APPENDIX

A1.1 ELECTRIC FIELD DUE TO CONTINUOUS CHARGE DISTRIBUTION

Consider the following charged object of irregular shape as shown in Figure A1.1. The entire charged object is divided into a large number of charge elements $\Delta q_1, \Delta q_2, \Delta q_3 ......\Delta q_n$ and each charge element Δq is taken as a point charge.

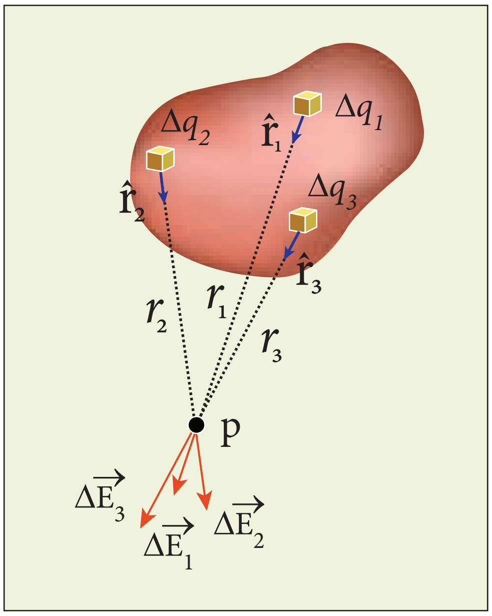

Figure A1.1 Continuous charge distributions

The electric field at a point P due to a charged object is approximately given by the sum of the fields at P due to all such charge elements.

$$\vec{E} \approx \frac{1}{4\pi\epsilon_0}\left(\frac{\Delta q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{\Delta q_2}{r_{2P}^2}\hat{r}_{2P} + ...+\frac{\Delta q_n}{r_{nP}^2}\hat{r}_{nP}\right)$$
$$\approx \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{\Delta q_i}{r_{iP}^2}\hat{r}_{iP} \qquad \text{(A1.1)}$$

Here Δq_i is the ith charge element, r_{iP} is the distance of the point P from the ith charge element and $\hat{r}_{iP}$ is the unit vector from ith charge element to the point P.

However the equation (A1.1) is only an approximation. To incorporate the continuous distribution of charge, we take the limit $\Delta q \to 0 (= dq)$. In this limit, the summation in the equation (A1.1) becomes an integration and takes the following form

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r^2}\hat{r} \qquad \text{(A1.2)}$$

Here r is the distance of the point P from the infinitesimal charge dq and $\hat{r}$ is the unit vector from dq to point P. Even though the electric field for a continuous charge distribution is difficult to evaluate, the force experienced by some test charge q in this electric field is still given by $\vec{F} = q\vec{E}$.

(a) If the charge Q is uniformly distributed along the wire of length L, then linear charge density (charge per unit length) is $\lambda = \frac{Q}{L}$. Its unit is coulomb per meter (Cm^{-1}).

The charge present in the infinitesimal length dl is $dq = \lambda dl$. This is shown in Figure A1.2 (a).

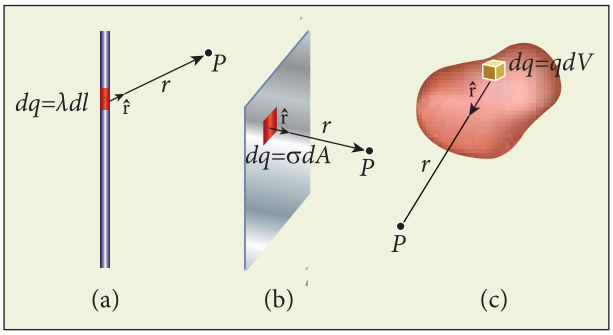

Figure A1.2 Line, surface and volume charge distribution

The electric field due to the line of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda dl}{r^2}\hat{r} = \frac{\lambda}{4\pi\epsilon_0}\int \frac{dl}{r^2}\hat{r}$$

(b) If the charge Q is uniformly distributed on a surface of area A, then surface charge density (charge per unit area) is $\sigma = \frac{Q}{A}$. Its unit is coulomb per square meter ($C\ m^{-2}$).

