii. Let E_2 be the event of getting at least 2 heads.

Then, E_2 is the event of getting 2 or 3 heads.

So, the favourable outcomes are

HHT, HTH, THH, HHH.

Number of favourable outcomes = 4.

∴ P(getting at least 2 heads) = $\frac{4}{8} = \frac{1}{2}$.

iii. Let E_3 be the event of getting at most 2 heads.

Then, E_3 is the event of getting 0 or 1 head or 2 heads.

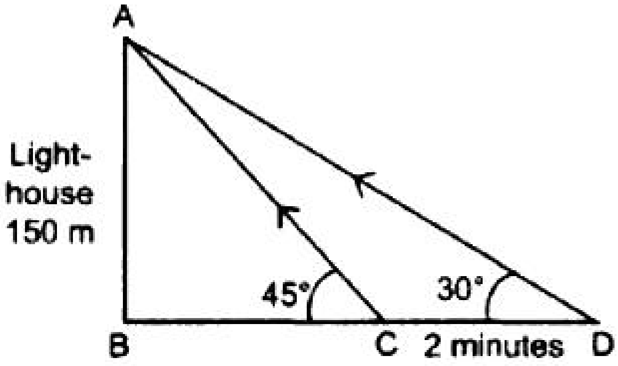
So, the favourable outcomes are

TTT, HTT, THT, TTH, HHT, HTH, THH.

Number of favourable outcomes = 7.

∴ P(getting at most 2 heads) $= P(E_3) = \frac{7}{8}$.

32.



AB = 150 m

Initially boat is at C and after 2 minutes it reaches at D.

In right $\triangle$ABC,

$\frac{\mathrm{AB}}{\mathrm{BC}} = \tan 45^\circ$

$\Rightarrow \frac{150}{\mathrm{BC}} = 1 \Rightarrow \mathrm{BC} = 150\mathrm{m}$

In right $\triangle$ABD, $\frac{\mathrm{AB}}{\mathrm{BD}} = \tan 30^\circ$

$\Rightarrow \frac{150}{\mathrm{BD}} = \frac{1}{\sqrt{3}} \Rightarrow \mathrm{BD} = 150\sqrt{3}$

Distance covered in 2 minutes $= BD - BC$

= $150\sqrt{3} - 150 = 150(\sqrt{3} - 1)m$

$\therefore$ speed = $\frac{\text{Distance covered}}{\text{time taken}} = \frac{150(\sqrt{3}-1)}{2}$

= $75 \times (1.732 - 1)$

$= 54.9\ m/min$

33. The assumed mean is 25.

Class Interval	Frequency(f_i)	Mid value x_i	Deviation $d_i = (x_i - 25)$	$(f_i \times d_i)$
0 - 10	12	5	-20	-240
10 - 20	18	15	-10	-180
20 - 30	27	25 = A	0	0
30 - 40	20	35	10	200
40 - 50	17	45	20	340
50 - 60	6	55	30	180
	$\Sigma f_i = 100$			$\Sigma(f_i \times d_i) = 300$

we know that, mean$= A + \frac{\sum(f_i \times x_i)}{\sum f_i}$

From table, $\Sigma f_i = 100$ and $\Sigma\left(f_i \times d_i\right)$= 300

Therefore, mean $\bar{x} = \left(25 + \frac{300}{100}\right)$

$= 25 + 3 = 28$

34. Let the sides containing the right - angle be x cm and (x - 7) cm

Since we know that area of a right-angled triangle is equal to $\frac{1}{2}$ (base × altitude)

But according to question area = 60 cm^2

$\therefore \frac{1}{2} \times x(x-7) = 60$

$\Rightarrow x(x-7) = 120$

$\Rightarrow x^2 - 7x - 120 = 0$

$\Rightarrow x^2 - 15x + 8x - 120 = 0$

$\Rightarrow x(x-15) + 8(x-15) = 0$

$\Rightarrow (x-15)(x+8) = 0$

$\Rightarrow x = 15$ [Neglecting $x = -8$]