The charge present in the infinitesimal area dA is dq = σdA. This is shown in the Figure A1.2 (b).

The electric field due to a of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_\circ}\int \frac{\sigma dA}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_\circ}\sigma\int \frac{dA}{r^2}\hat{r}$$

This is shown in Figure A1.2 (b).

(c) If the charge Q is uniformly distributed in a volume V, then volume charge density (charge per unit volume) is given by $\rho = \frac{Q}{V}$. Its unit is coulomb per cubic meter ($C\ m^{-3}$).

The charge present in the infinitesimal volume element dV is dq = ρdV. This is shown in Figure A1.2 (c).

The electric field due to a volume of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_\circ}\int \frac{\rho dV}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_\circ}\rho\int \frac{dV}{r^2}\hat{r}.$$

A3.1 EXPRESSION FOR TORQUE ON A CURRENT LOOP PLACED IN A UNIFORM MAGNETIC FIELD

Consider a single rectangular loop (this means the number of turns is one) PQRS kept in a uniform magnetic field $\vec{B}$. Let a and b be the length and breadth of the rectangular loop respectively. Let $\hat{n}$ be the unit vector normal to the plane of the current loop which completely describes the orientation of the loop. The direction of magnetic field $\vec{B}$ is shown in Figure A3.1.

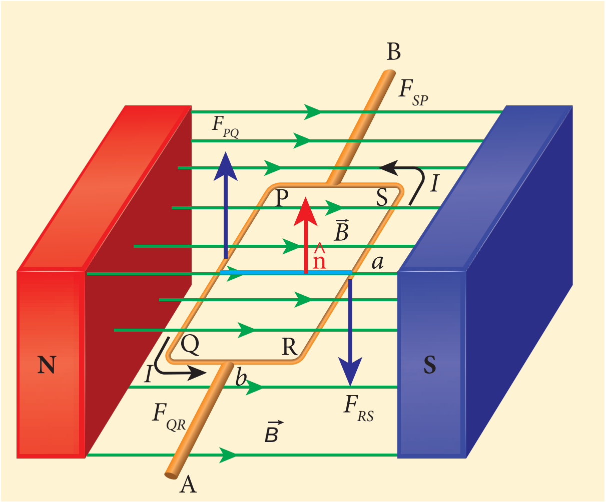

Figure A3.1 Rectangular coil placed in a magnetic field

When a steady current I passes through the loop PQRS, the net force acting on the loop is zero where as the net torque is not zero. For calculation purpose, we shall divide the rectangular loop into four sections PQ, QR, RS and SP. Now we shall consider how

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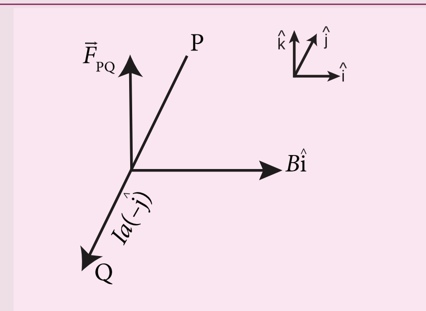
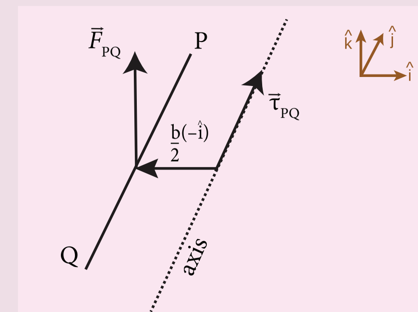
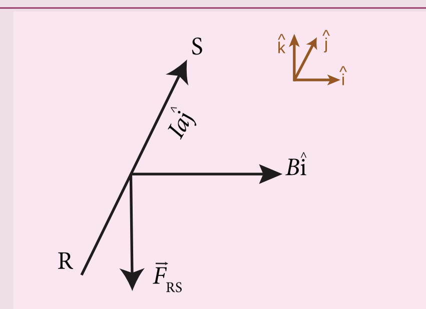
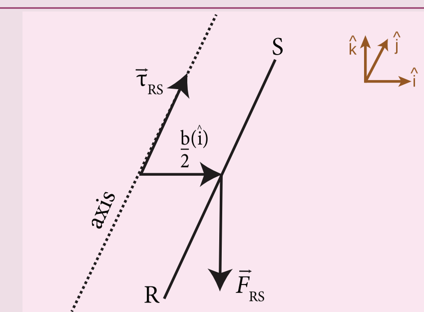
Table A3.1 Unit normal vector is perpendicular to magnetic field

Section	Force		Torque	
	$\vec{F} = I\vec{l} \times \vec{B}$	Explanatory diagram	$\vec{\tau} = \vec{r} \times \vec{F}$	Explanatory diagram
PQ	$\vec{F}_{PQ} = I(-a\hat{j}) \times B\hat{i}$ $= -IaB(\hat{j} \times \hat{i})$ $= IaB\hat{k}$		$\vec{\tau}_{PQ} = \frac{b}{2}(-\hat{i}) \times \vec{F}_{PQ}$ $= \left(-\frac{b}{2}\right)(IaB)(\hat{i} \times \hat{k})$ $= \frac{b}{2} IaB\hat{j}$	
QR	$\vec{F}_{QR} = Ib\hat{i} \times B\hat{i} = \vec{0}$		$\vec{\tau}_{QR} = \frac{a}{2}(-\hat{j}) \times \vec{F}_{QR} = \vec{0}$	
RS	$\vec{F}_{RS} = (Ia\hat{j}) \times B\hat{i}$ $= IaB(\hat{j} \times \hat{i})$ $= -IaB\hat{k}$		$\vec{\tau}_{RS} = \frac{b}{2}\hat{i} \times \vec{F}_{RS}$ $= \frac{b}{2}\hat{i} \times (-IaB\hat{k})$ $= \left(-\frac{b}{2}\right) IaB(\hat{i} \times \hat{k})$ $= \frac{b}{2} IaB\check{j}$	
SP	$\vec{F}_{SP} = Ib(-\hat{i}) \times B\hat{i} = \vec{0}$		$\vec{\tau}_{SP} = \frac{a}{2}\hat{j} \times \vec{F}_{SP} = \vec{0}$	

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to calculate torque when the plane of the loop is parallel to the direction of magnetic field $\vec{B}$, i.e., $\hat{n} \perp \vec{B}$

$\hat{n} \perp \vec{B}$ **(unit normal vector is perpendicular to magnetic field)**

Since the current carrying wire experiences a force in a magnetic field, we shall tabulate the force experienced by each section of the loop and also the torque about an axis passing through the centre (see Table (A3.1))

$$\therefore \text{Net force } \vec{F}_{net} = \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RS} + \vec{F}_{SP} = \vec{0}$$

And net torque

$$\vec{\tau}_{net} = \vec{\tau}_{PQ} + \vec{\tau}_{QR} + \vec{\tau}_{RS} + \vec{\tau}_{SP} = IabB\hat{j}$$

Thus, the net force on the rectangular loop is zero but net torque on the rectangular loop is not zero. Let A be the area of the rectangular loop (A = ab)

$$\text{Then } \vec{\tau}_{net} = ABI\hat{j}$$

If N be the number of turns of rectangular loop then

$$\vec{\tau}_{net} = NABI\hat{j}$$

Due to this torque, loop will start to rotate (here clockwise) and hence magnetic field $\vec{B}$ is no longer in the plane of the loop. Therefore, the above equation is a special case.