One side = 15 cm and other = (15 - 7) cm = 8 cm

Hypotenuse = $\sqrt{(15)^2 + (8)^2}\text{cm} = \sqrt{225 + 64}\text{cm}$

$= \sqrt{289}\text{cm} = 17\text{cm}$

∴ perimeter of triangle (15 + 8 + 17) cm = 40 cm

35. Suppose, speed of the train be x km/hr and the speed of taxi be y km/h.

time taken to cover 300 km by the train = $\frac{300}{x}$ hours

time taken to cover 200 km by the taxi = $\frac{200}{y}$ hours

Total time taken = $5\frac{30}{60}$ hours $= 5\frac{1}{2}$ hours $= \frac{11}{2}$ hours

$\therefore \frac{300}{x} + \frac{200}{y} = \frac{11}{2}$

$\Rightarrow \frac{600}{x} + \frac{400}{y} = 11$

Put $\frac{1}{x}$ = u and $\frac{1}{y}$ = v

⇒ 600u + 400v = 11 ...... (i)

time taken to cover 260 km by the train = $\frac{260}{x}$ hours

time taken to cover 240 km by the taxi = $\frac{240}{y}$ hours

Total time taken = $5\frac{36}{60}$ hours $= 5\frac{1}{2}$ hours $= \frac{11}{2}$ hours

⇒ 1300u + 1200v = 28 ...... (ii)

Multiplying (i) by 3 and subtracting (ii) from it,

$\Rightarrow 500u = 5 \Rightarrow u = \frac{5}{500} \Rightarrow u = \frac{1}{100}$

Substituting u = $\frac{1}{100}$ in (i), ⇒ v = $\frac{1}{80}$

$\therefore u=\frac{1}{100} \Rightarrow \frac{1}{x}=\frac{1}{100} \Rightarrow x=100$

$v=\frac{1}{80} \Rightarrow \frac{1}{y}=\frac{1}{80} \Rightarrow y=80$

∴ the speed of the train = 100 km/hr

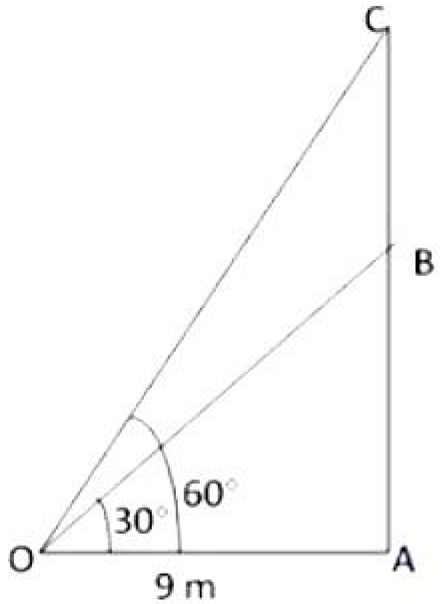
the speed of the taxi = 80 km/hr

36. Let us suppose that AB be the tower and BC be flagpole

Let us suppose that O be the point of observation. Then, OA = 9m

According to question it is given that

$\angle AOB=30^{\circ}$ and $\angle AOC=60^{\circ}$



From right angled $\triangle \mathrm{BOA}$

$\frac{AB}{OA}=\tan 30^{\circ}$

$\Rightarrow \frac{AB}{9}=\frac{1}{\sqrt{3}} \Rightarrow AB=3\sqrt{3}$

From right angled ΔOAC

$\frac{AC}{OA}=\tan 60^{\circ}$

$\frac{AC}{9}=\sqrt{3} \Rightarrow AC=9\sqrt{3}\mathrm{m}$

∴ BC = (AC - AB) $=6\sqrt{3}\mathrm{m}$

Thus $AB=3\sqrt{3}m=5.196m$ and $BC=6\sqrt{3}m=10.392m$

Hence, height of the tower is 5.196m and the height of the flagpole is 10.392 m