Note: When the plane of the loop is inclined to the direction of the magnetic field (i.e., $\hat{n} \not\perp \vec{B}$), then the torque is given by $\vec{\tau}_{net} = NABI\sin\theta\hat{j}$. In terms of magnetic dipole moment, $\vec{\tau}_{net} = \vec{p}_m \times \vec{B}$.

Special Cases:

(i) $\theta = 90^\circ, \vec{\tau}_{net} = NABI\hat{j} = \max imum$

$\vec{p}_m and \vec{B}$ are perpendicular to each other

(ii) $\theta = 0^\circ, \vec{\tau}_{net} = \vec{0}$

$\vec{p}_m$ and $\vec{B}$ are parallel

(iii) $\theta = 180^\circ, \vec{\tau}_{net} = \vec{0}$

$\vec{p}_m$ and $\vec{B}$ are anti-parallel

EXAMPLE A3.1

Show the time period of oscillation when a bar magnet is kept in a uniform magnetic field is $T = 2\pi\sqrt{\dfrac{I}{p_m B}}$ in second, where I represents moment of inertia of the bar magnet, p_m is the magnetic moment and B is the magnetic field.

Solution

The magnitude of deflecting torque (the torque which makes the object rotate) acting on the bar magnet which will tend to align the bar magnet parallel to the direction of the uniform magnetic field $\vec{B}$ is

$$|\vec{\tau}| = p_m B\sin\theta$$

The magnitude of restoring torque acting on the bar magnet can be written as

$$|\vec{\tau}| = I\frac{d^2\theta}{dt^2}$$

Under equilibrium conditions, both magnitude of deflecting torque and restoring torque will be equal but act in the opposite directions, which means

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$$I\frac{d^2\theta}{dt^2} = -p_m B\sin\theta$$

The negative sign implies that both are in opposite directions. The above equation can be written as

$$\frac{d^2\theta}{dt^2} = -\frac{p_m B}{I}\sin\theta$$

This is non-linear second order homogeneous differential equation. In order to make it linear, we use small angle approximation as we did in XI volume II (Unit 10 – oscillations, Refer section 10.4.4) i.e., $\sin\theta \approx \theta$, we get

$$\frac{d^2\theta}{dt^2} = -\frac{p_m B}{I}\theta$$

This linear second order homogeneous differential equation is a Simple Harmonic differential equation.

Comparing this equation with Simple Harmonic Motion (SHM) differential equation

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

where ω is the angular frequency of the oscillation. Therefore,

$$\omega^2 = \frac{p_m B}{I} \Rightarrow \omega = \sqrt{\frac{p_m B}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{p_m B}}$$

$$T = 2\pi\sqrt{\frac{I}{p_m B_H}} \text{ in second}$$

where B_H is the horizontal component of Earth's magnetic field.

Note

1. The current in circuit can be calculated from I = K tan θ, where K is called reduction factor of tangent Galvanometer, where

$$K = \frac{2RB_H}{\mu_\circ N}$$

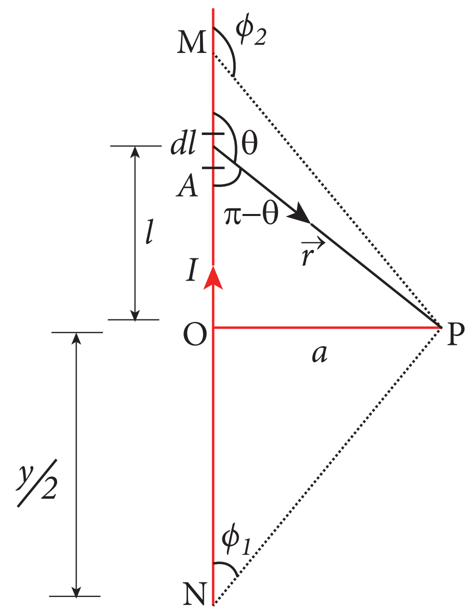
2. Sensitivity measures the change in the deflection produced by a unit current, mathematically

$$\frac{d\theta}{dI} = \frac{1}{K\left(1+\frac{I^2}{K^2}\right)}$$

3. The tangent Galvanometer is most sensitive at a deflection of 45°. Generally the deflection is taken between 30°

EXAMPLE A3.2

Calculate the magnetic field at a point P which is perpendicular bisector to current carrying straight wire as shown in figure.



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Solution

Let the length MN = y and the point P is on its perpendicular bisector. Let O be the point on the conductor as shown in figure.

Therefore, $OM = ON = \frac{y}{2}$, then

$$\cos\varphi_1 = \frac{\text{adjacent length}}{\text{hypotenuse length}} = \frac{ON}{PN}$$

$$= \frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4}+a^2}} = \frac{y}{\sqrt{y^2+4a^2}}$$

$$\cos(\pi-\varphi_2) = \frac{\text{adjacent length}}{\text{hypotenuse length}} = \frac{OM}{PM}$$

$$\cos\varphi_2 = -\frac{OM}{PM}$$

Using the equation,

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}(\cos\varphi_1 - \cos\varphi_2)\hat{n}$$

We get $\vec{B} = \frac{\mu_\circ I}{4\pi a}\frac{2y}{\sqrt{y^2+4a^2}}\hat{n}$

For long straight wire, $y \to \infty$,

$$\vec{B} = \frac{\mu_\circ I}{2\pi a}\hat{n}$$

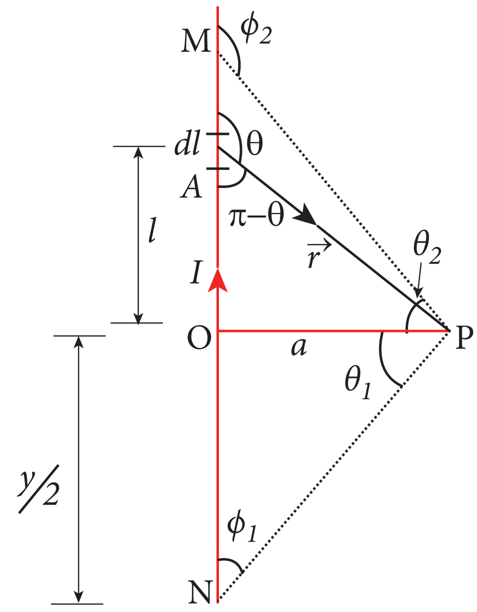
The result obtained is same as we obtained in equation (3.39).

EXAMPLE A3.3

Show that for a straight conductor, the magnetic field

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}(\cos\varphi_1 - \cos\varphi_2)\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}(\sin\theta_1 + \sin\theta_2)\hat{n}$$



Solution:

In a right angle triangle OPN, let the angle $\angle OPN = \theta_1$ which implies, $\varphi_1 = \frac{\pi}{2} - \theta_1$ and also in a right angle triangle OPM, $\angle OPM = \theta_2$ which implies, $\varphi_2 = \frac{\pi}{2} + \theta_2$

Hence,

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}\left(\cos\left(\frac{\pi}{2}-\theta_1\right) - \cos\left(\frac{\pi}{2}+\theta_2\right)\right)\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}(\sin\theta_1 + \sin\theta_2)\hat{n}$$

EXAMPLE A3.4

Consider a circular wire loop of radius R, mass m kept at rest on a rough surface. Let I be the current flowing through the loop and $\vec{B}$ be the magnetic field acting along horizontal as shown in Figure. Estimate the current I that should be applied so that one edge of the loop is lifted off the surface?

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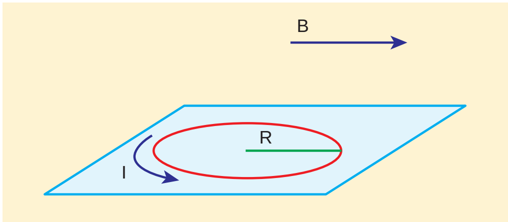
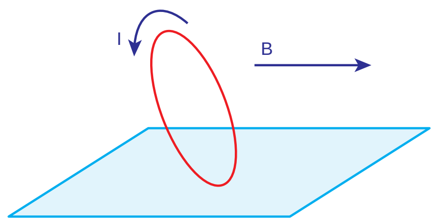


Solution



When the current is passed through the loop, the torque is produced. If the torque acting on the loop is increased then the loop will start to rotate. The loop will start to lift if and only if the magnitude of magnetic torque due to current applied equals to the gravitational torque as shown in Figure

$$\tau_{magnetic} = \tau_{gravitational}$$
$$IAB = mgR$$

But $p_m = IA = I\left(\pi R^2\right)$

$$\pi IR^2 B = mgR$$
$$\Rightarrow I = \frac{mg}{\pi RB}$$

The current estimated using this equation should be applied so that one edge of loop is lifted of the surface.

A4.1 MOTIONAL EMF FROM FARADAY'S LAW

Let us consider a rectangular conducting loop of width l in a uniform magnetic field $\vec{B}$ which is perpendicular to the plane of the loop

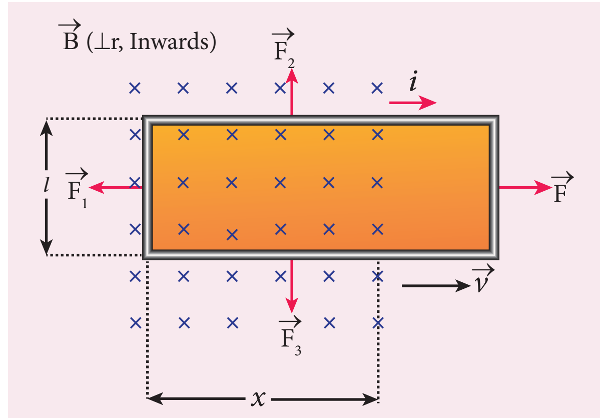


Figure A4.1 Motional emf from Faraday's law

and is directed inwards. A part of the loop is in the magnetic field while the remaining part is outside the field as shown in Figure A4.1.

When the loop is pulled with a constant velocity $\vec{v}$ to the right, the area of the portion of the loop within the magnetic field will decrease. Thus, the flux linked with the loop will also decrease. According to Faraday's law, an electric current is induced in the loop which flows in a direction so as to oppose the pull of the loop.

Let x be the length of the loop which is still within the magnetic field, then its area is lx. The magnetic flux linked with the loop is

$$\Phi_B = \int_A \vec{B}\cdot d\vec{A} = BA\cos\theta \quad = BA$$

Here $\theta = 0^\circ$ and $\cos 0^\circ = 1$

$$\Phi_B = Blx \qquad (A4.1)$$

As this magnetic flux decreases due to the movement of the loop, the magnitude of the induced emf is given by

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d}{dt}(Blx)$$

Here, both B and l are constants. Therefore,

$$\varepsilon = Bl\frac{dx}{dt} = Blv \qquad (A4.2)$$

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where $v = \frac{dx}{dt}$ is the velocity of the loop. This emf is known as motional emf since it is produced due to the movement of the loop in the magnetic field.

From Lenz's law, it is found that the induced current flows in clockwise direction. If R is the resistance of the loop, then the induced current is given by

$$i = \frac{\varepsilon}{R} = \frac{Blv}{R} \qquad (A4.3)$$

A4.2 ANALOGIES BETWEEN LC OSCILLATIONS AND SIMPLE HARMONIC OSCILLATIONS

Quantitative treatment

The mechanical energy of the spring-mass system is given by

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \qquad (A4.4)$$

The energy E remains constant for varying values of x and v. Differentiating E with respect to time, we get

$$\frac{dE}{dt} = \frac{1}{2}m\left(2v\frac{dv}{dt}\right) + \frac{1}{2}k\left(2x\frac{dx}{dt}\right) = 0$$

$$\text{or } m\frac{d^2x}{dt^2} + kx = 0 \qquad (A4.5)$$

$$\text{since } \frac{dx}{dt} = v \text{ and } \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

This is the differential equation of the oscillations of the spring-mass system. The general solution of equation (4.68) is of the form

$$x(t) = X_m \cos(\omega t + \phi) \qquad (A4.6)$$

where X_m is the maximum value of $x(t)$, ω the angular frequency and ϕ the phase constant.

Similarly, the electromagnetic energy of the LC system is given by

$$U = \frac{1}{2}Li^2 + \frac{1}{2}\left(\frac{1}{C}\right)q^2 = \text{constant} \qquad (A4.7)$$

Differentiating U with respect to time, we get

$$\frac{dU}{dt} = \frac{1}{2}L\left(2i\frac{di}{dt}\right) + \frac{1}{2C}\left(2q\frac{dq}{dt}\right) = 0$$

$$\text{or } L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \qquad (A4.8)$$

$$\text{since } i = \frac{dq}{dt} \text{ and } \frac{di}{dt} = \frac{d^2q}{dt^2}$$

The general solution of equation (A4.8) is of the form

$$q(t) = Q_m \cos(\omega t + \phi) \qquad (A4.9)$$

where Q_m is the maximum value of $q(t)$, ω the angular frequency and ϕ the phase constant.

A5.1 PROPERTIES OF ELECTROMAGNETIC WAVES

- The energy density (energy per unit volume) associated with an electromagnetic wave propagating in vacuum or free space is

$$u = \frac{1}{2}\varepsilon_\circ E^2 + \frac{1}{2\mu_\circ}B^2$$

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where, $\frac{1}{2}\varepsilon_{\circ}E^2 = u_E$ is the energy density in an electric field and $\frac{1}{2\mu_0}B^2 = u_B$ is the energy density in a magnetic field.

Since, $E = Bc \Rightarrow u_B = u_E$.

- The energy density of the electromagnetic wave is

$$u = \varepsilon_0 E^2 = \frac{1}{\mu_0}B^2$$

- The average energy density for electromagnetic wave,

$$\langle u \rangle = \frac{1}{2}\varepsilon_{\circ}E^2 = \frac{1}{2}\frac{1}{\mu_{\circ}}B^2.$$

- The energy crossing per unit area per unit time and perpendicular to the direction of propagation of electromagnetic wave is called the intensity.

Intensity, $= \frac{\text{Energy}}{\text{speed}} = \frac{U}{c}$. or

$$I = \frac{\text{total electromagnetic energy (U)}}{\text{Surface area (A)} \times \text{time (t)}}$$

$$= \frac{\text{Power (P)}}{\text{Surface area (A)}}$$

Note

For a point source, $I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$

For a line source, $I \propto \frac{1}{r}$

For a plane source, I is independent of r

- Like other waves, electromagnetic waves also carry energy and momentum. For the electromagnetic wave of energy U propagating with speed c has linear momentum which is given by $p = \frac{\text{Energy}}{\text{speed}} = \frac{U}{c}$. The force exerted by an electromagnetic wave on unit area of a surface is called radiation pressure.
- If the electromagnetic wave incident on a material surface is completely absorbed, then the energy delivered is U and momentum imparted on the surface is $p = \frac{U}{c}$.
- If the incident electromagnetic wave of energy U is totally reflected from the surface, then the momentum delivered to the surface is

$$\Delta p = \frac{U}{c} - \left(-\frac{U}{c}\right) = 2\frac{U}{c}.$$

- The rate of flow of energy crossing a unit area is known as Poynting vector for electromagnetic waves, which is $\vec{S} = \frac{1}{\mu_{\circ}}\left(\vec{E}\times\vec{B}\right) = c^2\varepsilon_{\circ}\left(\vec{E}\times\vec{B}\right)$. The unit for Poynting vector is W m^{-2}. The Poynting vector at any point gives the direction of energy transport from that point.

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